Polarized Light 2020

Laboratory Excercise

Våglära och optik FAFF30

Learning Outcomes

The aim of this laboratory exercise is for you to learn more about the transversal wave nature of light, *i.e.* the possibility to polarize the light. You will study several important concepts related to polarization; you will study how to creat polarized light and how to change this polarization.

This laboratory exercise focuses in particular at the following course aims.

Aim of the course

The course aims at developing the physics problem solving skills of the student. The course gives the students training in discussing physics, improves their experimental skills and introduces computer aided data analysis and graphical data representation. It treats wave propagation with particular emphasis on electromagnetic waves and optics. Electromagnetic radiation and wave propagation are central concepts in modern technology, but also connects directly to the wave function description of the quantum world. This course provides a foundation for understanding and developing the technology around us, and modern physics.

Learning outcomes

Knowledge and understanding
For a passing grade the student must

- know the basic physical principles of wave propagation,
- be able to relate abstract mathematical models and analogies to experiments and the reality,
- know how to analyze basic problems and calculations on waves and wave propagation.

Competences and skills
For a passing grade the student must

- have improved his/her ability to carry out laboratory work and analyze the results using computers,
- have improved his/her ability to present calculations and observations in written form, and discuss physical problems with collegues
- be able to solve simple physics problems in a structured way.

Judgement and approach
For a passing grade the student must

- be able to demonstrate an understanding of scientific methods and to realize possibilities and limitations of physics,
- be able to evaluate results of different experimental methods,
- be able to identify his/her further need of knowledge also within other areas.

Introduction

Polarization of light is the orientation of the electric field with respect to the propagation. In this lab you will work with several important concepts related to polarization. First you will study different optical components that create and change the polarization state of light. We create linearly polarized light with *filters*, built on the principle of *selective absorption* (*dichroism*). Linearly polarized light may then be changed into an arbitrary state using a *wave retarder*, built on the principle of *birefringence* in an *anisotropic media*.

By measuring the optical properties with good spectral resolution you will be able to determine the wavelength dependence of the refractive index, *i.e.* the *dispersion* of the media. This phenomenon explains for example the beautiful colors seen in the rainbow or refracted from a diamond ring.

Note: **it is really important to read the appendices 1 and 2 before doing the preparatory exercises**

Preparatory problems

1. Which state of polarization is described by the following expressions of the electric field:
   a) $\overline{E} = E_0 \cdot (\overline{e}_x \cdot \sin(kz - \omega t) + \overline{e}_y \cdot \cos(kz - \omega t))$
   b) $\overline{E} = E_0 \cdot (\overline{e}_x \cdot \sin(kz - \omega t) - \overline{e}_y \cdot \cos(kz - \omega t))$
   c) $\overline{E} = 5 \cdot \overline{e}_x \cdot \sin(kz - \omega t + \pi/2) + 3 \cdot \overline{e}_y \cdot \sin(kz - \omega t)$
   d) $\overline{E} = E_0 \cdot (\overline{e}_x \cdot \sin(kz - \omega t + \pi/3) + \overline{e}_y \cdot \sin(kz - \omega t))$
2. The transmitted intensity of linear light through an ideal linear polarizer is given by the law of Malus. Formulate this law.
3. What does it mean that a material is birefringent?
4. What happens if <u>un</u>polarized light passes through a birefringent crystal perpendicular to the optical axis?
5. White light reflects off a plane reflection grating with 300 lines/mm at an angle of 15° to the normal. At which angle do you observe 550 nm light in the first spectral order?

Experiments

1. The Spectrometer

The optical layout of the spectrometer, from Acton Research Corporation, is shown in Figure 1. The exit slit assembly has been removed in our set-ut to allow a direct visual inspection of the spectrum through a lens. Each registered spectrum will be saved as 1024 binary numbers, your supervisor will save the data for you.

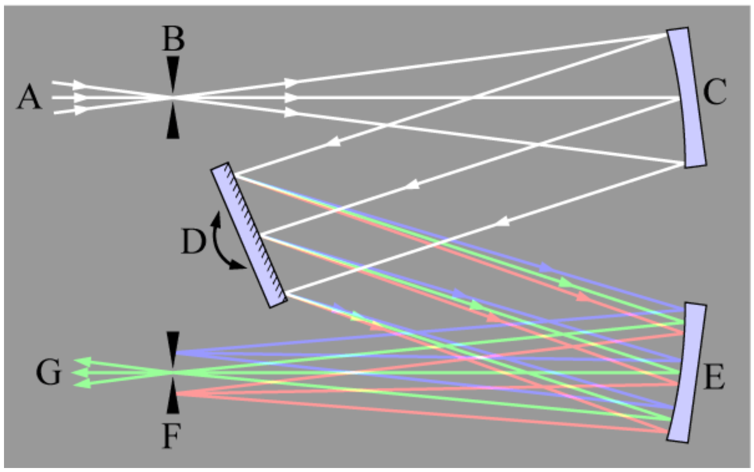

Figure 1. *Czerny-Turner spectrometer SpectraPro 300i. White light from A enters through the entrance slit B (to have a point source). The light is then reflected on the curved mirror C which collimates it and sends it to the grating (D) where the spectral components are dispersed and the mirror E focus them on the slit F.*

2 Birefringence - wave retarder.

2.1 Determine $\Delta n = n_e - n_o$ in a direction perpendicular to the optic axis in quartz

Use the spectrum registered by the supervisor. According to Appendix 2 (Eq. A2.2) there will be an intensity maximum in the spectrum whenever the phase difference $\delta = (2m+1)\cdot\pi$, *i.e.* when:

$$d \cdot \Delta n(\lambda) = (2m+1) \cdot \frac{\lambda}{2}\,. \tag{1}$$

Since the difference in refractive index, Δn, is almost constant from one maximum to the next we know that ***m* must decrease** by one for each new maximum when we move towards **longer wavelengths**. To calculate the refractive index as a function of wavelength with very high accuracy we could use a *Sellmeier equation* with empirically determined parameters. However, a much simpler formula, which is sufficiently accurate for our purposes, is that of Cauchy:

$$n = A + \frac{B}{\lambda^2}\,. \tag{2}$$

Here the constants A and B are slightly different for the ordinary and extraordinary rays, hence $\Delta n = n_e - n_o$ is also given by the same type of equation. If you combine equation 1 and 2 you may determine $\Delta n(\lambda)$ for quartz in a direction perpendicular to the optical axis from the measured spectrum.

Choose 3 of the observed maxima with integer differences p and q, as in Figure 3. Use MATLAB to determine the wavelengths ($\lambda_1 < \lambda_2 < \lambda_3$) of the maxima.

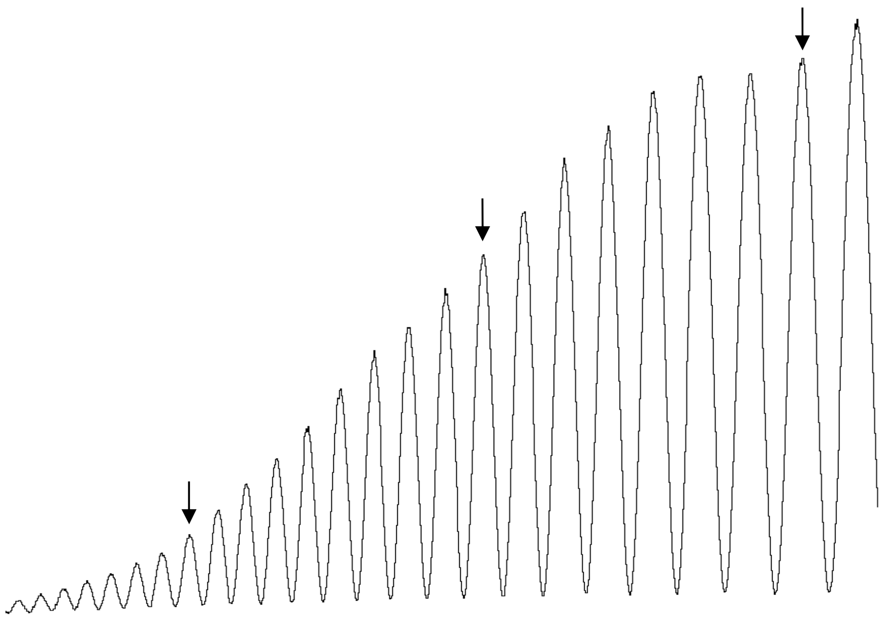

Figure 3. *At the wavelengths λ_2 and λ_3 the value of m is reduced by p and q, respectively, relative to the peak at λ_1. In the Figure $p = 9$ and $q = 16$.*

From the 3 wavelengths you obtain the following system of equations:

$$\begin{cases} d \cdot (A + \dfrac{B}{\lambda_1^2}) = (2m+1)\dfrac{\lambda_1}{2} \\ d \cdot (A + \dfrac{B}{\lambda_2^2}) = (2(m-p)+1)\dfrac{\lambda_2}{2} \\ d \cdot (A + \dfrac{B}{\lambda_3^2}) = (2(m-q)+1)\dfrac{\lambda_3}{2} \end{cases}$$

From this you may determine m and the constants A and B in Cauchy's formula for the difference Δn.

Draw a diagram of Δn as a function of wavelength.

Appendix 1 Description of polarized light

Here we use an explicit way of describing the light as a superposition of 2 orthogonal oscillations with a relative phase, δ. Thus, the electric field of a polarized wave propagating along the z-axis may be written

$$\overline{E}(x,y,z,t) = E_{0x} \cdot \overline{e}_x \cdot \sin(kz - \omega t + \delta) + E_{0y} \cdot \overline{e}_y \cdot \sin(kz - \omega t)\,. \qquad \text{(A1.1)}$$

The light is in general elliptically polarized, this is shown in Figure A1-1 for the special case when $E_{0x} = E_{0y}$.

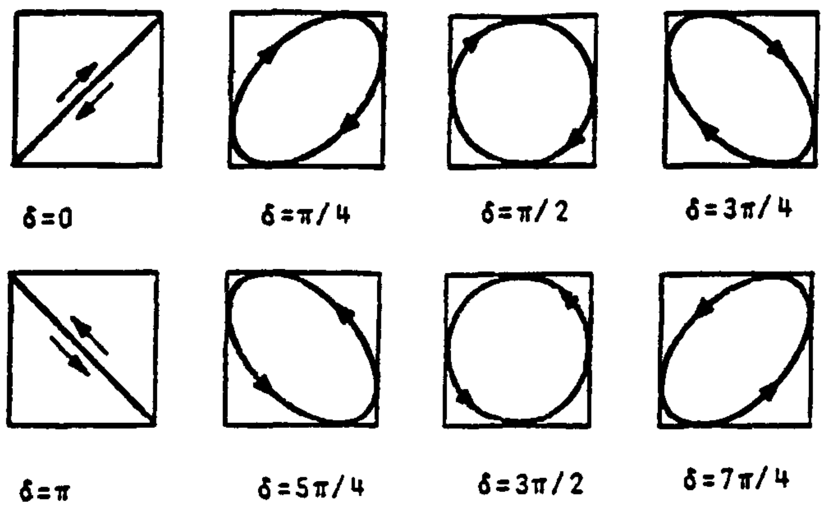

Figure A1-1. *State of polarization for different relative phases, δ, when $E_{0x} = E_{0y}$.*

When the 2 components have a relative phase $\delta = m\cdot\pi$ the light is said to be linearly polarized. Another special case occurs for $\delta = (2m + 1)\cdot\pi/2$ <u>and</u> $E_{0x} = E_{0y}$ then the light is circularly polarized.

It is important to note that linearly polarized light can be written as a superposition of 2 circularly polarized waves. Using the notation R (right) and L (left) we find from Eq. A1.1 with $\delta = \pi/2$:

$$\begin{aligned} \overline{E}_L &= E_0 \cdot (\overline{e}_x \cos(kz - \omega t) + \overline{e}_y \sin(kz - \omega t)) \\ \overline{E}_R &= E_0 \cdot (-\overline{e}_x \cos(kz - \omega t) + \overline{e}_y \sin(kz - \omega t)) \end{aligned} \qquad \text{(A1.2)}$$

and that

$$\overline{E} = \overline{E}_L + \overline{E}_R = 2E_0 \cdot \overline{e}_y \sin(kz - \omega t)\,, \qquad \text{(A1.3)}$$

which represents a linearly polarized wave oscillating parallel to the y-axis.

Appendix 2 Light in anisotropic media.

2.1 Light propagating perpendicular to the optical axis in uniaxial crystals, wave retarders.

Light incident on a quartz plate perpendicular to the optical axis is transmitted as 2 orthogonally polarized linear waves inside the plate, as shown in Figure A2-1.

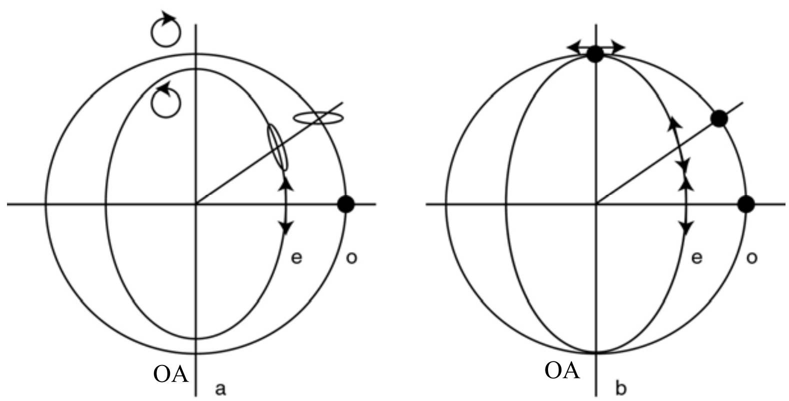

Figure A2-1. *The ordinary and extraordinary wave fronts in a positive crystal (n_e - n_o > 0). The difference between the 2 fronts is greatly exaggerated. The polarization state of the transmitted waves in some directions is also indicated. a) Optically active crystal b) Not optically active crystal*

The 2 waves propagate with different velocities corresponding to different indices of refraction. Thus after passing through the plate with thickness *d* the 2 waves have acquired a phase difference of:

$$\boxed{\delta = \frac{2\pi}{\lambda_0} \cdot d \cdot \Delta n} \tag{A2.1}$$

Here λ_0 is the vacuum wavelength. A plate used in this configuration is called a wave retarder. If the incoming light is linearly polarized the 2 components will initially be in phase but shifted by δ at the exit. Thus, a retarder typically converts linear light to elliptical light. However, there are some important special cases:

$\delta = 2m\cdot\pi$ Full wave plate. No noticeable effect on the light

$\delta = (2m+1)\cdot\pi$ Half wave plate. The outgoing light is still linearly polarized but the plane of oscillation is rotated.

$\delta = (2m+1)\cdot\pi/2$ Quarter wave plate. If the incoming light oscillates at 45° to the optical axis the outgoing light is circularly polarized

If a retarder plate is placed between 2 crossed polarizers we will in general retrieve some of the light, except for those wavelengths where the plate acts as a full wave plate. This will be clearly seen in the lab.

You will also use a quantitative measurement of the transmitted intensity as a function of wavelength to determine how δ and, from Eq. A2.1, Δn depend on λ.

The set-up for this experiment is shown in Figure A2-2.

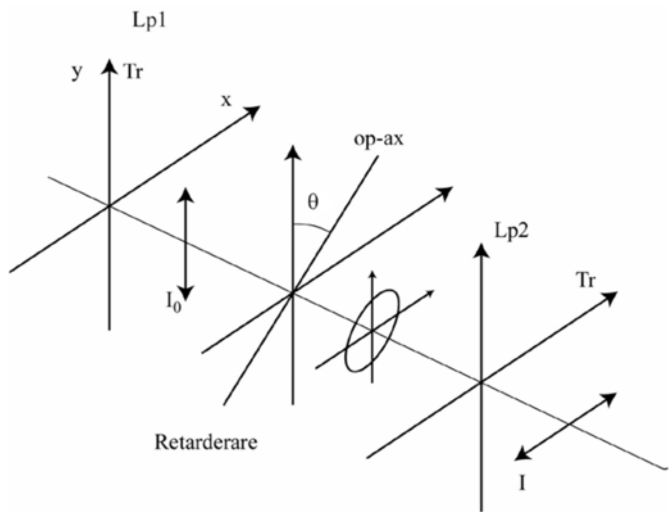

Figure A2-2. *Set-up to investigate a retarder plate.*

We will now derive how the intensity, *I,* after the second polarizer depends on the intensity I_0 before the plate, the angle θ, and the induced phase delay δ (Figure A2-2).

The simplest representation of the vertically polarized light after the first polarizer would be:

$$\overline{E} = E_0 \cdot \overline{e}_y \cdot \sin(kz - \omega t).$$

This is, however, not convenient when we describe the action of the retarder plate. Instead we describe the vertically polarised light as a superposition of 2 waves parallel ($\overline{e}_{//}$) and perpendicular ($\overline{e}_{\perp}$) to the optical axis, as shown in Figure A2-3.

$$\overline{E} = E_0 \cos(\theta) \cdot \overline{e}_{//} \cdot \sin(kz - \omega t) + E_0 \sin(\theta) \cdot \overline{e}_{\perp} \cdot \sin(kz - \omega t)$$

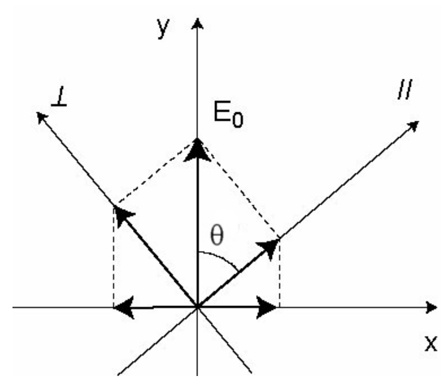

Figure A2-3.

The effect of the retarder plate is to introduce a phase delay, δ, between these components. The observable wave at the exit of the plate will thus be:

$$\overline{E} = E_0 \cos(\theta) \cdot \overline{e}_{//} \cdot \sin(kz - \omega t + \delta) + E_0 \sin(\theta) \cdot \overline{e}_{\perp} \cdot \sin(kz - \omega t).$$

Only the projection of these fields on the transmission axis of the second polarizer will contribute to the final intensity. According to Figure A2-3 the field after the analyzer is:

$$\begin{aligned}\overline{E} &= E_0 \cos(\theta) \cdot \cos(90 - \theta) \cdot \overline{e}_x \cdot \sin(kz - \omega t + \delta) - \\ &\quad E_0 \sin(\theta) \cdot \cos(\theta) \cdot \overline{e}_x \cdot \sin(kz - \omega t) = \\ &\quad \frac{1}{2} E_0 \sin(2\theta) \cdot \overline{e}_x \cdot \big(\sin(kz - \omega t + \delta) - \sin(kz - \omega t)\big) = \\ &\quad E_0 \sin(2\theta) \cdot \overline{e}_x \cdot \cos(kz - \omega t + \delta/2) \cdot \sin(\delta/2)\end{aligned}$$

This field gives a light intensity of:

$$I \propto \left\langle \overline{E}^2 \right\rangle = E_0^2 \cdot \sin^2(2\theta) \cdot \sin^2(\delta/2) \cdot \left\langle \cos^2(kz - \omega t + \delta/2) \right\rangle =$$

$$\frac{1}{2} E_0^2 \cdot \sin^2(2\theta) \cdot \sin^2(\delta/2)$$

With $I_0 \propto \frac{1}{2} E_0^2$ we obtain the final result:

$$\boxed{I = I_0 \cdot \sin^2(2\theta) \cdot \sin^2(\delta/2)} \tag{A2.2}$$

Some consequences:

1. If $\theta = 0$ or 90°, *i.e.* if the incident light is parallel or perpendicular to the optical axis the final intensity is zero.
2. Maximum intensity is obtained for $\theta = 45°$.
3. If $\delta = 2m \cdot \pi$, i.e. if the plate is a full wave plate the intensity is zero.
4. If $\delta = (2m + 1) \cdot \pi$, i.e. if the plate is a half wave plate the intensity is maximized.