Homework 0

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Due Monday, January 22, 2018

Prerequisites for this course include some exposure to probability or statistics and linear algebra. This homework will give you an opportunity to test your background. All questions have been designed so that you can solve them *without using a computer.*

If you need some review, you may find the following resources helpful:

- Zico Kolter's brief review of Linear Algebra (http://cs229.stanford.edu/section/cs229-linalg.pdf)

- Joe Blitzstein's Harvard statistics course (http://projects.iq.harvard.edu/stat110)

If you find that you are able to answer all questions easily, the first several weeks of this class may be easy for you but the subsequent material should be useful. If you are able to answer most questions, but need to think about some of them or look up information, you are well prepared for the class. If you need to look up information for most questions, you will be able to take this course but some extra self-study may be required, particularly in the first weeks. If you are unable to answer most of these questions, this class will be challenging for you and you should not enroll in the class without talking to me or the TA.

Grading and Submission Instructions

Please turn in your answers to this course as a `.pdf` file on Courseworks using the `Assignments` tab. You can type your answers (for example using LaTeX) or write clearly by hand and scan them (please make sure the scanned document is legible!)

You will recieve full credit for this homework if you submit complete answers, by the due date, and following the submission instructions, regardless of how many questions you answer correctly.

1 LINEAR ALGEBRA

Let us define

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} k \\ \ell \end{bmatrix} \tag{1.1}$$

We will also use the notation $(\cdot)^T$ to mean the transpose of $(\cdot)$. The transpose is sometimes written as $(\cdot)'$ but this can lead to confusion and we will avoid writing it this way.

1. What is $A_{2,1}$?

2. What is A^T?

3. What is AB?

4. What is $x^T A x$?

5. What is $x^T x$?

6. What is xx^T?

Now let C be a matrix of shape 10×2.

7. Is AC defined? If so what shape is the resulting matrix?

8. Is CA defined? If so what shape is the resulting matrix?

2 PROBABILITY AND RANDOM VARIABLES

Define y and w as random variables

$$y \sim \mathcal{N}(\mu_y, \sigma_y) \tag{2.1}$$
$$w \sim \mathcal{N}(\mu_w, \sigma_w) \tag{2.2}$$

where $\mathcal{N}(\mu, \sigma)$ denotes a normal variable with mean μ and standard deviation σ.

9. What is the expected value of y^2, written $\mathbb{E}[y^2]$?

10. If y and w are independent, what is the distribution of $y + w$?

Now let us consider a random 5-card poker hand delt from a standard 52-card deck[1] What is the probability that the 5-card hand is:

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[1]This is a deck with 4 suits, each of which has 13 cards: the numbers 2-10, plus the jack (J), queen (Q), king (K), and ace (A). See https://en.wikipedia.org/wiki/Standard_52-card_deck.

11. A pair (two cards of the same number but different suit)

12. A flush[2] (all cards of the same suit).

Also consider the following longer problems

13. A spam filter is designed based on common words and phrases.[3] Suppose that 75% of email is spam (and so 25% of email is not spam). In 25% of spam emails, the phrase "congratulations" is used, whereas this phrase is used in only 10% of non-spam emails. A new email has just arrived, which includes the word "congratulations". What is the probability that this email is spam?

14. A coin is tossed repeatedly until it lands Tails for the first time. Let Y be the number of tosses that are required (including the final one that lands Tails). If it is a fair coin (equal probability of heads or tails), what is the probability that:

    a) $Y = 1$

    b) $Y = 2$

    c) $Y = 100$ (write the expression you would solve, but do not attempt to write it as a decimal)

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[2]Do not exclude a royal flush
[3]This would be a very primitive spam filter!