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Course: MATH 649 - Abstract Algebra
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Homework - Week 7
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Problem 19.3.18
If R is an integrally closed domain with quotient field $\mathbb{F}$, and $f, g \in \mathbb{F}[x]$ are monic with $fg \in \mathbb{R}[x]$, then $f, g \in R[x]$.

Solution:

Problem 19.4.8
Show that the conclusion of the Incomparability theorem fails for the ring extension $\mathbb{F}[x] \subseteq \mathbb{F}[x, y]$.

Solution:

Problem 19.4.13
True or false? Let $A \supseteq R$ be an integral ring extension, with A being a domain. If every non-zero prime ideal of R is a maximal ideal, then every non-zero prime ideal of A is also maximal.

Solution:

Problem 19.4.15
Consider the ring extension $\mathbb{Z} \subset \mathbb{Z}[\sqrt{5}]$.

(1) Find all prime ideals of $\mathbb{Z}[\sqrt{5}]$ which lie over the prime ideal (5) of $\mathbb{Z}$.

(2) Find all prime ideals of $\mathbb{Z}[\sqrt{5}]$ which lie over the prime ideal (3) of $\mathbb{Z}$.

(3) Find all prime ideals of $\mathbb{Z}[\sqrt{5}]$ which lie over the prime ideal (2) of $\mathbb{Z}$.

Solution:

Problem 20.1.5
If the ring R is noetherian, then so is the ring $R[[x_1, \ldots, x_n]]$ of formal power series.

Solution: