Use Noise as a Resource for Simulating Open Quantum Dynamics

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Introduction

- **Trapped-ion quantum processor:** internal states as qubits, shared motional states as quantum bus

--Noise: environmental noise (motional dephasing), imperfect experimental control (time miscalibration)

--Two-qubit entangling gate: Mølmer-Sørensen (MS) gate [1]

- **Non-Markovian processes:** growth of distinguishability of states, reversed flow of information from the environment to the open system [2]

--non-divisible map: the map for the whole process is completely positive (CP), but cannot be divided into small arbitrary parts that are themselves CP [2]

--non-CP map: the initial correlation with environment can lead to non-CP map [3]

Question

- What is the behavior of MS gate with time miscalibration and over time sequence?
- Can the repetitive MS gate with time miscalibration be exploited to construct non-CP map?
- If so, how can such non-CP maps be further employed to simulate non-Markovian dynamics?

Mølmer-Sørensen Hamiltonian and propagator

- **Interaction Hamiltonian**

In the interaction picture, for a pair of trapped ions,

$$H = -\eta\Omega J_y (a e^{-\mathrm{i}\delta t} + a^\dagger e^{\mathrm{i}\delta t})$$

$J_y = \sigma_y^{(1)} + \sigma_y^{(2)}$: collective spin operator

$a^\dagger(a)$: creation (annihilation) operator of the motional mode

δ: detuning to the first-order sideband

Ω: Rabi frequency

η: Lamb-Dicke parameter

- **Propagator**

$$U(t) = \exp\left[\mathrm{i}\sqrt{2}\frac{\eta\Omega}{\delta}\sin\delta t J_y x\right]\exp\left[\mathrm{i}\sqrt{2}\frac{\eta\Omega}{\delta}(1-\cos\delta t)J_y p\right]\exp\left[\mathrm{i}\frac{\eta^2\Omega^2}{\delta}\left(t-\frac{1}{2\delta}\sin 2\delta t\right)J_y^2\right]$$

$x = (a^\dagger + a)/\sqrt{2}$: dimensionless position operator

$p = \mathrm{i}(a^\dagger - a)/\sqrt{2}$: dimensionless momentum operator

Mølmer-Sørensen gate with time miscalibration

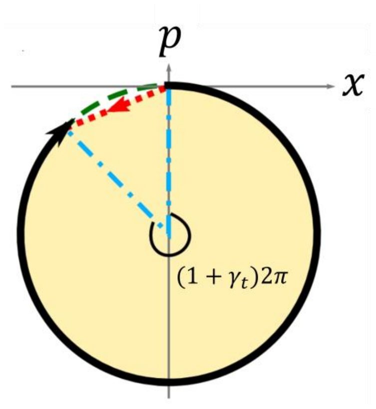
- The MS gate with time miscalibration can be decomposed into

$$U_{\mathrm{MS}}(\Delta t) = V_m(\Delta t)\, V_q(\Delta t)\, U_{\mathrm{MS}}$$

$$V_m(\Delta t) = \exp\left[\mathrm{i}\sqrt{2}\frac{\eta\Omega}{\delta}J_y\left(x\sin\delta\Delta t + p(1-\cos\delta\Delta t)\right)\right]$$

$$V_q(\Delta t) = \exp\left[\mathrm{i}\sqrt{2}\frac{\eta^2\Omega^2}{\delta^2}J_y^2(\delta\Delta t - \sin\delta\Delta t)\right]$$

$$U_{\mathrm{MS}} = \exp\left[\mathrm{i}\frac{\pi}{4}\sigma_y^{(1)}\sigma_y^{(2)}\right]$$



$V_m(\Delta t)$: error gate induced by entanglement with the motional mode

$V_q(\Delta t)$: error gate induced by an effective additional self-interaction

U_{MS}: ideal MS gate

- We define a dimensionless parameter γ_t to describe the absolute time error

$$\gamma_t \equiv \delta\Delta t/2\pi$$

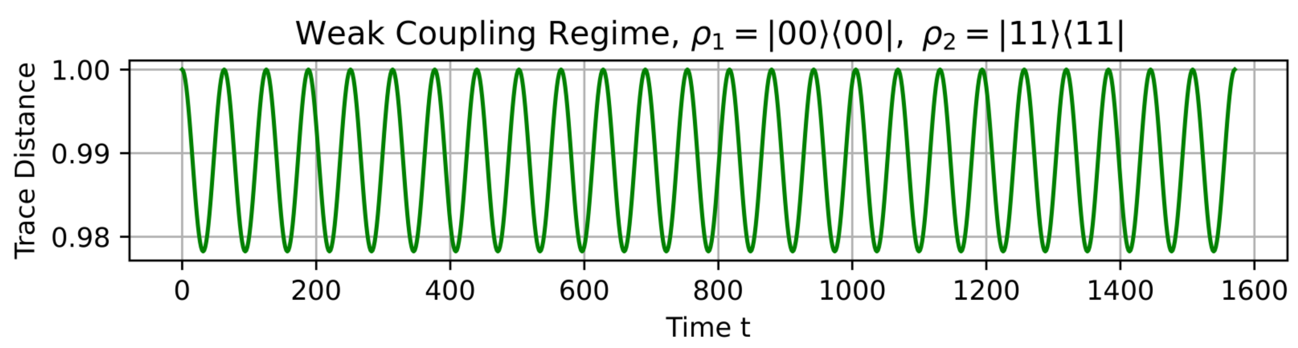
The information flow in Mølmer-Sørensen evolution

- The trace distance [4]

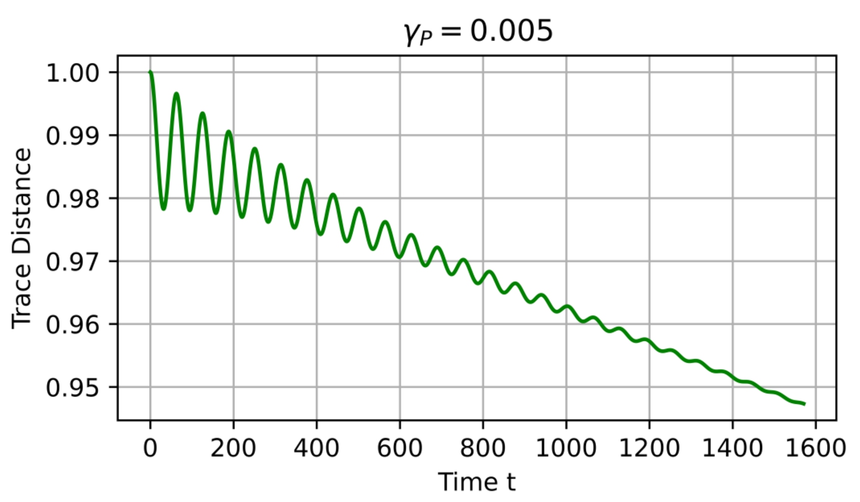
$$D(\rho_1,\rho_2) = \frac{1}{2}\mathrm{Tr}|\rho_1 - \rho_2|$$

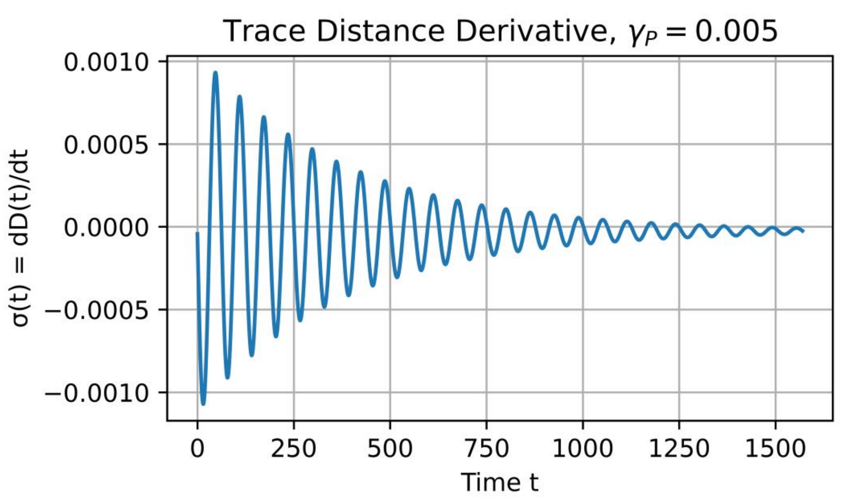
describes the distinguishability of quantum states, the change of which reflects the flow of information between open system and environment

- Trace distance D versus time t for MS evolution without any dissipation



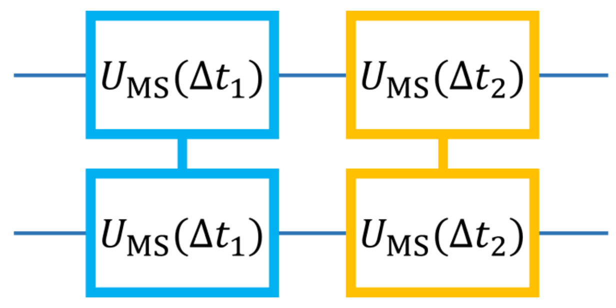
- Consider motional dephasing





Construct non-Completely Positive map

- Consider two repetitive MS gate with time miscalibration Δt_1, Δt_2



- Define map

$$\mathcal{E}_1[\rho_q(0)] \equiv \mathrm{Tr}_m\left[U_{\mathrm{MS}}(\Delta t_1)\,\rho_q(0)\otimes\rho_m(0)U_{\mathrm{MS}}^\dagger(\Delta t_1)\right]$$

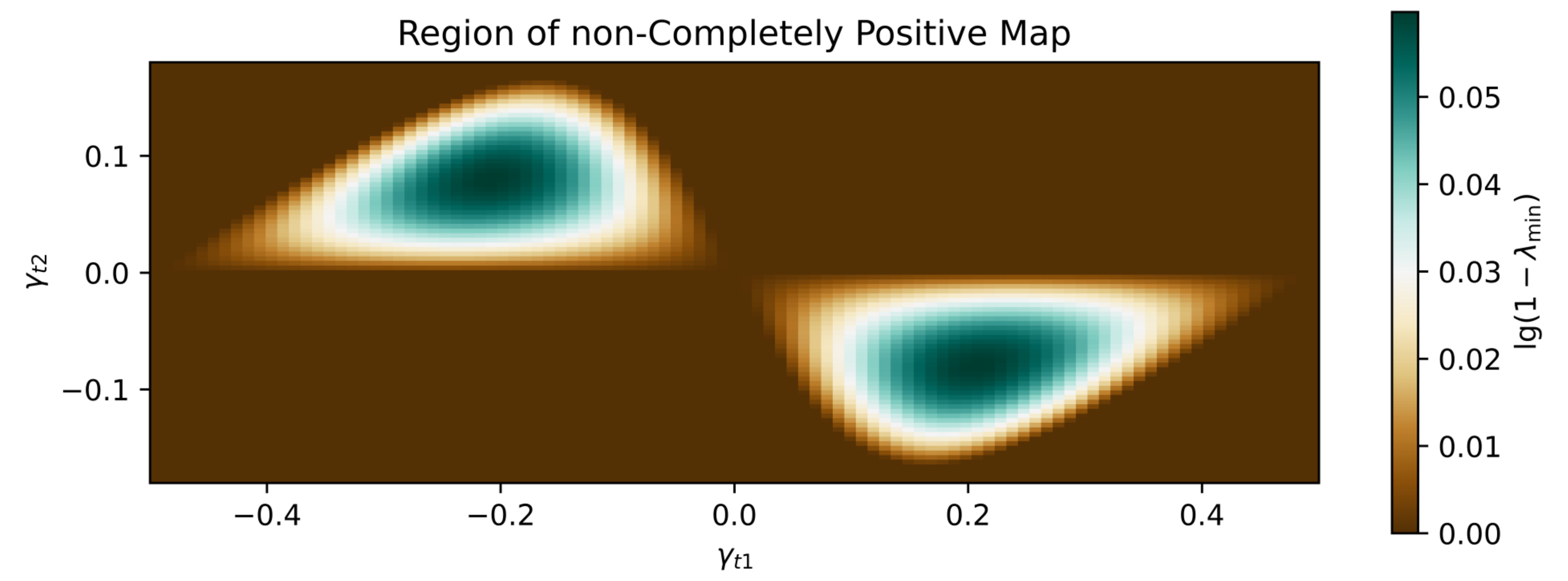
$$\mathcal{E}_2[\rho_q(0)] \equiv \mathrm{Tr}_m\left[U_{\mathrm{MS}}(\Delta t_2)U_{\mathrm{MS}}(\Delta t_1)\,\rho_q(0)\otimes\rho_m(0)U_{\mathrm{MS}}^\dagger(\Delta t_1)U_{\mathrm{MS}}^\dagger(\Delta t_2)\right]$$

Construct the effective map

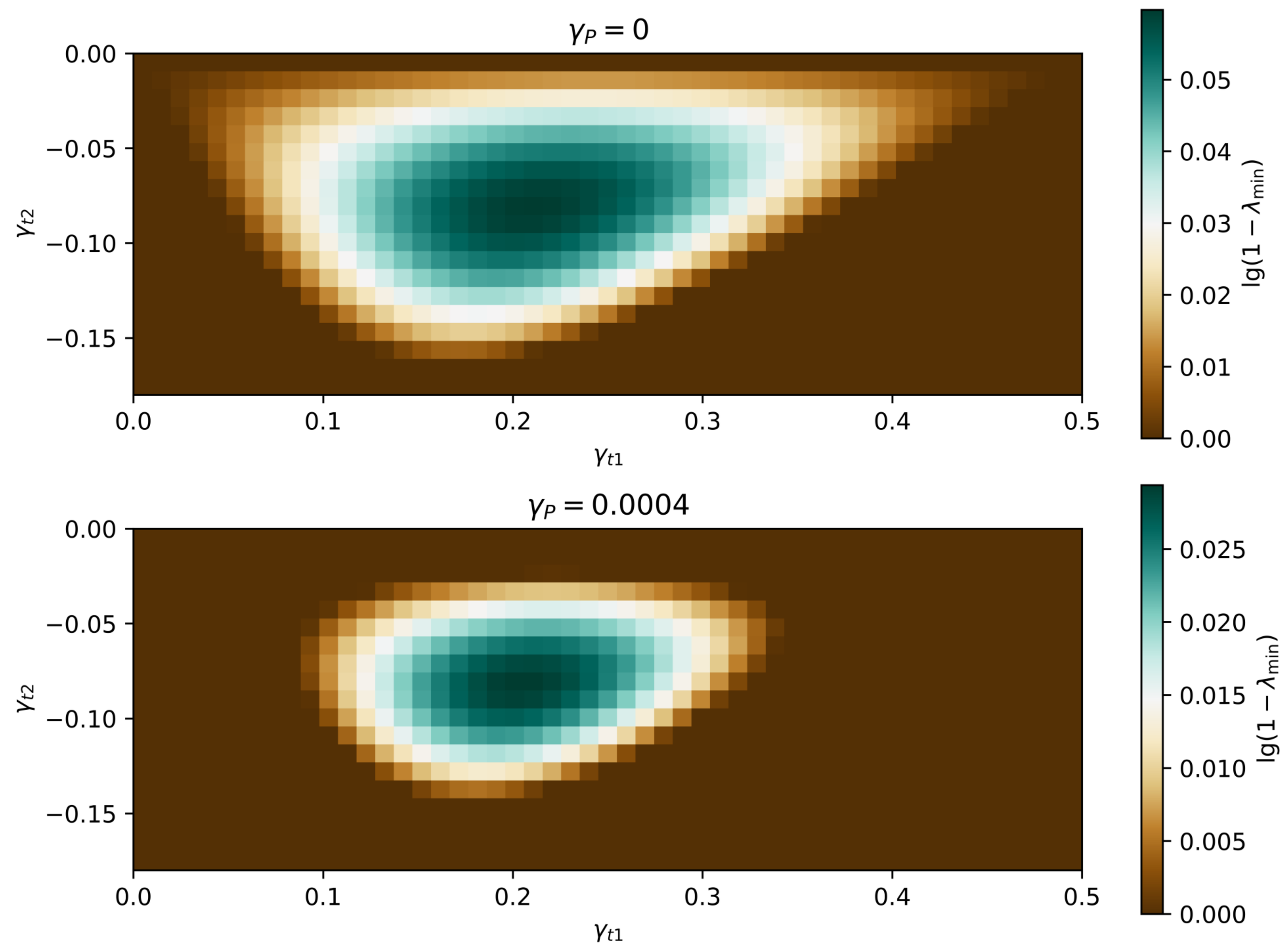
$$\mathcal{E}:\ \mathcal{E}_1[\rho_q(0)] \to \mathcal{E}_2[\rho_q(0)]$$

Define a measurement $\lg(1-\lambda_{\min})$ to describe the extent of non-Complete Positivity (the deviation from Complete Positivity), where $\lambda_{\min}$ is the minimum eigenvalue of Choi matrix $\mathcal{J}(\mathcal{E})$

- **The region of non-CP map in the parameter space**



- **The effect of dissipation on the region of non-CP map**



Summary

- We derive the propagator of the MS gate with time miscalibration.
- We demonstrate the presence of information flow from the motional mode to the qubit system during the MS evolution, indicating that the qubit dynamics is non-Markovian. Moreover, the presence of dissipation not only suppresses this non-Markovianity, but also renders the qubit dynamics fully Markovian after finite time.
- We demonstrate that, for two repetitive MS gates with time miscalibration, the presence of residual entanglement can render the dynamical map associated with the second gate non-CP for certain parameter settings. We identify such regions in the parameter space and quantify the degree of deviation from complete positivity within them. Furthermore, we show that dissipation will breaks the non-Complete Positivity.
- Our results suggest that it may be feasible to simulate non-Markovian dynamics by designing specific quantum circuits that exploit the non-CP maps generated by repetitive MS gates with time miscalibration.

References:
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[4] Nielsen, M. A. & Chuang, I. L. *Quantum Computation and Quantum Information*. (Cambridge University Press, Cambridge ; New York, 2010).