Institute of Computing

Application of Deep and Reinforcement Learning to Boundary Control Problems

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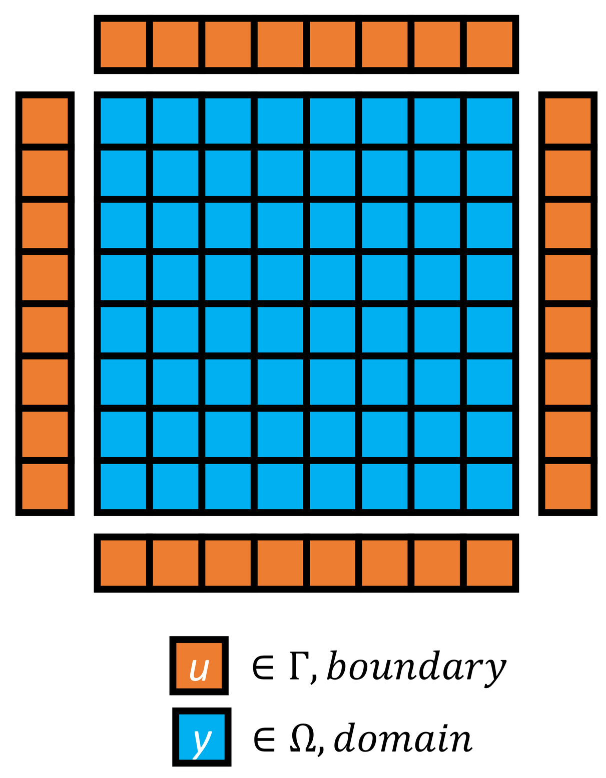
ABSTRACT

The goal of the boundary control problem is, in essence, to find the optimal values for the boundaries such that the values for the enclosed domain are as close as possible to desired values. Traditionally, the solution is obtained using nonlinear optimization methods, such as interior point, wherein the computational bottleneck is introduced by the large linear systems. Our objective is to use Deep Learning (DL) and Reinforcement Learning (RL) methods to solve boundary control problems with Dirichlet boundary condition faster than traditional solvers.

What is an elliptic boundary control problem?

- Domain values follow an elliptic PDE.
- Domain cells have desired values.
- Both domain and boundary cells have upper and lower bound constraints.
- Boundary cell values are controllable.



Find the most optimal values for the boundaries such that the domain would be closest to their desired values, without violating their constraints.

In mathematical sense,

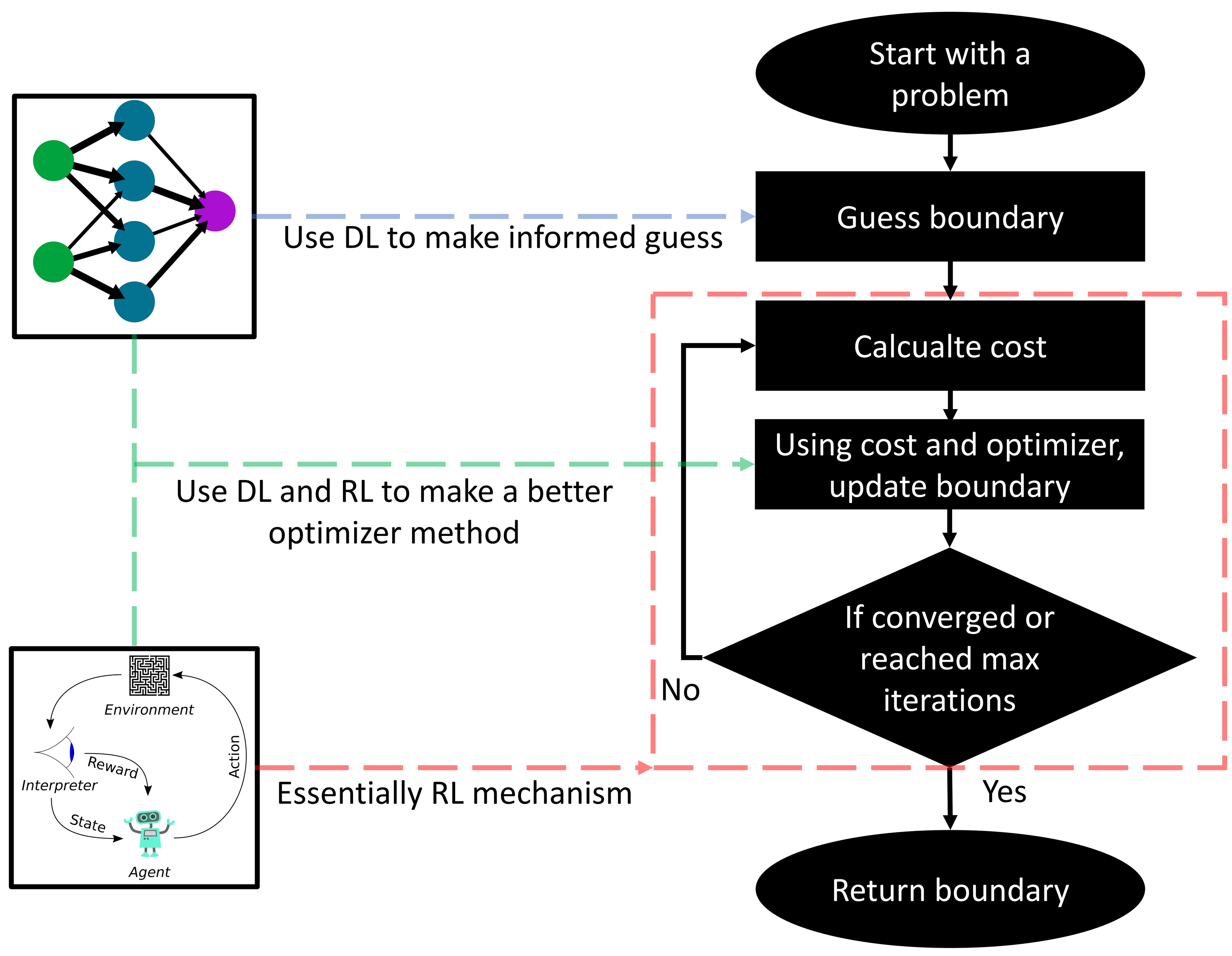
Minimize: $$F(y,u) = \frac{1}{2}\int_{\Omega}\left(y(x) - y_d(x)\right)^2 dx + \frac{\alpha}{2}\int_{\Gamma}\left(u(x) - u_d(x)\right)^2 dx$$

Subject to: $\nabla^2 y = c \;,\; y_{min} < y < y_{max} \;,\; u_{min} < u < u_{max}$

- y: domain values
- y_d: desired domain values
- α: non-negative constant
- u: boundary values
- u_d: desired boundary values
- c: is a constant sourcing term

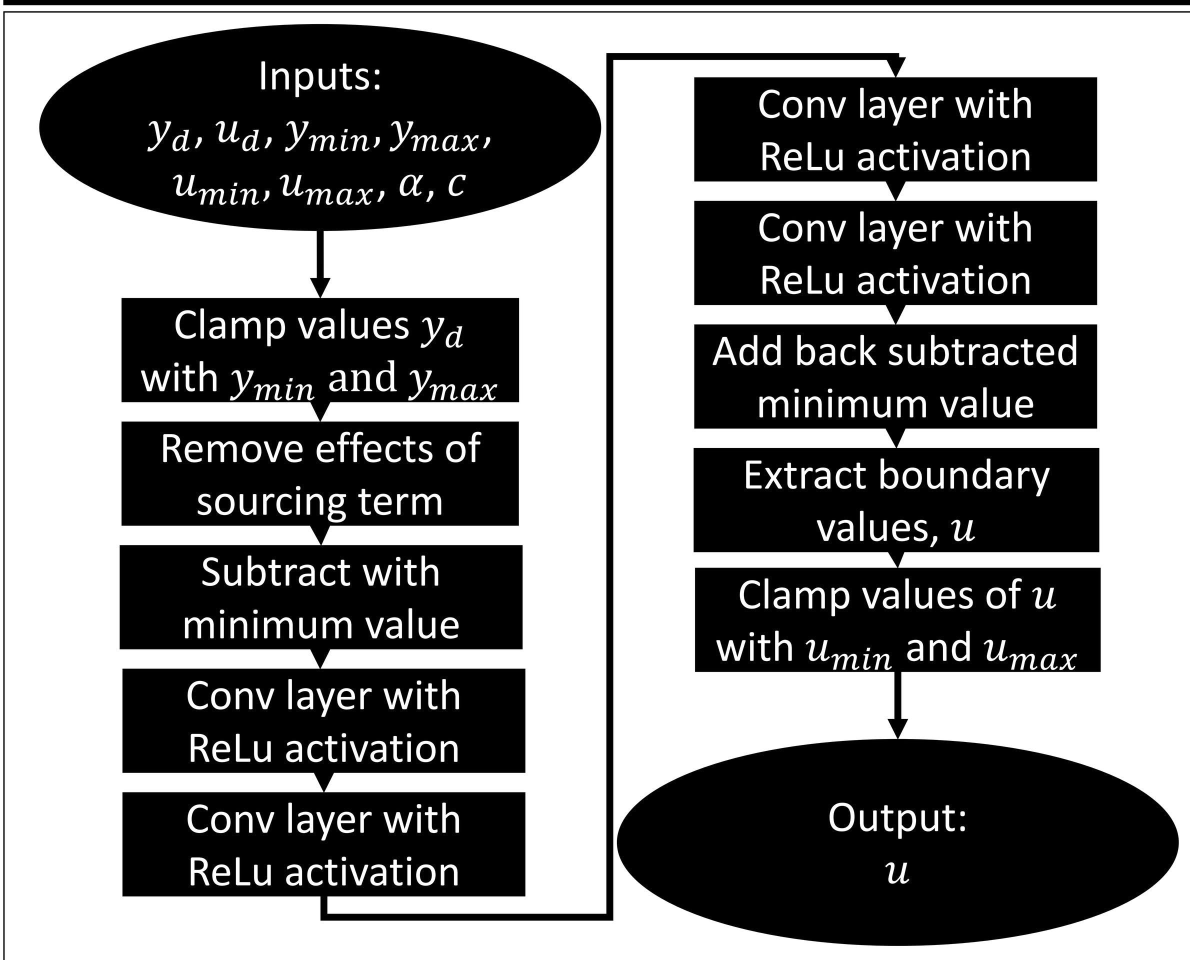
Applications: Heat transfer, fluid mechanics, acoustics, electromagnetism, structural mechanics, image processing.

Where/how to use DL and RL?

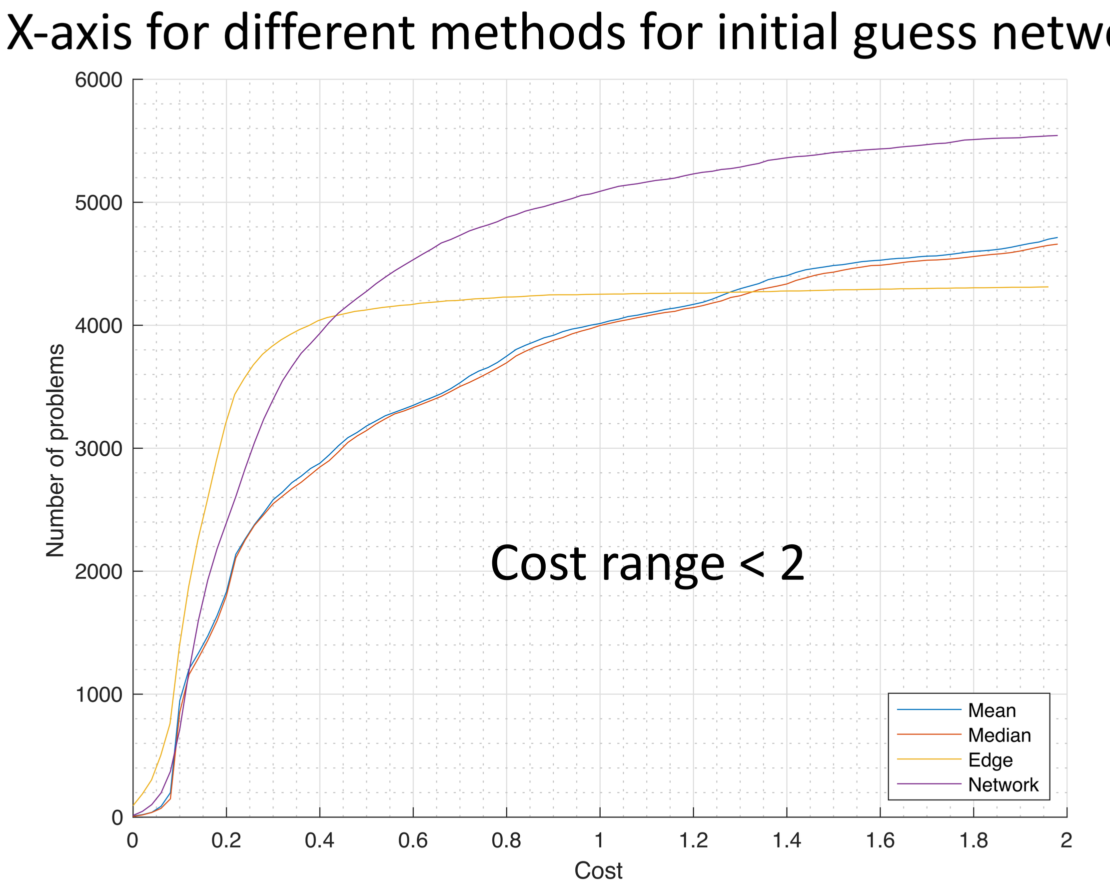


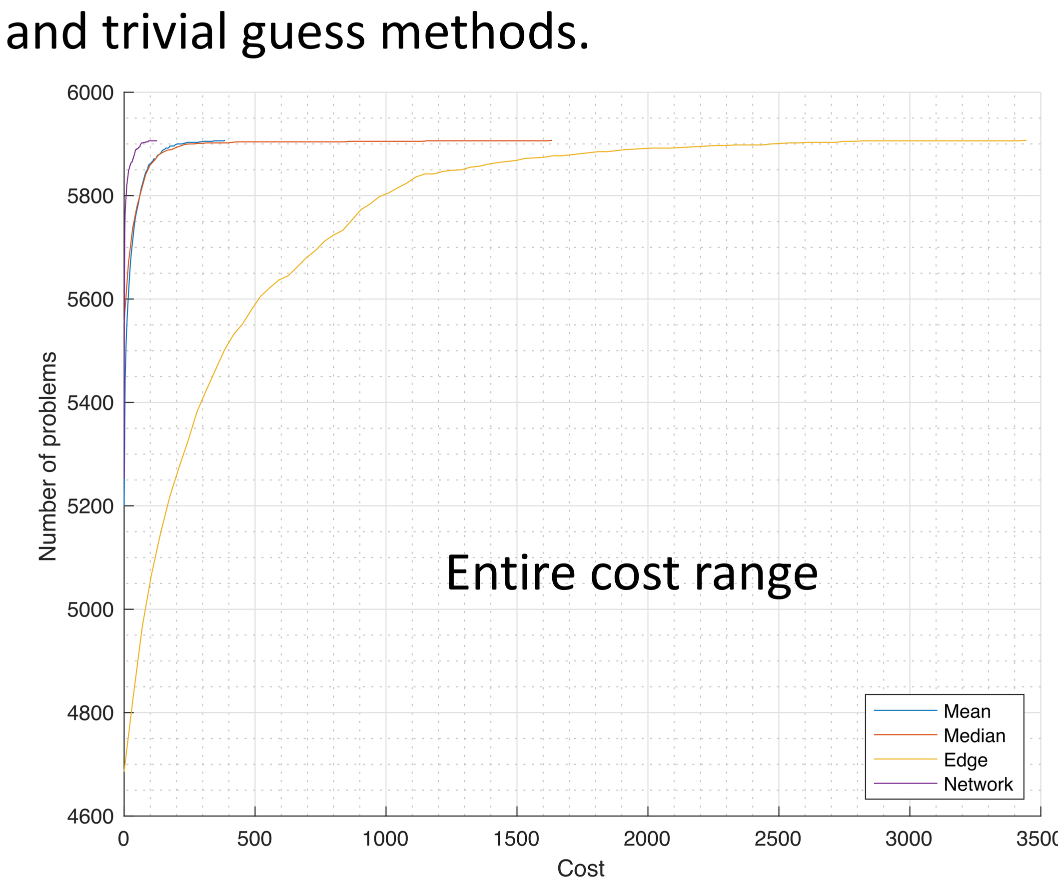
Note: While designing a neural network, ensure that it can work with different input/output sizes, as the size of the domain/boundary can vary.

Designing a method to make informed guess

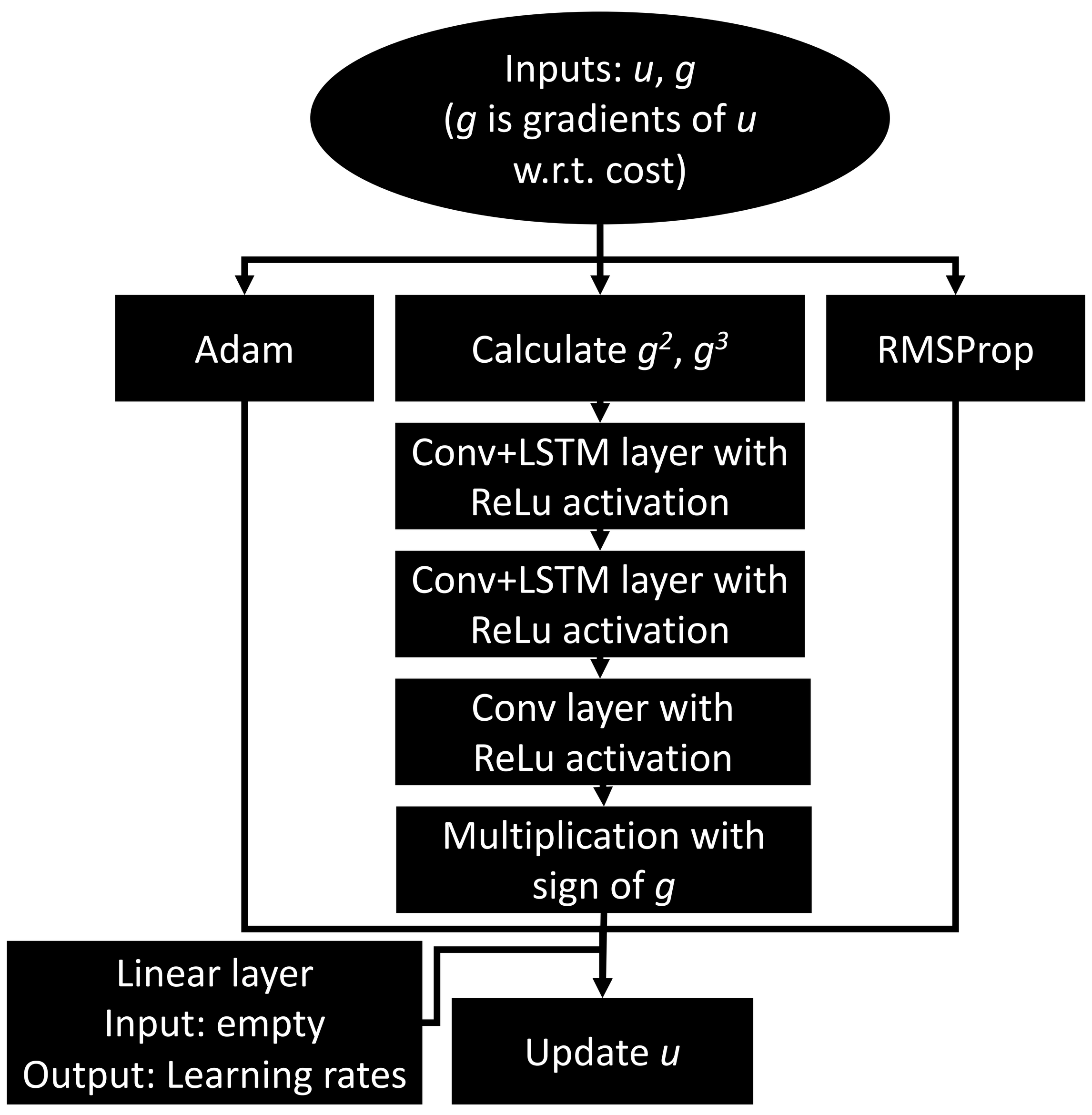


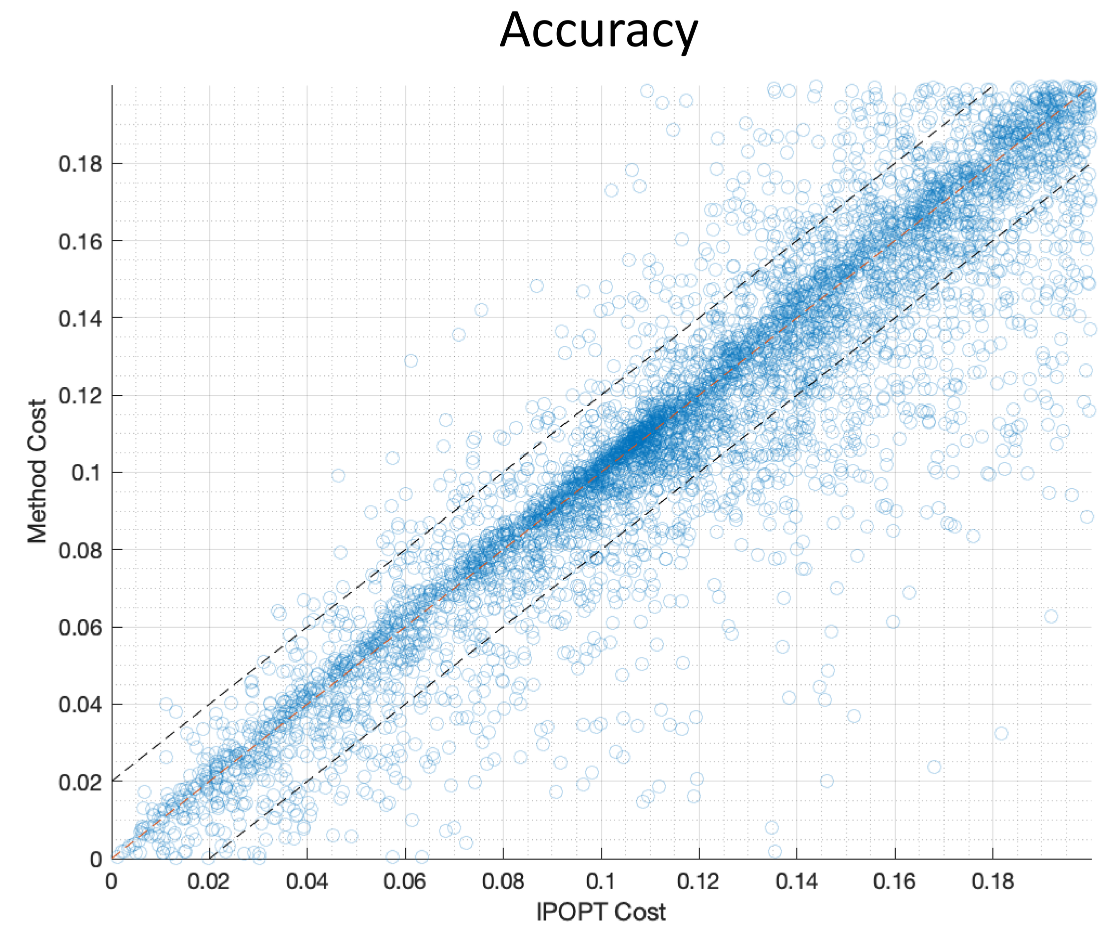
Cumulative plots showing the number of problems for which the cost value is less than the value in the X-axis for different methods for initial guess network and trivial guess methods.

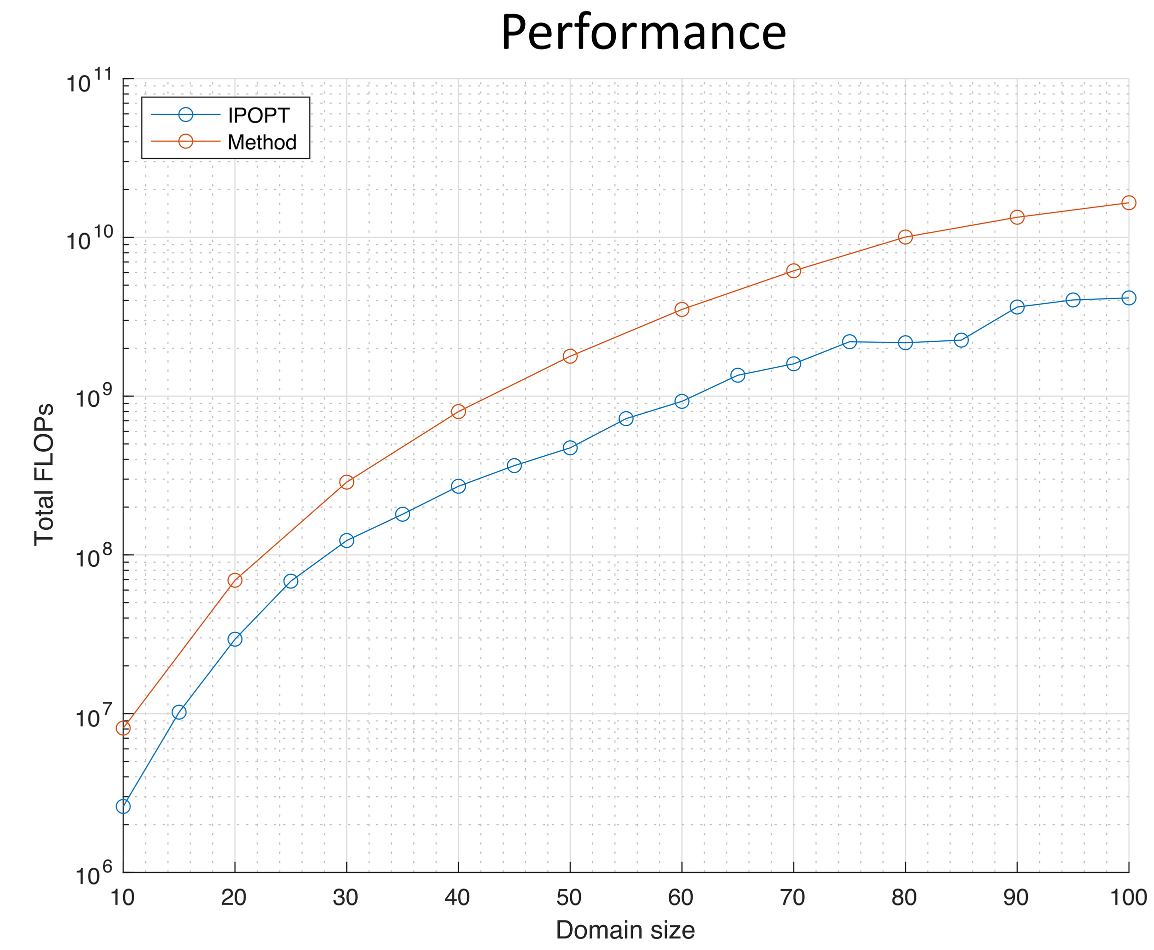




Designing a method to iteratively perform optimization







A total of 10,000 problems were synthetically generated out of which 5907 problems had feasible solutions. Cost predicted by the network is lower than IPOPT for 3011 problems (50.97% of the problems).

REFs

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- Maurer, Helmut, and Hans D. Mittelmann. "Optimization techniques for solving elliptic control problems with control and state constraints: Part 1. Boundary control." Computational Optimization and Applications 16 (2000): 29-55.

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