

Homework 2

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1. P is a Knight and Q is a Knave.

There are four potential cases:

- (a) $P = \text{knight}$ and $Q = \text{knight}$
- (b) $P = \text{knave}$ and $Q = \text{knight}$
- (c) $P = \text{knight}$ and $Q = \text{knave}$
- (d) $P = \text{knave}$ and $Q = \text{knave}$

Case 1 is impossible because if P is a knight then Q must be a knave.

In cases 2 and 4, if P is a knave, then what he says is untrue, therefore they are impossible. This leaves only case 3, which is the only possible combination because if P is a knight then Q must be a knave since knights do not lie.

2. A is Knave and B is a Knight.

There are four potential cases:

- (a) $A = \text{knight}$ and $B = \text{knight}$
- (b) $A = \text{knave}$ and $B = \text{knight}$
- (c) $A = \text{knight}$ and $B = \text{knave}$
- (d) $A = \text{knave}$ and $B = \text{knave}$

Case 1 and 3 are impossible because if A is a knight, then whatever he says is true.

Case 4 is impossible because if B is a knave, he always lies.

Therefore, case 2 is the only possible combination. If A is knave then B must be a knight.

1 Logical Identities

1. $\neg(p \rightarrow (q \rightarrow p))$
 $= \neg p \rightarrow \neg(q \rightarrow p)$
 $= \neg p \rightarrow (\neg q \rightarrow \neg p)$

2. $\neg((p \wedge q) \rightarrow (q \vee p))$
 $= \neg(p \wedge q) \rightarrow \neg(q \vee p)$
 $= (\neg p \vee \neg q) \rightarrow (\neg q \wedge \neg p)$

2 Logical Equivalences

1.

p	q	r	$p \rightarrow (q \rightarrow r)$	$(p \wedge q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

The propositional statements are equivalent because the final column of the truth table receives a value of one for all cases, meaning $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$ is always true. Therefore, the pair is equivalent.

2.

p	q	r	$p \rightarrow (q \rightarrow r)$	$(p \rightarrow q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

The propositional statements are not equivalent because the final column of the truth table, $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$, does not receive a value of one for all cases.

3 Logical Consequence

1. This is a valid argument because we have no way of knowing if the conclusion is false or not.
2. This is a valid argument since Puerto Rico is surrounded by water and since all islands are surrounded by water, then Puerto Rico must be an island.

4 Collaboration

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