Assignment 9

Computer Vision
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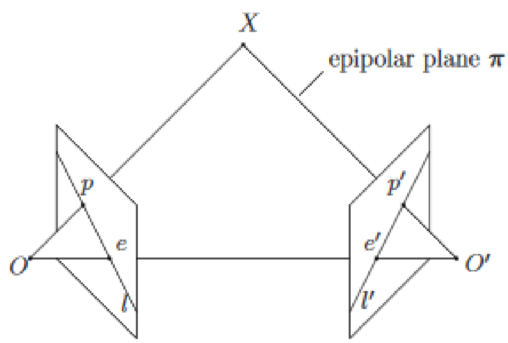
1 Exercise 1



From the image let's set:

$$\overrightarrow{O'P'} = x' \tag{1}$$

We also know that

$$\overrightarrow{O'O} = t = (t_1, t_2, t_3)^T \tag{2}$$

From the second camera's (O') coordinates:

$$\overrightarrow{OP} - \mathbf{Rx}, where \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{3}$$

We then substitute these into equation (1) where we can easily get:

$$\mathbf{x'} : (t \times \mathbf{Rx}) = 0 \tag{4}$$

Let's recall the property: $(\mathbf{a} = (a_x, a_y, a_z)^T and \mathbf{b} = (b_x, b_y, b_z)^T$

$$a \times b = \begin{bmatrix} 0 & a_z & a_y \\ a_z & 0 & -a_x \\ a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [a_x]b \tag{5}$$

Thanks to (3) we can get:

$$\mathbf{x'} \cdot (t \times \mathbf{Rx}) = \mathbf{x'}^T (r \times \mathbf{Rx}) = \mathbf{x'}^T [t_x] \mathbf{Rx} = \mathbf{x'}^T \ \mathbf{Ex} \tag{6}$$

The matrix $\mathbf{E}$ is the essential matrix.

2 Exercise 2

a. We know that:

$$\text{disparity} = x - x' = \frac{B \cdot f}{z}$$

Then:

$$z = \frac{B \cdot f}{d} = \frac{0.06 \cdot 0.01}{0.01} = 0.06m$$

b. Assuming that the smallest value for disparity is 0.01 mm the corresponding z for one pixel is:

$$z_1 = \frac{0.06 \cdot 0.01}{0.00001} = 60m$$

Thus going for d smaller than 0.01 mm, the interval will be between 60 m and ∞ so $]60; \infty[$ m.

c. In order to obtain the point coordinates with respect of the camera on the left we have to multiply the real world point coordinates by the **P** of the corresponding camera. The point must be in homogeneous coordinates. In this case:

$$\mathbf{P} \cdot \mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \tag{7}$$

The third component of the point is his scaling so the corresponding point in the image is: (3/3,0,3/3) = (1,0,1) so (1,0) in 2D coordinates.

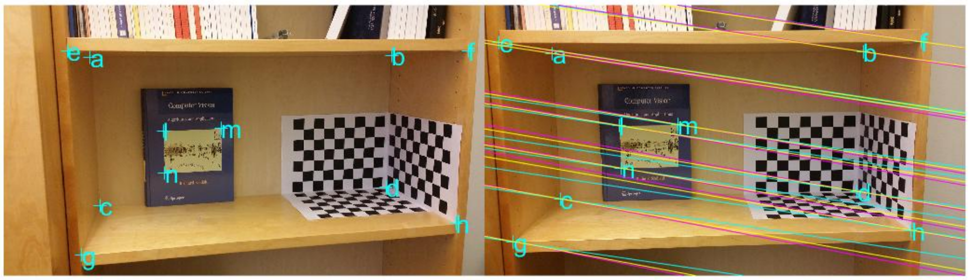
The epipolar line is defined as $\mathbf{E} \cdot \mathbf{x}'$:

$$\mathbf{t} \times \mathbf{R}\mathbf{x} \tag{8}$$

As we stated above the Essential matrix can be written as: $[t_x]Rx$ so:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix} \tag{9}$$

3 Exercise 3



In the image are showed the results of the normalized eight points algorithm (yellow) with respect to the results of the eight point algorithm (magenta).
In order to implement the normalized eight points algorithm I centered the data at the origin and scaled it so the mean squared distance between the origin and the data points is 2 pixels. Then I called the function implemented for the eight points algorithm and then the fundamental matrix in original coordinates has been computed as $T'^{T}FT$.