Zhengdong Zhang
Email: zhengz@uoregon.edu
Course: MATH 649 - Abstract Algebra
Instructor: Professor Sasha Polishchuk

Homework - Week 5
ID: 952091294
Term: Spring 2025
Due Date: 7^{th} May 2025

Problem 13.3.5
The 5th cyclotomic field $\mathbb{Q}(\zeta_5)$ contains $\sqrt{5}$.

Solution:

Problem 13.3.7
If p is a prime then

$$\Phi_{p^n}(x) = 1 + x^{p^{n-1}} + x^{2p^{n-1}} + \cdots + x^{(p-1)p^{n-1}}.$$

Solution:

Problem 13.3.9
$\mathbb{Q}(\sqrt[3]{2})$ is not a subfield of any cyclotomic extension of $\mathbb{Q}$.

Solution:

Problem 13.5.2
Let $\mathbb{K}/\mathbb{k}$ be a field extension. If $\alpha_1, \ldots, \alpha_n \in \mathbb{K}$ is algebraically independent over $\mathbb{k}$, and $\alpha \notin \mathbb{k}$ is the element of $k(\alpha_1, \ldots, \alpha_n)$, then α is transcendental over $\mathbb{k}$.

Solution:

Problem 13.5.4
If β is algebraic over $\mathbb{k}(\alpha)$ and β is transcendental over $\mathbb{k}$ then α is algebraic over $\mathbb{k}(\beta)$.

Solution:

Problem 13.5.19
Let $\mathbb{k} \subsetneq \mathbb{F} \subseteq \mathbb{k}(x)$ be field extensions, with x transcendental over $\mathbb{k}$. Then $\mathbb{k}(x)/\mathbb{F}$ is finite.

Solution:

Problem 13.6.5 (Newton's identities)
Let $x_1, \ldots, x_n$ be variables, and define power sum symmetric functions

$$p_k = p_k(x_1, \ldots, x_n) = x_1^k + \cdots + x_n^k \qquad (k \in \mathbb{Z}_{>0}).$$

Prove the *Newton identities*:

$$ke_k = \sum_{i=1}^{k} (-1)^{i-1} e_{k-i} p_i$$

where e_k are the elementary symmetric functions interpreted 1 if $k = 0$ and as 0 if $k > n$. Deduce that every elementary symmetric function e_k can be written down as a polynomial in $p_1, \ldots, p_k$ with rational coefficients. Deduce that every symmetric polynomial can be written down as a polynomial in the power sum symmetric functions.

Solution: