Applied Statistical Analysis I/Quantitative Methods I
POP77003/77051
Fall 2024

Week 2

Yao (Sara) HAN
hany3@tcd.ie

Department of Political Science
Trinity College Dublin
18 September, 2024

Today's Agenda

(1) Lecture recap

(2) Tutorial exercises

Null-hypothesis significance testing

What are the five steps of null-hypothesis significance testing?

Null-hypothesis significance testing

TABLE 6.1: The Five Parts of a Statistical Significance Test

1. **Assumptions**
   Type of data, randomization, population distribution, sample size condition
2. **Hypotheses**
   Null hypothesis, H_0 (parameter value for "no effect")
   Alternative hypothesis, H_a (alternative parameter values)
3. **Test statistic**
   Compares point estimate to H_0 parameter value
4. ***P*-value**
   Weight of evidence against H_0; smaller P is stronger evidence
5. **Conclusion**
   Report P-value
   Formal decision (optional; see Section 6.4)

(Agresti and Finlay 2009, 147)

Null-hypothesis significance testing

Step 1: Assumptions

- <u>Type of data</u>: Continuous or categorical data
- <u>Sampling method</u>: Data randomly obtained (e.g., random sample)
- <u>Population distribution</u>: Variable assumed to follow certain distribution (e.g., normal distribution)
- <u>Sample size</u>: Validity improves with increasing sample size

(Agresti and Finlay 2009, 144)

Null-hypothesis significance testing

Step 2: Hypotheses

- Hypothesis: "a statement about a population. It takes the form of a prediction that a parameter takes a particular numerical value or falls in a certain range of values" (Agresti and Finlay 2009, 143).
- Null (H_0) and alternative hypothesis (H_a): "The null hypothesis is a statement that the parameter takes a particular value. The alternative hypothesis states that the parameter falls in some alternative range of values. Usually the value in the null hypothesis corresponds, in a certain sense, to no effect. The values in the alternative hypothesis then represent an effect of some type" (Agresti and Finlay 2009, 144).
- Can be one-sided ($<$, $>$, $\geq$, $\leq$) or two-sided ($=$, $\neq$)

Null-hypothesis significance testing

Step 3: Test statistic

- Test statistic: "The test statistic summarizes how far that estimate falls from the parameter value in H_0. Often this is expressed by the number of standard errors between the estimate and the H_0 value" (Agresti and Finlay 2009, 145).
- Depending on probability distribution, test statistic is also called z-statistic or t-statistic.

Null-hypothesis significance testing

Step 4: P-value

- P-value: "The P-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_a. It is calculated by presuming that H_0 is true. The smaller the P-value, the stronger the evidence is against H_0" (Agresti and Finlay 2009, 145).

Null-hypothesis significance testing

Step 5: Conclusion

		Null hypothesis (H_0) is	
		True	**False**
Decision about Null hypothesis (H_0)	**Don't reject**	Correct inference (true negative) (probability = $1-\alpha$)	Type II Error (false negative) (probability = β)
	Reject	Type I Error (false positive) (probability = α)	Correct inference (true positive) (probability = $1-\beta$)

$\rightarrow$ most concerned about Type I Error, probability of obtaining a false positive result, α (error probability), p-value.

Null-hypothesis significance testing

Step 5: Conclusion

- Validate whether the obtained test statistic is unlikely to occur, under the assumption that the null hypothesis is true. $\rightarrow$ p-value, if probability is low, we reject H_0.
- More precisely, select α-level, which indicates acceptable probability of Type I error (usually 0.05, 0.01). If p-value $< \alpha$, we reject H_0. $\rightarrow$ proof by contradiction (disprove the null)
- Careful when making conclusion, there is still a probability that we falsely reject H_0, even if p-value $< \alpha$. $\rightarrow$ We use a certain language

Two types of hypothesis tests for today

1. Significance test for a mean
2. Significance test for a difference in means

Significance test for a mean (t-test)

TABLE 6.3: The Five Parts of Significance Tests for Population Means

1. **Assumptions**
   Quantitative variable
   Randomization
   Normal population (robust, especially for two-sided H_a, large n)
2. **Hypotheses**
   H_0: $\mu = \mu_0$
   H_a: $\mu \neq \mu_0$ (or H_a: $\mu > \mu_0$ or H_a: $\mu < \mu_0$)
3. **Test statistic**
   $t = \dfrac{\bar{y} - \mu_0}{se}$ where $se = \dfrac{s}{\sqrt{n}}$
4. ***P*-value**
   In t curve, use
   P = Two-tail probability for H_a: $\mu \neq \mu_0$
   P = Probability to right of observed t-value for H_a: $\mu > \mu_0$
   P = Probability to left of observed t-value for H_a: $\mu < \mu_0$
5. **Conclusion**
   Report P-value. Smaller P provides stronger evidence against H_0 and supporting H_a. Can reject H_0 if $P \leq \alpha$-level.

(Agresti and Finlay 2009, 155)

Significance test for a mean (t-test)

TABLE 6.2: Responses of Subjects on a Scale of Political Ideology

Response	Race		
	Black	White	Other
1. Extremely liberal	10	36	1
2. Liberal	21	109	13
3. Slightly liberal	22	124	13
4. Moderate, middle of road	74	421	27
5. Slightly conservative	21	179	9
6. Conservative	27	176	7
7. Extremely conservative	11	28	2
	$n = 186$	$n = 1073$	$n = 72$

H_0: mean political ideology is moderate $\rightarrow \mu = 4.0$
H_a: mean political ideology falls in liberal or conservative direction
$\rightarrow \mu \neq 4.0$, ($\mu < 4$ or $\mu > 4$).

(Agresti and Finlay 2009, 149)

Significance test for a mean (t-test)

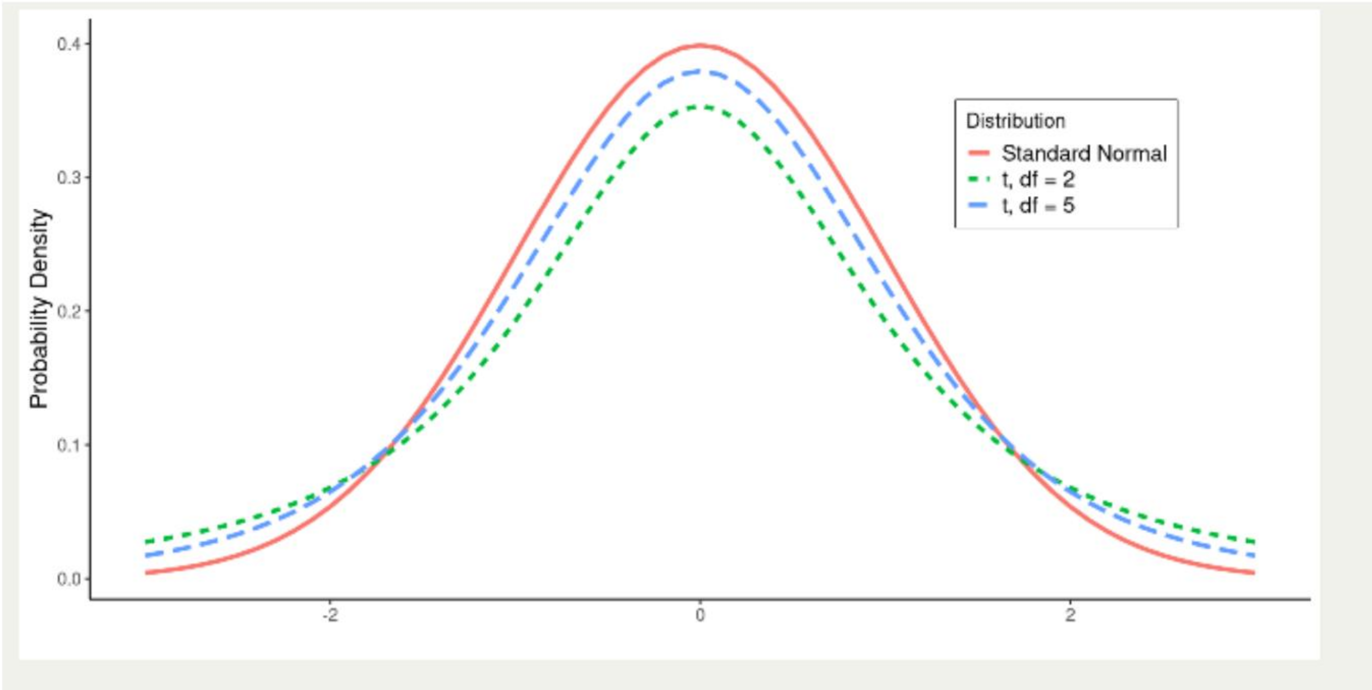
With $\bar{y} = 4.075$, $s = 1.512$, we calculate $se = \frac{s}{\sqrt{n}} = \frac{1.512}{\sqrt{186}} = 0.111$

and then $t = \frac{\bar{y} - \mu_0}{se} = \frac{4.075 - 4.0}{0.111} = 0.68$

How to interpret this value? How likely are we to observe data in sample (this test statistics), under the assumption that H_0 is true?
$\rightarrow$ Probability distribution

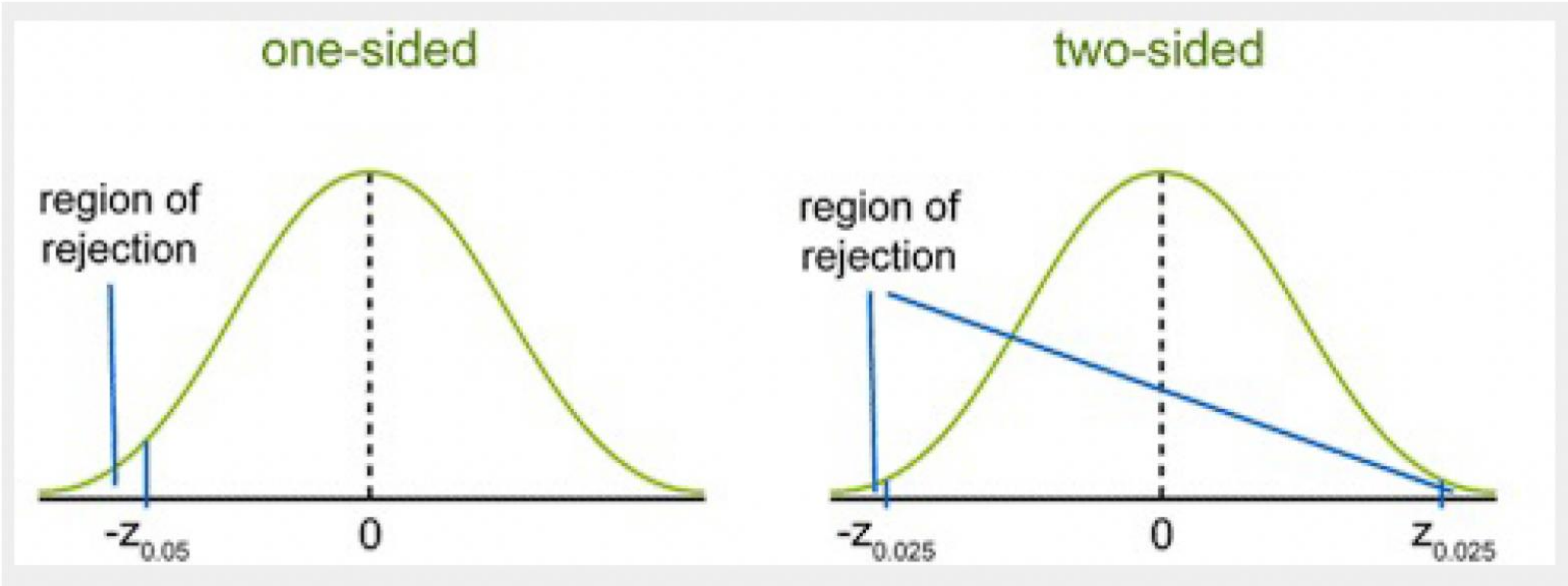
(Agresti and Finlay 2009, 150)

Significance test for a mean (t-test)



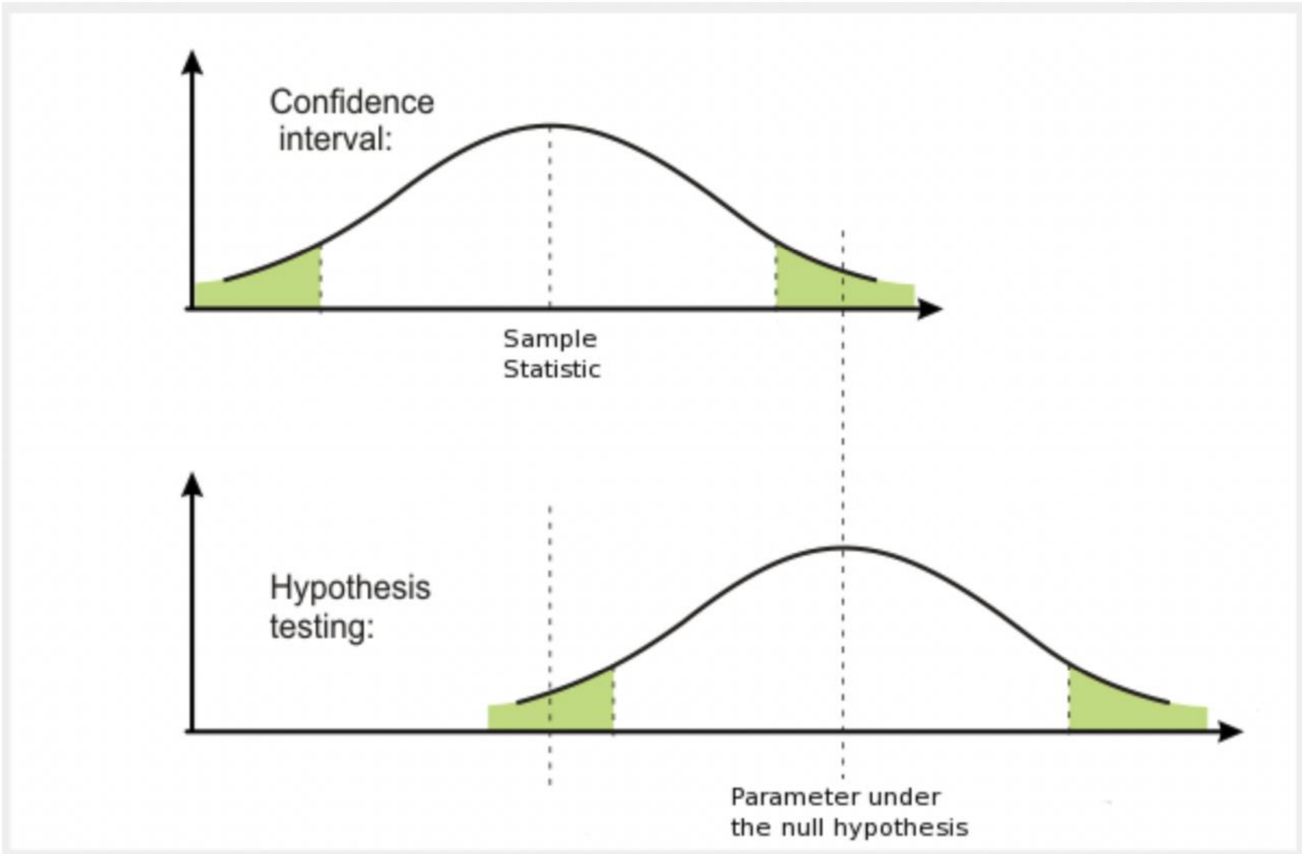
What is the conclusion? P-value=0.50, and is not < 0.05, thus we cannot reject $H_0 \rightarrow$ It is plausible that the population mean is 4.0, and therefore moderate.

One-sided versus two-sided test



"In most research articles, significance tests use two-sided P-values" (Agresti and Finlay 2009, 153). Why? But can also be two-sided, always depends on research question.

Confidence intervals and NHST



Does confidence intervals contain parameter under the null hypothesis?

Significance test for a difference in means (t-test)

What is a t-test for the difference in means?

- Null and alternative hypothesis: (Step 2) The means of two groups are identical, $\bar{y}_1 = \bar{y}_2$ or $\bar{y}_1 - \bar{y}_2 = 0$ (H_0), the means of two groups are different, $\bar{y}_1 \neq \bar{y}_2$ (H_a).
- Test statistics: (Step 3) "measures the number of standard errors between the estimate and the H_0 value" (Agresti and Finlay 2009, 192).

  $t = \frac{\textit{Estimate of parameter} - \textit{Null hypothesis value of parameter}}{\textit{Standard error of estimate}}$

  $t = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{se}$, H_0 assumes $\bar{y}_2 - \bar{y}_1 = 0$, $se = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Significance test for a difference in means (t-test)

What is a t-test for the difference in means?

TABLE 7.1: Cooking and Washing Up Minutes, per Day, for a National Survey of Men and Women Working Full Time in Great Britain

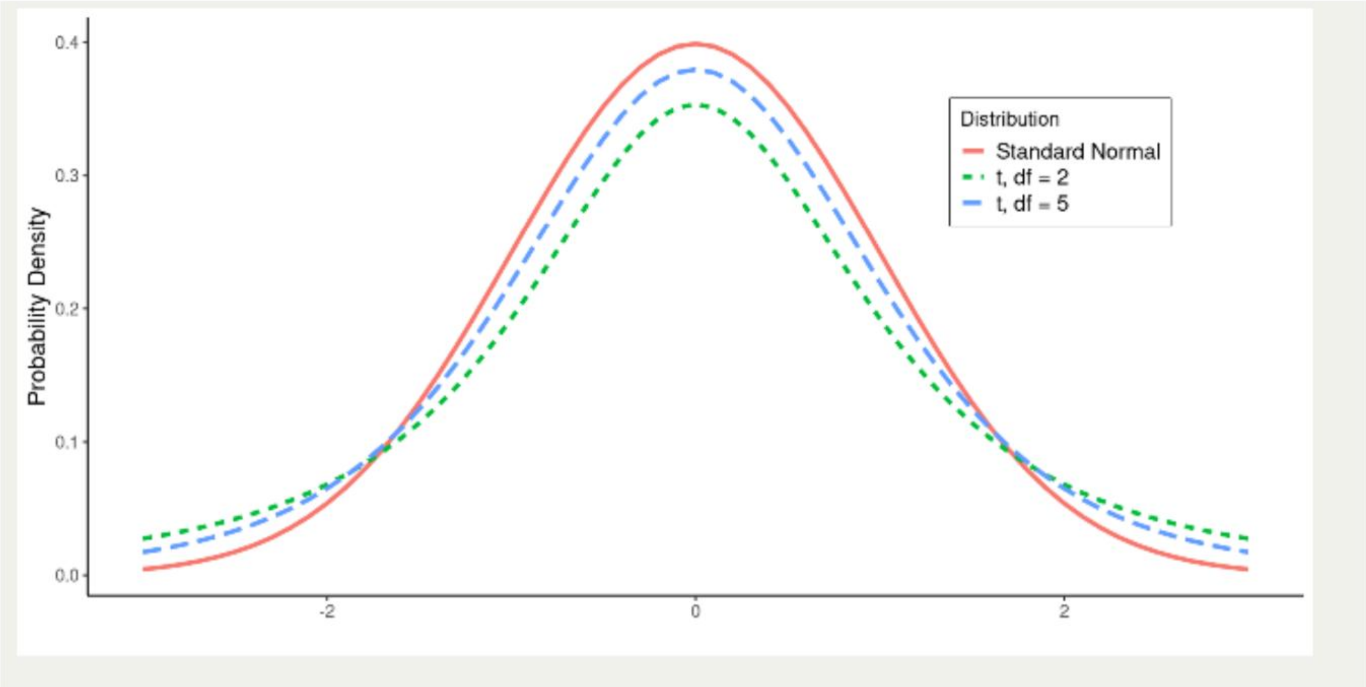
		Cooking and Washing Up Minutes	
Sex	Sample Size	Mean	Standard Deviation
Men	1219	23	32
Women	733	37	16

$$t = \frac{(\bar{y}_2 - \bar{y}_1) - 0}{se} = \frac{(37 - 23)}{1.09} = 12.8$$

How to interpret this value? How likely are we to observe data in sample (this test statistics), under the assumption that H_0 is true? $\rightarrow$ Probability distribution

(Agresti and Finlay 2009, 192)

Significance test for a difference in means (t-test)



What is the conclusion? P-value < 0.05, We can reject H_0 with an error probability (p-value) of essentially 0%. $\rightarrow$ Suggests that population means differ, and sample means show that women have a higher mean household work time than men

Is the Relationship Causal?

(Bueno de Mesquita and Fowler 2021, 157)

Causation

- Outcome Y_i
- $T = 1$ Treated
- $T = 0$ Untreated

- Y_{1i} = outcome for unit i if $T = 1$
- Y_{0i} = outcome for unit i if $T = 0$

- Effect of $T=1$ on $Y_i = Y_{1i} - Y_{0i}$

(Bueno de Mesquita and Fowler 2021, 164)

What is the fundamental problem of causal inference?

- “we can only observe, at most, one of the two quantities–Y_{1i} or Y_{0i}–for any individual at a particular point in time” (Bueno de Mesquita and Fowler 2021, 164).

Average Treatment Effect (ATE)

- $\overline{Y}_1$ = average outcome if all units had T = 1
- $\overline{Y}_0$ = average outcome if all units had T = 0

- ATE = $\overline{Y}_1$ - $\overline{Y}_0$

(Bueno de Mesquita and Fowler 2021, 164)

The difference in average outcome comparing two counterfactual scenarios — one where everyone in the population is treated and one where everyone in the population is untreated.

(Bueno de Mesquita and Fowler 2021, 187)

Population Difference in Means and Sample Difference in Means

- $\overline{\mathrm{Y}}_{1_\mathrm{T}}$ = average outcome if all units had T = 1
- $\overline{\mathrm{Y}}_{0_\mathrm{U}}$ = average outcome if all units had T = 0

- Population Difference in Means = $\overline{\mathrm{Y}}_{1_\mathrm{T}}$ - $\overline{\mathrm{Y}}_{0_\mathrm{U}}$

- Sample Difference in Means = $\overline{\mathrm{Y}}_{1_\mathrm{T}}$ - $\overline{\mathrm{Y}}_{0_\mathrm{U}}$ + Noise

(Bueno de Mesquita and Fowler 2021, 165)

Average Treatment Effect on the Treated (ATT)

ATT = $\overline{Y}_{1T} - \overline{Y}_{0T}$

(Bueno de Mesquita and Fowler 2021, 165)

- The difference in average outcome comparing the scenario where everyone in the subgroup of people who in fact received treatment is treated and the counterfactual scenario where everyone in that subgroup is untreated.

(Bueno de Mesquita and Fowler 2021, 187)

Average Treatment Effect on the Untreated (ATU)

ATU = $\overline{Y}_{1U} - \overline{Y}_{0U}$

(Bueno de Mesquita and Fowler 2021, 165)

- The difference in average outcome comparing the counterfactual scenario where everyone in the subgroup of people who did not receive treatment is treated and the scenario where everyone in that subgroup is untreated.

(Bueno de Mesquita and Fowler 2021, 187)

- ATE is a weighted average of the ATT and the ATU, where the weights depend on the size of each group.

- Just like the ATE, the ATT and ATU are both fundamentally unobservable.

(Bueno de Mesquita and Fowler 2021, 165)

Estimate = Estimand + Bias + Noise

Sample Difference in Means = Population Difference in Means + Noise,
Population Difference in Means = $\overline{\mathrm{Y}}_{1\mathrm{T}} - \overline{\mathrm{Y}}_{0\mathrm{U}}$

(Bueno de Mesquita and Fowler 2021, 166)

- Sample Difference in Means = ATT + Bias_{ATT} + Noise
- Sample Difference in Means = $\overline{Y}_{1T}$ - $\overline{Y}_{0T}$ + $\overline{Y}_{0T}$ - $\overline{Y}_{0u}$ + Noise

- Sample Difference in Means = ATU + Bias_{ATU} + Noise
- Sample Difference in Means = $\overline{Y}_{1u}$ - $\overline{Y}_{0u}$ + $\overline{Y}_{1T}$ -$\overline{Y}_{1u}$+ Noise

(Bueno de Mesquita and Fowler 2021, 166-7)

Bias

What are the sources of bias? And how can we overcome bias?

Bias

What are the sources of bias?

- Baseline differences: "[d]ifference in the average potential outcome between two groups (e.g., the treated and untreated groups), even when those two groups have the same treatment status" $\rightarrow$ Confounders may cause baseline differences, which may cause bias (*omitted variable bias*)

(Bueno de Mesquita and Fowler 2021, 187)

Bias

What are the sources of bias?

1. Confounder: "A feature of the world that (1) has an effect on treatment status and (2) has an effect on the potential outcome over and above the effect it has through its effect on treatment status".

$$
\begin{tikzcd}
\text{Treatment (X)} \arrow[rr] & & \text{Outcome (Y)} \\
& \text{Confounder (Z)} \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
\text{Poverty (X)} \arrow[rr] & & \text{Civil War (Y)} \\
& \text{Regime Type (Z)} \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

(Bueno de Mesquita and Fowler 2021, 187)

Bias

What are the sources of bias?

2. Other sources of bias: Reverse causality, unobserved unit heterogeneity (special type of omitted variable bias), post-treatment bias

Bias

And how can we overcome bias?

1. Control for confounders:
   * But "there will still be unobservable confounders that we can't control for, reverse causation, or variables that are part confounder and part mechanism" (Bueno de Mesquita and Fowler 2021, 215).
2. Randomized experiments: Randomly assigning units to treatment and control group eliminates baseline differences
   * But "the ideal experiment that we'd like to run is often impractical, infeasible, or unethical" (Bueno de Mesquita and Fowler 2021, 239).
3. Causal inference methods (e.g., Difference-in-Difference Design, Instrumental Variables)

References I

Agresti, Alan, and Barbara Finlay. 2009. *Statistical methods for the social sciences.* Essex: Pearson Prentice Hall.

Bueno de Mesquita, Ethan, and Anthony Fowler. 2021. *Thinking clearly with data: A guide to quantitative reasoning and analysis.* Princeton: Princeton University Press.