ME206, Experiment 2

MOTION OF AN OBJECT WITH RESPECT TO A ROTATING FRAME OF REFERENCE

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Introduction

This experiment was conducted to exhibit and measure the velocity and acceleration of a moving particle with respect to a rotating frame of reference. To show this, the experimental setup which was devised is described as follows:

- The setup consists of two parts, the rotating frame, and the translating object (whose motion have to analyse with respect to the rotating frame).
- **The rotating frame:**
  - It consists of a disc, which has a ball-bearing fixed to its centre to enable it to rotate freely.
  - There is a stationary frame which serves as the support or stand for the rotating frame.
  - The disc system is attached to the rotating frame by a rod which passes through the ball bearing and the stationary frame, securing them together as well as allowing the disc to rotate freely.
- **The translating object:**
  - This is the object whose motion we have to analyse with respect to the rotating frame.
  - For this experiment, we have used a simple table tennis ball as our object.
  - To translate it, we used a groove, in the form of the rail on which a window slides. We rolled the ball on the groove to give it a translational velocity.
- The translating object and the rotating frame were brought onto the same level. This was done by placing a desk and a stack of books below the rotating frame.
- Then, a phone was used to record the motion of both the rotating frame and the translating object simultaneously.

Experimental Design

AIM OF THE EXPERIMENT: Device an experimental setup to exhibit the velocity and acceleration of a moving particle with respect to a rotating frame of reference ($\vec{\Omega}, \vec{\dot{\Omega}}$).

For physically creating the setup, we made use of a lot of different tools and software available so that we could make a design which is fine-tuned to the requirements of our experiments and is able to achieve what we want it to do.

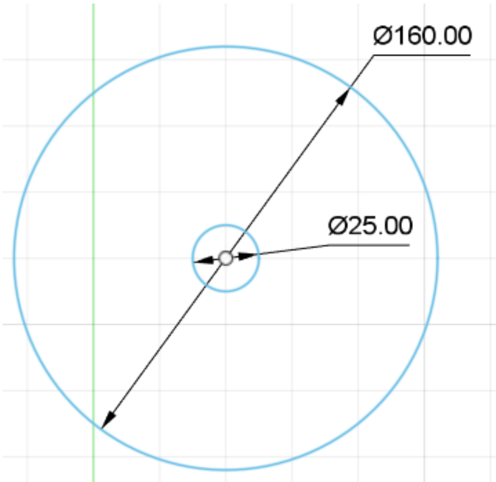
We used Autodesk's Fusion360 software to model the parts required for our setup, and them exported them as .dxf files so that they could be edited in Trocen's LaserCAD software for use in the laser cutting machine. More about this is in the *Fabrication Details* section below.

We also used a phone to record the motion of the ball and the rotating disc from the top and then analysed the video recordings to obtain the velocity and position of the ball. Both the rotational motion of the disc and translational motion of the ball were initiated by hand.

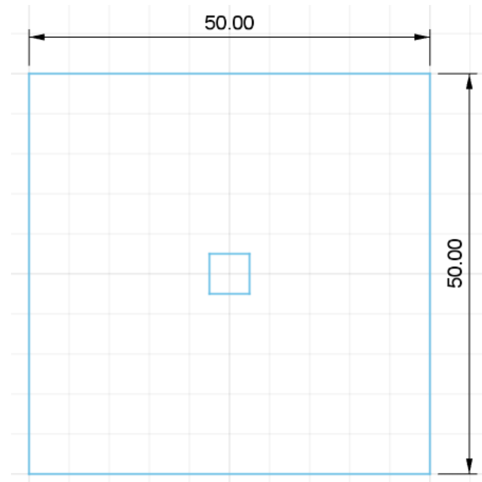
Engineering Drawings

CAD Models

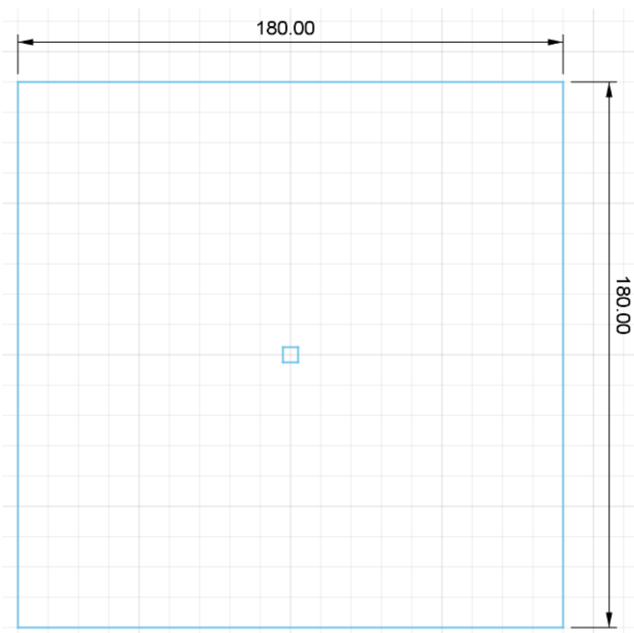
Below are the images of the models of the parts designed in Autodesk's Fusion360 CAD modeling software.

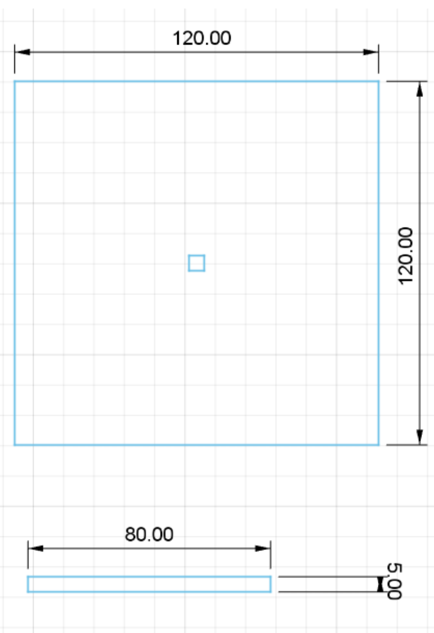


The disc of the rotating frame.



A part of the stationary frame.





Other parts of the stationary frame. The 80mm long rod passes through the 5mm wide square holes in the square parts above to secure the stationary frame.

Material Data

The material used in the making of the setup for this experiment is fairly straightforward and somewhat easily available. Below is the list of the material that was used in the making of our experimental setup:

1. 5mm thick MDF sheet ($667.153cm^2$)
2. Ball-Bearing (Outer diameter 2.5cm, inner diameter 1cm, height 7.5mm)

Fabrication Details

The setup for this experiment was fabricated in three parts. The first part was performed using software, like Fusion360 and LaserCAD. The second part was the cutting of the CAD designs on the MDF sheet. The third part was the entire assembly of the parts.

To cut out our designs, we utilised the laser-cutting machine available in the Tinkerers' Lab at IIT Gandhinagar.

- The Fusion360 sketches in CAD were first exported as .dxf files.
- These files were then opened in the LaserCAD software and edited so that the space between the different elements was minimized (to prevent wastage of material).
- Then, the obtained file was loaded onto the computer connected to the laser-cutting machine which then downloaded the file onto the machine.
- After that, the machine was switched on and the parts were cut out.

After obtaining the parts, they were systematically joined together.

- First, the ball bearing was fit into the inner hole of the disc.
- Then, the stationary frame was assembled by stacking the squares on top of one another (with the largest one being at the bottom and the smallest one at the top) and placing the two small discs on top of the stack, and then passing the cubical rod through the square holes in the centres of all the elements to secure them. tem was hammered onto the small discs so the ball bearing was secured to the frame and the structure was complete.

Theoretical and Mathematical Analysis

The following expressions were used to calculate the velocity and acceleration of the object with respect to the rotating reference frame:

$$\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\Omega} \times \vec{r}_{AB}$$

$$\frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d^2\vec{r}_{AB}}{dt^2}\right)_{Bxyz} + \vec{\dot{\Omega}} \times \vec{r}_{AB} + 2\vec{\Omega} \times \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{AB})$$

The video recordings of the experiment we conducted were thoroughly analysed by us, and and our methods of analysis can be found in the *Measurement Techniques* section below.

$$\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\Omega} \times \vec{r}_{AB}$$

$$\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{x_1 - x_0}{2\Delta t}\right)\hat{\imath} + \left(\frac{y_1 - y_0}{2\Delta t}\right)\hat{\jmath} + \vec{\Omega} \times (x_{0.5}\hat{\imath} + y_{0.5}\hat{\jmath}) \;.\;.\;.\;.\;.\; (i)$$

$$\frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d^2\vec{r}_{AB}}{dt^2}\right)_{Bxyz} + \vec{\dot{\Omega}} \times \vec{r}_{AB} + 2\vec{\Omega} \times \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}_{AB}\right)$$

$$\frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{x_1 - 2x_{0.5} + x_0}{\Delta t^2}\right)\hat{\imath} + \left(\frac{y_1 - 2y_{0.5} + y_0}{\Delta t^2}\right)\hat{\jmath} + 0$$
$$+ 2\vec{\Omega} \times \left(\left(\frac{x_1 - x_0}{2\Delta t}\right)\hat{\imath} + \left(\frac{y_1 - y_0}{2\Delta t}\right)\hat{\jmath}\right) + \vec{\Omega} \times \left(\vec{\Omega} \times (x_{0.5}\hat{\imath} + y_{0.5}\hat{\jmath})\right) \;.\;.\;.\;.\;.\; (ii)$$

The subscripts for x and y indicate the time instant at which that particular value of x or y was measured. Here are the measured values for $\vec{r}_{AB}$:

1st set of readings:
$\vec{\Omega} = -5.06\hat{k}\ rad/s$
$At\ t = 0s$: $x_0 = 19.99cm, y_0 = 14.67cm, \vec{r}_{AB} = (19.99\hat{\imath} + 14.67\hat{\jmath})cm$
$At\ t = 0.5s$: $x_{0.5} = -17.26cm, y_0 = -6.24cm, \vec{r}_{AB} = (-17.26\hat{\imath} - 6.74\hat{\jmath})cm$
$At\ t = 1s$: $x_1 = 15.64cm, y_1 = 3.28cm, \vec{r}_{AB} = (15.64\hat{\imath} + 3.28\hat{\jmath})$

2nd set of readings:
$\vec{\Omega} = -8.38\hat{k}\ rad/s$
$At\ t = 0s$: $x_0 = 13.89cm, y_0 = 14.74cm, \vec{r}_{AB} = (13.89\hat{\imath} + 14.74\hat{\jmath})cm$
$At\ t = 0.5s$: $x_{0.5} = 7.6cm, y_0 = -14.8cm, \vec{r}_{AB} = (7.6\hat{\imath} - 14.8\hat{\jmath})\text{cm}$
$At\ t = 1s$: $x_1 = -12.44cm, y_1 = 7.99cm, \vec{r}_{AB} = (-12.44\hat{\imath} + 7.99\hat{\jmath})cm$

3rd set of readings:
$\vec{\Omega} = -5.76\hat{k}\ rad/s$
$At\ t = 0s$: $x_0 = 19.20cm, y_0 = 14.40cm, \vec{r}_{AB} = (19.20\hat{\imath} + 14.40\hat{\jmath})cm$
$At\ t = 0.5s$: $x_{0.5} = -13.47cm, y_0 = -11.79cm, \vec{r}_{AB} = (-13.47\hat{\imath} - 11.79\hat{\jmath})cm$
$At\ t = 1s$: $x_1 = 4.21cm, y_1 = 14.74cm, \vec{r}_{AB} = (4.21\hat{\imath} + 14.74\hat{\jmath})cm$

After substituting these obtained values into the equations (i) and (ii), we get the following values for the velocity and acceleration of the object at $t = 0.5s$:

1st set of readings:
$\vec{\dot{r}}_{AB} = (-0.2975\hat{\imath} + 0.7594\hat{\jmath})\,m/s$
$\vec{\ddot{r}}_{AB} = (6.07\hat{\imath} + 2.48\hat{\jmath})\,m/s^2$

2^{nd} set of readings:
$\vec{\dot{r}}_{AB} = (-1.504\hat{\imath} - 0.704\hat{\jmath})\, m/s$
$\vec{\ddot{r}}_{AB} = (-7.02\hat{\imath} + 16.90\hat{\jmath})\, m/s^2$

3^{rd} set of readings:
$\vec{\dot{r}}_{AB} = (-0.53\hat{\imath} + 0.78\hat{\jmath})\, m/s$
$\vec{\ddot{r}}_{AB} = (6.52\hat{\imath} + 7.75\hat{\jmath})\, m/s^2$

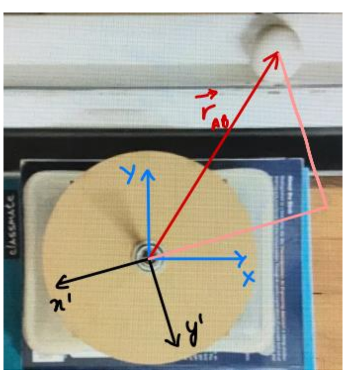
Measurement Techniques

For this experiment, we employed fairly simple measurement techniques. We performed the experiment three different times.

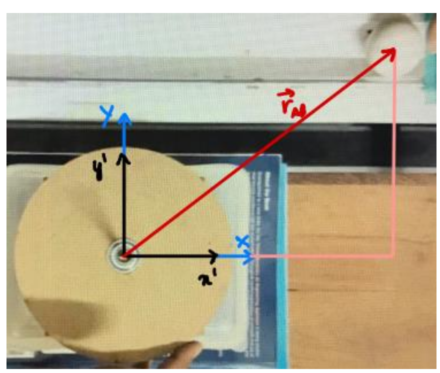
To obtain the scale for the system, we first measured the radius of the disc on screen, and divided the value of the actual radius by the measured value. We then multiplied all the values of displacements obtained by the scale.

$$Scale = \frac{measured\ value\ on\ model}{measured\ value\ on\ screen}$$

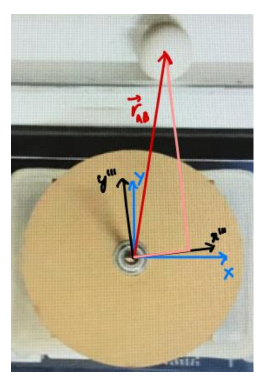
We assumed the acceleration and the velocity of the translating object (the ball) to be constant for the very short period of time in which we conducted our measurements. Assuming the motion of the ball to start at $t = 0s$, we measured its velocity at $t = 0.5s$. For this, we first obtained the displacement vectors at $t = 0s, t = 0.5s$ and $t = 1s$ respectively. We then used the Taylor series expansion to approximate the values of the derivative and double derivative of the displacement obtained.



At $t = 0s$



At $t = 0.5$s



At $t = 1s$

Results

	1st set of readings	2nd set of readings	3rd set of readings
$\lvert r_{AB} \rvert$ (cm)	18.35	16.4	24
$\lvert \dot{r}_{AB} \rvert$ (m/s)	0.815	0.894	0.94
$\lvert \ddot{r}_{AB} \rvert$ (m/s^2)	6.56	18.3	10.13

From the table, we can see that there is an increasing trend in the values of $|r_{AB}|$ and $|\dot{r}_{AB}|$, but however, this does not hold true for $|\ddot{r}_{AB}|$.

Discussions

In the context of this experiment, our primary aim was to both demonstrate and quantify the velocity and acceleration of a moving particle within the confines of a rotating frame of reference. The experimental apparatus, composed of a rotating frame and a translating object, was thoughtfully devised to provide a practical avenue for delving into the intricacies of such a dynamic system. We emploed advanced tools like Autodesk's Fusion360 for meticulous part modeling and Trocen's LaserCAD for efficient laser cutting. Central to our setup was the use of a table tennis ball as the translating object, guided along a groove that was the sliding rail of a window. This configuration facilitated the emulation of translational motion within the rotating frame. Subsequently, we used a smartphone to record the motion, followed by a meticulous analysis of the data to extract valuable velocity and position information.

The data we acquired, encompassing velocity and acceleration measurements across three distinct trials, has illuminated the behaviour of the translating object within the rotating frame. These datasets have unveiled diverse patterns of motion, effectively illustrating how the interplay between translational and rotational motion influences the velocity and acceleration of the particle. Upon close examination, it becomes apparent that the velocity vectors of the particle manifest varying magnitudes and orientations in each trial. Similarly, the acceleration vectors exhibit different magnitudes and directions, showing the interplay between centripetal and tangential acceleration components. This experiment, therefore, stands as an instructive showcase of the principles governing motion within rotating frames of reference.

Scope for Improvement

There is always scope for improvement for an experiment. Here are some of the improvements which can be made to our experimental setup so that we can get more accurate results:

1. **Measurement Accuracy:**
   - Utilize more precise measurement instruments like digital calipers or laser sensors to meticulously gauge the dimensions of the components and the translating object's position.
   - Employ a more advanced approach for measuring the rotational velocity (Ω) of the rotating frame, such as the utilization of an optical encoder or a tachometer, to achieve higher accuracy.
2. **Data Recording:**
   - Shift from relying on a smartphone camera to capture motion and instead explore the use of high-speed cameras or dedicated video analysis software. These options can offer more precise data, especially when tracking small and swiftly moving objects like a table tennis ball.
   - Enhance data collection by integrating sensors like accelerometers or gyroscopes. These sensors can directly measure the acceleration and angular velocity of the rotating frame, providing more accurate insights into your experiment.
3. **Educational Value:**
   - Consider how this experiment can be used as an educational tool. Provide a clear explanation of the scientific principles involved and their real-world applications. Consider creating educational materials, such as a lab manual or instructional video, to help students understand the experiment better.

References

[1] MIT OpenCourseWare, "Velocity and Acceleration in a Rotating Frame of Reference," YouTube. [Online]. Available:
https://youtu.be/IOcrHOc23N4?si=AQq2OKxjZX1CHQOK
[Accessed: 28/09/2023].

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Last but not least, we would like to thank our peers and fellow students who collaborated and shared insights to make this experiment a success.