Balancing a Ball on a Beam

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1. Problem Statement

As shown in the figure below, a ball is placed on a beam, where it rolls under the influence of gravity. The ball has one degree of freedom along the length of the beam. The beam's angle is adjusted using a lever arm connected to the beam at one end and to a servo motor gear at the other. As the servo gear rotates by an angle θ about its axis, the lever arm angle changes by α from the horizontal.

2. Assumptions

The ball remains in contact with the beam at all times. The ball performs pure rolling motion (rolling without slipping).

3. Objective

Design a controller to manipulate the ball's position on the beam, with the input being the servo gear angle and the output being the ball's position.

4. Task 1: Define the System

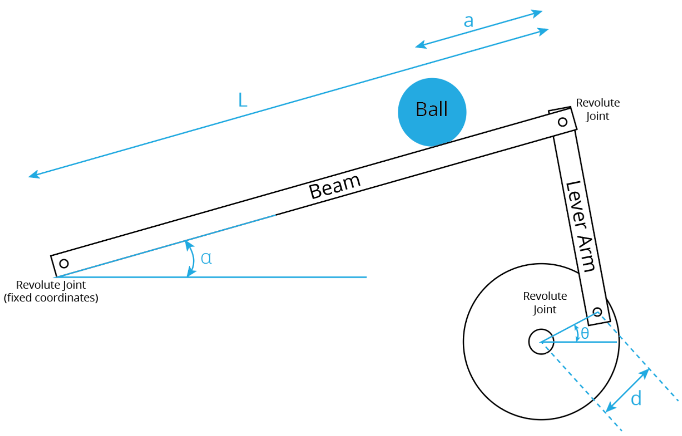


Figure 1. *Model Schematic*

4.1. System Dynamics Equations (Equations of Motion)

The above figure shows the schematic diagram of the system. We obtain the equation of motion of the system using the Lagrangian method, and it turns out to be:

$$\ddot{a}\left(m + \frac{I}{r^2}\right) + mg\sin\alpha - ma\dot{\alpha}^2 = 0$$

Where the symbols are as mentioned in figure 1, and I is the moment of inertia of the ball, and r is the radius of the ball. Now, we substitute the moment of inertia of a hollow sphere as $I = \frac{2}{3}mr^2$ and the angle α as $\frac{d}{L}\theta$. Simplifying, we get:

$$\ddot{a} + 0.6g\frac{d}{L}\sin\theta - 0.6a\frac{d}{L}\dot{\theta}^2 = 0 \tag{1}$$

4.2. System Transfer Function

To obtain the transfer function of the system, we take the Laplace Transform of the linearised equation 3 with zero initial conditions:

$$A(s)s^2 = -0.6g\frac{d}{L}\Theta(s)$$

Therefore the transfer function $T(s)$ becomes:

$$T(s) = \frac{A(s)}{\Theta(s)} = -\frac{0.6gd}{Ls^2} \tag{2}$$

4.3. Linearised Equation and State Space Representation

For linearising equation 1, we make the small angle approximation i.e. $\sin\theta = \theta$, and also for very small angles $\dot{\theta}$ is very small hence negligible.
The linearised equation of motion then comes out to be:

$$\ddot{a} = -0.6g\frac{d}{L}\theta \tag{3}$$

Equation 3 can be represented in state space as:

$$\begin{bmatrix}\dot{a}\\ \ddot{a}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -0.6g\frac{d}{L}\end{bmatrix}\theta \tag{4}$$

5. Task 2: Analysis of the System

Suitable values for parameters in Transfer function based on the model that we have created physically.

$$\begin{aligned}
\text{Mass of the ball, } m &= 0.0027\,\text{kg}\\
\text{Radius of the ball, } R &= 0.02\,\text{m}\\
\text{Lever arm offset, } d &= 0.073\,\text{m}\\
\text{Length of the beam, } L &= 0.25\,\text{m}\\
\text{Ball's moment of inertia, } J &= 7.2\times10^{-7}\,\text{kg}\cdot\text{m}^2\\
\text{Ball's position on the beam, } a &= 0.1\,\text{m}\\
\text{Acceleration due to gravity, } g &= 9.81\,\text{m/s}^2\\
\text{Beam angle and servo angle} &= \theta
\end{aligned}$$

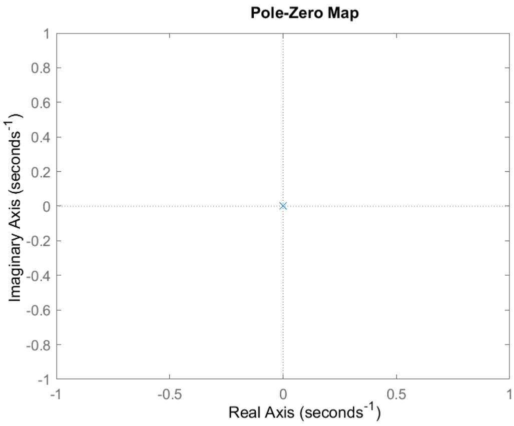
5.1. Matlab analysis based on assumed parameters:

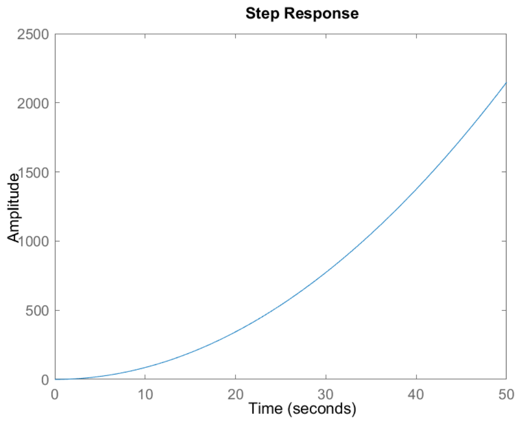
Substituting d and L in equation 2, the required transfer function for our system is:

$$T(s) = \frac{1.719}{s^2}$$

5.2. Observations from the obtained plots:

This is a second-order system with no zeros and two poles located at the origin (since the denominator has s^2, both poles are at $s = 0$). This indicates that the system is a Type 2 system, as it has two integrators (poles at zero) in the open-loop transfer function.





1. Since there are no zeros and the poles are at the origin, this indicates that the system has a pure integrator behavior, which results in an unbounded response over time for any non-zero input.
2. The amplitude increases quadratically with time, meaning the system will not reach a steady state but instead continue to grow without bound.This also denotes that the system is unstable as it will not reach to a steady state.

6. Task 3: PID Control

3.1 Design a controller with unity feedback and plot the performance of the following controllers for varying gains:

Proportional Controller. Now, let's study the response of the system when using a proportional controller.

Now, the transfer function for a proportional controller is given by

$$C(s) = K_\mathrm{p} \tag{5}$$

Thus, we have plotted the step response of the closed loop transfer function for four different values of $\mathrm{K_p}$ as shown below:

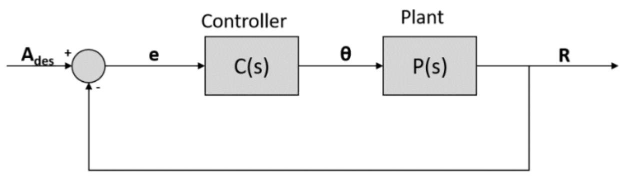
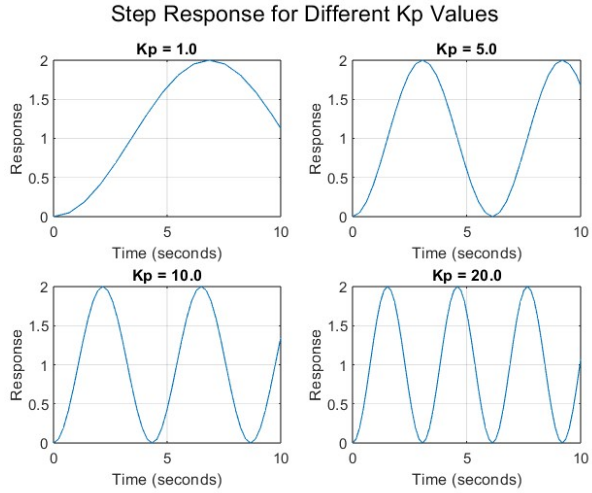
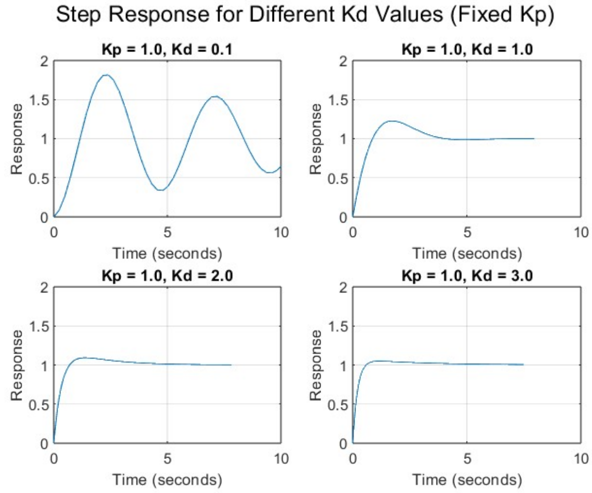


Figure 2. *System Representation*





Proportional-derivative controller. Now, lets add a derivative term to the controller. Therefore, the transfer function for proportional-derivative control is given by

$$C(s) = K_\mathrm{p} + K_\mathrm{d}s \tag{6}$$

Thus, we plot the transfer function for a proportional-derivative controller for varying values of $\mathrm{K_d}$ and a fixed value of $\mathrm{K_p}$. The plot for this is shown below:

Proportional-integral-derivative controller. Finally, we add an integral term to the controller. So the transfer function becomes

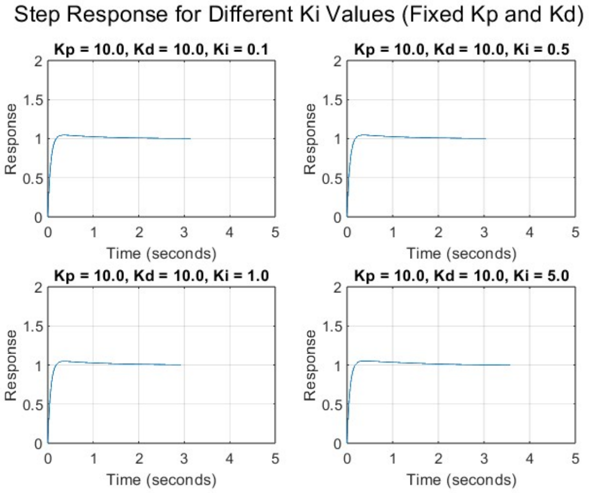
$$C(s) = K_\mathrm{p} + \frac{K_\mathrm{i}}{s} + K_\mathrm{d}s \tag{7}$$

6.1. Mention your observations on all three controller types and how they affect transient response, steady-state response, and set point error.

Observations :

Proportional Controller (P):.

- **Transient Response:** The settling time tends to decrease very slightly with higher K_p values.



- **Steady-State Response:** Steady-state errors remain if the system has a non-zero reference input, particularly for constant disturbances (e.g., steady-state error).
- **Set Point Error:** The proportional controller does not eliminate steady-state error for constant inputs.

Proportional-Derivative Controller (PD):.

- **Transient Response:** Adding a derivative term helps improve the transient response, reducing the settling time and overshoot compared to the P controller. It effectively anticipates future behavior based on the rate of error change.
- **Steady-State Response:** Similar to P controllers, PD controllers do not eliminate steady-state error.
- **Set Point Error:** Can be less pronounced compared to the P controller alone but still exists.

Proportional-Integral-Derivative Controller (PID):.

- **Transient Response:** PID controllers generally offer better performance in terms of reducing both settling time and overshoot compared to P and PD controllers. The integral term improves the ability to eliminate steady-state errors.
- **Steady-State Response:** The integral component helps eliminate steady-state error for constant inputs, making the system more accurate.
- **Set Point Error:** The PID controller can achieve zero steady-state error in the presence of constant disturbances.

3.3 Design a controller to follow the following criteria - settling time less than 3 seconds and overshoot less than 5%

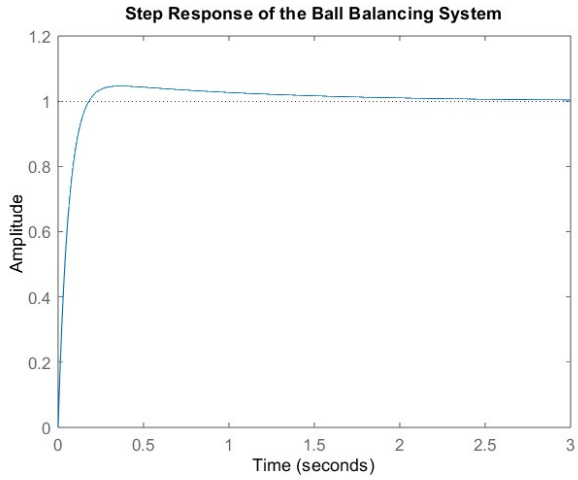
After tuning the PID controller for values of K_p,K_i and K_d we get the following values,

$$K_p = 10$$

$$K_i = 1$$

$$K_d = 10$$

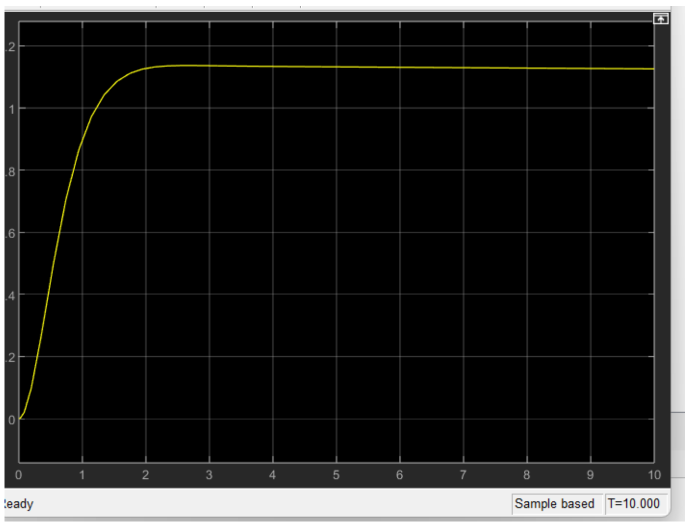
For these values, we get the settling time and overshoot as **1.2937 seconds** and **4.6251%** respectively. The step response of the closed loop transfer function is given below:



Task 4: Simulation of the Ball and Beam in MATLAB

The ball and beam system was simulated in MATLAB and Simulink. The files for those have been submitted separately.

The output is given as follows:



Task 5: Physical System

After the process of PID tuning, the gains obtained were:

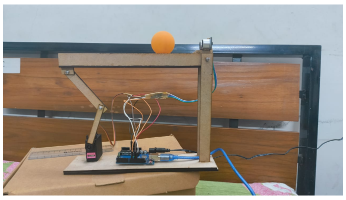
$$K_p = 9$$

$$K_i = 0.008$$

$$K_d = 1$$

The image of the system is as shown: **Process:**

- Initially, small values of K_p were taken, and K_i and K_d were taken as 0.



- The value of K_p was increased until the setpoint was reached.
- The values of K_i and K_d were then changed to satisfy the required criteria of maximum overshoot and settling time.

Observations

- During the experiment we observe that when we were increasing the value of Kp then system was unstable as it was varying with high frequency.
- So we decreased the value of Kp and stopped at certain point. Then we start changing the values of Kd keeping the Ki constant. In the end our system become stable.
- after trying many times our ball was staying at same position so we were able to conclude that it's stable.

7. Challenges Faced

- While making the setup, some things were missed which were only realised during assembly, and hence the parts needed to be recut.
- The model in simscape was particularly hard to make, since we were using simscape for the first time.
- The PID control was very hard and time consuming owing to the trial and error nature of the process. The gains took a lot of time to finally get right.

References

[1] C. T. for MATLAB and S. (CTMS), *Ball and beam control*, https://ctms.engin.umich.edu/CTMS/index.php?example=BallBeam&section=SimulinkControl, Accessed: 2024-10-20, 2024.

Control Systems: Project part II

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Task 6: Root Locus and Bode Plot of the Ball-Beam System

6.1: Plotting graphs for the system's open-loop transfer function

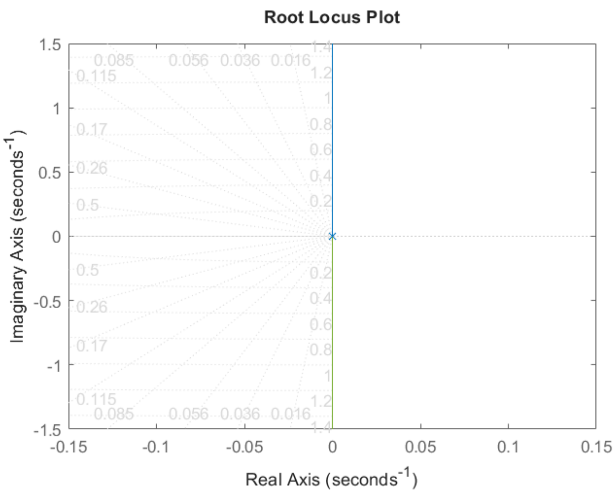

Figure 1. *Root locus plot of the open-loop transfer function*

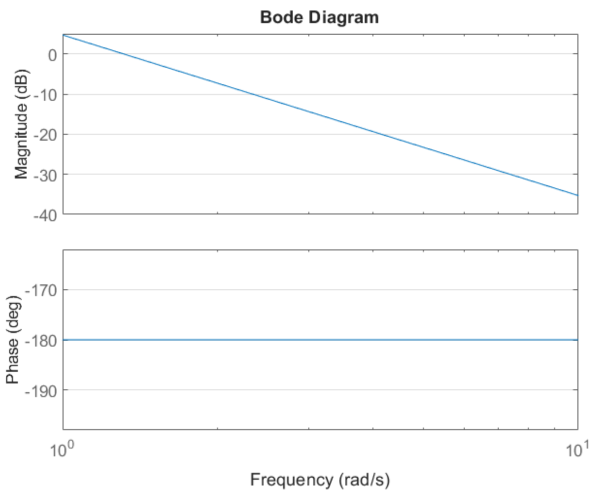

Figure 2. *Bode plot of the open-loop transfer function*

Obtained gain margin (K_g)= 1 dB
Obtained phase margin (ϕ_m)= 0 degrees

6.2: Considering the given design criteria and proceeding accordingly

Given design criteria:

$$M_p(\%) \leq 5\% \text{ or } M_p \leq 0.05$$

$$t_s \leq 3s$$

Where M_p is the maximum overshoot and t_s is the settling time. From the equations below, the system parameters were obtained, after which the dominant closed loop poles were obtained.
Maximum overshoot:

$$M_p = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

Settling time (2% criterion):

$$t_s = \frac{4}{\zeta\omega_n}$$

Putting the values of M_p and t_s, we get:

$$\zeta = 0.69$$

$$\omega_n = 1.932$$

Now to obtain the closed loop poles, we solve the following characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Solving the above equation, the dominant closed loop poles were obtained to be:

$$s = 1.333 \mp j1.398$$

Also the desired phase margin was calculated using the following formula:

$$\phi_p \approx \arctan\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^2}}}\right)$$

From the above equation, the desired phase margin came out to be:

$$\phi_m = 61.42°$$

So, our system needs to have a phase margin above this to be sufficiently stable.

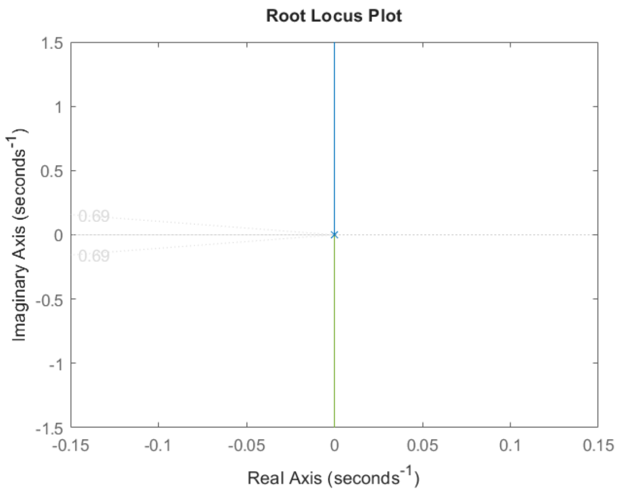

Figure 3. *Root locus of the open-loop transfer function of the system with the design criteria*

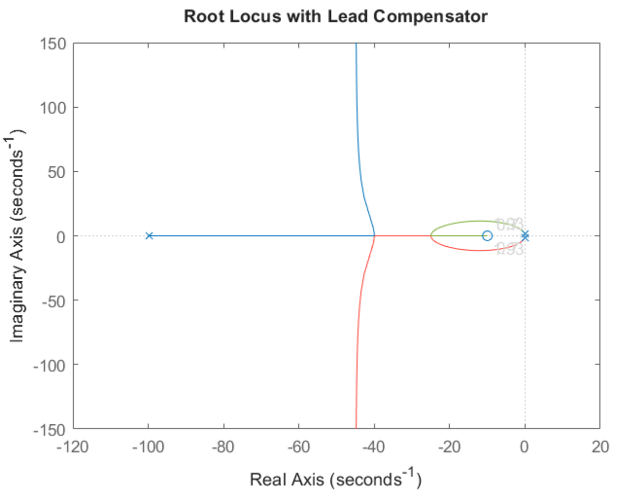

Figure 4. *Root locus of the designed system*

Compensator recommendation:

- The design criteria suggest a need for better damping and settling time, so a *lead compensator* might be necessary to improve the system's response by obtaining the dominant closed loop poles.

- The lead compensator can help shift the poles to improve the damping ratio and increase the natural frequency.

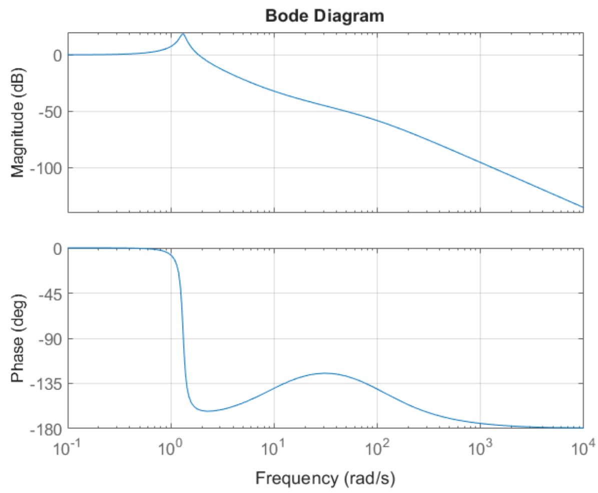

Figure 5. *Bode plot of recommended design*

Analysis of the Bode Plot:

- The Phase Margin gives an indication of the stability of the system and directly correlates to the overshoot. For less than 5% overshoot, a phase margin above 50-60 degrees is needed.

- The Gain Margin indicates how much gain can be increased before the system becomes unstable.

- If the phase margin is insufficient, a lead compensator can be used to increase the phase margin and improve stability.

Task 7: Controller Design using Root Locus and Bode Plot Approaches

The phase margin of the uncompensated system was zero degrees and was hence unstable and needed a positive phase margin for stability, hence a lead compensator was chosen.

7.1: Design of a first-order lead/lag compensator to meet the design criteria

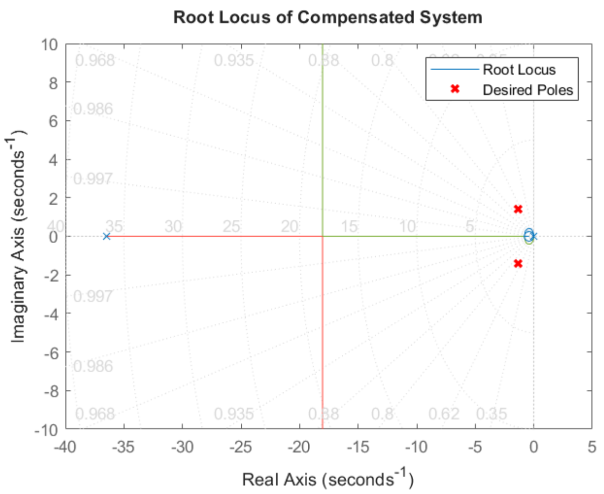

Figure 6. *Root locus of the compensated system*

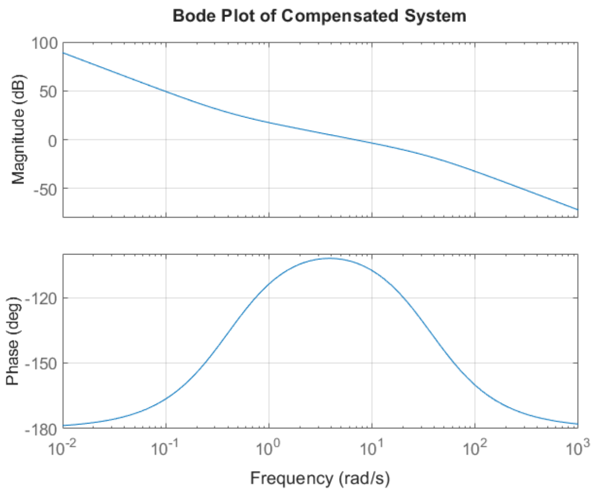

Figure 7. *Bode plot of the compensated system*

7.2: Effect of the compensator on the pole-zero placement

Placement Strategy:

- Since a lead compensator is being used, we added a pole and a zero for the lead compensator such that the pole is farther away from the origin compared to the zero to give the required phase lead to the system.

- The placements of poles and zeros and the determination of the value of the gain K was done using trial and error until the required design criteria were met.

Effects on the root locus:

- The addition of the zero near the required dominant closed loop poles pulls the root locus towards the left half of the s-plane increasing stability.

- Increase in stability improves the transient response specifications, reducing the settling time and maximum overshoot.

Effects on the Bode plot:

- A positive phase boost is reflected in the system since the lead compensator increases the gain and phase margins of the system. The gain margin.

- The overall phase margin is increased since the zero adds a phase lead and the pole reduces the phase lead at higher frequencies.

Therefore the obtained lead compensator was:

$$G_c(s) = 147\frac{s+0.41}{s+36.5}$$

Hence the final open loop transfer function comes out to be:

$$G_c(s)G(s) = 252.693\frac{s+0.41}{s^2(s+36.5)}$$

7.3: Plotting and verification of the closed-loop response of the system with the compensator

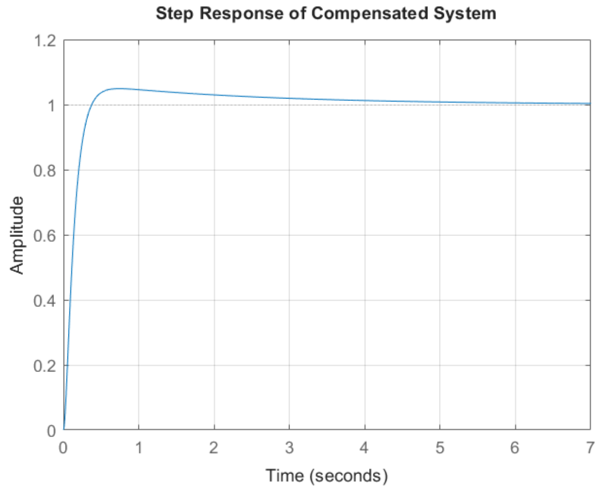

Figure 8. *Step response of the compensated system*

Verification using the root locus approach:

- The transient response specifications were satisfied by our system. The settling time obtained was 2.9052 which is less than the required 3 seconds and the maximum overshoot is 4.89% which is less than the required 5%.

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Verification using the Bode plot approach:

- We obtained the phase margin to be 75.98 degrees which is over our desired value of 61.42 degrees.

Task 8

This is the physical demonstration of our project.