Homework 6

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1 Modular Exponentiation

1. $2019 = 2(1009) + 1$
   $1009 = 2(504) + 1$
   $504 = 2(252) + 0$
   $252 = 2(126) + 0$
   $126 = 2(62) + 0$
   $63 = 2(31) + 1$
   $31 = 2(15) + 1$
   $15 = 2(7) + 1$
   $7 = 2(3) + 1$
   $3 = 2(1) + 1$
   $1 = 2(0) + 1$
   Therefore, the binary expansion of 2019 = 11111100011

2. Using the above binary expansion for 2019.
   $(13^{2^{10}+2^9+2^8+2^7+2^6+2^5+2^1+2^0})\ mod\ 37$
   $= (13^{2^{10}} * 13^{2^9} * 13^{2^8} * 13^{2^7} * 13^{2^6} * 13^{2^5} * 13^{2^1} * 13^{2^0})\ mod\ 37$
   $= (13^{2^{10}} mod\ 37) * (13^{2^9} mod\ 37) * (13^{2^8} mod\ 37) * (13^{2^7} mod\ 37) * (13^{2^6} mod\ 37) * (13^{2^5} mod\ 37) *$
   $(13^{2^1} mod\ 37) * (13^{2^0} mod\ 37)$
   $13^{2^0} mod\ 37 = 13$
   $13^{2^1} mod\ 37 = (13^{2^0})^2\ mod\ 37 = 169\ mod\ 37 = 21$
   $13^{2^2} mod\ 37 = (13^{2^1})^2\ mod\ 37 = 441\ mod\ 37 = 34$
   $13^{2^3} mod\ 37 = (13^{2^2})^2\ mod\ 37 = 1156\ mod\ 37 = 9$
   $13^{2^4} mod\ 37 = (13^{2^3})^2\ mod\ 37 = 81\ mod\ 37 = 7$
   $13^{2^5} mod\ 37 = (13^{2^4})^2\ mod\ 37 = 49\ mod\ 37 = 12$
   $13^{2^6} mod\ 37 = (13^{2^5})^2\ mod\ 37 = 144\ mod\ 37 = 33$
   $13^{2^7} mod\ 37 = (13^{2^6})^2\ mod\ 37 = 1089\ mod\ 37 = 16$
   $13^{2^8} mod\ 37 = (13^{2^7})^2\ mod\ 37 = 256\ mod\ 37 = 34$
   $13^{2^9} mod\ 37 = (13^{2^8})^2\ mod\ 37 = 1156\ mod\ 37 = 9$
   $13^{2^{10}} mod\ 37 = (13^{2^9})^2\ mod\ 37 = 81\ mod\ 37 = 7$
   Therefore:
   $(13^{2019})\ mod\ 37 = 13 * 21 * 12 * 33 * 16 * 34 * 9 * 7 = 370577376$

2 Greatest Common Divisor

1. gcd(288, 126)
   The factors of $288 = 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, 36, 48, 72, 96, 144, 288$
   The factors of $126 = 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126$
   Therefore the greatest common divisor is: 18

2. gcd(899, 703)
   The factors of $899 = 1, 29, 31, 899$
   The factors of $703 = 1, 19, 37, 703$
   Therefore the greatest common divisor is: 1