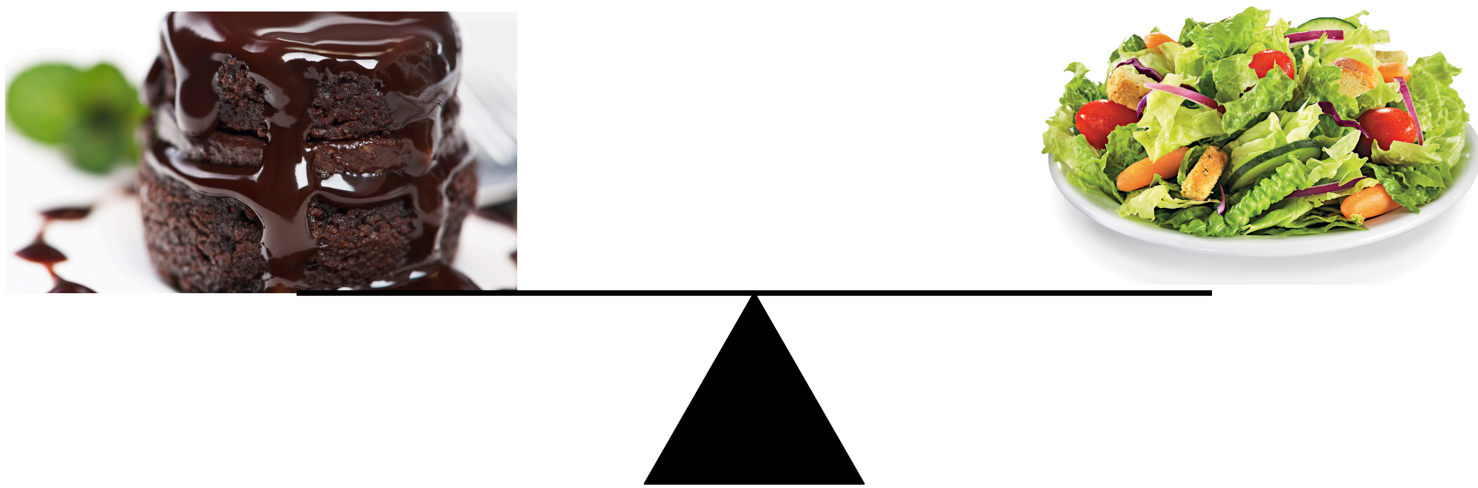
Discounting of Non-Unitary Rewards

04/03/2015

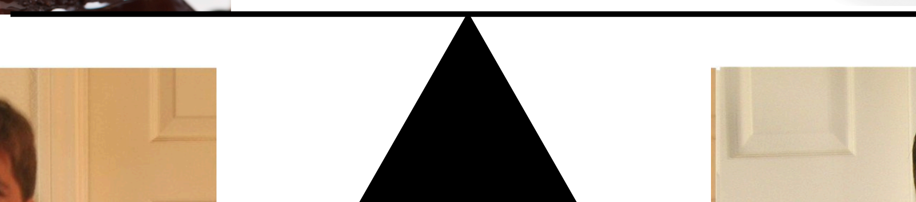
Ayse Zeynep Enkavi

Introduction

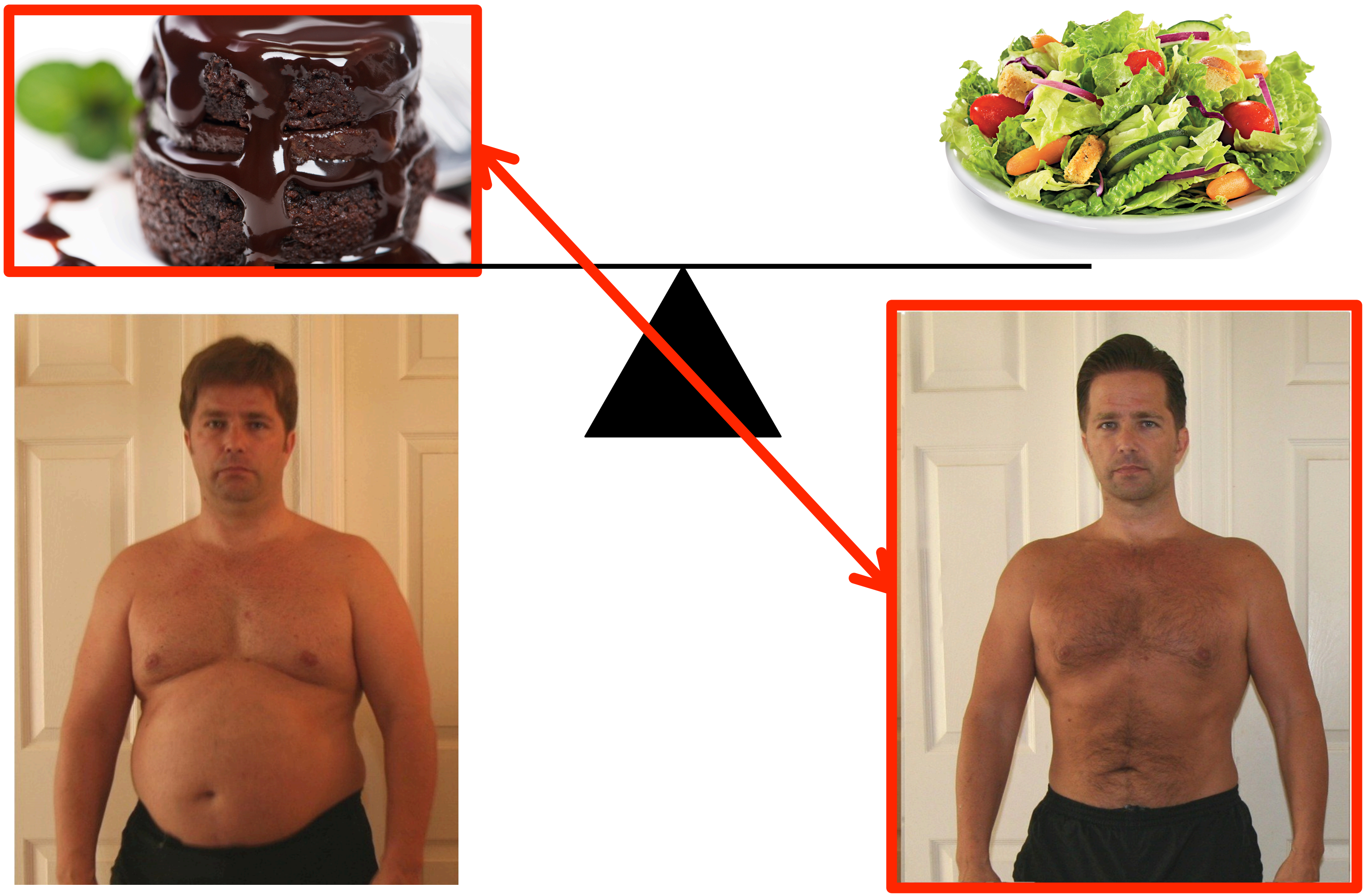
Intertemporal Choice: Unitary outcomes



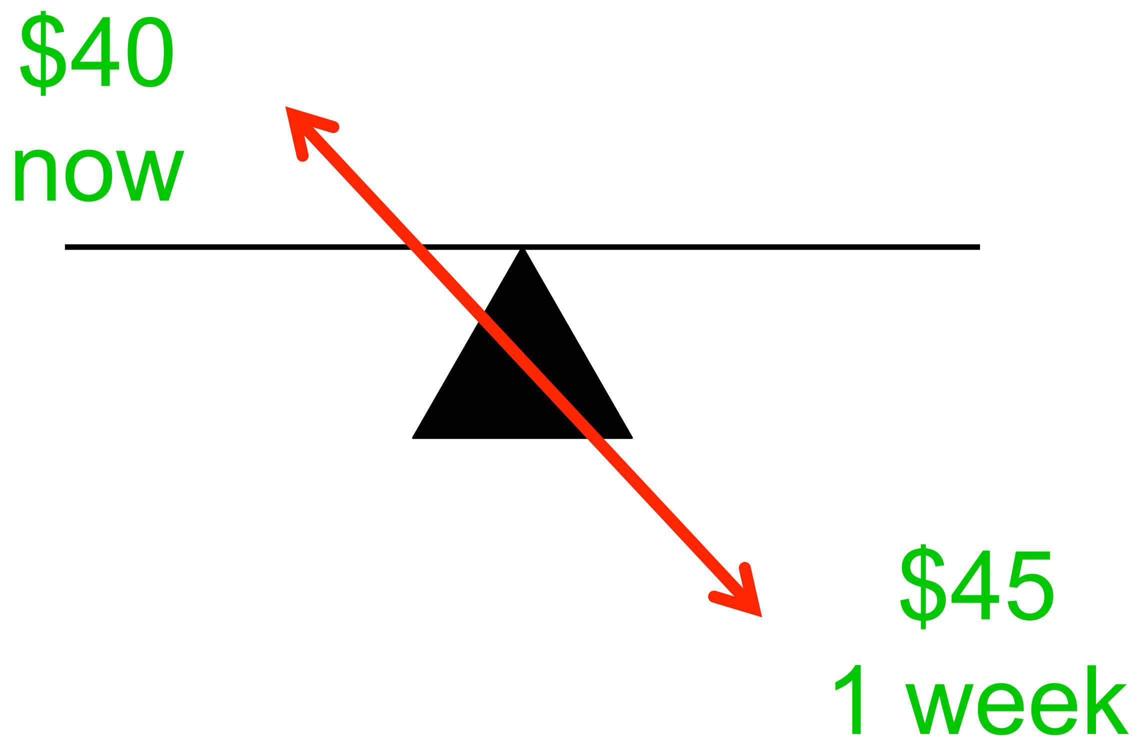
Intertemporal Choice in REAL Life: Non-Unitary outcomes



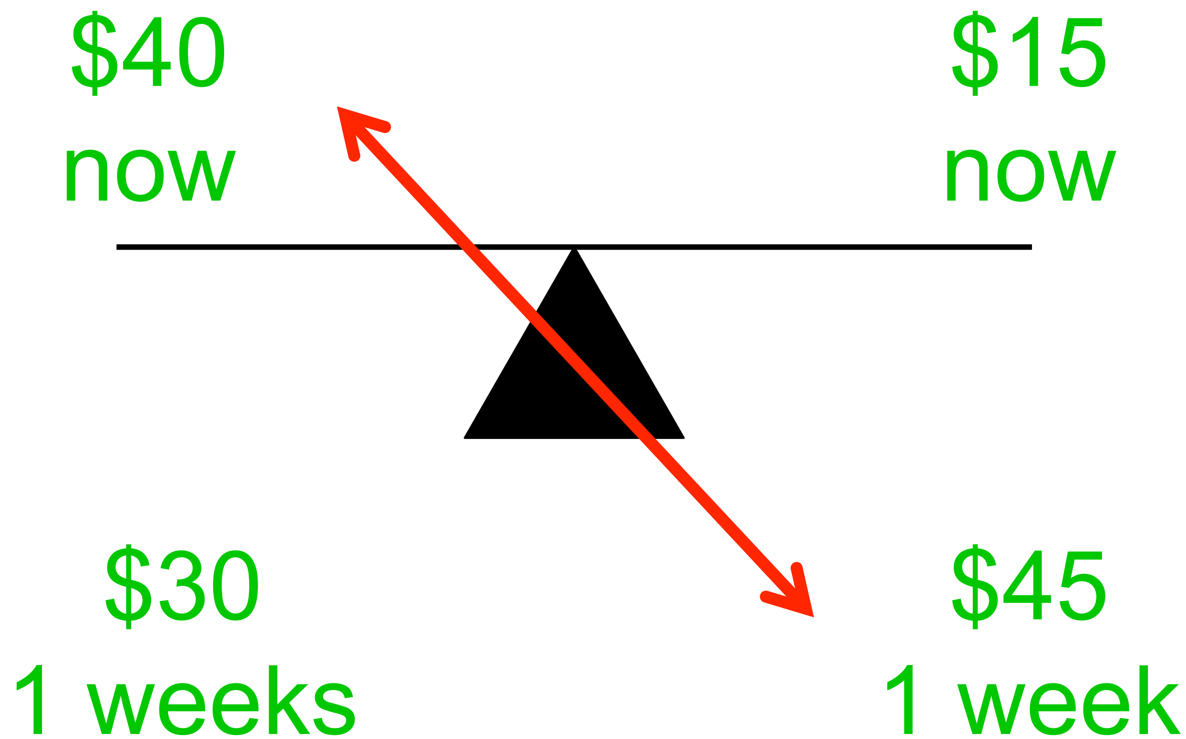
Intertemporal Choice in REAL Life: Non-Unitary Outcomes



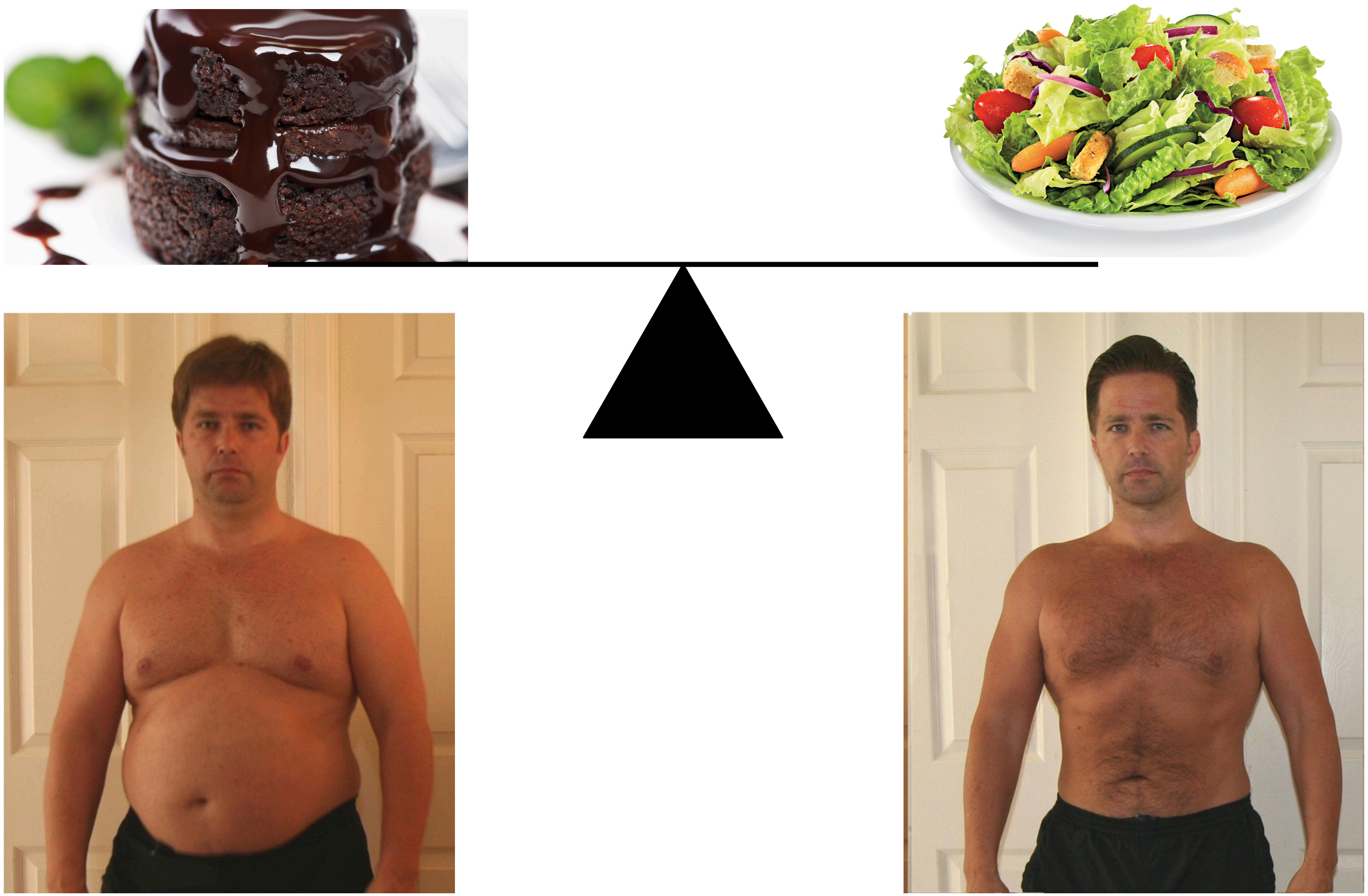
Intertemporal Choice in Lab: Unitary Outcomes



Intertemporal Choice in Lab: Non-Unitary Outcomes



Intertemporal Choice in REAL Life..



Discounted Utility Framework

$$U^t(c_t,\ldots,c_T) = \sum_{k=0}^{T-t} D(k)u(c_t+k)$$

Discounted Utility Framework

$$U^t(c_t, \ldots, c_T) = \sum_{k=0}^{T-t} \boxed{D(k)}\, u(c_t + k)$$

Discounted Utility Framework

$$U^t(c_t,\ldots,c_T) = \sum_{k=0}^{T-t} D(k)u(c_t + k)$$

1. Assumed independence / Linearity
2. IS THIS A REASONABLE ASSUMPTION?

Now

1 week

Now

66%

1 week

34%

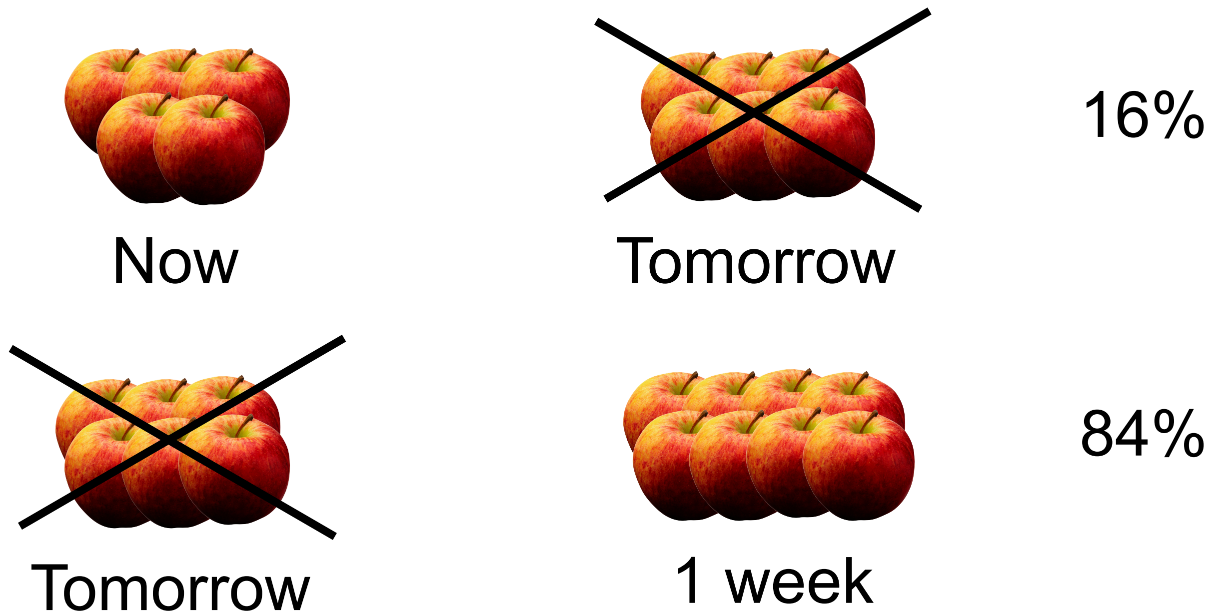
Rao & Li, 2011

Now

Tomorrow

Tomorrow

1 week



Rao & Li, 2011

Now

66%

1 week

34%

$u(5,0) > u(8, 7)$

Rao & Li, 2011

Now

Tomorrow

16%

Tomorrow

1 week

84%

$$u(5,0) > u(8, 7)$$

$$u(5,0) + u(-6,1) < u(8, 7) + u(-6,1)$$

Rao & Li, 2011

-1

Now

Tomorrow

16%

+2

Tomorrow

1 week

84%

Do people prefer the larger total amount?

Rao & Li, 2011

Revising discounted utility

$$U^{t}(c_{t},\ldots,c_{T}) = \sum_{k=0}^{T-t} D(k)u(c_{t}+k) + F(c_{t},\ldots,c_{T})$$

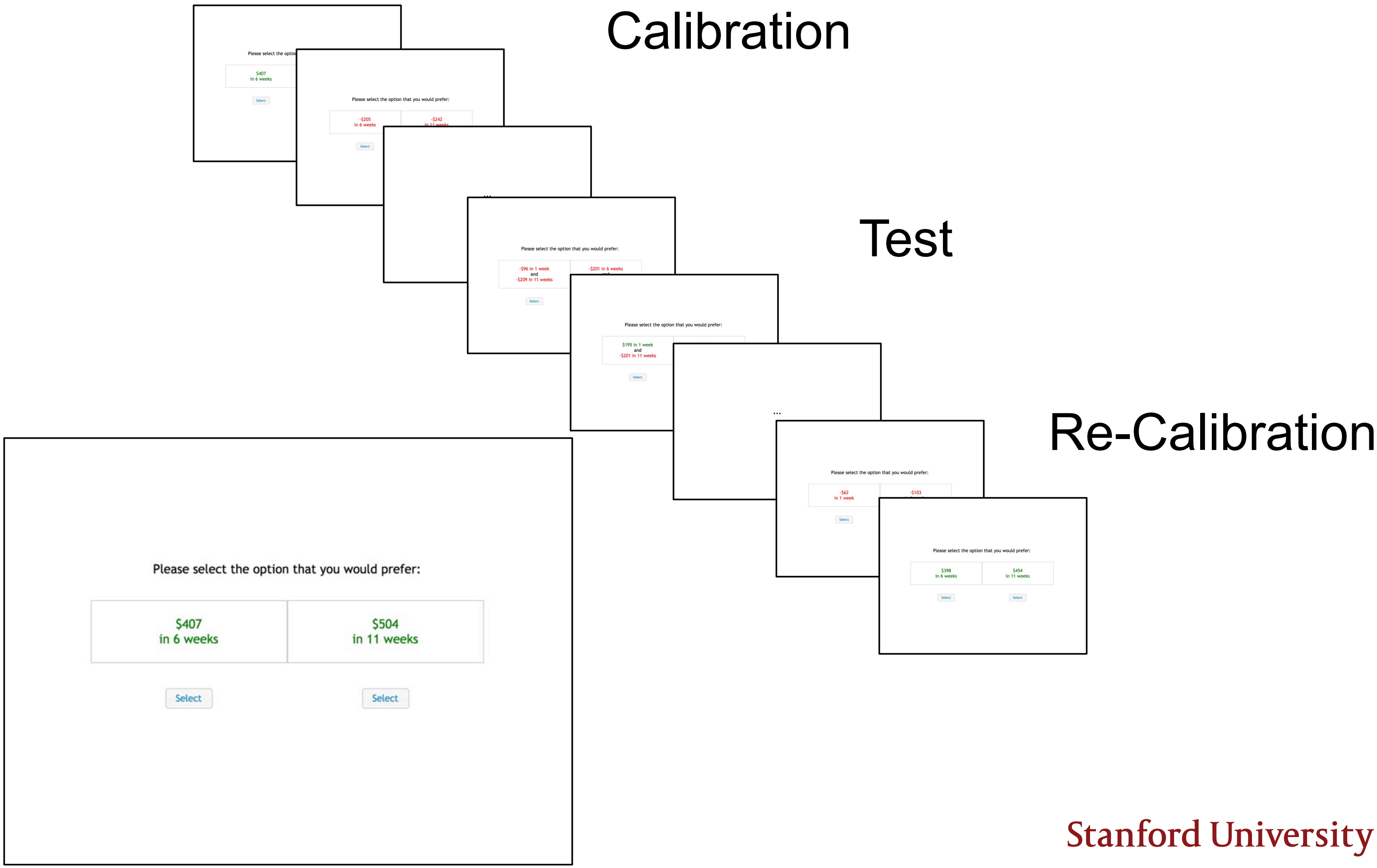
Revising discounted utility

$$U^t(c_t,\ldots,c_T) = \sum_{k=0}^{T-t} D(k)u(c_t+k) + F(c_t,\ldots,c_T)$$

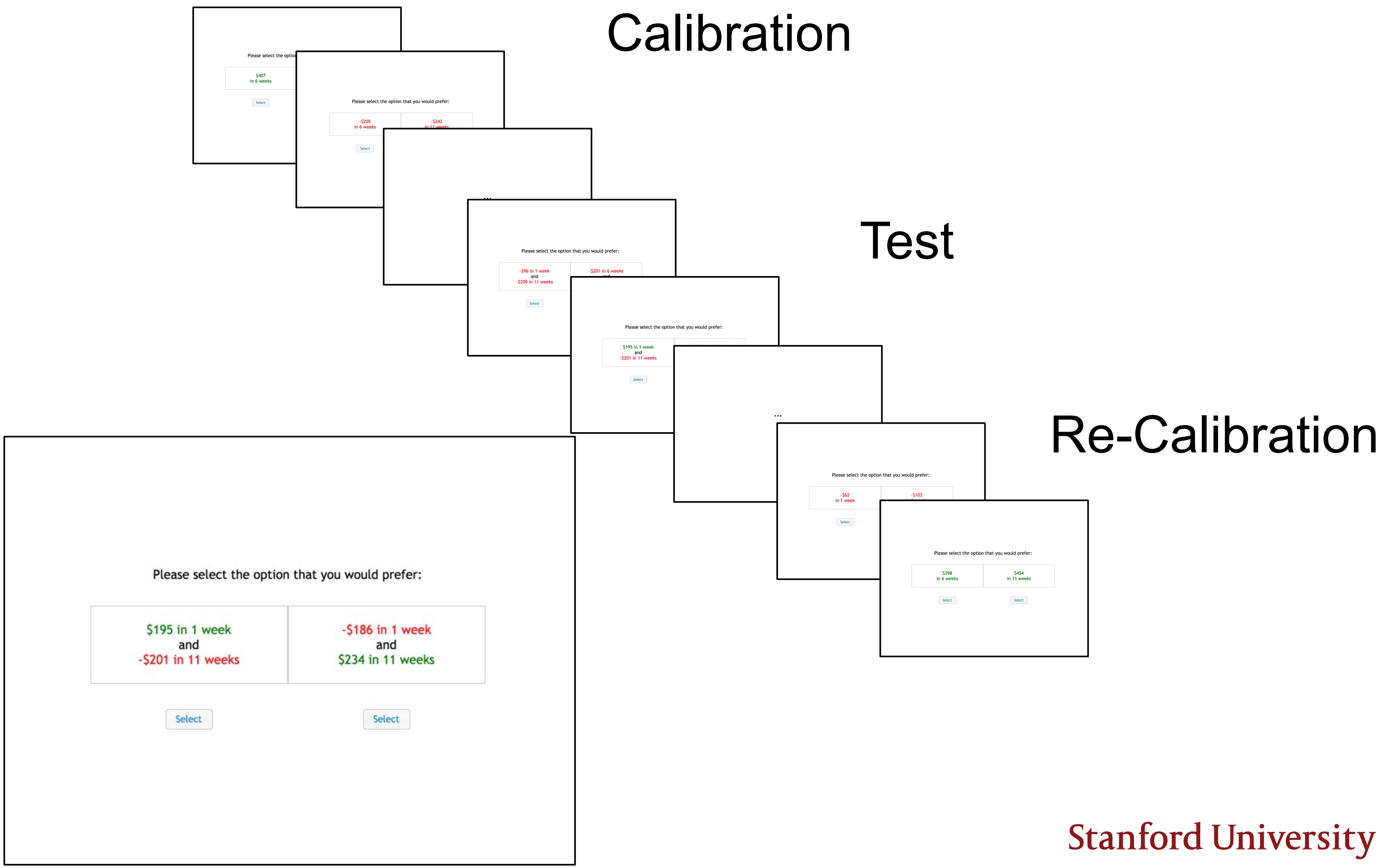
1. Create options at indifference wrt delay discounting
2. Combine and test whether indifference is preserved
3. If not preserved, investigate components of F
4. Confirm we were at indifference

Experiment

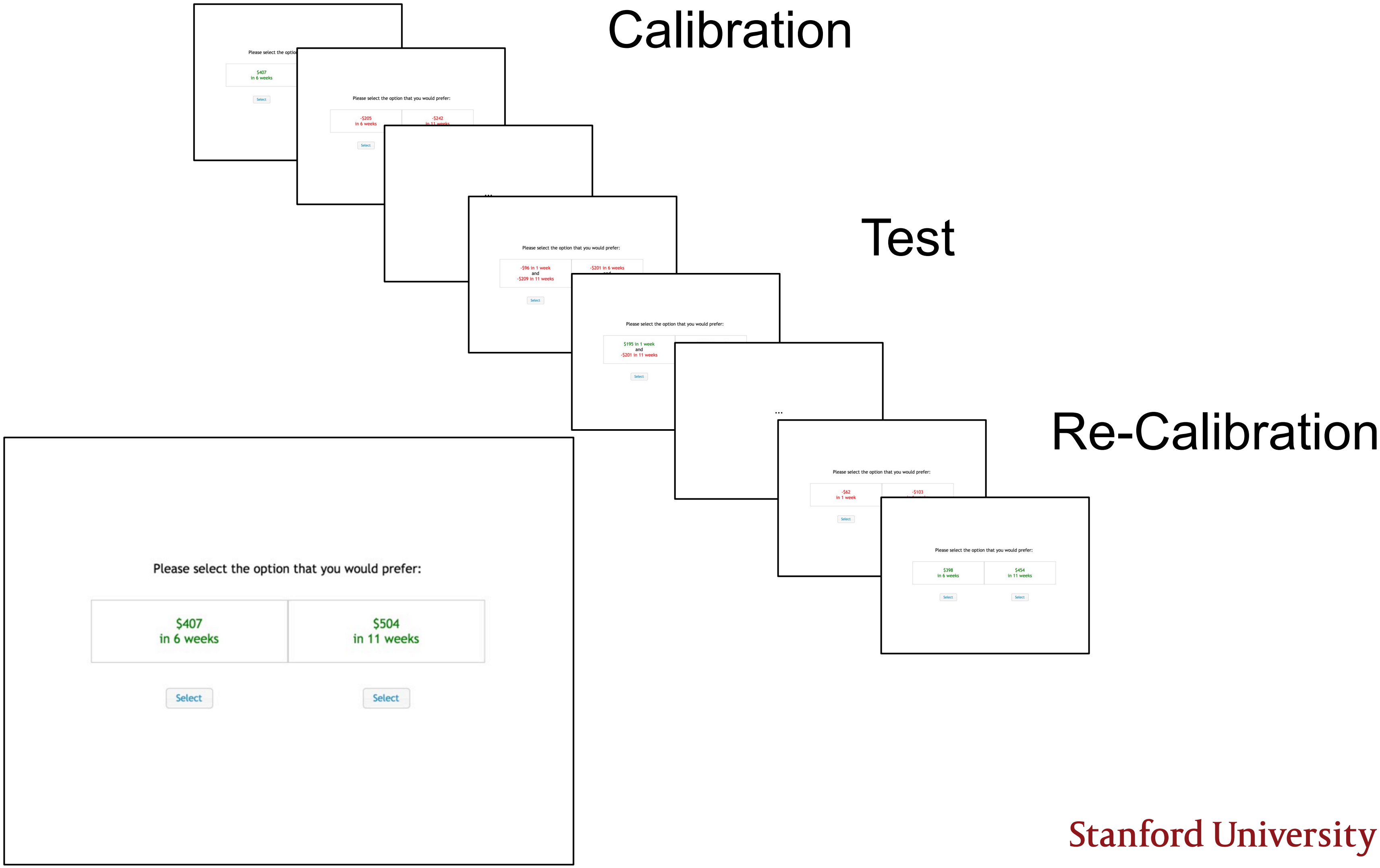
Design


Calibration
Test
Re-Calibration
Please select the option that you would prefer:
$407
in 6 weeks
$504
in 11 weeks
Select
Select

Design


Calibration
Test
Re-Calibration
Please select the option that you would prefer:
$195 in 1 week
and
-$201 in 11 weeks
-$186 in 1 week
and
$234 in 11 weeks
Select
Select

Design

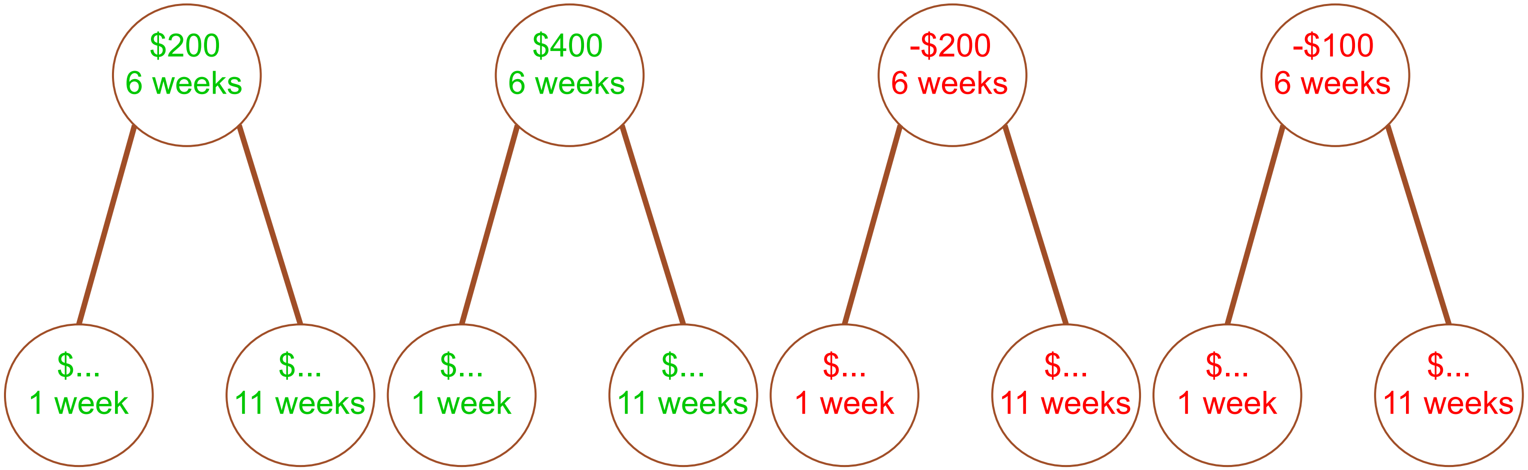

Calibration
Test
Re-Calibration
Please select the option that you would prefer:
$407
in 6 weeks
$504
in 11 weeks
Select
Select

Calibration

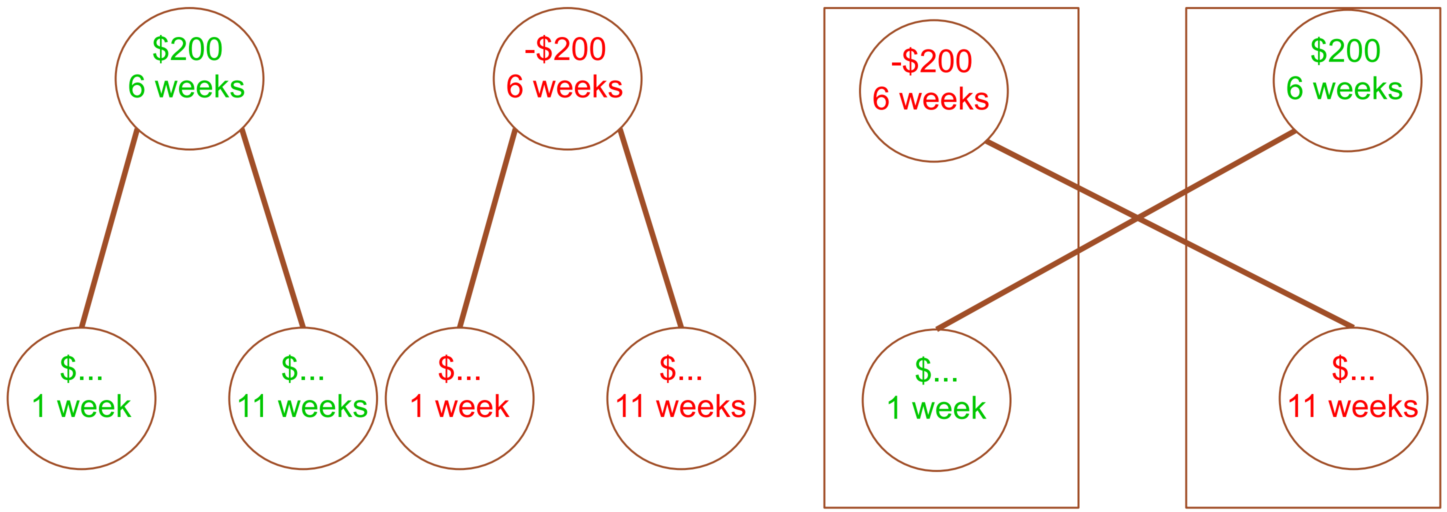
- Gain and loss outcomes (hypothetical)
- Fixed intervals of 5 weeks (i.e. 1 vs. 6 weeks and 6 vs. 11 weeks)
- Staircase procedure to find indifference
- 8 indifference pairs

$...	$200
1 week	6 weeks

Calibration



Calibration



Discount factor

$$\delta = \left(\frac{x_1}{x_2}\right)^{\frac{1}{t_2 - t_1}}$$

Discount factor

$$\delta = \left(\frac{x_1}{x_2}\right)^{\frac{1}{t_2 - t_1}}$$

$20 now	$35 1 week
$20 5 weeks	$35 6 weeks

Delay effect: Higher discount factors for later delays

Discount factor

$$\delta = \left(\frac{x_1}{x_2}\right)^{\frac{1}{t_2 - t_1}}$$

\$20 now	\$35 1 week
\$200 now	\$350 1 week

Magnitude effect: Higher discount factors for larger amounts

Discount factor

$$\delta = \left(\frac{x_1}{x_2}\right)^{\frac{1}{t_2 - t_1}}$$

\$20	\$35
now	1 week
-\$20	-\$35
now	1 week

Sign effect: Higher discount factors for losses than gains

Discount factor

$$\delta = \left(\frac{x_1}{x_2}\right)^{\frac{1}{t_2 - t_1}}$$

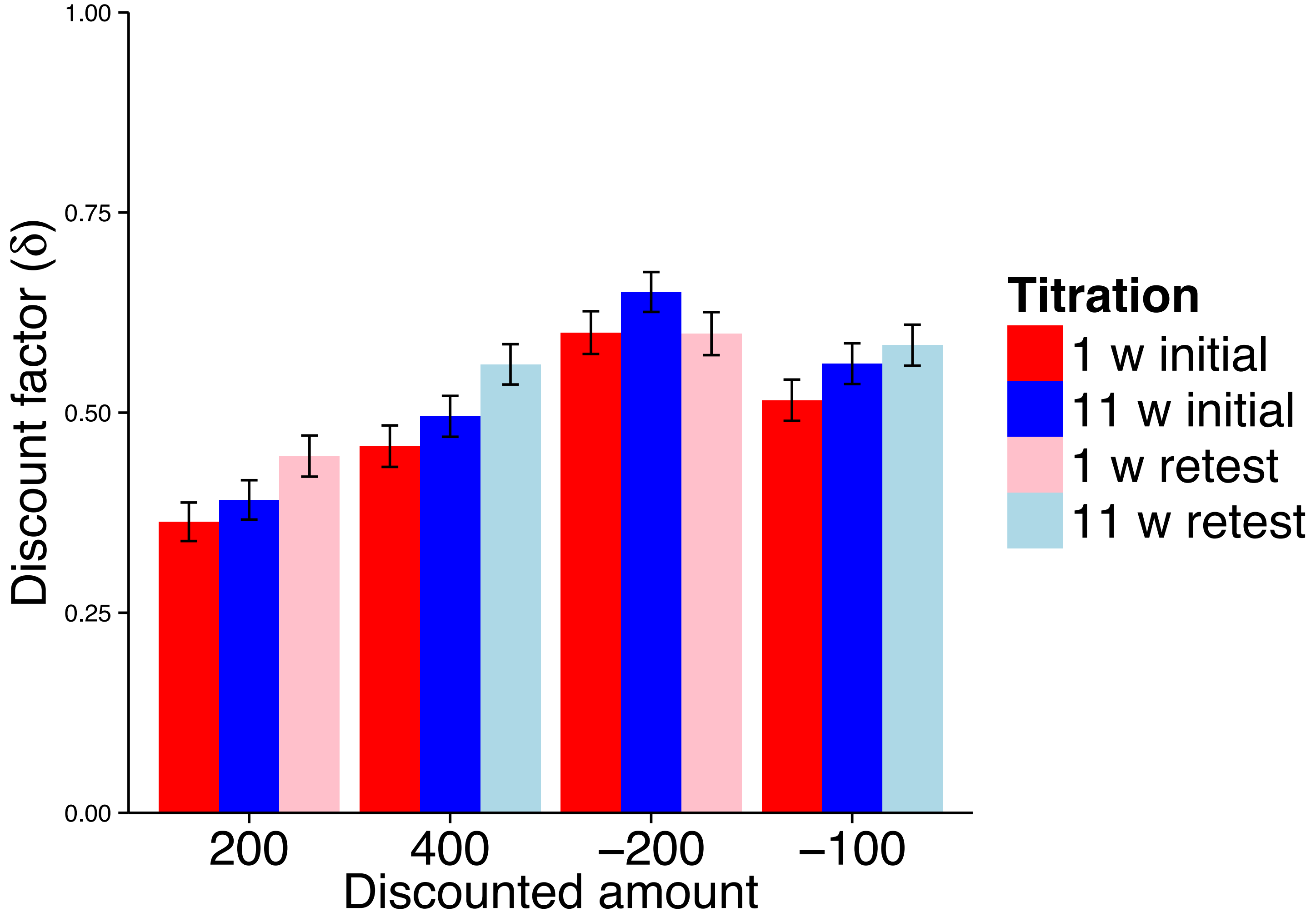
Expected from delay discounting literature:

Delay effect: Higher discount factors for later delays

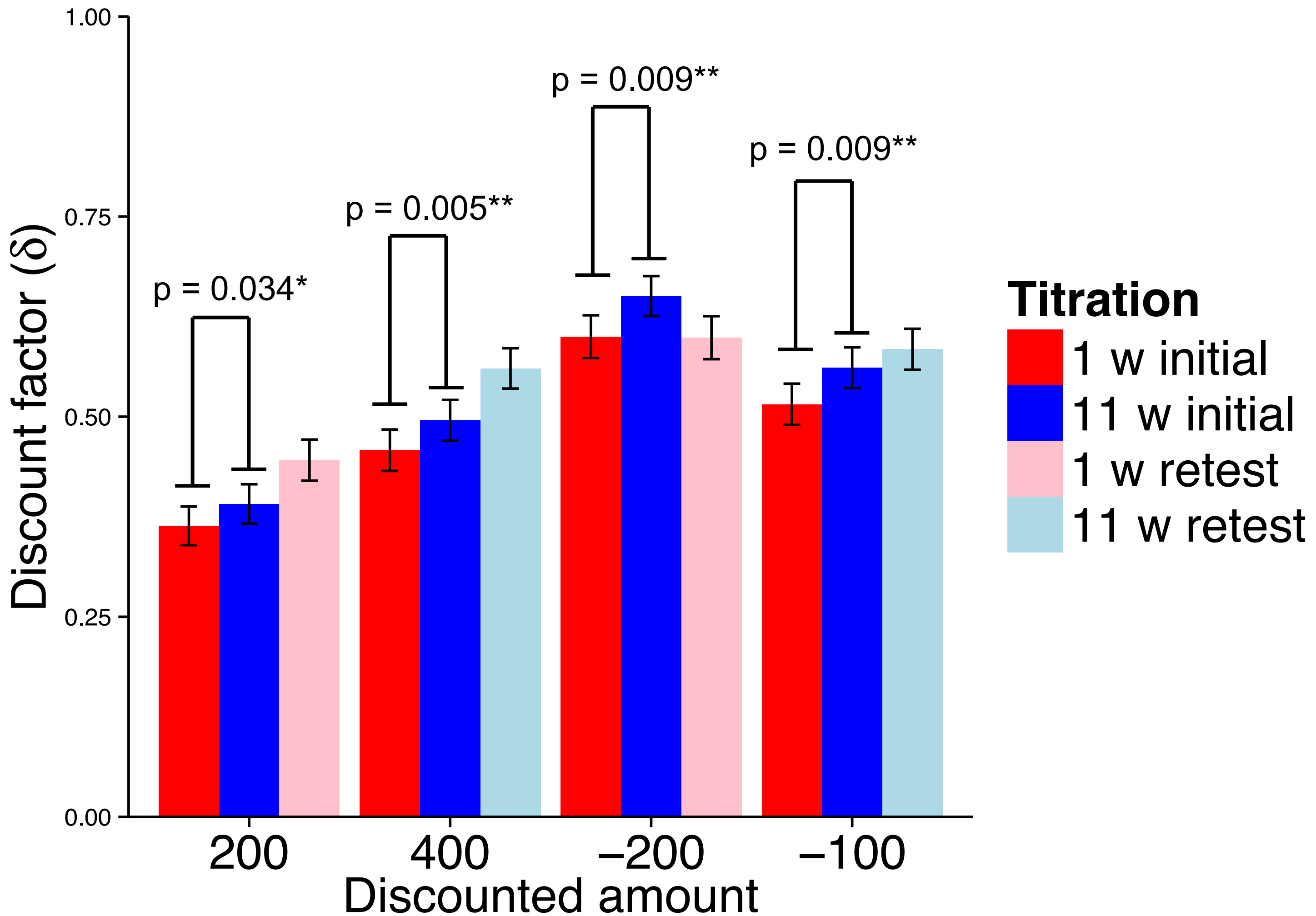
Magnitude effect: Higher discount factors for larger amounts

Sign effect: Higher discount factors for losses than gains

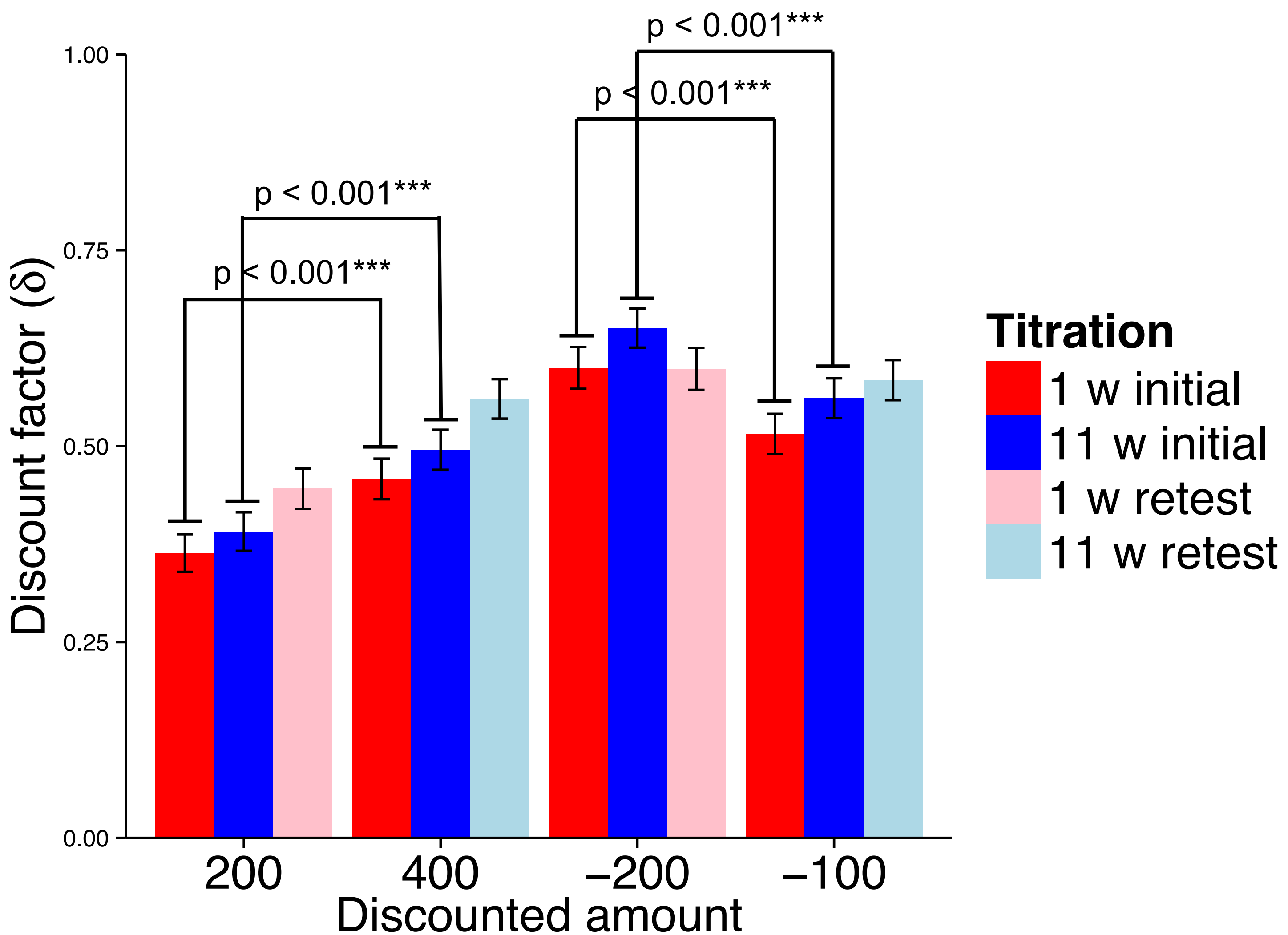
Calibration (n = 199)



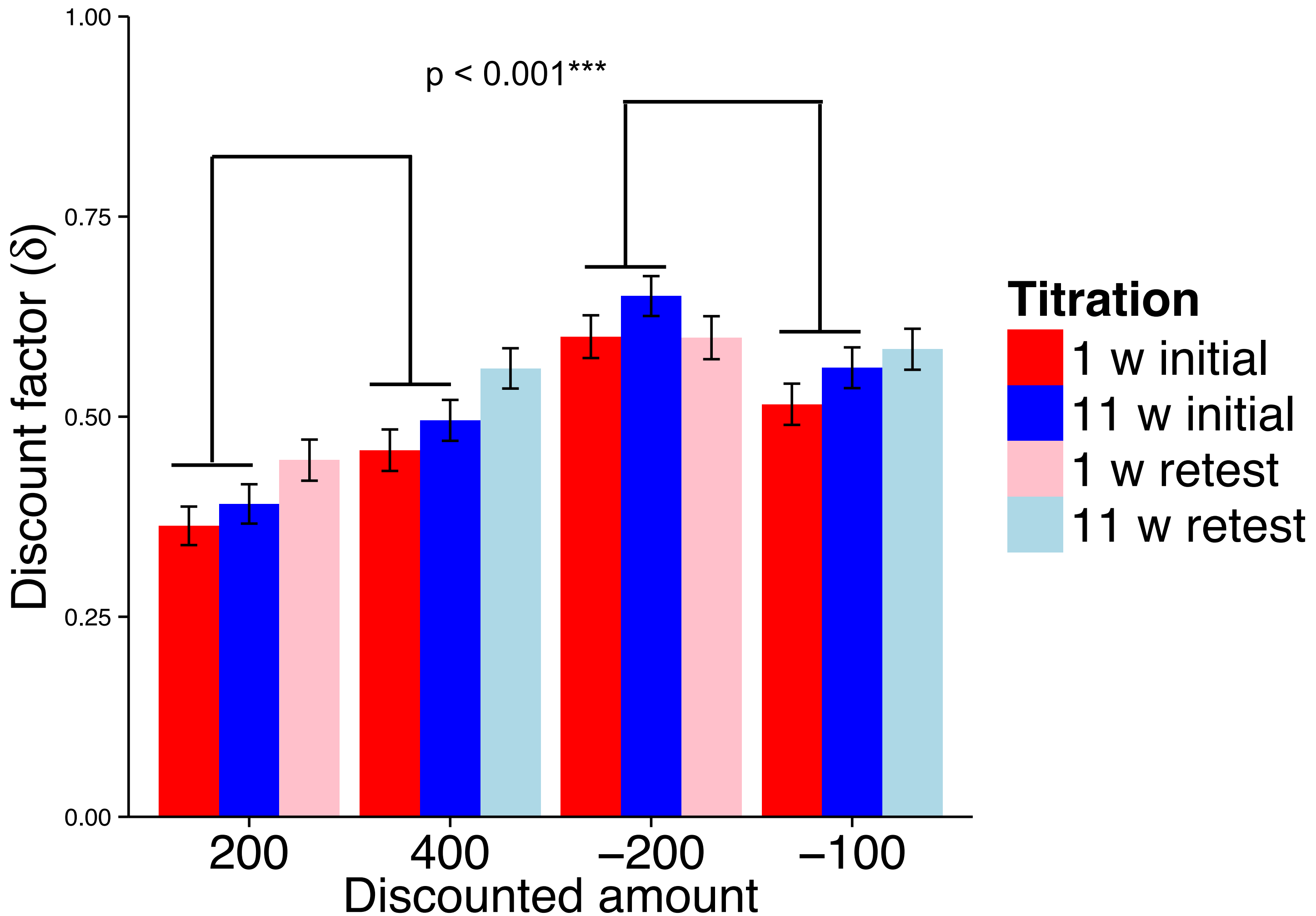
Calibration: Delay effect



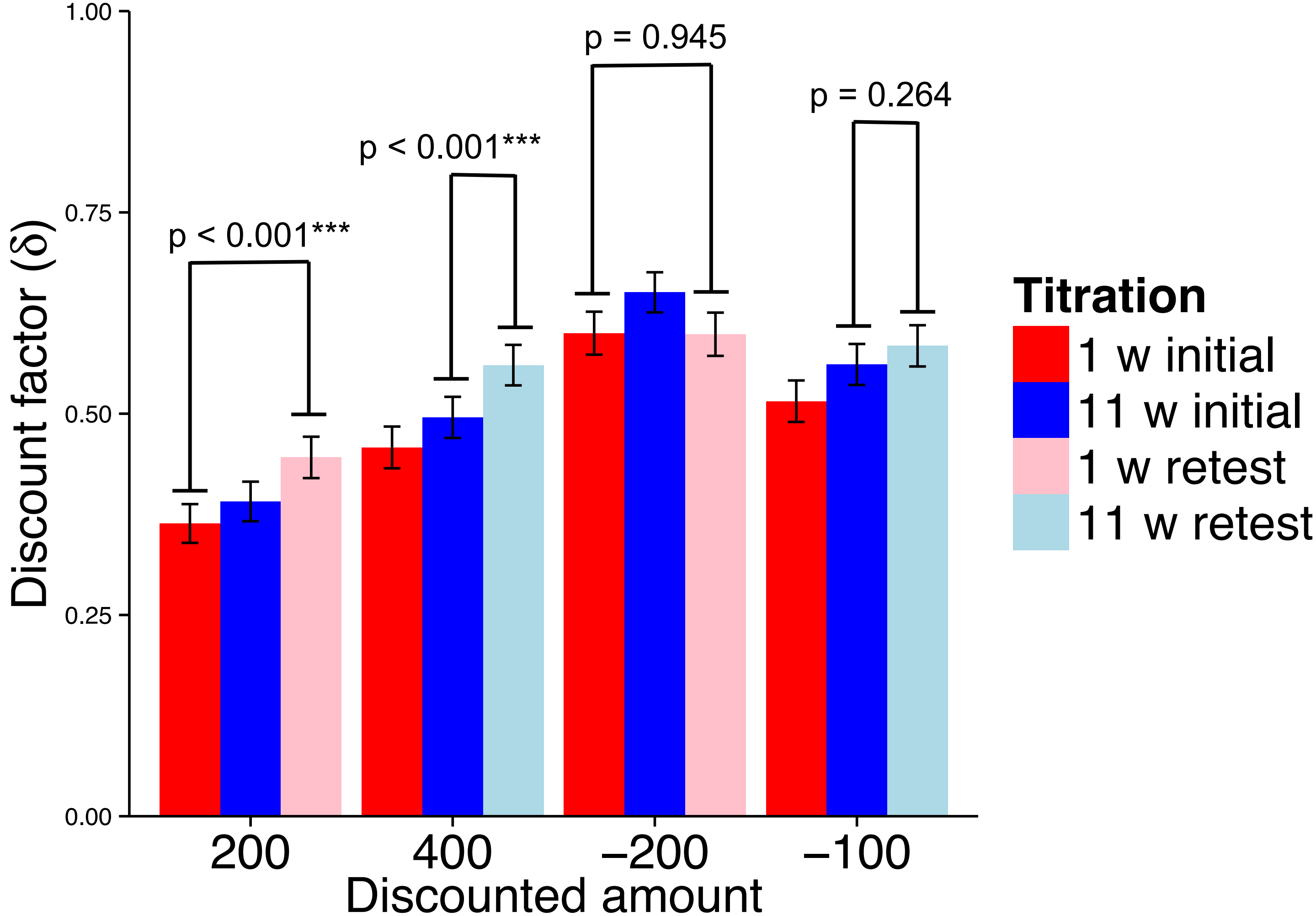
Calibration: Magnitude effect



Calibration: Sign effect



Calibration: Retest stability



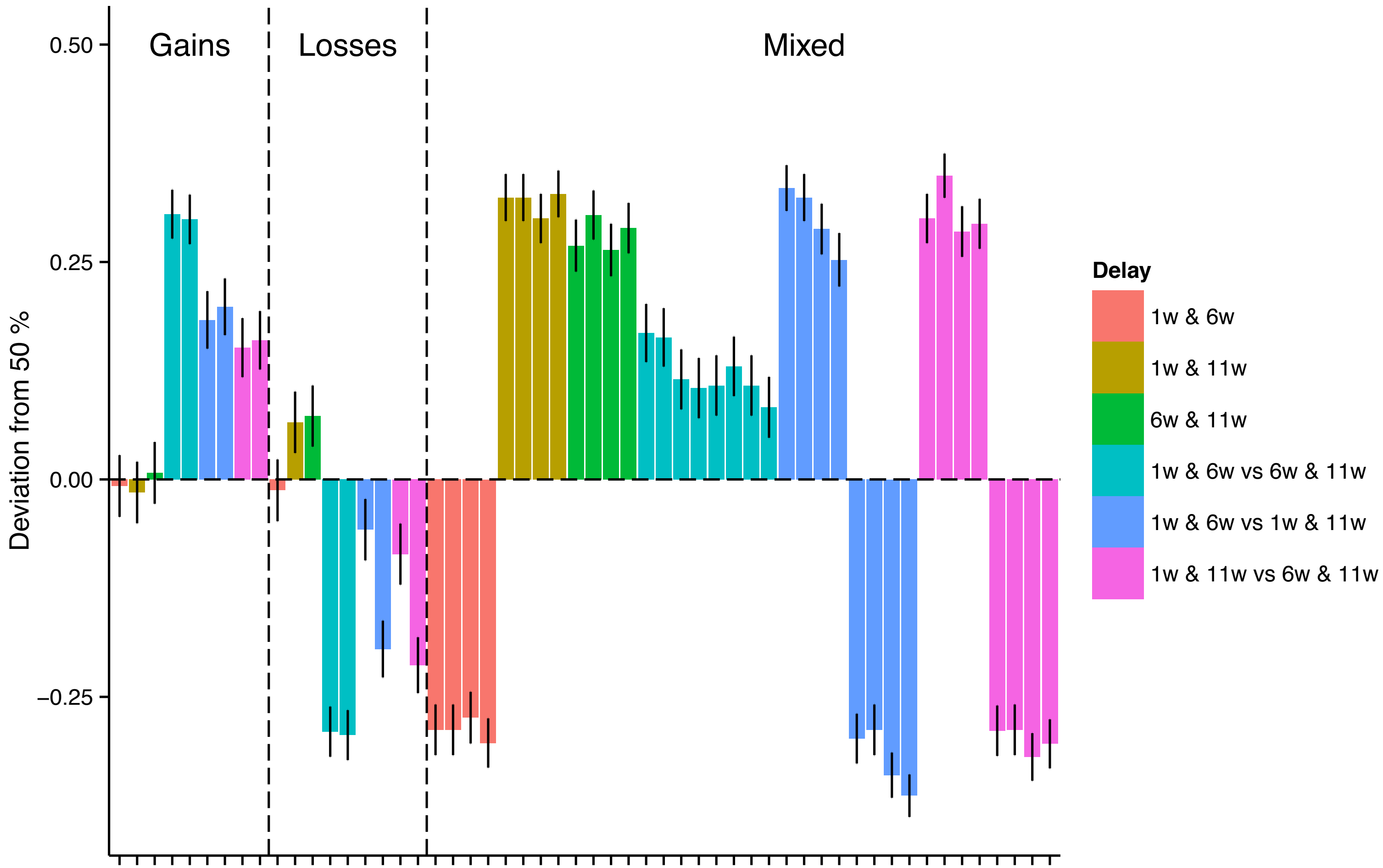
Sequences

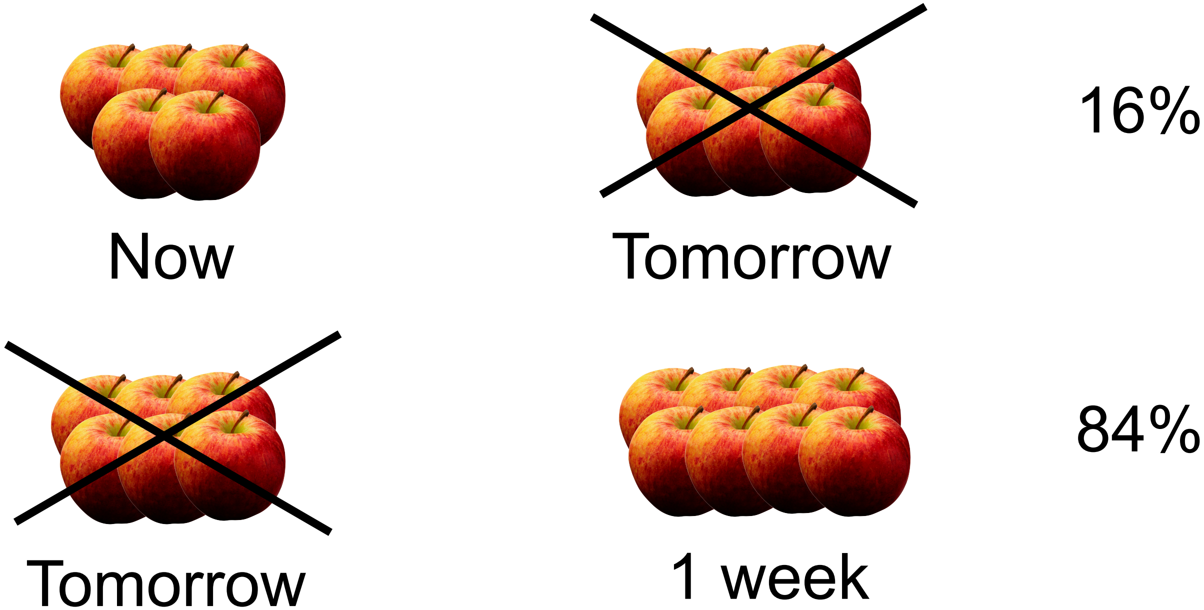
54 test sequence pairs

- 3 reward types (gains, losses, mixed)
- 6 different delay quartets
- 6 all improving, 6 all declining and 42 improving + declining

$394 in 1 week	$188 in 1 week
and	and
$204 in 6 weeks	$413 in 11 weeks

Independence Violated (n = 199)





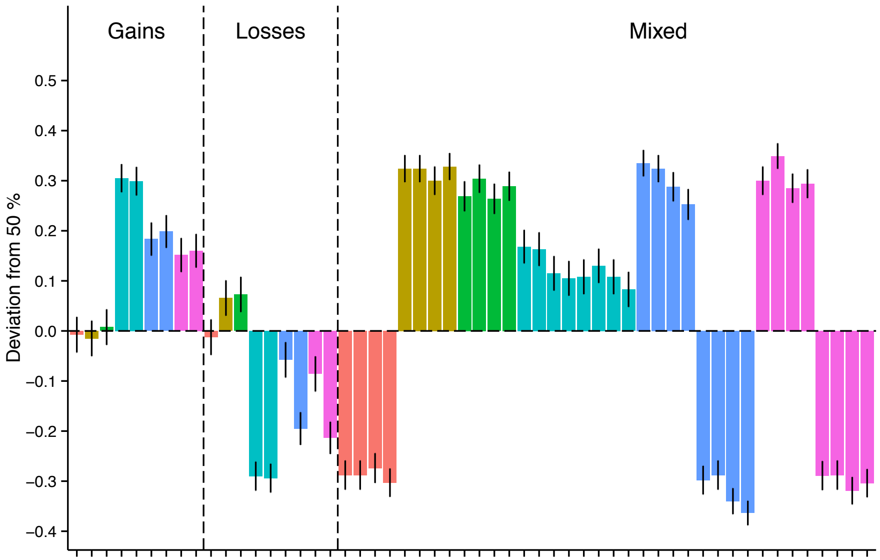
Do people prefer the larger total amount?

$$\Delta_T = r_1 + r_2 - (r_1' + r_2')$$

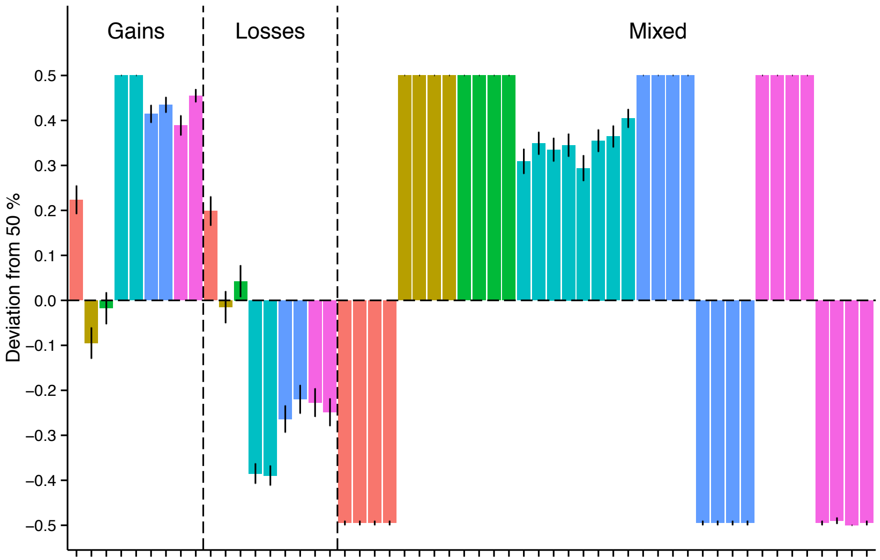
Rao & Li, 2011

Independence Violated (n = 199)

Empirical



Choose max Δ_T



$$Choice \sim \Delta_T$$

$$z_{\Delta_T} = -27.512, p < 0.001$$

Preference for improvement

Imagine you have two possible payment schemes:

\$35,000 1 year	\$36,000 2 years	\$37,000 3 years	\$38,000 4 years
\$38,000 1 year	\$37,000 2 years	\$36,000 3 years	\$35,000 4 years

Preference for improvement

Imagine you have two possible payment schemes:

\$35,000 1 year	\$36,000 2 years	\$37,000 3 years	\$38,000 4 years
\$38,000 1 year	\$37,000 2 years	\$36,000 3 years	\$35,000 4 years

$$\Delta_M = \frac{r_1 + r_2}{d_1 + d_2} - \frac{r_1' + r_2'}{d_1' + d_2'}$$

Preference for improvement

$$\Delta_M = \frac{r_1 + r_2}{d_1 + d_2} - \frac{r'_1 + r'_2}{d'_1 + d'_2}$$

$$Choice \sim \Delta_T \qquad z_{\Delta_T} = -27.512, p < 0.001$$

$$Choice \sim \Delta_T + \Delta_M \qquad z_{\Delta_T} = -13.925, p < 0.001$$

$$z_{\Delta_M} = -25.029, p < 0.001$$

Revising discounted utility

$$U^{t}(c_{t},\ldots,c_{T}) = \sum_{k=0}^{T-t} D(k)u(c_{t}+k) + F(c_{t},\ldots,c_{T})$$

Conclusion

- Intertemporal choice is more complicated than discounting of unitary outcomes
  - Independence assumption does not hold!

Conclusion

- Intertemporal choice is more complicated than discounting of unitary outcomes
  - Independence assumption does not hold!
- So far we've been considering heuristics but we aim to create process accounts
  - e.g. Attention to total amounts implies attention away from delays

Conclusion

- Intertemporal choice is more complicated than discounting of unitary outcomes
  - Independence assumption does not hold!
- So far we've been considering heuristics but we aim for process-level accounts
  - e.g. Attention to total amounts implies attention away from delays
- Long term goal: Neuroimaging

Thank you!

Improving != Higher total reward

$393 in 1 week
and
$203 in 6 weeks

$188 in1 week
and
$398 in 6 weeks

Improving != Higher total reward

$393 in 1 week and $203 in 6 weeks	$188 in1 week and $398 in 6 weeks
$596	$586

How to quantify/categorize choice?

Preference for one sequence that is..

Preference for improving sequence

Preference for higher reward sequence

Preference for sooner delivery (e.g. 1 & 6 w instead of 6 & 11 w)

Preference for less deviation from uniform distribution

Preference for integrated sequence (Read-Scholten style)

????

Next steps

- Titration data
- Run gains and losses only
- Real-world equivalents
- Testing the Read & Scholten model
- Improving the model
- Process data?
- Fixing titration? Or is there something more interesting going on with people switching to become more patient with more trials?
- Disentangling different motives more carefully (e.g. pref for improvement vs. pref for larger payout)
- What are people doing when they aren't choosing based on these simple heuristics (Read and Scholten style integration models)

Independence axiom

$$\textit{if}$$
$$X \succ Y$$
$$\textit{then}$$
$$pX + (1-p)Z \succ pY + (1-p)Z$$

¥ 10,000

Now

¥ 30,000

1 year

Rao & Li, 2011

¥ 10,010,000

Now

¥ 10,000,000	¥ 30,000
Now	1 year

Rao & Li, 2011

¥ 10,000		33.3%
Now		
	¥ 30,000	66.7%
	1 year	

Rao & Li, 2011

¥ 10,010,000		51.1%
Now		
¥ 10,000,000	¥ 30,000	48.9%
Now	1 year	

1. Not entirely total amounts
2. Relative amounts?

Rao & Li, 2011

€ 1,010

Now

€ 1,030

2 months

Read & Scholten, 2012

€ 2,010

Now

€ 1,000	€ 1,030
Now	2 months

Read & Scholten, 2012

€ 1,010		81%
Now		
	€ 1,030	19%
	2 months	

Read & Scholten, 2012

€ 2,010		54%
Now		
€ 1,000	€ 1,030	46%
Now	2 months	

Read & Scholten, 2012

“The most controversial assumption in the von Neumann-Morgenstern theory is the “independence axiom.” (sometimes referred to as the substitution axiom). Other axioms involve properties, such as transitivity, completeness, and continuity, that are needed to ensure the existence of a utility functional defined on a space of lotteries, but it is the independence axiom that results in the expected-utility representation that is linear in the probabilities” (Holt, 1986)

Experiment I

$ 10

Now

$ 30

2 months

Read & Scholten, 2012

$ 10	$ 0
Now	2 months
$ 0	$ 30
Now	2 months

Read & Scholten, 2012

$ 510

Now

$ 500	$ 30
Now	2 months

Read & Scholten, 2012

$ 510

Now

$ 530

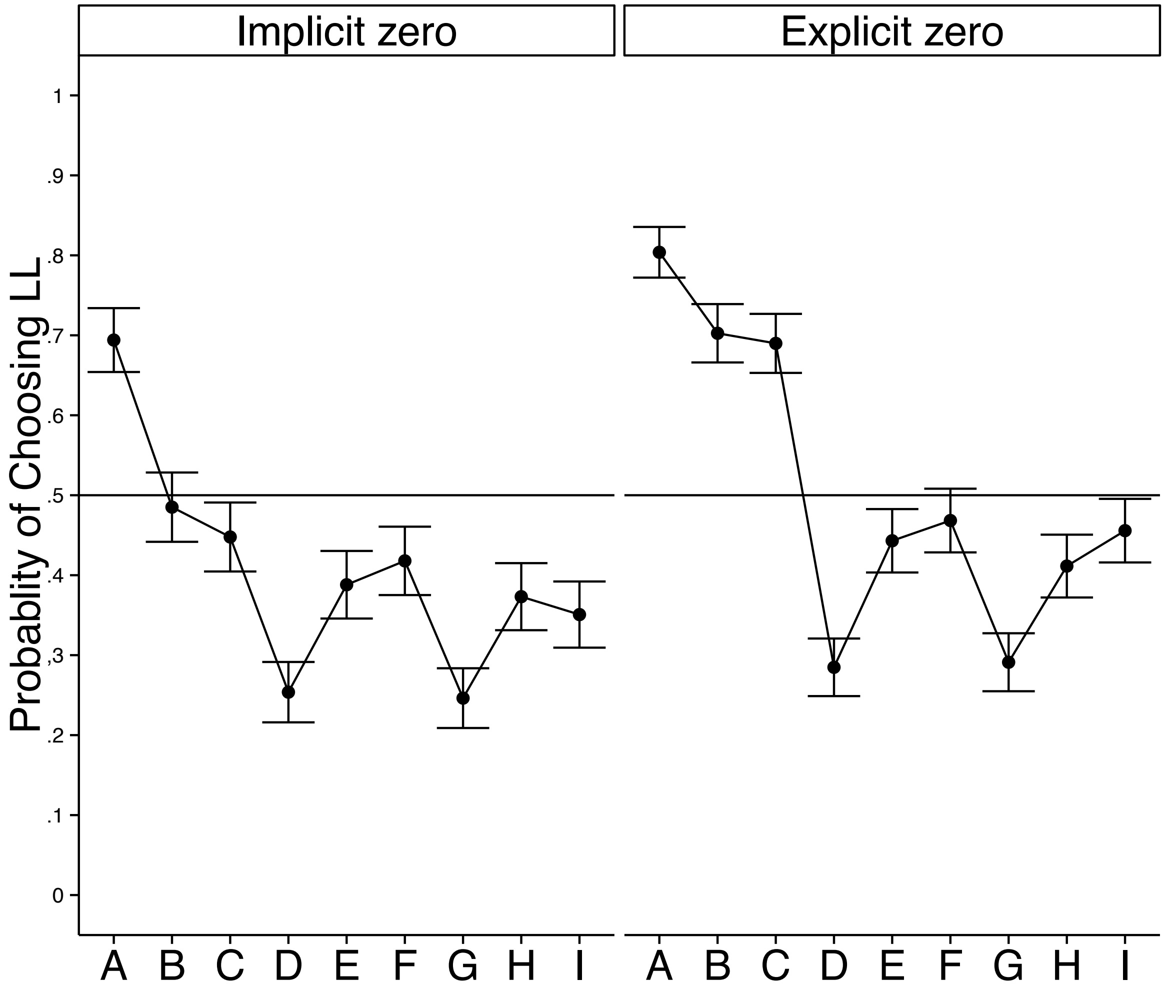
2 months

Read & Scholten, 2012

Design

	$A_{0:2}$		
	$0	$500	$1000
A_0	**A**	**D**	**G**
$0	($10, 0; $0, 2) ($0,0; $30,2)	($510, 0; $0, 2) ($0,0; $530,2)	($1010, 0; $0, 2) ($0,0; $1030,2)
	B	**E**	**H**
$500	($510, 0; $0, 2) ($500,0; $30,2)	($1010, 0; $0, 2) ($500,0; $530,2)	($1510, 0; $0, 2) ($500,0; $1030,2)
	C	**F**	**I**
$1000	($1010, 0; $0, 2) ($1000,0; $30,2)	($1510, 0; $0, 2) ($1000,0; $530,2)	($2010, 0; $0, 2) ($1000,0; $1030,2)

Results (n = 292)



$$\log(\Omega) = \frac{1}{\varepsilon}[v(x_{L_1} + x_{L_2}) - v(x_{S_1} + x_{S_2}) - \sigma(d(x_{L_1}, x_{L_2}) - d(x_{S_1}, x_{S_2})) - \kappa(w(\hat{t}_L) - w(\hat{t}_S))]$$