1.4 Mathemagic on Britain's Got Talent?

Got Talent[a] is a televised british talent show competition. The *Got Talent* franchise spawned[b] *spin-offs* in over 69 countries, including France's *La France a un incroyable talent.*

The British original show aired 14 seasons since 2007, averages 10 million viewers and has a huge following on youtube. Fans enjoy a wide variety of performances as well as 4 charismatic judges: comedian David Walliams, singer Alesha Dixon, actress Amanda Holden, and Simon Cowell, the show's creator.

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[a]original idea by mathsamoi.com

[b]*to spawn* : to produce or deposit large number of eggs.

On 30th May 2020, the jury was *wonderstruck* by a mathemagic trick performed by Damien O'Brien. Let's watch his performance (5' YouTube clip youtu.be/qt-tjFFHTfg). Can you find out how the mathemagic trick works?

1) Write down the number picked by Simon, Amanda and Alesha and express each as a product of prime factors.

Simon chose _____ because ....

Amanda's number is _____. It represents ....

Alesha picked _____. ....

A prime number is a natural number which has exactly two different factors.

The *fundamental theorem of arithmetic* states that every composite number can be written as the product of prime factors in exactly one way (ignoring order).

2) Write down the prime factorization of the product of the three chosen numbers.

3) At the 2'25" mark, video shows the result of David's multiplication. The number is 157,612,840.

   a) Use your calculator to write down the prime factorization of this number.

   b) David mistyped at least one of the three numbers. Can you guess which one? Explain your answer.

   c) Find the number he typed.

4) After dividing by his pin number, David gets 71,059 which we will see is a prime number. Explain how we know the calculator app used is part of the trickery. Explain some of the diversion tactics used by the magician.

5) We want to use a short python code to check that 71,059 is a prime number. The following code should display all factors of a natural number **n** but instead, runs into a `ZeroDivisionError`. Can you correct it and show all factors of 12? [1]

```python
n = 12        # once corrected, use 71059
for i in range(n) :
    if n % i == 0 :
        print(i)
```

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[1]in python, **range(n)** returns a sequence of numbers starting from 0 by default and increments by 1 (by default), and stops before **n**

solution.

1) $1402 = 2 \times 701$ is the birthday of Simon's son.

   $1012 = 2^2 \times 11 \times 23$ is anniversary of Amanda's wedding.

   $110 = 2 \times 5 \times 11$ is the birthday of Alesha's daughter.

2) The product of $156070640 = 1402 \times 1012 \times 110 = 2^4 \times 5 \times 11^2 \times 23 \times 701$.

3) $157612840 = 2^3 \times 5 \times 7 \times 11 \times 73 \times 701$. David mistyped Amanda's number because there is no 23 among prime factors.

   He might have typed the number $1022 = 157612840 \div 1402 \div 110$.

4) 157612840 divided by any integer cannot yield 71059 as 71059 isn't a factor of 157612840!

   The app was designed to show 71059, picked by the magician ahead of the act.

   By asking David to enter his pin number, the magician was sure that the camera wouldn't show the number used. The public believed the computations were legit. Unfortunately, the final and expected number was a prime.

5)
```python
n = 71059        # once corrected, use 71059
for i in range(2, n) :
    if n % i == 0 :
        print(i)
```

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