

2.3 Place-Value Fractions

A small notation change will make it easier to see how the Babylonians wrote numbers less than 1. Instead of using their symbols for the numbers 1 through 59, we'll use our own, with commas to separate the groups.

For example, we will write $\llcorner \lrcorner \lrcorner \llcorner \lrcorner$, which means $11 \cdot 60^2 + 2 \cdot 60 + 21$ as 11, 2, 21.

- Write each of these Babylonian numerals using our symbols. Then write it as a sum of powers of 60, as in the example above.

a) $\lrcorner \lrcorner \llcorner \llcorner \lrcorner \lrcorner \lrcorner \llcorner \llcorner \lrcorner$

b) $\llcorner \llcorner \lrcorner \lrcorner \llcorner \lrcorner \lrcorner \lrcorner \lrcorner$

c) $\lrcorner \llcorner \llcorner \llcorner \llcorner \llcorner \lrcorner \lrcorner$

To write fractions, the Babylonians started with 60ths. Nobody knows for sure why their system is based on 60, but some historians think it is related to their system of money: 60 shekels = 1 mina; 60 minas = 1 talent

This is not very different from our own base-ten system for money:

10 pennies = 1 dime; 10 dimes = 1 dollar

One big advantage of using 60 is that it has a lot of factors (numbers that divide it without remainder), so many different fractions can be expressed as multiples of $1/60$.

- List all the factors of 60

Fractions were written by putting symbol groups of to the right of the ones place, just as we do with decimals. The first group was for 60ths, the next for 3600ths, etc. For example :

$$1; 30 = 1 + \frac{30}{60} = 1\frac{1}{2} \quad 5; 12, 40 = 5 + \frac{12}{60} + \frac{40}{60^2} = 5 + \frac{1}{5} + \frac{1}{90} = 5\frac{19}{90}$$

(If you think of the units as hours, then the first place to the right would be minutes and the next would be seconds.)

- Explain $7\frac{1}{4} = 7; 15$ and $5\frac{3}{8} = 5; 22, 30$.

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- How would we write these numbers today ?

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|--------------------|---------------------|
| a) 1; 20 | e) 0; 03, 45 |
| b) 2; 30, 30 | f) 0; 02, 30 |
| c) 0; 05 | g) 3; 24, 36 |
| d) 0; 04 | h) 4; 1, 1, 1 |

A place-value system makes adding and subtracting fractions easy. The Babylonians just added the whole numbers place by place. If a sum was more than 60, they “exchanged” it for a 1 in the next place to the left. For example,

$$1; 45 + 1; 20 = 3; 5$$

This is just like “carrying” 10 when we add decimals in our system.

4. Check the example above by converting it to common fractions

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5. Add 2; 40 and 3; 50 in the Babylonian system. Then check by converting to common fractions.

Sum : Check :

6. By converting to common fractions, compute the reciprocal (the multiplicative inverse) of the following babylonian numbers :

- a) 1; 30
 b) 2; 40