Assignment 8

Computer Vision
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June 16, 2020

Exercise 1

a. What could be the main reasons why most of the features are not tracked very long in case *b*) above?
   From the video we can notice that when the image is rotated, the number of tracked features drops by half. In addition, when the speed of movement of the camera increases the number of detected features drops again, because the points move too much from a frame to the next one.

b. In order to improve the algorithm we could use the Gaussian pyramid frame by frame. Thus iteratively apply the KLT algorithm for each level of the pyramid, being able to compare the images at different scale. An improvement that does not affect the algorithm could be the restriction of rotation and curve movements of the camera.

Exercise 2

W(x;p) denotes the parameterized set of allowed warps, where $\mathbf{p} = (p_1, ..., p_n)^T$ is a vector of parameters. The warp takes the pixel **x** into the coordinate frame of the template T and maps it to the subpixel location **W(x;p)** in the coordinate frame of the image I.
In our case the wrap is a traslation:

$$W(x,p) = \begin{bmatrix} x+u \\ y+v \end{bmatrix} \text{ and } \nabla p = \begin{bmatrix} u \\ v \end{bmatrix}$$

The Jacobian of the wrap is:

$$\frac{\delta W}{\delta p} = \begin{bmatrix} \frac{\delta W_x}{\delta u} & \frac{\delta W_x}{\delta v} \\ \frac{\delta W_y}{\delta u} & \frac{\delta W_y}{\delta v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Substituting it in the equation (10) we obtain:

$$\nabla p = H^{-1} \sum_x [\nabla I \frac{\delta W}{\delta p}]^T [\nabla I \frac{\delta W}{\delta p}] =$$

Where **H** is the n $\times$ n (Gauss Newton approximation to the) Gaussian matrix:

$$\mathbf{H} = \sum_x [\nabla I \frac{\delta W}{\delta p}]^T [\nabla I \frac{\delta W}{\delta p}]$$

$$= \sum_x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\delta I}{\delta x} \\ \frac{\delta I}{\delta y} \end{bmatrix} \begin{bmatrix} \frac{\delta I}{\delta x} & \frac{\delta I}{\delta y} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \sum \begin{bmatrix} \frac{\delta I}{\delta x}\frac{\delta I}{\delta x} & \frac{\delta I}{\delta x}\frac{\delta I}{\delta y} \\ \frac{\delta I}{\delta y}\frac{\delta I}{\delta x} & \frac{\delta I}{\delta y}\frac{\delta I}{\delta y} \end{bmatrix}$$

$$= \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Returning to the equation (10)

$$\nabla p = H^{-1} \sum_x [\nabla I \frac{\delta W}{\delta p}]^T [T(x) - I(W(x;p))]$$

$$H \nabla p = \sum_x [\nabla I \frac{\delta W}{\delta p}]^T [T(x) - I(W(x;p))]$$

We consider that T(x) is an extracted sub-region of the image for t = 1 and I(x) is the image at t=2, thus:

$$H \nabla p = \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} [-I_t]$$

$$H \nabla p = \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

Substituting **H**:

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$