Assignment 5

Computer Vision
Zeno Sambugaro 785367

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Exercise 1

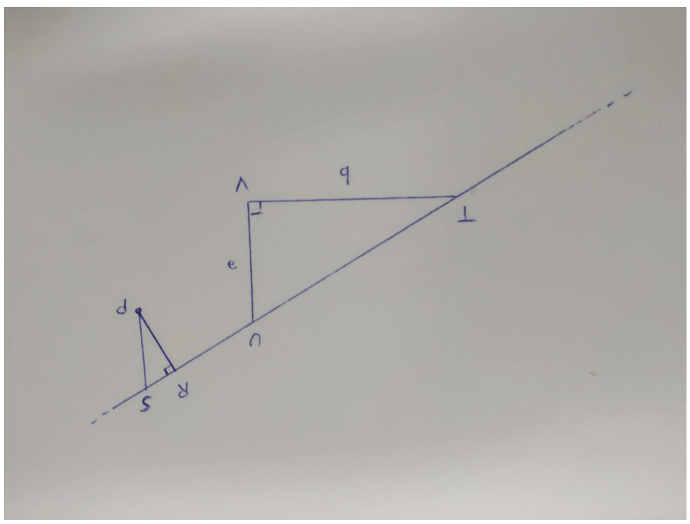
a. 

Figure 1: Similar triangles

$$ax + by - d = 0 \tag{1}$$

The two triangles we can see from the figure, δPRS, δTVU are similar triangles, Hence $\frac{PR}{PS} = \frac{TV}{TU}$

$$Distance = |PR| = \frac{|PS| \cdot |TV|}{|TU|} = \frac{|y_i - m| \cdot |b|}{\sqrt{a^2 + b^2}} \tag{2}$$

We let S to be (x_i, m) since S is on line L, it follows $ax_i + b_m - d = 0$
$m = \frac{-ax_i + d}{b}$

$$distance = |y_i - \frac{-ax_i + d}{b}| \times |\frac{|b|}{\sqrt{a^2 + b^2}} = |\frac{by_i + ax_i - d}{b}| \times |\frac{b}{\sqrt{a^2 + b^2}}| = |\frac{ax_i + by_i - d}{\sqrt{a^2 + b^2}}|$$

Since $a^2 + b^2 = 1$, distance $= |ax_i + by_i - d|$

b. Let's compute the partial derivative of E with respect of d:

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2 \tag{3}$$

$$\frac{\delta E}{\delta d} = \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0$$

$$a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} d = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i - nd$$

$$nd = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i$$

$$d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i = a\hat{x} + b\hat{y}$$

c. Substituting the obtained formula in the E equation we obtain:

$$E = \sum_{i=1}^{n} (ax_i + by_i - b\hat{y} - a\hat{x})^2 = \sum_{i=1}^{n} (a(x_i - \hat{x}) + b(y_i - \hat{y}))^2 \tag{4}$$

We define U as the matrix containing the differences $x_i - \hat{x}$ and $y_i - \hat{y}$:

$$U = \begin{bmatrix} x_1 - \hat{x} & y_1 - \hat{y} \\ x_2 - \hat{x} & y_2 - \hat{y} \\ \vdots & \vdots \\ x_n - \hat{x} & y_n - \hat{y} \end{bmatrix} \tag{5}$$

$$E = \left\| \begin{bmatrix} x_1 - \hat{x} & y_1 - \hat{y} \\ x_2 - \hat{x} & y_2 - \hat{y} \\ \vdots & \vdots \\ x_n - \hat{x} & y_n - \hat{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = \|UN\|^2 = (UN)^T(UN) = N^T U^T U N \tag{6}$$

d. Starting from the expression obtained above:

$$U = \begin{bmatrix} x_1 - \hat{x} & y_1 - \hat{y} \\ x_2 - \hat{x} & y_2 - \hat{y} \\ \vdots & \vdots \\ x_n - \hat{x} & y_n - \hat{y} \end{bmatrix} \tag{7}$$

$$U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \hat{x})^2 & \sum_{i=1}^{n} (x_i - \hat{x})(y_i - \hat{y}) \\ \sum_{i=1}^{n} (x_i - \hat{x})(y_i - \hat{y}) & \sum_{i=1}^{n} (y_i - \hat{y})^2 \end{bmatrix}$$

Starting from the second moment matrix $U^T U$, we can obtain the following:

$$E = (UN)^T(UN) = N^T U^T U N = [ab] \begin{bmatrix} \sum_{i=1}^{n} (x_i - \hat{x})^2 & \sum_{i=1}^{n} (x_i - \hat{x})(y_i - \hat{y}) \\ \sum_{i=1}^{n} (x_i - \hat{x})(y_i - \hat{y}) & \sum_{i=1}^{n} (y_i - \hat{y})^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

We can minimize E by:

$$\frac{\delta E}{\delta N} = 2(U^T U)N = 0$$

solve for:

$$(U^T U)N = 0$$

Since we are interested in the trivial solution h = 0 and we have the constraint: $a^2+b^2 = 1$, we can reach the solution using the Single Value Decomposition (SVD):
A can be rewritten as USV^T, where U and V are both orthonormal. Since their orthonormality, we have that:

$$|USV^T N| = |SV^T N| \tag{8}$$

$$|V^T N| = |N| \tag{9}$$

We can now define $y = V^T N$ which has same constraint as N. This leads as to a situation of a constrained problem. S is obtained from the SVD so we know that is a diagonal matrix, where eigenvalues are in decreasing order. As consequence, the minimum array N we can choose to minimize E is the eigenvector associated with the minimum eigenvalue.

Exercise 2

- The number of samples s was stated in the assignment text.
- The threshold value T has been found in an empirical manner, and thus set to 5.
- The number of samples N has been set to 300 in an empirical manner.

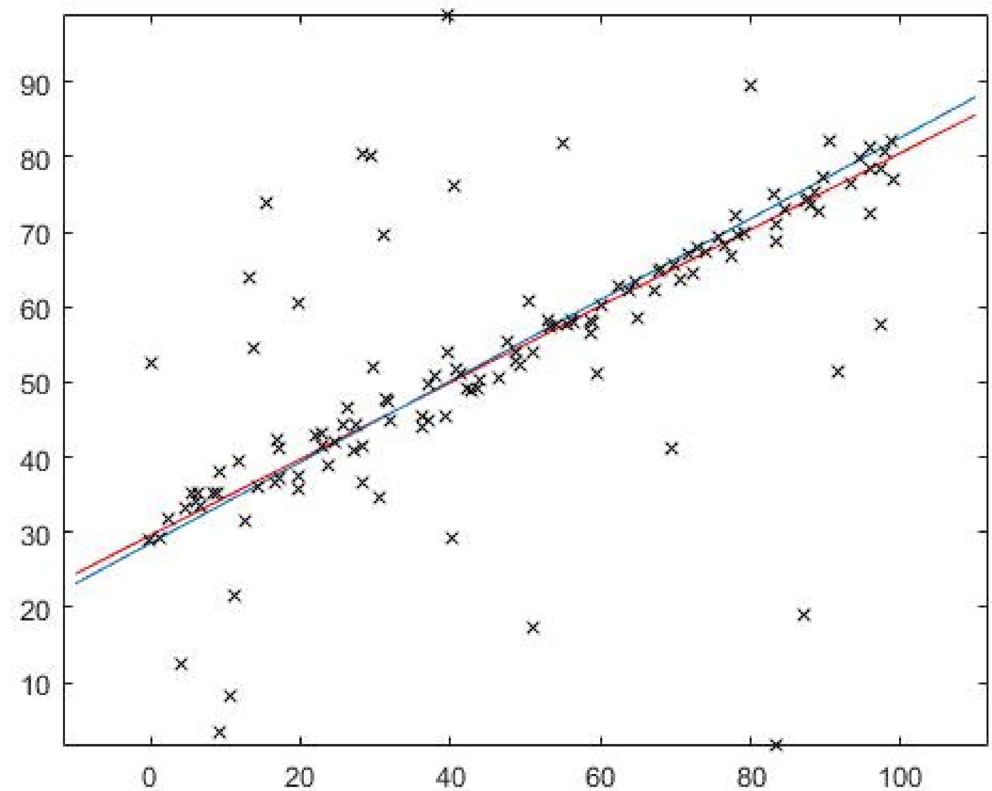


Figure 2: Line with most inliners in blue, total least square fitting of inliners in red.