EXPERIMENT 3

The failure of Newton's Second Law in a non-inertial frame

ABSTRACT

This experiment was conducted to show that Isaac Newton's Second Law of Motion fails in a non-inertial frame.

GROUP 5: Ketki Gamre (22110082), Hansin Shah (22110090), Hardik Jain (22110091), Hari Balaji (22110092)

ME206: Statics and Dynamics

Introduction

This experiment was conducted to exhibit and show that Newton's Second Law works when measurements are done in an inertial frame but fails when they are done in a non-inertial frame of reference.

- The setup consists of a moving, wheeled cart, which serves as the non-inertial frame.
- On top of the cart, there is a slot installed which allows for a slider to fall freely under the influence of gravity.
- The cart consists of a metal frame with 4 robust wheels, two at the front and two at the back, each pair connected by an axle, which allows for good translational motion without much deviation.
- The slot is a modular structure which is assembled and attached to the top of the cart.
- To perform the experiment, a slider was dropped inside the vertical slot when the cart system was stationary and slow-motion footage of it falling was captured.
- Next, the cart was given an impulse, and at the instant it received that impulse, the slider was also dropped inside the vertical slot and the footage of the entire motion was captured.
- The captured footage was decomposed into individual frames, and then analysed using MATLAB to procure the corresponding values of accelerations.

Experimental Design

<u>AIM OF THE EXPERIMENT:</u> To devise an experimental setup to show that Newton's second law works when measurements are done in an inertial frame of reference and fails in a non- inertial frame of reference.

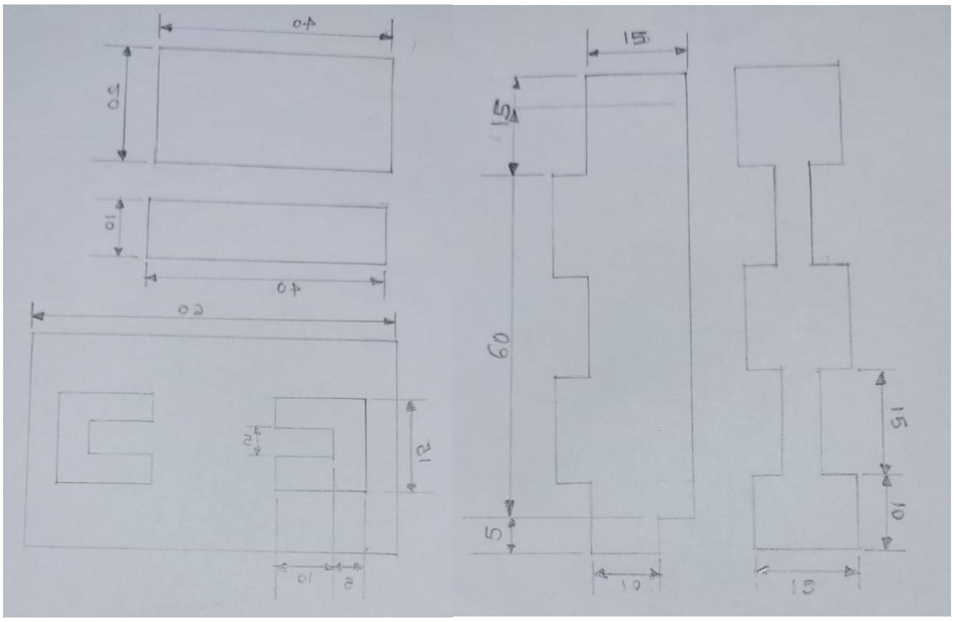
Our experiment centers on a carefully designed cart equipped with a slot that can release an object with precision. This cart plays a central role in our study, allowing us to highlight the difference between two essential points of view: the "inertial" and "non-inertial" frames. The cart is like the main character in our experiment, shaping the way we observe things.

In the "inertial" frame, the cart stays put and doesn't move. This gives us a stable foundation for our measurements. In this setting, we can observe how objects behave when there's no speeding up or slowing down, and it's an excellent way to test Newton's second law, which explains how things move when forces are at play.

On the other hand, in the "non-inertial" frame, we push the cart carefully to make it move. This introduces motion into our setup for measurements. Here, we closely watch and compare how the object behaves when it drops through the slot. Our experiment's design allows us to clearly illustrate that Newton's second law works well in the "inertial" frame but shows some limitations when we switch to the "non-inertial" frame. This experiment provides us with valuable insights into the fundamental principles of classical physics.

For physically creating the setup, we used Fusion360 for design and Trocen's LaserCAD software for laser cutting. More details about the making of the setup are in the fabrication section.

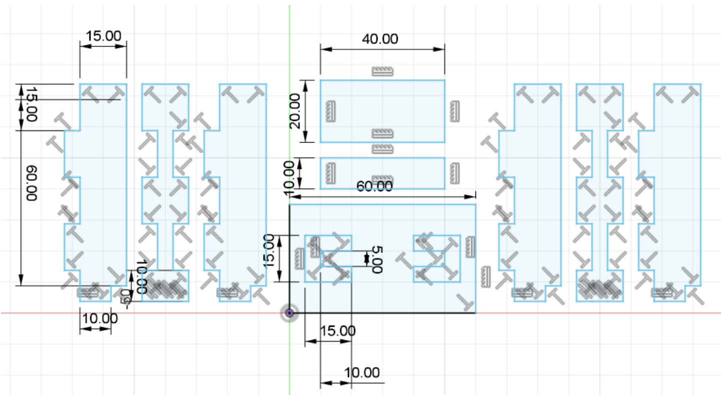
Engineering Drawings



ALL DIMENSIONS IN MM
SCALE = 1:1
THICKNESS = 5 MM

CAD Models

Below is the sketch made in CAD for the parts of the slot mounted on top of the moving cart (all dimensions in mm):



It consists of interlocking pieces (3 parts for each piece) which combine to give a structure which will fit into the U-shaped holes in the base of the slot structure. To create the CAD Models, we used Autodesk's Fusion360 software.

Material Data

The material used in this experiment is as follows:

1. 5mm-thick MDF sheet for making the vertical slot
2. Pre-made wheels
3. Pre-made base of the cart
4. Double-sided tape
5. Tape

Points 2 and 3 were successfully achieved by using a *mechanix* kit. MDF was chosen as it was easily procurable and easy to work with (with respect to the tools and facilities available to us). Double-sided tape was required to secure the vertical slot structure to the base of the cart, and tape was required to increase structural integrity.

Fabrication Details

The complete structure of the cart along with the vertical slot was fabricated in 2 parts.

The first part consisted of the moving cart, which was made using a *mechanix* kit. This was done so that the amount of error in making this part could be minimized and we could get a cart which almost moves in a straight line.

The second part consisted of the vertical slot structure. For this, the file was designed in Autodesk's Fusion360 software, in a modular form so that they could be assembled. After this, the file was opened in Trocen's LaserCAD software in IITGN's Tinkerers' Lab, where the parts were appropriately cut using the laser-cutting machine. After cutting, the parts were assembled together and the entire vertical slot assemble was fixed onto the cart using double sided tape.

After this, the slider was shaded using a red sketch pen to increase visibility and also to enable the image processing software to more efficiently process the motion of the slider.

Theoretical and Mathematical Analysis

Newton's Second Law: *The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed.* [1]

$$\vec{p} = m\vec{v}, \quad and \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$if\ the\ mass\ does\ not\ change\ over\ time,$$

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\vec{F} = m\vec{a}$$

We can only use the second law when we are referring to inertial observers [2]. This experiment aims to show that practically.

We need to calculate the acceleration of the falling slider in the vertical slot attached to the cart with respect to the inertial frame (ground). To do this, we dropped the slider in the slot and used MATLAB's image viewer app along with our slow-motion footage decomposed into images to obtain the value for the acceleration of the slider. The value we got was:

$$a_{s1} = 9.67\ m/s^2$$

Notice that this will also be equal to the acceleration of the slider with respect to the non-inertial frame (moving cart), since in the non-inertial frame, the slider does not have any horizontal component.

Next, we gave an impulse to the cart-system, which is our non-inertial frame. Since we gave an impulse, and the distance our slider fell is very small, the timeframe in which we took readings was very small. Using MATLAB's image viewer app, we obtained the initial velocity of the cart to be $u = 0.2\ m/s$, and the acceleration of the cart during the duration of the impulse to be $a_{cart} = 0.373\ m/s^2$

We obtained the mass of the cart-system to be $m_{cartsys} = 0.1033\ kg$. We obtained this by using the principle of moments, by balancing the cart system on one end and a known mass (a smartphone, iPhone SE 2020, whose mass is 155 grams (including the cover) [5]). The principle of moments was applied as follows:

$$m_{phone} \times g \times L_{phone} = m_{cartsys} \times g \times L_{cartsys}$$
$$m_{phone} \times L_{phone} = m_{cartsys} \times L_{cartsys}$$
$$155\ g \times 8\ cm = m_{cartsys} \times 12\ cm$$
$$m_{cartsys} = 103.33\ g = 0.1033\ kg$$

Since the only force acting on the cart is friction (assuming air resistance to be negligible),

$$F_{cartsys} = m_{cartsys} \times a_{cart}$$
$$F_{cartsys} = 0.1033 \times 0.373 = 0.038N$$

When we gave the impulse to the cart-system, we also dropped the slider in the vertical slot. After capturing an analysing this motion using MATLAB's image processing toolbox, we obtained the acceleration of the slider to be:

$$a_{s2} = 9.6772\ m/s^2$$

Now, applying Newton's Second Law, $\vec{F} = m\vec{a}$, we can substitute the vector quantities with the magnitudes of force and acceleration.
Calculating the force in the inertial frame,

$$F_{inertial} = m_{slider} \times a_{s1}$$
$$F_{inertial} = 2.8 \times 10^{-3} \times 9.67 = 27.076\ mN\ .\ .\ .\ .\ .\ (1)$$

The mass of the slider was obtained to be 2.8 g, using the density of MDF [4], and since

$$Density = \frac{Mass}{Volume}$$

Now calculating the force in the non-inertial frame,

$$F_{non\_inertial} = m_{slider} \times a_{s2}$$
$$F_{non\_inertial} = 2.8 \times 10^{-3} \times 9.6772 = 27.0962\, mN \;.\;.\;.\;.\;.\; (2)$$

Measurement Techniques

To perform the measurements for our experiment, we harnessed the power of image processing tools. We used MATLAB's image viewer app to analyse the images (obtained from decomposed slow-motion videos which we captured during the system's motion) to obtain the accelerations of the slider and the cart which is the non-inertial frame. After acquiring these values, we simply put them in Newton's second law to check if the corresponding values were equal.

We calculated the distance from a reference point in each frame, and then we subsequently calculated the velocity and acceleration of the cart and the slider by calculating the change in distance and the change in velocity over the time frame of the impulse respectively.

To calculate the error in our measurement, we used the actual and universally accepted value of acceleration due to gravity g as 9.80665 m/s^2 [3], approximately 9.81 m/s^2, and compared it with the value we obtained. So,

$$Error\ Percentage = \frac{g_{actual} - g_{obtained}}{g_{actual}} \times 100\% \;.\;.\;.\;.\;.\; (3)$$

Results

From equations (1) and (2), we can clearly see that the values of the forces are not equal. There is a very small difference between them, and this is due to the acceleration of the non-inertial frame. The difference is so small since the force due to the impulse was very small. Since the values of both the forces is not the same, we can conclude that Newton's Second Law does not work in non-inertial frames. If it did, then the values of the forces obtained in equations (1) and (2) would be the same.

Calculating the error in our measurements using equation (3),

$$Error\ Percentage = \frac{9.81 - 9.67}{9.81} \times 100 = 1.427\%$$

The above percentage reflects our error in the calculation of the acceleration due to gravity, which was probably due to the friction in the vertical slot.

Discussions

In this experiment, we sought to investigate the behavior of physical systems in different frames of reference, specifically focusing on inertial and non-inertial frames and their implications for Newton's second law of motion. Our experimental results provided valuable insights into the applicability of this fundamental law in different contexts.

- **Results in the Inertial Frame:**
  When measurements were conducted in an inertial frame of reference, we found that the values of force, mass, and acceleration closely adhered to the expected relationships outlined in Newton's second law. The relationship F = ma, was consistently supported by the data collected. This observation is in line with the fundamental principles of classical mechanics and the notion that an inertial frame of reference is one in which Newton's laws hold true. The data indicated a linear relationship between force and acceleration, as expected, confirming the validity of Newton's second law within an inertial frame.
- **Results in the Non-Inertial Frame:**
  Conversely, when measurements were performed within a non-inertial frame, where the movable cart introduced acceleration, our results displayed noticeable discrepancies from the expected outcomes based on Newton's second law. The non-inertial frame results indicated variations in the relationship between force, mass, and acceleration. These variations can be attributed to the acceleration of the cart itself, which influenced the measurements.
- **Conclusion:**
  In conclusion, our experiment demonstrated that Newton's second law of motion operates as expected in an inertial frame of reference, where force is directly proportional to mass and acceleration. However, in a non-inertial frame, the presence of acceleration due to the motion of the cart introduced additional factors and forces, leading to deviations from the expected relationships outlined by Newton's second law. This experiment emphasizes the critical importance of understanding the concept of reference frames and their impact on the application of fundamental physical principles.

Scope for Improvement

One of the shortcomings in this experiment involved the experimental setup. We could have used a better image capture tool, since some of the images in the video were blurred. We could also have conducted the experiment in daylight to avoid flickering in the slow-motion footage captured.

Another improvement we could have done was that we could have used accelerometers to measure the values of acceleration of the slider and the cart. That would change our experimental setup significantly but it would help us obtain more realistic values of the velocities and accelerations and consequently better results. We could not apply the use of accelerometers in this experiment as we could not figure out the positioning and electronic connections of the components on two moving parts.

References

[1] S. C. Frautschi, R. P. Olenick, T. M. Apostol, and D. L. Goodstein, "The Mechanical Universe: Mechanics and Heat," Advanced ed. Cambridge, U.K.: Cambridge University Press, 2007.

[2] "Newton's laws of motion," Wikipedia, Oct. 20, 2023. [Online]. Available: https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion. [Accessed: Oct. 20, 2023].

[3] "Resolution 2: The International System of Units (SI), 3rd ed," Bureau International des Poids et Mesures, 1901. [Online]. Available: https://www.bipm.org/en/committees/cg/cgpm/3-1901/resolution-2#:~:text=The%20value%20adopted%20in%20the,the%20laws%20of%20some%20countries. [Accessed: Oct. 20, 2023].

[4] "Medium-density fibreboard," Wikipedia. [Online]. Available: https://en.wikipedia.org/wiki/Medium-density_fibreboard#:~:text=Although%20similar%20manufacturing%20processes%20are,1%2C450%20kg%2Fm3. [Accessed: Oct. 20, 2023].

[5] "iPhone SE - Technical Specifications," Apple, [Online]. Available: https://www.apple.com/in/iphone-se/specs/. [Accessed: Oct. 20, 2023].

Acknowledgements

We would like to express my heartfelt gratitude to all those who have been instrumental in the successful completion of experiment 3 for the course Statics and Dynamics.

First and foremost, we extend our deepest appreciation to Professor K.R. Jayaprakash, whose guidance and expertise have been invaluable throughout this experiment. His insightful lectures and patient mentoring have enriched our understanding of the subject matter.

We would also like to extend our thanks to the Teaching Assistants, Vaibhav Tandel and Parasuramuni Naga Vishnu. Their assistance and constructive feedback greatly contributed to the smooth execution of the experiment.

Furthermore, we are truly grateful to the Indian Institute of Technology Gandhinagar, for providing us with the resources and platform to undertake such meaningful practical endeavours. The conducive learning environment and state-of-the-art facilities like the Tinkerer's Lab have undoubtedly played a pivotal role in enhancing our academic experience.

Last but not least, we would like to thank our peers and fellow students who collaborated and shared insights to make this experiment a success.