Unit

Babylonian mathematics 2

Notes

The material is taken from [1, 2]. Plimpton 322 analysis is adapted from original research article [4]. Oral comprehension part has potential link to Jefferson and modern era *cultural diplomacy* [3]

Introduction should be expedited quickly, with preferably no knowledge of Babylonian numerals. Students work for 10-20 min in pairs to discover the tablet. If they are unable to guess, tell them to compare lines 7 and 8, and that it is about squares of counting numbers. One can also show table 2.1.

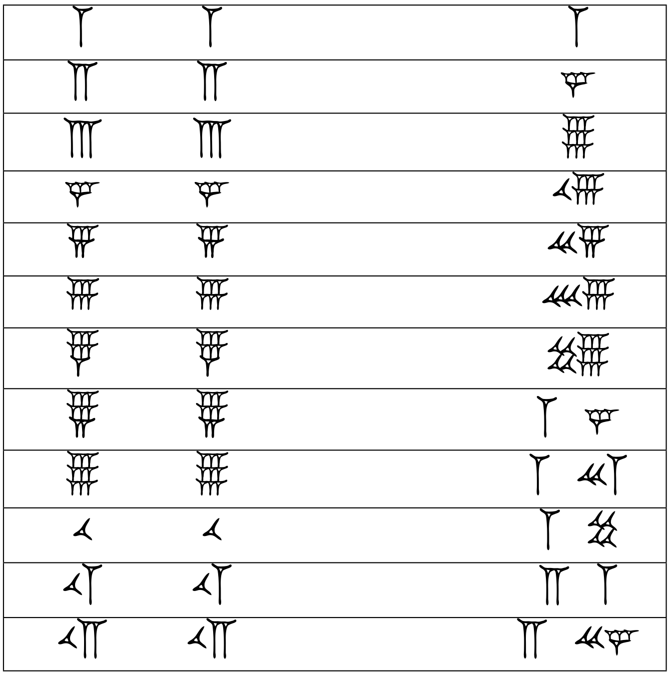


Table 2.1 – Recreated version of the numerals in the table, with no obfuscation.

2.1 Babylonian Numeration

The mathematics of Mesopotamia from 2500 BCE[1] to 300 BCE is usually called “Babylonian”. This oversimplifies the history of that region. During that very long period of time, a variety of civilizations occupied that area. The Sumerians first invented a unique method of writing sometime during the third millennium BCE, and successive civilizations like the Babylonians adopted it.

They didn’t use paper, but instead used a small stylus tool to put marks into clay tablets. The clay[2] pieces were often small enough to fit in one hand. The stick made two different wedge-shaped marks 𒌋 or 𒁹 when pressed into the clay in two different ways.[3]

The writing is sometimes called *cuneiform* (“kyoo-nee-uh-form”) which means ”wedge-shaped”.

[1] Before Common Era

[2] clay : *argile*

[3] Writing cuneiform https://youtu.be/HbZ2asfyHcA

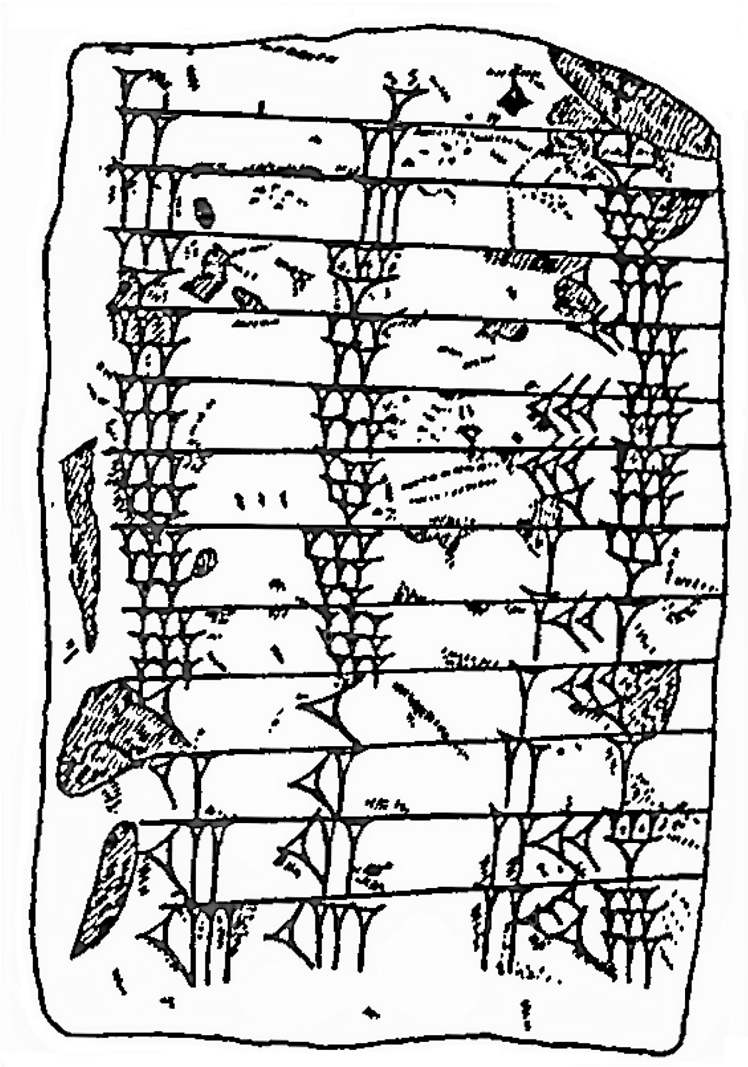
On the next page you will find a recreation of a real tablet, presented close to its actual size. It was discovered in the Sumerian city of Nippur (in modern-day Iraq), and dates to around 1500 BCE. We’re not completely sure what this is, but most scholars suspect that it is a **homework exercise**, preserved for thousands of years. When the sun hardened the clay, these tablets became permanent records. Thousands of them still exist in museums around the world.

Of course, it’s not preserved perfectly, and dealing with the parts that have become damaged is part of the challenge (and part of the fun) of working with old texts like this. If you study the picture closely, you may discover that you can figure out a lot about Babylonian numerals.

Meet a Babylonian Tablet

Study the image below an try to answer

1. How do Babylonian numerals work?[1]
2. Describe the mathematics on this tablet
3. Write the number 72 in Babylonian numerals.
4. Why can this tablet remind us of a school homework?



𒁹	𒁹	𒁹
𒈫	𒈫	𒐉
𒐈	𒐈	𒑆
𒐉	𒐉	𒌋𒐋
𒐊	𒐊	𒎙𒐊
𒐋	𒐋	𒌍𒐋
𒑂	𒑂	𒐏𒑆
𒑄	𒑄	𒁹 𒐉
𒑆	𒑆	𒁹 𒎙𒁹
𒌋	𒌋	𒁹 𒐏
𒌋𒁹	𒌋𒁹	𒈫 𒁹
𒌋𒈫	𒌋𒈫	𒈫 𒎙𒐉

Figure 2.1 – Left is image 26 on Plate 16 in Hermannn Vollrat Hilprecht, The Babylonian Expedition of the University of Pennsylvania (1906). Right is a recreated version of the numerals in the table, with no obfuscation

[1] symbols 𒁹 and 𒌋 can be written 𒁹 and 𒌋

2.2 Babylonian numerals

Almost 4000 years ago the Babylonians wrote numbers using a place-value system based on sixty. They wrote in soft clay with a stick that made two different wedge-shaped marks, for one 𒁹 and for ten 𒌋.

> To write the numbers from 1 to 59, they put together ones and tens. For instance, thirty-two was 𒌋𒌋𒌋𒁹𒁹.

1. What are these numbers?

   a) 𒌋𒌋𒁹𒁹𒁹𒁹 .... | b) 𒌋𒌋𒌋 ....

2. Write each number in Babylonian.

   a) 17 .... | b) 53 ....

> For 60 to 3599, they put a second group of these symbols to the left of the first one, separated by a space. The value of the whole thing was the value of second group multiplied by 60 and added to the value of the first group. For instance,
>
> 𒁹𒁹 𒌋𒁹𒁹 is $2 \cdot 60 + 12 = 132$

3. What are these numbers?

   a) 𒁹 𒌋𒌋𒁹 ....

   b) 𒌋𒁹 𒌋𒁹𒁹𒁹𒁹 ....

   c) 𒌋 𒌋𒁹𒁹 𒌋𒌋𒁹𒁹𒁹 ....

4. Write each number in Babylonian.

   a) 125 ....

   b) 792 ....

   c) 3,154 ....

5. In our notation, how much is 60? ....

   How much is $60^2 = 3{,}600$? ....

   How much is 60^3? ....

6. What is missing in Babylonian numerals?

Numbers from 3600 on were written by using more groups farther to the left, multiplied by 602, 603, and so on. For instance,

$$\text{𒌋𒁹 𒁹𒁹 𒌋𒌋𒁹} \quad \text{is} \quad 11 \cdot 60^2 + 2 \cdot 60 + 21 = 39{,}741$$

7. Explain how 7,883 is 𒁹𒁹 𒌋𒁹 𒌋𒌋𒁹𒁹𒁹

....

....

8. Write each number in Babylonian

   a) 50,000..

   b) 11,425..

The Babylonians' place-value system let them write large numbers easily with only two symbols. But it had one big flaw: they left a place empty for zero. There was no way to show that a place had been skipped!

For instance, 𒁹 could mean 1, or 60, or $60^2 = 3{,}600$ or something even bigger. The only way to know was to figure out what made sense for the situation

9. A clay tablet says that the total number of some things is 𒁹𒁹 𒌋𒁹𒁹𒁹, but the part that says what is being counted is broken off.

   a) If this is a shepherd counting his sheep, what number is it likely to be?

   b) If this is King Hammurabi counting his soldiers, what numbers are more likely?

2.3 Place-Value Fractions

> A small notation change will make it easier to see how the Babylonians wrote numbers less than 1. Instead of using their symbols for the numbers 1 through 59, we'll use our own, with commas to separate the groups.
>
> For example, we will write 𒌋𒁹 𒁹𒁹 𒌋𒌋𒁹, which means $11 \cdot 60^2 + 2 \cdot 60 + 21$ as $11, 2, 21$.

1. Write each of these Babylonian numerals using our symbols. Then write it as a sum of powers of 60, as in the example above.

   a) 𒁹𒁹 𒌋𒌋𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒁹 ....

   b) 𒌋𒌋𒌋 𒁹 𒌋𒁹𒁹𒁹𒁹𒁹𒁹𒁹 ....

   c) 𒁹 𒌋𒁹 𒌋𒌋𒌋𒌋𒌋 𒌋𒁹𒁹 ....

   ....

> To write fractions, the Babylonians started with 60ths. Nobody knows for sure why their system is based on 60, but some historians think it is related to their system of money: 60 shekels = 1 mina; 60 minas = 1 talent
>
> This is not very different from our own base-ten system for money:
> 10 pennies = 1 dime; 10 dimes = 1 dollar

One big advantage of using 60 is that it has a lot of factors (numbers that divide it without remainder), so many different fractions can be expressed as multiples of 1/60.

2. List all the factors of 60 ....

   ....

> Fractions were written by putting symbol groups of to the right of the ones place, just as we do with decimals. The first group was for 60ths, the next for 3600ths, etc. For example :
>
> $$1;30 = 1 + \frac{30}{60} = 1\frac{1}{2} \qquad 5;12,40 = 5 + \frac{12}{60} + \frac{40}{60^2} = 5 + \frac{1}{5} + \frac{1}{90} = 5\frac{19}{90}$$
>
> (If you think of the units as hours, then the first place to the right would be minutes and the next would be seconds.)

3. Explain $7\frac{1}{4} = 7;15$ and $5\frac{3}{8} = 5;22,30$.

   ....

   ....

   ....

4. How would we write these numbers today ?

a) $1;20$ ....

b) $2;30,30$ ....

c) $0;05$ ....

d) $0;04$ ....

e) $0;03,45$ ....

f) $0;02,30$ ....

g) $3;24,36$ ....

h) $4;1,1,1$ ....

A place-value system makes adding and subtracting fractions easy. The Babylonians just added the whole numbers place by place. If a sum was more than 60, they "exchanged" it for a 1 in the next place to the left. For example,

$$1;45+1;20=3;5$$

This is just like "carrying"10 when we add decimals in our system.

4. Check the example above by converting it to common fractions

....

....

5. Add $2;40$ and $3;50$ in the Babylonian system. Then check by converting to common fractions.

Sum : .... Check : ....

6. By converting to commun fractions, compute the reciproqal (the multiplicative inverse) of the following babylonian numbers :

   a) $1;30$ ....

   b) $2;40$ ....

2.4 Plimpton 322

Plimpton 322 is one of the world's most famous ancient mathematical artefacts (figure 2.2). It is just one of several thousand mathematical documents surviving from ancient Iraq. In its current state, it comprises a four-column, fifteen-row table of Pythagorean triples, written in cuneiform script on a clay tablet measuring about 13 by 9 by 2 cm.

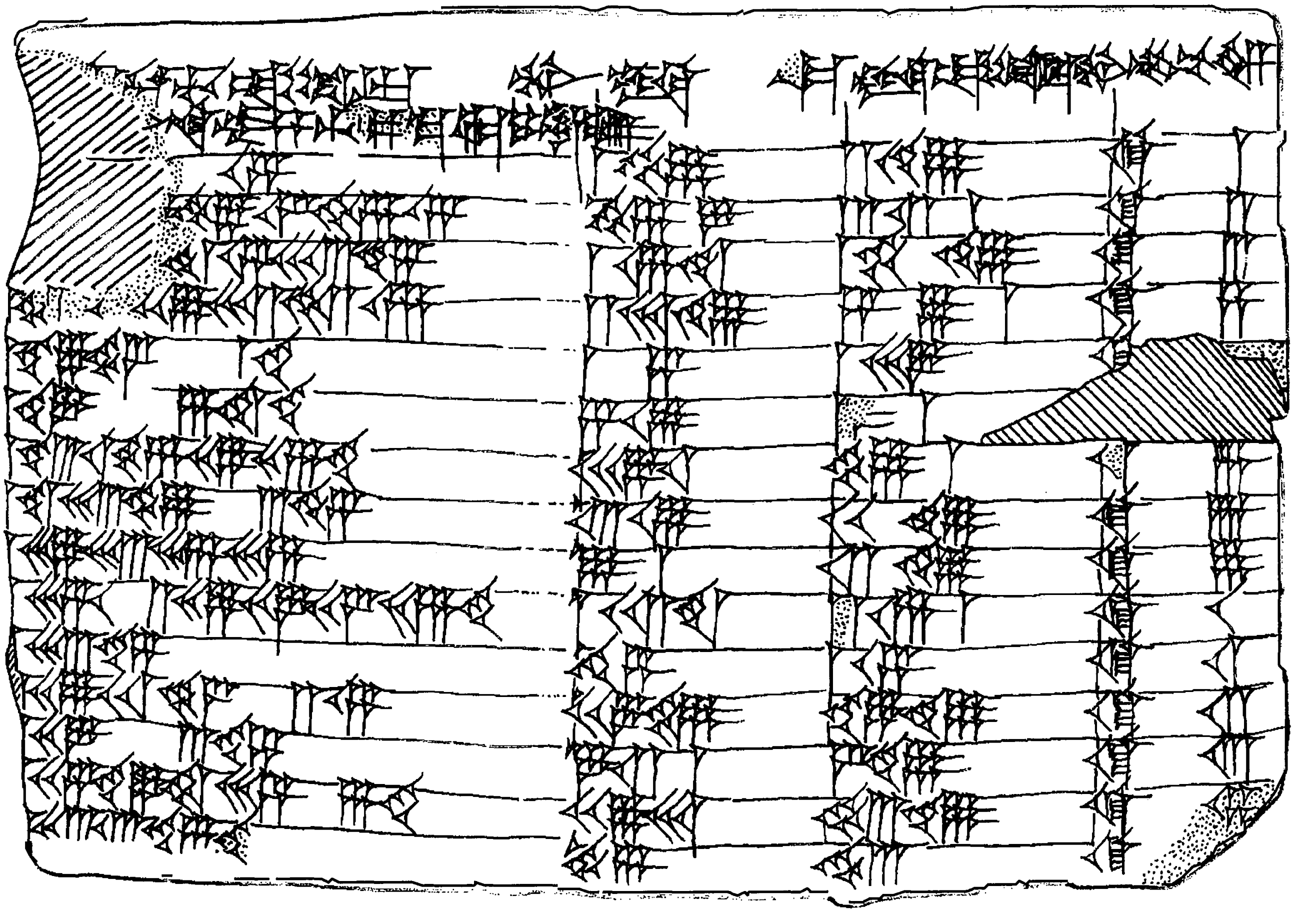


Figure 2.2 – Plimpton 322. Drawing by Eleanor Robson

Eleanor Robson analyzed the mathematical content of this tablet in its historical, archaeological and linguistical context[4]. Comparing the mathematics to other tablets around the same period gave general information about the tablet as well as clear insight to the mathematical methods used.

For instance, we are certain that our author was a male : all known female scribes from ancient Mesopotamia lived and worked much further north, in central and northern Iraq. The author must also have been someone who used literacy, arithmetic, and mathematical skills in the course of his working life (a teacher for example). He is also familiar with the format of documents used by temple and palace administrators.

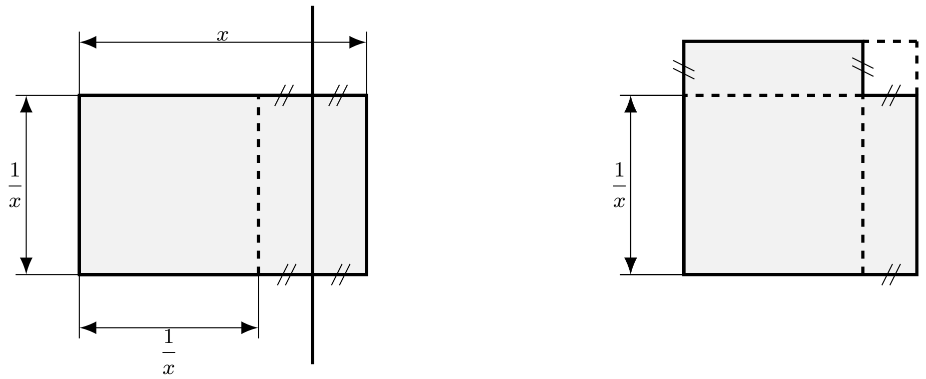
On the next page, we shall explain the mathematical method the author most likely used to fill this tablet.

1. Prove algebraically that for all non zero x :

$$1 = \left(\frac{x + 1/x}{2}\right)^2 - \left(\frac{x - 1/x}{2}\right)^2$$

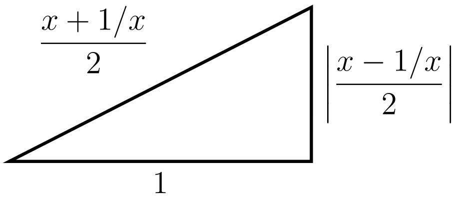
   Hint: Which might be easier: expanding or factorizing each side?

2. To fill the tablet, the author didn't use algebra, but most likelly used a known algorithm of **completing the square**. The algorithm is illustrated in the visual below. Explain why the visual can show the equation in question 1?



   Hint: What is the area of the shaded region?

3. Explain why for any positive value x, the triangle below is a right angle triangle with $\dfrac{x+1/x}{2}$ the length of the hypotenuse.



Our understanding of Plimpton 322 is :

1. There are 2 missing columns on the left. They should feature reciprocal pairs x and $\dfrac{1}{x}$ each four places long or shorter. The values chosen are among pairs frequently used in other Babylonian tablets around that time.
2. Column I shows $\left(\frac{x+1/x}{2}\right)^2$
3. Columns II and III show length s and d of a scaled up triangle similar to the triangle in question 3.

$$d\colon l\colon s = \tfrac{x+1/x}{2} : 1 : \tfrac{x-1/x}{2}$$

4. Last column is the line number.

Missing columns		**Column I**	**Column II**	**Column III**	**Column IV**
x	$\frac{1}{x}$	$\left(\frac{x+1/x}{2}\right)^2$	s	d	
2; 24	0; 25	1; 59, 00, 15	1; 59	2; 49	line 1
....					
2;	0; 30	1; 33, 45	0; 45	1; 15	line 11
....					

Table 2.2 – Partial explanation of Plimpton 322. All values are given in Babylonian numeration.

4. Using table 2.2, check the algorithm for lines 11 then 1. Find the scale factor.

2.5 The Cyrus cylinder

Oral comprehension https://youtu.be/nRMzrzu0wRw?t=56 (from 0'56" to 5'00")

Our story begins ______________, with Cyrus the Great, ______________________________________. After uniting the Persian and Median tribes, Cyrus came into conflict with King Croesus of Lydia in what's now modern-day Turkey. He defeated Croesus in battle. Cyrus then clashed with the powerful Babylonian Empire to the west. In 539 BC, he besieged their capital, Babylon. It fell with barely a fight. Persia under Cyrus became a superpower of the ancient world.

Cyrus' conquest of Babylon had been predicted and was ____________________________________ by the Jewish prophet Daniel :

> That very night, the Babylonian regent, Belshazzar had held a feast. He feasted using the sacred gold vessels from the temple of Jerusalem. And then suddenly, miraculously, there was a hand writing on the wall, and the writing said : you have been weighed in the balance, you have been found wanting, and your kingdom will be given to the Medes and the Persians.

So from this we have the expression ________________________________. And we have Rembrand's great painting "Belshazzar's Feast". Within hours, the great empire of Babylon was no more.

Cyrus justified his invasion to the Babylonian people in a ______________ written on a clay tablet such as this one, which are likely to have widely distributed. Surviving fragments, along with the Cyrus Cylinder, carry identical wording.

Because the Cyrus Cylinder was meant to a Babylonian audience which is a Semitic tongue related to the modern languages of Hebrew and Arabic and Aramaic. The writing system which Cyrus' officials used was the traditional __________________ which had been invented in ancient Iraq well before 3,000 B.C. It is written by pressing a stylus, something a bit like a chopstick, into the surface of the clay which is nearly dry. And the signs which convey the sound of the language consist of different arrangement of these strokes. They are written one by one, and the reader has to join them up and the sound emerges from the clay. This is the line that says "I am Kurash, King of the world, the Great King, King of Babylon" and so it goes on.

[...Irving Finkel describes how to write cuneiform script...]

The early lines on the cylinder extol[2] Cyrus' virtues and his reasons for invading Babylon. But it's line 30 to 35 that set Cyrus up as a great leader in the modern liberal sense.

One of Cyrus' first acts after his conquest was to ____ by the Babylonians. This was a mark of his style of rule. And it's this very event that's recorded on the cylinder. So this was a key moment in the history of the Jewish people. They had spent many years weeping by the rivers of Babylon and now finally they were allowed to return to Jerusalem and then to rebuild the temple.

Questions

1) What was the main purpose of the Cyrus cylinders?

2) Explain the meaning of the phrase "The writing is on the wall".

Listen to some examples of how you can use this expression[5].

a) My auntie is really sad because she had to close her shop. To be honest, the writing has been on the wall for her business for a long time.

b) You know Mary. She's a party girl that loves the city but I heard

. . . .

The writing is definitely on the wall for their marriage.

c) .

. . . .

My teachers told me if I don't try harder the writing is on the wall.

[2] to extol, to praise

2.5.1 Partial script of the video

https://youtu.be/nRMzrzu0wRw?t=56 (from 0'56" to 5'00").

Our story begins 550 years BC, with Cyrus the Great, founder of the Persian Empire. After uniting the Persian and Median tribes, Cyrus came into conflict with King Croesus of Lydia in what's now modern-day Turkey. He defeated Croesus in battle. Cyrus then clashed with the powerful Babylonian Empire to the west. In 539 BC, he besieged their capital, Babylon. It fell with barely a fight. Persia under Cyrus became a superpower of the ancient world.

Cyrus' conquest of Babylon had been predicted and was described in the Old Testament by the Jewish prophet Daniel. That very night, the Babylonian regent, Belshazzar had held a feast. He feasted using the sacred gold vessels from the temple of Jerusalem. And then suddenly, miraculously, there was a hand writing on the wall, and the writing said, you have been weighed in the balance, you have been found wanting, and your kingdom will be given to the Medes and the Persians. So from this we have the expression "the writing is on the wall". And we have Rembrand's great painting "Belshazzar's Feast".

Within hours, the great empire of Babylon was no more. Cyrus justified his invasion to the Babylonian people in a proclamation written on a clay tablet such as this one, which are likely to have widely distributed. Surviving fragments, along with the Cyrus Cylinder, carry identical wording.

Because the Cyrus Cylinder was meant to a Babylonian audience which is a Semitic tongue related to the modern languages of Hebrew and Arabic and Aramaic. The writing system which Cyrus' officials used was the traditional cuneiform script which had been invented in ancient Iraq well before 3,000 B.C. It is written by pressing a stylus, something a bit like a chopstick, into the surface of the clay which is nearly dry. And the signs which convey the sound of the language consist of different arrangement of these strokes. They are written one by one, and the reader has to join them up and the sound emerges from the clay.

This is the line that says "I am Kurash, King of the world, the Great King, King of Babylon" and so it goes on.

[Irving Finkel describes how to write cuneiform script]

The early lines on the cylinder extol Cyrus' virtues and his reasons for invading Babylon. But it's line 30 to 35 that set Cyris up as a great leader in the modern liberal sense.

One of Cyrus' first acts after his conquest was to release those who had been made captive by the Babylonians. This was a mark of his style of rule. And it's this very event that's recorded on the cylinder. So this was a key moment in the history of the Jewish people. The had spent many years weeping by the rivers of Babylon and now finally they were allowed to return to Jerusalem and then to rebuild the temple.

> **Cultural Diplomacy**
>
> In 2012, British Museum lent the Cyrus cylinder to Iran. The following year, it went on a tour in US cities with one of the 2 existing copies of *Cyropaedia*, Xenophon's book on Cyrus (Greek

military leader and historian). In it, Xenophon describes how Cyrus ruled a diverse society based on tolerance.

The book became popular during the Enlightenment among political thinkers in Europe and America, including those who drafted the US Constitution in 1787. Thomas Jefferson, the third president of the US studied the book in detail.

References

[1] William P. Berlinghoff and Fernando Q. Gouvêa. *Pathways from the Past*. Colby College: Oxton House Publishers, 2010. URL: https://bit.ly/3wsj92r (see p. 1).

[2] Dominic Klyve. "Babylonian Numeration". In: *Number Theory* 2 (2017). URL: https://bit.ly/3kbS0fk (see p. 1).

[3] BBC News. *Cyrus Cylinder: How a Persian monarch inspired Jefferson*. 2013. URL: https://www.bbc.com/news/world-us-canada-21747567 (see p. 1).

[4] Eleanor Robson. "Words and Pictures: New Light on Plimpton 322". In: *The American Mathematical Monthly* 109.2 (2002), pages 105–120. URL: http://www.jstor.org/stable/2695324 (see pp. 1, 8).

[5] The English We Speak. *The writing is on the wall*. 2013. URL: https://www.bbc.co.uk/worldservice/learningenglish/language/theenglishwespeak/2013/02/130226_tews_112_the_writing_is_on_the_wall.shtml (see p. 12).