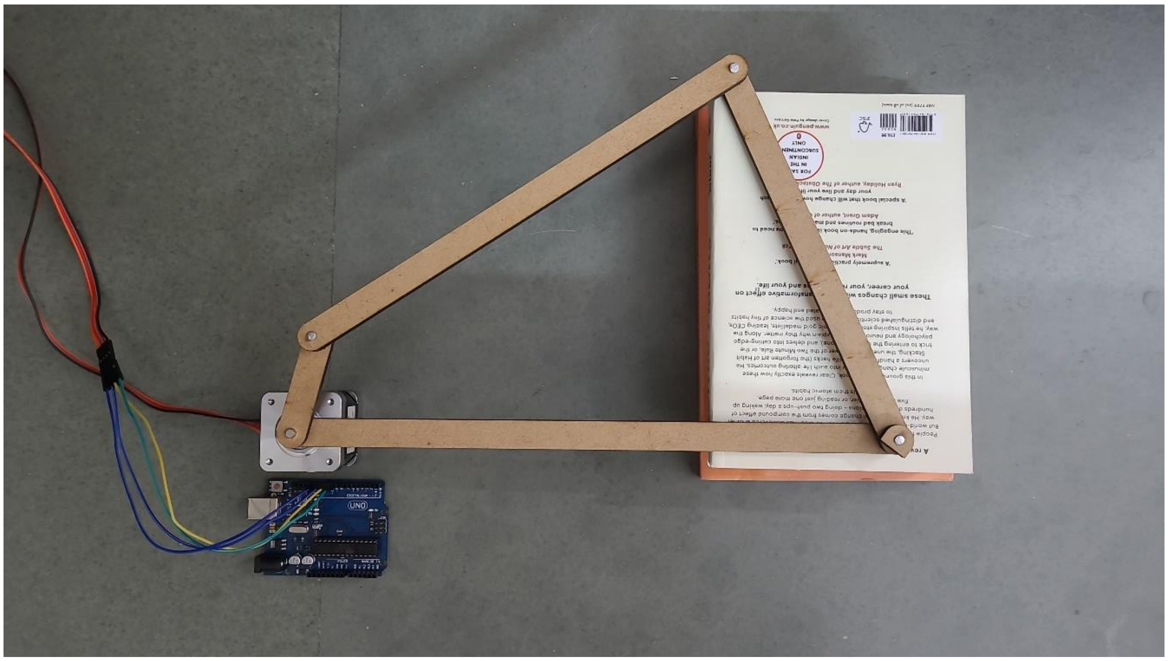

ME 206, Experiment 4

ANALYSING THE MOTION OF A FOUR -BAR LINKAGE MECHANISM

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Group 5 | 20th November, 2023

Introduction

The study of planar four-bar mechanisms is integral to the field of mechanical engineering, providing insights into the design and analysis of mechanical systems widely used in various applications. This experiment focuses on the investigation of a specific four-bar mechanism composed of a crank, rocker, and coupler linkages.

A planar four-bar mechanism is characterized by four rigid links connected by revolute joints. In this experiment, the mechanism is designed to have one link (crank) rotate at a constant angular velocity while the opposite link (rocker) undergoes partial rotation. The objective is to analyze the velocity of the center of mass of the coupler link, which connects the crank and the rocker, as well as the angular velocity of the coupler itself.

The experiment involves evaluating these velocities at different angular positions of the crank, specifically at intervals of 0, $\pi/3$, $2\pi/3$, and so on. This systematic exploration allows for a comprehensive understanding of the dynamic behavior of the four-bar mechanism throughout its complete revolution.

The analysis will consist of 3 parts:

- **The Theoretical Part:** The angular velocity of the centre of mass of the coupler link will be found using the basic principles and mathematical analysis of the four-bar linkage system.
- **The Experimental Part:** The motion of the coupler link will be recorded and analysed with the help of software to experimentally determine the velocity of its centre of mass.
- **The Simulation:** The four-bar linkage mechanism will be designed in the ADAMS simulation software, and the velocity of the centre of mass of the coupler link will be found using the simulated motion of the linkage.

All three parts of the analysis will then be compared to obtain comprehensive results about the motion of the coupler.

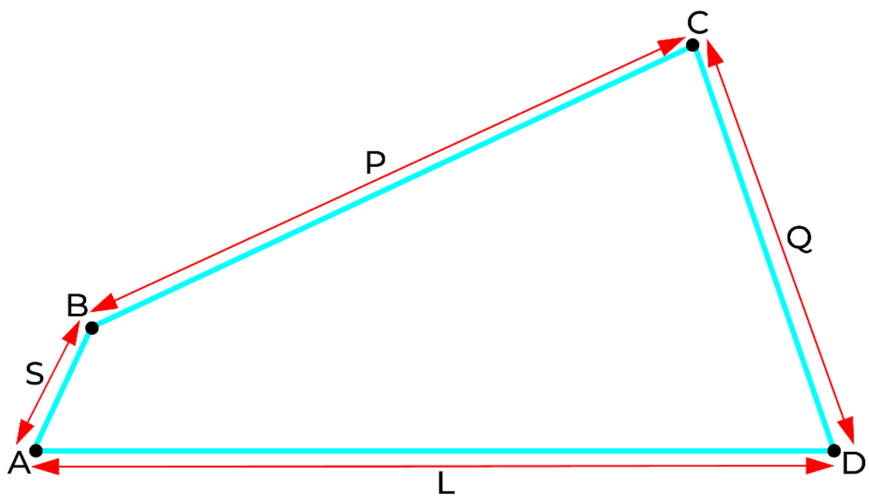
By conducting this experiment, we aim to deepen our understanding of the kinematics of planar four-bar mechanisms and gain practical insights into the correlation between theoretical predictions, experimental results, and simulations. This knowledge is crucial for engineers and researchers involved in the design and optimization of mechanical systems for diverse applications in industries ranging from automotive to robotics.

Experimental Design

AIM OF THE EXPERIMENT: To design a planar four-bar mechanism with one link (crank) that can rotate (at a constant angular velocity) completely and the opposite link (rocker) rotates partially. Find the velocity (analytically, experimentally and in ADAMS) of the center of mass of the coupler link (that connects crank and the rocker) and the angular velocity of the coupler for at least six different angles (0, π/3, 2π/3, etc.) of the crank.

To create the setup for the experiment, we were required to make a four-bar linkage mechanism. For this, we used Autodesk's Fusion360 software to model the parts required for our linkage.

The lengths of the four bars in the linkage were decided arbitrarily, but they followed the Grashof Condition.



According to the Grashof Condition, $\boldsymbol{S + L \leq P + Q}$.

Hence, we chose the lengths to be:
$S = 5cm$
$L = 30cm$
$P = 25cm$
$Q = 20cm$
These lengths obey Grashof's Law, and our four-bar linkage operates as expected.

The designs were then exported as .dxf files to allow them to be edited and prepared for laser-cutting in Trocen's LaserCAD software. The finished files were then laser cut using the Laser-Cutter in IITGN's Tinkerers' Lab, and then rivets were used to connect the linkages with each other. A motor was attached to the crank to rotate it with a constant angular velocity.

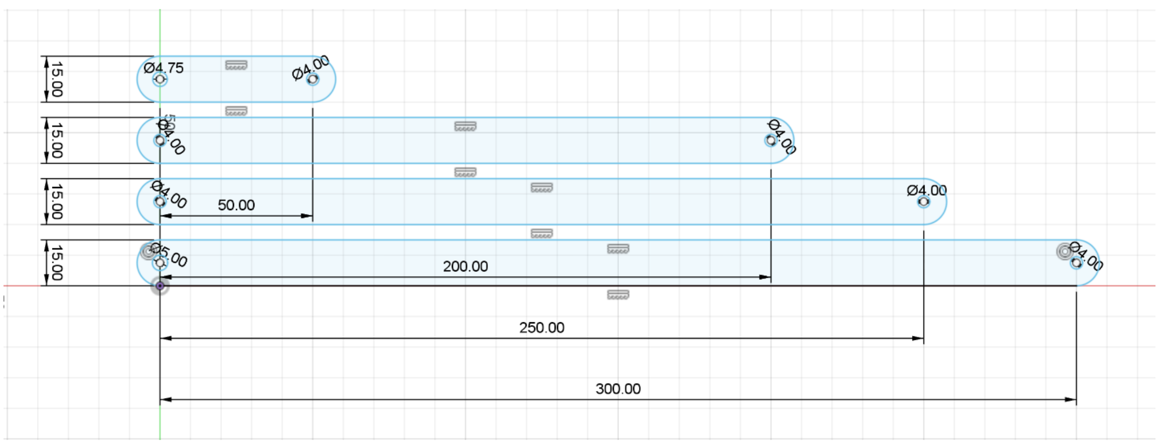
The motion was analysed using the basic principles of the motion of rotating bodies, and an analytical solution was obtained.

A phone was used to record the motion of the crank. The resulting video was analysed using MATLAB's image processing toolbox to obtain the results for the experimental part of the analysis.

We also made use of MSC's ADAMS software to simulate the mechanism on a computer and analyse it by giving values appropriate and corresponding to our experiment to complete the simulative part of our analysis.

CAD Models

Below are the images of the models of the parts designed in Autodesk's Fusion360:



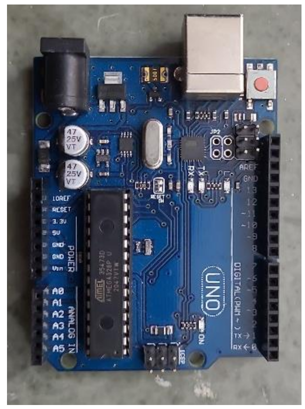
Material Data

The materials and parts used in the making of the setup for this experiment are as follows:

1. 5mm thick MDF sheet (400cm^2)
2. 6 aluminium rivets (5mm diameter, 10mm length)
3. Stepper Motor of maximum torque 2 kg-cm
4. Arduino Uno
5. Jumper Cables

This experiment utilises a stepper motor controlled by an Arduino so that different values of constant angular velocities may be given to the crank to increase the experimental data obtained.

Stepper Motor



Arduino Uno

Fabrication Details

The setup for this experiment was fabricated in three phases. The first phase was performed using software, like Fusion360 and LaserCAD. The second phase was the cutting of the CAD designs on the MDF sheet. The third phase was the entire assembly of the parts.

To cut out our designs, we utilised the laser-cutting machine available in the Tinkerers' Lab at IITGN.

- Firstly, the dimensions of the parts required were determined.
- Then, the parts were designed in Fusion360.
- The Fusion360 sketches in CAD were exported as .dxf files.
- These files were then opened in the LaserCAD software and edited so that the space between the different elements was minimized (to prevent wastage of material).
- Then, the obtained file was loaded onto the computer connected to the laser-cutting machine which then downloaded the file onto the machine.
- After that, the machine was switched on and the parts were cut out.

After obtaining the parts, they were systematically joined together.

- The parts were connected by using aluminium rivets which were passed through the holes in the cut parts and hammered to prevent dismantling.

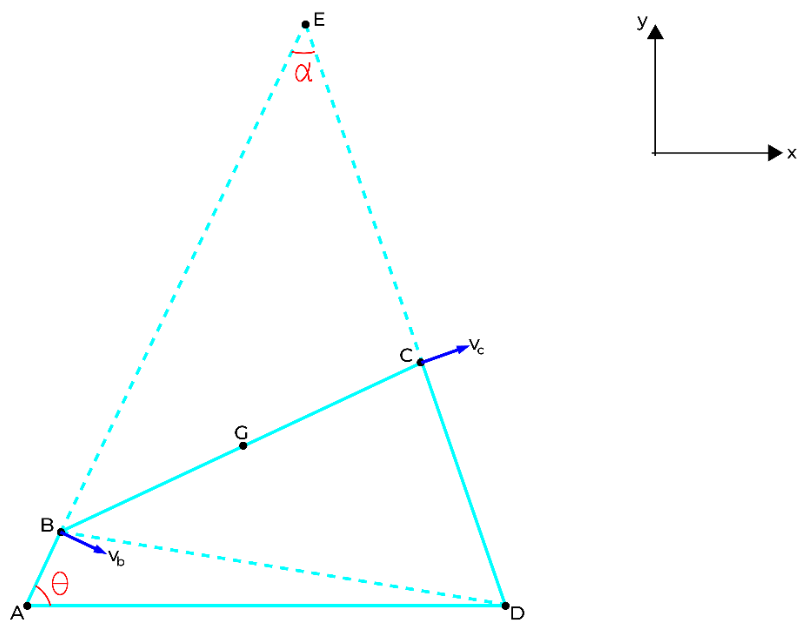
- The crank was connected to the motor.
- The four-bar linkage and the motor system together were affixed to a base plate.
- The Arduino was connected to the Stepper motor and the required program was uploaded into the Arduino.

Theoretical and Mathematical Analysis

Theoretical/Analytical Approach:

Case 1: $\boldsymbol{\theta < \pi}$

We know the angle BAD and we also know the lengths BA, AD, CD and BC respectively.



BA is the crank, BC is the coupler and CD is the rocker.
Point E is the instantaneous centre of rotation of the coupler.
The angular velocity given to the crank is ω, and the angular velocity of the coupler is denoted as Ω.

$$\cos BAD = \frac{(BA)^2 + (AD)^2 - (BD)^2}{2(AD)(BA)} \qquad (using\ the\ cosine\ rule)$$

$$therefore, BD = \sqrt{(\mathrm{BA})^2 + (\mathrm{AD})^2 - \cos BAD \cdot 2(AD)(BA)} \ .\ .\ .\ .\ .(i)$$

$$\cos BCD = \frac{(BC)^2 + (CD)^2 - (BD)^2}{2(BC)(CD)} \ .\ .\ .\ .\ .(ii)$$

$$BCE = \pi - BCD\ .\ .\ .\ .\ .(iii)$$

$$\cos ABD = \frac{(AB)^2 + (BD)^2 - (AD)^2}{2(AB)(BD)} \quad . \; . \; . \; . \; . \; (iv)$$

$$\cos DBC = \frac{(BD)^2 + (BC)^2 - (CD)^2}{2(BD)(BC)} \quad . \; . \; . \; . \; . \; (v)$$

$$EBC = \pi - ABD - DBC \quad . \; . \; . \; . \; . \; (vi)$$

$$BEC = \pi - EBC - BCE \quad . \; . \; . \; . \; . \; (vii)$$

using the sine rule,

$\frac{\sin BEC}{BC} = \frac{\sin EBC}{CE} = \frac{\sin BCE}{BE}$, *and we obtain the lengths BE and CE using the sine rule.*

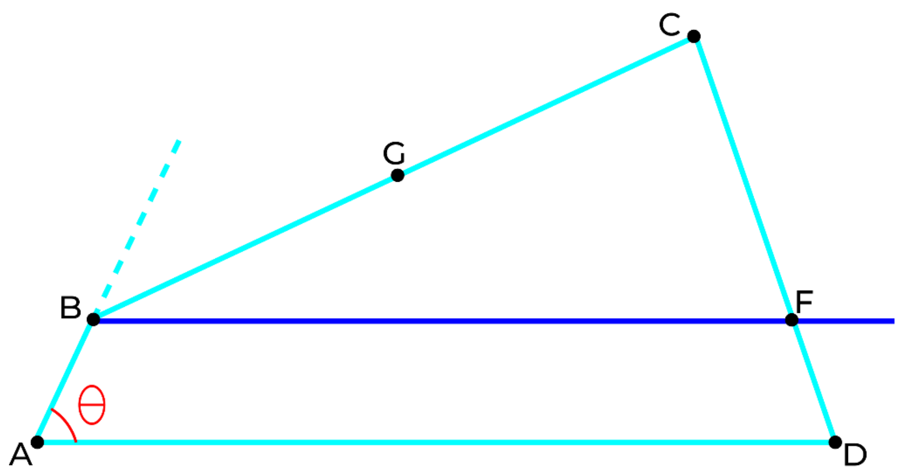
We know, $\vec{v}_b = \vec{\omega} \times \overrightarrow{BA}$. *Since E is the instantaneous centre of rotation,* $\vec{v}_b$ *is also equal to* $\vec{\Omega} \times \overrightarrow{EB}$.

so, $\vec{\omega} \times \overrightarrow{BA} = \vec{\Omega} \times \overrightarrow{EB}$, *and so we obtain* $\vec{\Omega}$.

Also, $\vec{v}_c = \vec{\Omega} \times \overrightarrow{EC}$, *and we obtain* $\vec{v}_c$

To obtain $\vec{v}_G$, *we apply the principles of relative motion.*

$$\vec{v}_G = \vec{v}_b + \vec{\Omega} \times \overrightarrow{GB}$$



To obtain $\overrightarrow{GB}$, *we construct line BF parallel to AD. Since we know angle EBC (point E not shown here, please refer to above diagrams) and angle EBF is equal to angle BAD (corresponding angles of parallel lines),* $CBF = EBF - EBC$.

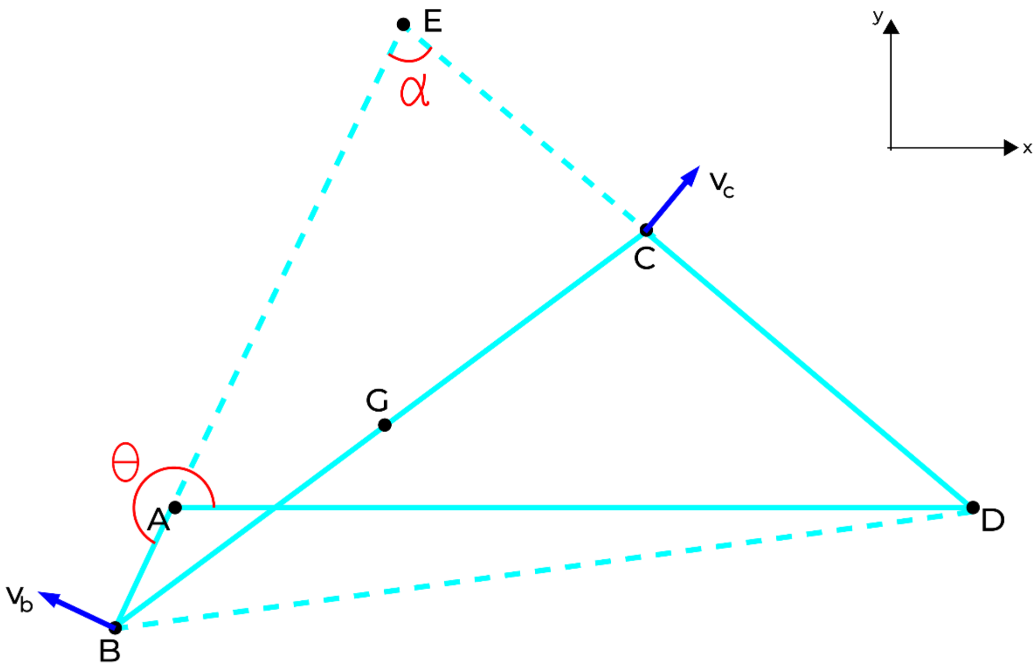
$$We\ also\ know\ |GB| = \frac{|BC|}{2}$$

$$So, \overrightarrow{GB} = |GB| \cos CBF\,\hat{\imath} + |GB| \sin CBF\,\hat{\jmath}$$

we can then calculate $\vec{v}_G = \vec{v}_b + \vec{\Omega} \times \overrightarrow{GB}$

Case 2: $\boldsymbol{\theta \geq \pi}$

We know the angle BAD and we also know the lengths BA, AD, CD and BC respectively.



BA is the crank, BC is the coupler and CD is the rocker.

Point E is the instantaneous centre of rotation of the coupler.

The angular velocity given to the crank is ω, and the angular velocity of the coupler is denoted as Ω.

$$DAB = 2\pi - \theta$$

$$\cos DAB = \frac{(BA)^2 + (AD)^2 - (BD)^2}{2(AD)(BA)} \qquad (using\ the\ cosine\ rule)$$

$$therefore, BD = \sqrt{(\mathrm{BA})^2 + (\mathrm{AD})^2 - \cos DAB \cdot 2(AD)(BA)}\ .\ .\ .\ .\ .(i)$$

$$\cos BCD = \frac{(BC)^2 + (CD)^2 - (BD)^2}{2(BC)(CD)}\ .\ .\ .\ .\ .(ii)$$

$$BCE = \pi - BCD\ .\ .\ .\ .\ .(iii)$$

$$\cos ABD = \frac{(AB)^2 + (BD)^2 - (AD)^2}{2(AB)(BD)}\ .\ .\ .\ .\ .(iv)$$

$$\cos DBC = \frac{(BD)^2 + (BC)^2 - (CD)^2}{2(BD)(BC)}\ .\ .\ .\ .\ .(v)$$

$$EBC = ABD - DBC\ .\ .\ .\ .\ .(vi)$$

$$BEC = \pi - EBC - BCE\ .\ .\ .\ .\ .(vii)$$

using the sine rule,

$$\frac{\sin BEC}{BC} = \frac{\sin EBC}{CE} = \frac{\sin BCE}{BE}, and\ we\ obtain\ the\ lengths\ BE\ and\ CE\ using\ the\ sine\ rule.$$

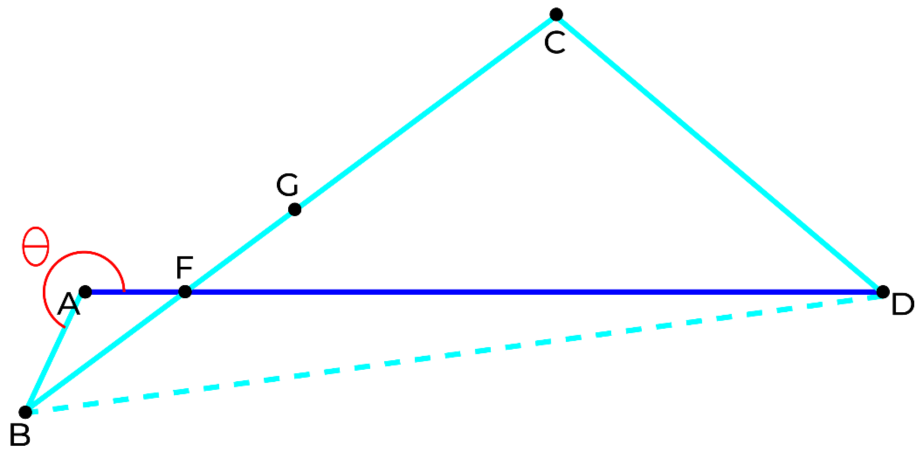
$We\ know, \vec{v}_b = \vec{\omega} \times \overrightarrow{BA}.\ Since\ E\ is\ the\ instantaneous\ centre\ of\ rotation, \vec{v}_b\ is\ also\ equal\ to\ \vec{\Omega} \times \overrightarrow{EB}.$

$$so, \vec{\omega} \times \overrightarrow{BA} = \vec{\Omega} \times \overrightarrow{EB},\ \ and\ so\ we\ obtain\ \vec{\Omega}.$$

$$Also, \vec{v}_c = \vec{\Omega} \times \overrightarrow{EC}, and\ we\ obtain\ \vec{v}_c$$

$$To\ obtain\ \vec{v}_G, we\ apply\ the\ principles\ of\ relative\ motion.$$

$$\vec{v}_G = \vec{v}_b + \vec{\Omega} \times \overrightarrow{GB}$$



$To\ obtain\ \overrightarrow{GB},\ \ we\ mark\ pont\ F\ at\ the\ intersection\ of\ BC\ and\ AD.\ We\ know$
$EBC = ABC.$
$also,\ \ AFB = \pi - ABC - DAB.\ Since\ AFB\ and\ GFD\ are\ vertically\ opposite\ angles,$
$AFB = GFD.$
$We\ also\ know\ |GB| = \frac{|BC|}{2}$
$So, \overrightarrow{GB} = |GB| \cos GFD\, \hat{\imath} + |GB| \sin GFD\, \hat{\jmath}$
$we\ can\ then\ calculate\ \vec{v}_G = \vec{v}_b + \vec{\Omega} \times \overrightarrow{GB}$

For both the cases, python programs were created to solve the equations and obtain solutions and the values of v_G and Ω.

For Case 1:

```python
import numpy as np
import math

# Constants for link lengths (in cm)
BA_LENGTH = 5
AD_LENGTH = 30
BC_LENGTH = 25
CD_LENGTH = 20

# Distance between centers of crank and rocker
DISTANCE_CENTERS_CRANK_ROCKER = 30

theta = [0, math.pi / 3, 2 * math.pi / 3, math.pi]

for angle in theta:
    bad = angle
    w = np.array([0, 0, 2.932])
    bd = math.sqrt(BA_LENGTH**2 + AD_LENGTH**2 - 2 * np.cos(bad) * AD_LENGTH * BA_LENGTH)
    bcd = np.arccos((BC_LENGTH**2 + CD_LENGTH**2 - bd**2) / (2 * BC_LENGTH * CD_LENGTH))
    bce = math.pi - bcd
    abd = np.arccos((BA_LENGTH**2 + bd**2 - AD_LENGTH**2) / (2 * BA_LENGTH * bd))
    dbc = np.arccos((bd**2 + BC_LENGTH**2 - CD_LENGTH**2) / (2 * bd * BC_LENGTH))
    ebc = math.pi - abd - dbc
    bec = math.pi - ebc - bce

    k = np.sin(bec) / BC_LENGTH
    be = (np.sin(bce) / k)
    ce = (np.sin(ebc) / k)
```

```python
    cda = 2 * math.pi - bcd - abd - dbc - bad
    cd = math.sqrt(CD_LENGTH**2 + AD_LENGTH**2 - 2 * np.cos(cda) * CD_LENGTH * AD_LENGTH)

    ebf = bad
    cbf = ebf - ebc
    gb = BC_LENGTH / 2

    a = np.array([0, 0, 0])
    b = np.array([BA_LENGTH * np.cos(angle), BA_LENGTH * np.sin(angle), 0])
    c = np.array([AD_LENGTH - cd * np.cos(cda), cd * np.sin(cda), 0])
    d = np.array([AD_LENGTH, 0, 0])
    e = np.array([(BA_LENGTH + be) * np.cos(angle), (BA_LENGTH + be) * np.sin(angle), 0])
    g = np.array([b[0] + gb * np.cos(cbf), b[1] + gb * np.sin(cbf), 0])

    vba = b - a
    vbe = b - e

    def mag(vec):
        return np.sqrt(vec[0]**2 + vec[1]**2 + vec[2]**2)

    lba = mag(vba)
    lbe = mag(vbe)

    v = np.cross(w, vba)
    Wm = mag(np.cross(w,vba)) / lbe
    W=[0,0,-Wm]
    print(mag(v+np.cross(W,g-b)))
    vg=mag(v+np.cross(W,g-b))

    print(f"Angle: {angle * 180 / math.pi} degrees, Angular Velocity: {Wm * 180 / math.pi} degrees/s")
```

For Case 2:

```python
import numpy as np
import math

# Constants for link lengths (in cm)
BA_LENGTH = 5
AD_LENGTH = 30
BC_LENGTH = 25
CD_LENGTH = 20

# Distance between centers of crank and rocker
DISTANCE_CENTERS_CRANK_ROCKER = 30

theta = [math.pi, 4 * math.pi / 3, 5 * math.pi / 3]

for angle in theta:

    dab = angle
    w = np.array([0, 0, 2.932])
    bd = math.sqrt(BA_LENGTH**2 + AD_LENGTH**2 - 2 * np.cos(dab) * AD_LENGTH * BA_LENGTH)
    bcd = np.arccos((BC_LENGTH**2 + CD_LENGTH**2 - bd**2) / (2 * BC_LENGTH * CD_LENGTH))
    bce = math.pi - bcd
    abd = np.arccos((BA_LENGTH**2 + bd**2 - AD_LENGTH**2) / (2 * BA_LENGTH * bd))
    dbc = np.arccos((bd**2 + BC_LENGTH**2 - CD_LENGTH**2) / (2 * bd * BC_LENGTH))
    ebc = abd - dbc
    bec = math.pi - ebc - bce

    k = np.sin(bec) / BC_LENGTH
    be = (np.sin(bce) / k)
    ce = (np.sin(ebc) / k)
```

```python
abc=ebc
afb=math.pi-abc-dab
gfd=afb

gb = BC_LENGTH / 2

a = np.array([0, 0, 0])
b = np.array([BA_LENGTH * np.cos(angle), BA_LENGTH * np.sin(angle), 0])
c = np.array([b[0] + BC_LENGTH * np.cos(gfd), b[1] + BC_LENGTH * np.sin(gfd), 0])
d = np.array([AD_LENGTH, 0, 0])
e = np.array([(be-BA_LENGTH) * np.cos(angle-math.pi), (be-BA_LENGTH) * np.sin(angle-math.pi), 0])
g = np.array([b[0] + gb * np.cos(gfd), b[1] + gb * np.sin(gfd), 0])

vba = b - a
vbe = b - e

def mag(vec):
    return np.sqrt(vec[0]**2 + vec[1]**2 + vec[2]**2)

lba = mag(vba)
lbe = mag(vbe)

v = np.cross(w, vba)
Wm = mag(np.cross(w,vba)) / lbe
W=[0,0,-Wm]
print(mag(v+np.cross(W,g-b)))
print(f"Angle: {angle * 180 / math.pi} degrees, Angular Velocity: {Wm * 180 / math.pi} degrees/s")
```

Experimental Approach and Measurement Techniques

For the experimental part of this experiment, we had to measure the instantaneous angular velocity of the body and the velocity of its centre of mass for at least 6 different angles. The angles we chose were: 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$.

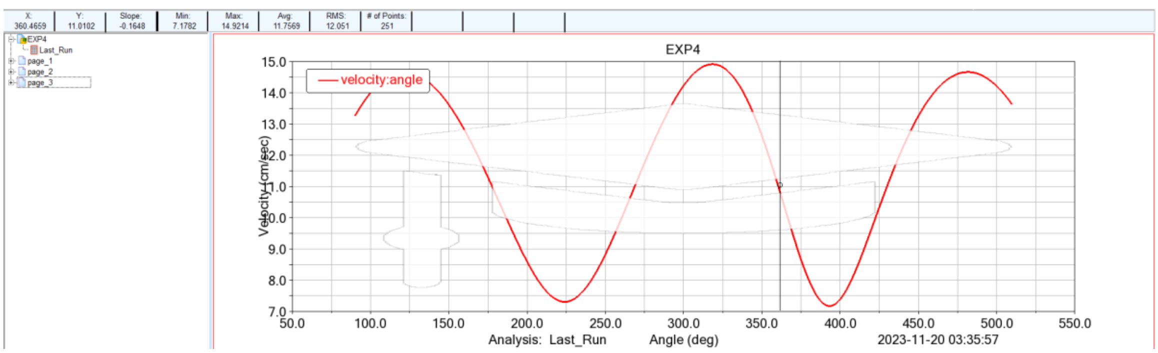
We recorded the motion of the system using a phone camera and then used the individual frames in which the crank was making the chosen angles with the base and analysed those screenshots in MATLAB's Image Processing Toolbox

We attempted to do the abovementioned, due to time constraints and lack of concrete results, we were unable to produce values for our experiment.

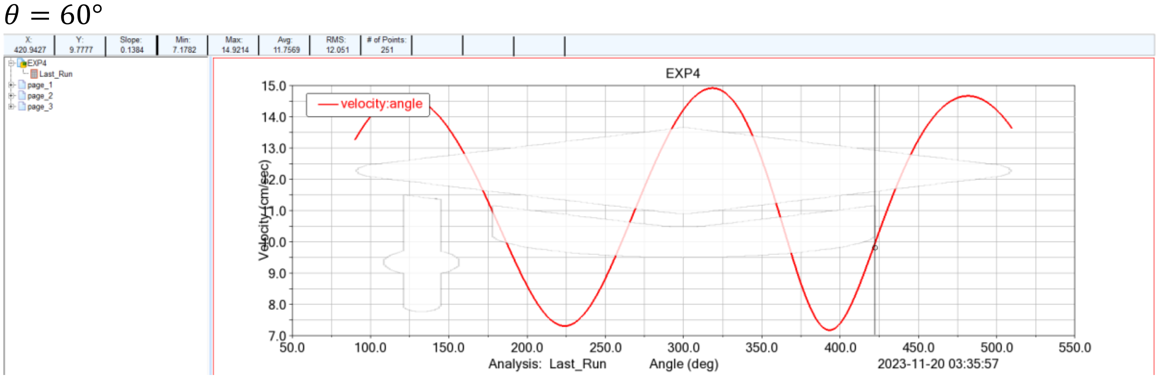
Simulation in ADAMS

Graphs for the velocity of the centre of mass of the coupler link (v_g):

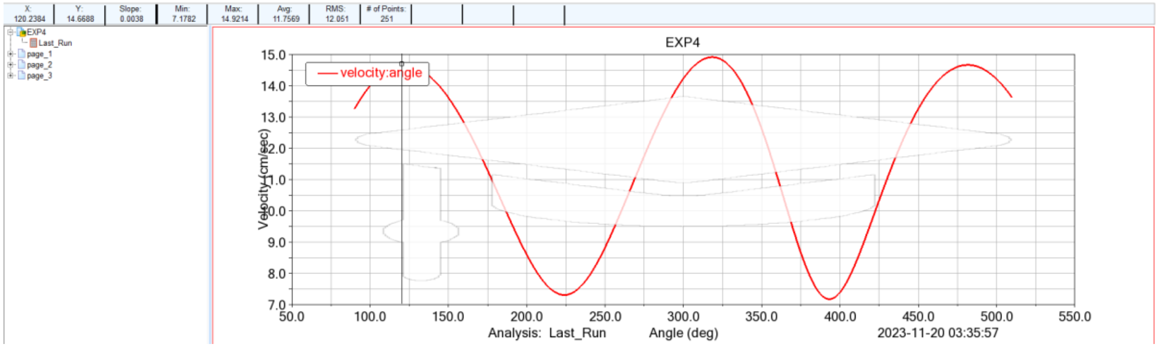
$\theta = 0°$



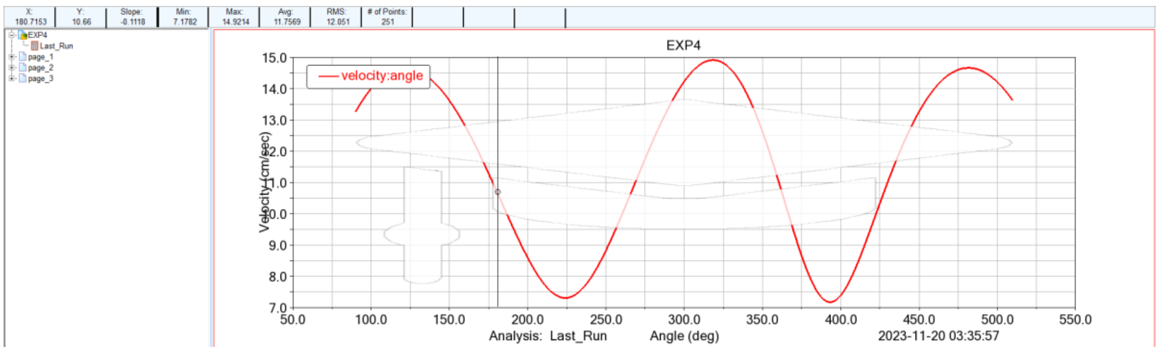
$\theta = 60°$



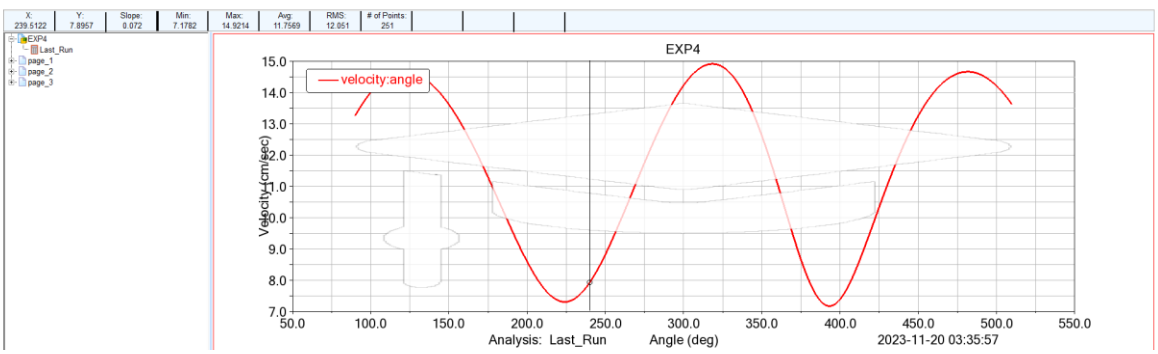
$\theta = 120°$



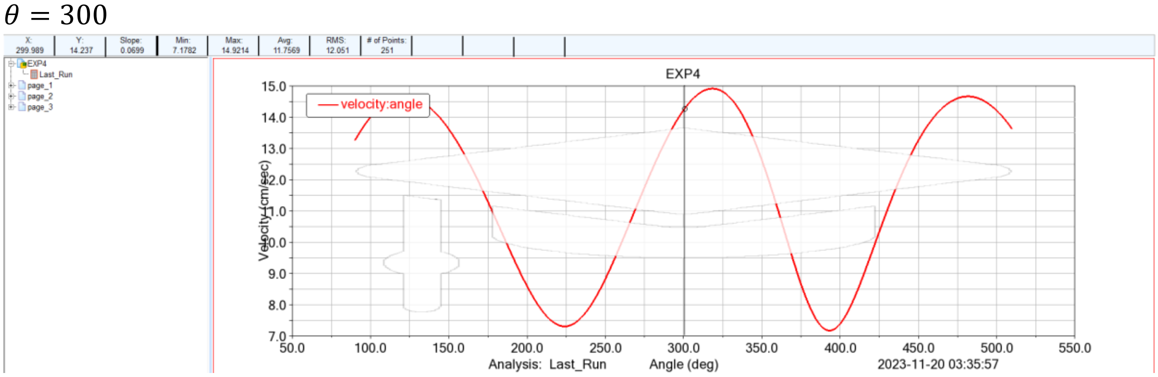
$\theta = 180°$



$\theta = 240°$

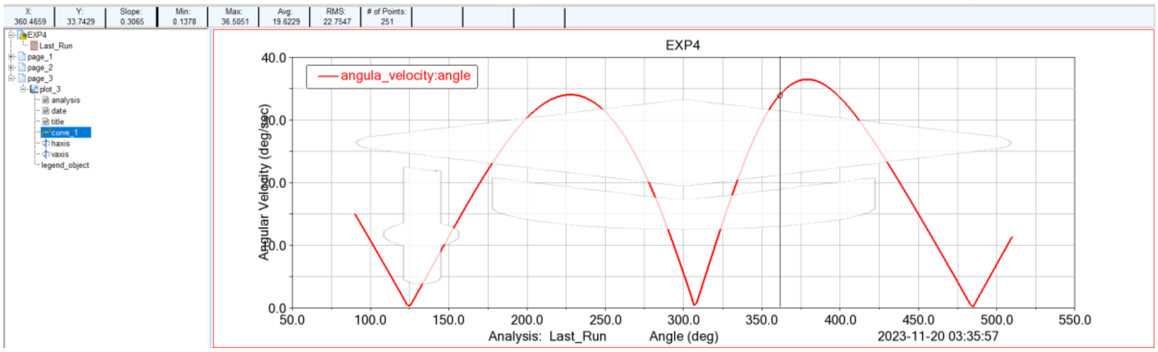


$\theta = 300$

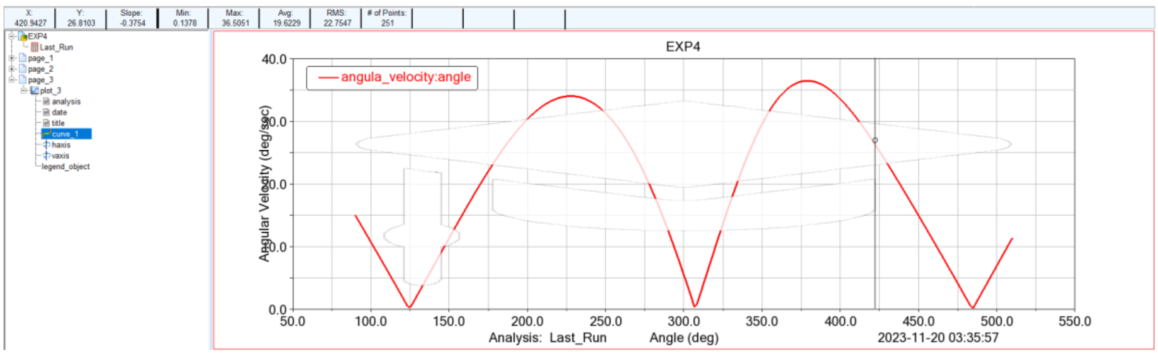


Graphs for the angular velocity of the of the coupler link (Ω):

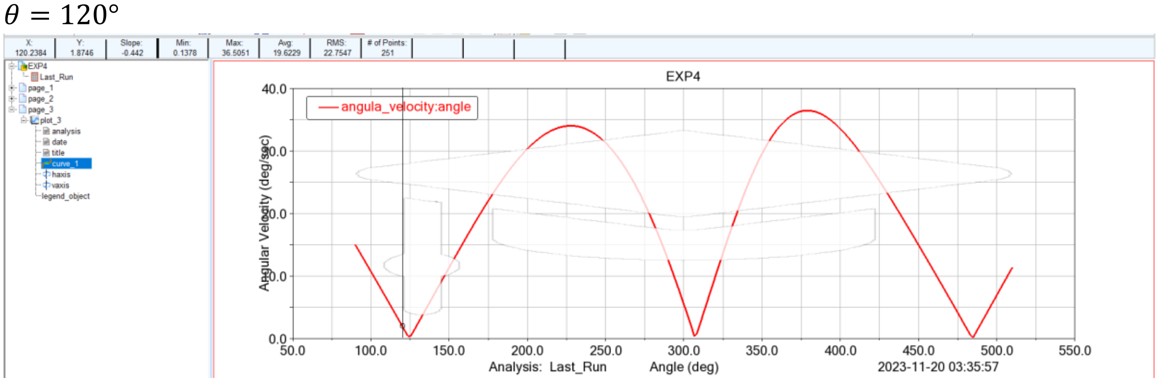
$\theta = 0°$



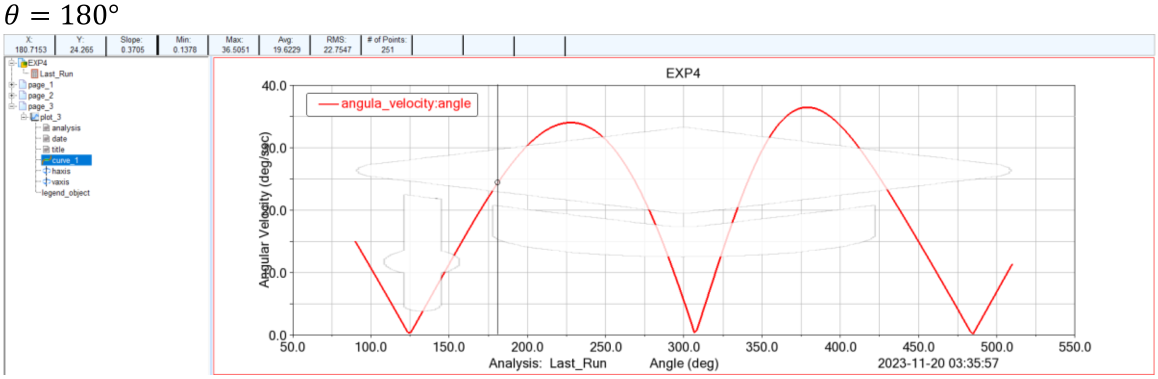
$\theta = 60°$



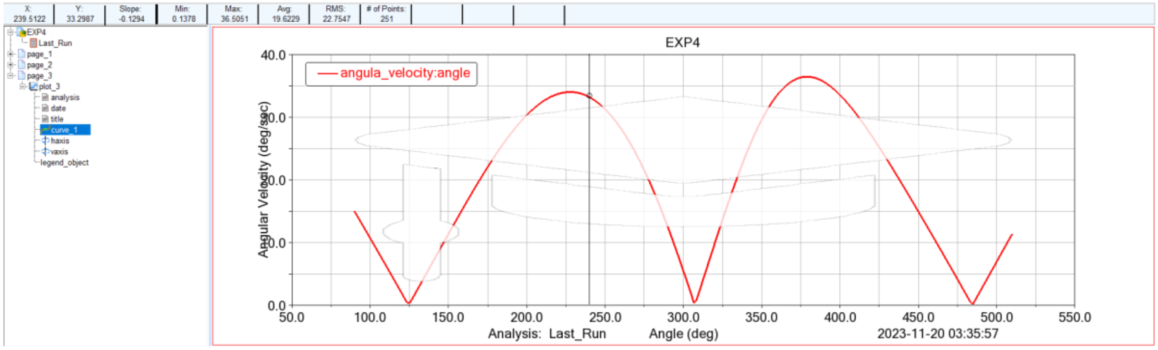
$\theta = 120°$



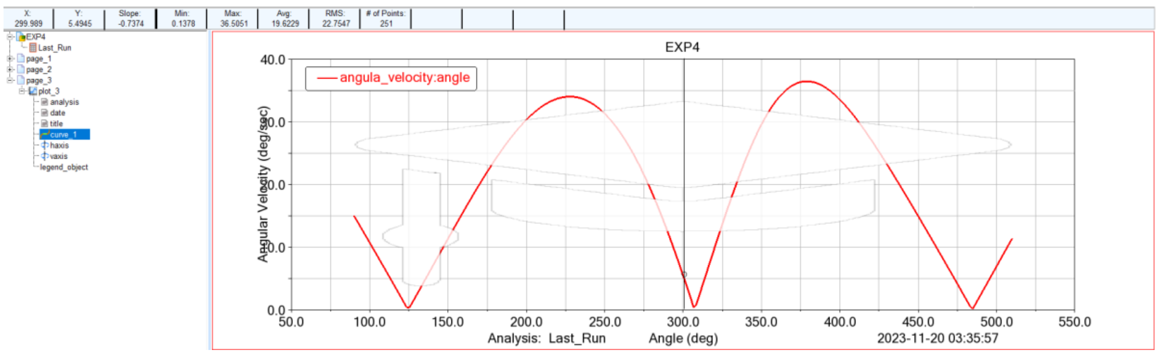
$\theta = 180°$



$\theta = 240°$

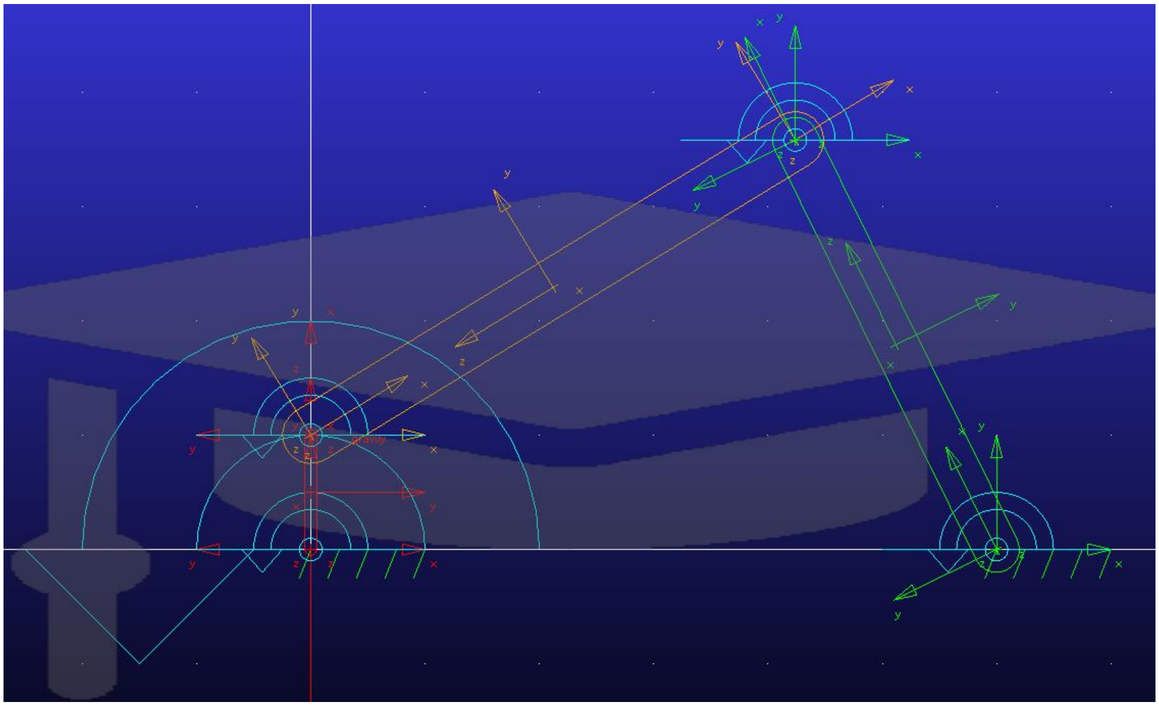


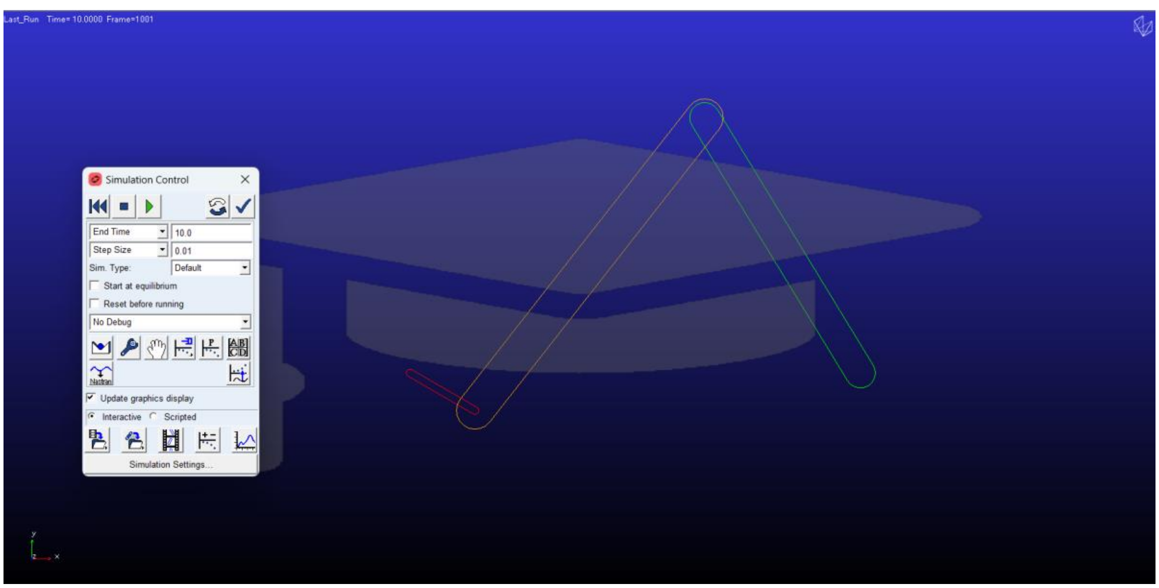
$\theta = 300°$



In both the sets of graphs, the x-axis represents the angle of the crank with the +ve x direction in the simulation's coordinate system, and the y-axis represents the value of the velocity/angular velocity respectively at that instant.

Model in ADAMS:





Results

Comparison of results:
Values of the angular velocity of the coupler (in deg/s):

Angle	Analytical	Experimental	Using ADAMS
0	33.59		33.74
60°	26.83		26.81
120°	1.65		1.87
180°	23.99		24.265
240°	32.90		33.299
300°	5.16		5.49

Values of the magnitude of the velocity of the coupler (in cm/s):

Angle	Analytical	Experimental	Using ADAMS
0	11.06		11.01
60°	9.66		9.77
120°	14.66		14.66
180°	10.54		10.66
240°	7.79		7.89
300°	14.09		14.23

Discussions

In this experiment, we designed a planar four-bar mechanism with specific constraints on its motion to investigate the velocity of the centre of mass of the coupler link and the angular velocity of the coupler for various crank angles. The mechanism consisted of a crank, rocker, and a coupler link connecting them.

- Design of the Mechanism:
  The four-bar mechanism was designed with one link (the crank) rotating at a constant angular velocity, while the opposite link (rocker) underwent partial rotation. The objective was to achieve specific angular positions (0, π/3, 2π/3, ...) of the crank and observe the corresponding motion of the coupler link.

- Theoretical Analysis:
  Analytically, the velocity of the center of mass of the coupler link and the angular velocity of the coupler were determined using kinematic equations. The relationship

between the crank angle and the resulting velocities was established through mathematical modeling. This theoretical analysis served as a baseline for comparison with experimental and simulation results.

- Experimental Analysis:
  The experiment involved constructing the physical four-bar mechanism and measuring the velocities at different crank angles. We used video capture and image processing software (MATLAB's Image Processing Toolbox) to analyse the motion of the mechanism. The collected data were then compared with the analytical predictions. Discrepancies between the theoretical and experimental results were analyzed to understand the limitations of the theoretical model and to identify potential sources of error in the physical setup

- Simulation using ADAMS:
  To further validate the results, a simulation was conducted using ADAMS (Automatic Dynamic Analysis of Mechanical Systems), a software tool for dynamic analysis of mechanical systems. The simulated model replicated the physical setup, and the motion of the mechanism was observed for the specified crank angles. The velocities obtained from the simulation were compared with both analytical and experimental results to assess the accuracy and reliability of the simulation.

- Discussion of Results:
  The comparison of results from analytical, experimental, and simulation approaches provided valuable insights into the behavior of the four-bar mechanism. The simulation results helped corroborate the findings and identify potential improvements in the experimental design.

- Conclusion:
  The study of the planar four-bar mechanism with a rotating crank and partially rotating rocker yielded a comprehensive understanding of the motion of the coupler link. The integration of analytical, experimental, and simulation approaches allowed for a thorough analysis of the system's behavior. This experiment contributes to the broader field of mechanical engineering, providing insights into the complexities of real-world mechanical systems and the importance of considering practical factors in theoretical models.

Scope for Improvement

1. Enhanced Experimental Setup:
   - Improve the precision of sensors and measurement devices to minimize experimental errors.
   - Consider implementing advanced sensing technologies to capture more accurate angular positions and velocities of the moving links.
2. Friction Analysis:
   - Conduct a detailed analysis of frictional effects within the mechanism, both theoretically and experimentally.
   - Explore methods to reduce friction, such as using lubricants or optimizing link geometries to minimize contact forces.
3. Real-world Constraints Consideration:
   - Introduce real-world constraints such as manufacturing tolerances and material properties in the analytical model to enhance its accuracy.
   - Investigate the impact of imperfections in the physical setup on the mechanism's performance and incorporate these considerations into the analysis.
4. Simulation Model Validation:
   - Validate the simulation model by comparing its predictions with additional experimental data beyond the original set of angles.
   - Consider incorporating dynamic factors, such as inertial effects and external forces, into the simulation for a more comprehensive analysis.
5. Parameter Sensitivity Analysis:
   - Perform sensitivity analysis on key parameters, such as link lengths and initial conditions, to assess their impact on the system's behavior.
   - Identify critical parameters that significantly influence the mechanism's performance and focus on optimizing these for improved overall operation.

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