Zhengdong Zhang **Homework - Week 4** Email: zhengz@uoregon.edu ID: 952091294 Course: MATH 649 - Abstract Algebra Term: Spring 2025 Instructor: Professor Sasha Polishchuk Due Date: 30^{th} April , 2025

Problem 12.16
Let char $k \neq 2$ and $f \in \mathbb{k}[x]$ be a cubic whose discriminant has a square root in $\mathbb{k}$, then f is either irreducible or splits in $\mathbb{k}$.

Solution:

Problem 12.4.9
Let $\mathbb{K}/\mathbb{k}$ be a finite Galois extension and $\alpha \in \mathbb{K}$. Consider the $\mathbb{k}$-linear operator $A_\alpha : x \mapsto \alpha x$ on the $\mathbb{k}$-vector space $\mathbb{K}$. Then $\det A_\alpha = N_{\mathbb{K}/\mathbb{k}}(\alpha)$ and tr $A_\alpha = T_{\mathbb{K}/\mathbb{k}}(\alpha)$.

Solution:

Problem 12.4.11
Let $a, b \in \mathbb{Q}$.

(a) $a^2 + b^2 = 1$ is equivalent to $N_{\mathbb{Q}(i)/\mathbb{Q}}(a + ib) = 1$.

(b) Use Hilbert's Theorem 90 to prove that the rational solutions of $a^2 + b^2 = 1$ are of the form $a = (s^2 - t^2)/(s^2 + t^2)$ and $b = 2st/(s^2 + t^2)$ for $s, t \in \mathbb{Q}$.

Solution:

Problem 13.2.9
True or false? Let $\mathbb{K}/\mathbb{F}_q$ be a finite extension, and $\mathbb{L}$, $\mathbb{M}$ be two intermediate subfields. Then either $\mathbb{L} \subseteq \mathbb{M}$ or $\mathbb{M} \subseteq \mathbb{L}$.

Solution:

Problem 13.2.12
Let p be a prime. Then there are exactly $(q^p - q)/p$ monic irreducible polynomials of degree p in $\mathbb{F}_q[x]$ (q is not necessarily a power of p).

Solution:

Problem 13.2.13
What is $\sum_A A^{100}$, where the sum is over all 17×17 matrices A over $\mathbb{F}_{17}$?

Solution: