# ENGI 9872

# Digital Communication

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# Project Report


Group 2:

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## Abstract

The digital communication important concepts are addressed in this project, such as power spectral density, bit error rate, diversity, and phase estimation of the wireless communication system. This report provides detailed information about the transmission process. The transmission process is based on the Bit Source, Transmitter, Channel, Receiver, and Bit Sink. Quadrature Phase Shift Keying (QPSK) technique is used as the baseband modulation. The root-raised cosine filter is used at the transmitter side for pulse shaping. Two channels are investigated as Rayleigh fading and Additive White Gaussian Noise (AWGN). Phase estimation and antenna diversity are being used to mitigate fading effects and noise. All fours problems given in the project description are addressed that are simulated using MATLAB software. To address the first problem, plotted the transmitted signal Power Spectral Density (PSD) for different roll-off factor values of root raised cosine filter, and comments on results are also available. The second problem provides the signal constellations study at the receiver side for the different SNR values in the AWGN channel. The observation after adding noise and effects of error while estimating the phase is also completed. After that, problem three includes plots of BER as a function of SNR by using only one antenna at the receiver side for both AWGN and Rayleigh fading channels.

Furthermore, BER vs. SNR was calculated for multiple antennas at a receiver such as L=2,3,4 and compared the theoretical results with the simulation results. The last problem employed the blind phase estimation, and it is investigated for all receivers with multiple antennas of L=1,2,3,4. Finally, a designed system conclusion is provided.

# 1. Introduction

Currently, digital communication systems are widely used, and they provide data transfer services using different types of transmission channels. The transmitter, channel, and receiver are the main parts of any communication system. Each part has its functions that are described below. The digital transmission system block diagram is shown in figure (1).

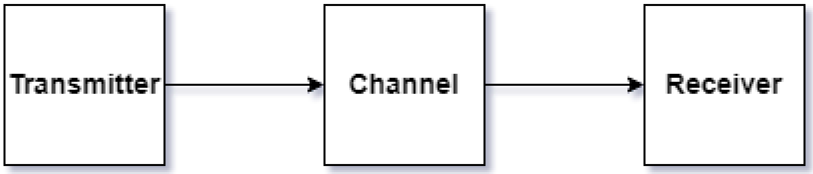


*Figure 1 Digital Communication System Block Diagram*

## 1.1 Transmitter

To covert the electrical signal into binary form, the transmitter is used. It changes the electrical signal into an appropriate format to transmit it over any transmitting media. In general, the transmitter uses a range of frequencies for transmission, and some functions are performed on the transmitted signal by it to match the range of frequency. A process named ‘modulation’ is used for matching purposes. The most commonly used modulation techniques are phase, frequency, and amplitude modulation [1].

## 1.2 Channel

The physical media used for signal transmission is called a channel. The Wireless communication system has no physical media, but free space is used for transmitting the signal. However, the wired communication system uses physical media such as coaxial cables, twisted paired cables, and optical fibers. Due to these types of physical media, signal degradation occurs in channels by interference through the noise. The most common noise is additive noise, also called thermal noise, and some other types of noise can be atmospheric noise and artificial noise that can cause degradation in signal. While designing the digital communication system, to characterize the interference by noise, the designers use some statistical models and mathematical models. However, there is a need to adopt a robust approach for designing channels to cope with any interference. This report includes the Flat block Rayleigh fading and Additive White Gaussian Noise (AWGN), and both channels examine signal distortion.

### 1.3 Receiver

The receiver in the digital communication system is used to extract the transmitted information. The receiver uses demodulation and detection to receive the message. The receiver uses noise suppression and signal filtering functions to remove unwanted components in the signal.

# 2. Background Theory

## 2.1 Root Raised Cosine Filter

There are two ways: pulse shaping and line coding to control the power spectral density in a digital communication system. As the bandlimited channel raises the inter-symbol interference (ISI). To eliminate ISI, Nyquist proposed two methods, such as root-raised cosine spectrum and Nyquist criteria for zero ISI.

A rectangular shape is formed by information such as 0's and 1's generated by the QPSK modulation. The pulse-shaping filter is used to reduce the ISI. The channel will be from the ISI if the Nyquist pulse is the same as the ideal low pass with the bandwidth Wc and it has the maximum rate T equal to 1 / 2Wc where side lobes decay at 1/t. The root-raised cosine filter minimizes the ISI and has the side lobes decay at 1/ t^3. The root-raised cosine filter requires excess bandwidth when compared with the Wc. It has a bandwidth equal to (1 + α) Wc. Where α is the role of factor has the range of 0 ≤ α ≥ 1. The impulse response of root raised cosine filter is given [2].

$$h(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \frac{cos\,\frac{\pi \alpha t}{T}}{1 - \left(\frac{2\alpha t}{T}\right)^2}$$

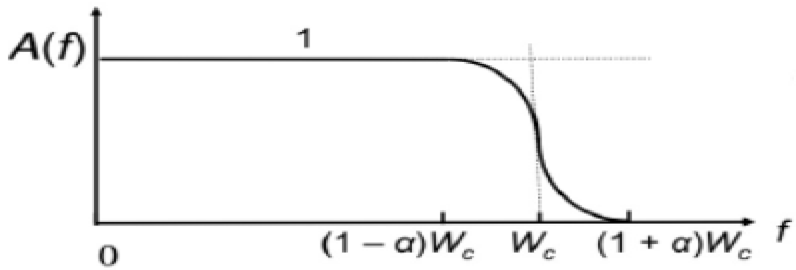


Figure 2 *Root raised cosine filter impulse response*

The value of α can be increased to reduce the ISI. After demodulation, this filter can also be used at the transmitter side to minimize the degradation added by the fading and noisy channel. A root-raised cosine filter also maximizes the SNR for better system performance [3].

## 2.2 Rayleigh Fading

The small-scale fading effects transmitted signal amplitude and phase. This type of fading is called Rayleigh fading when the multiple reflective paths are more significant in number. It has no line of sight (LOS) component between the source and destination in the propagation environment. The channel with no line of sight (NLOS) is used for multipath fading, and for this model of Rayleigh, fading is used. The received signal experiences high variations due to this type of fading. Some propagation paths occur due to scattering, reflections, and diffraction. The PDF of Rayleigh fading is given below.

$$f_X(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right), \ x \geq 0$$

If there is a dominant signal path due to LOS, then the Rician model envelopes the non-zero mean complex values Gaussian process. This type of channel is called the Rician fading channel [4].

## 2.3 Diversity

The diversity provides multiple independent copies of the transmitted signal to a receiver, improving communication system performance. The diversity order tells how many signal copies are available, and the BER curve vs. SNR can show it. Three ways can be used: space, frequency, and time to implement diversity. The signal is transmitted in different slots of time in the time diversity. And, for the frequency diversity, the transmitted signal has multiple carriers. The multiple antennas are used at the receiver side in the spatial diversity, and the distance between two antennas is ten wavelengths. The gain of diversity can be defined as improving signal power to achieve the required BER. The antenna gain provides information about the improvement in the strength of the signal when multiple antennas are used. The antenna gain can be expressed as SNR.

## 2.4 MRC and Bit Error Rate

The maximal ratio combiner (MRC) improves the SNR and combines the output received at multiple antennas. It combines the signals from multiple antennas, and the signal received at each independent antenna depends on the channel's strength and phase. The MRC multiplies the signal with a weight factor proportional to the signal's amplitude. The MRC is also helpful to improve the system's performance in terms of BER vs. SNR for the Rayleigh fading and AWGN channel by using multiple antennas at the receiver side. The equation of the received signal is given if the N number of antennas are used.

$$y_i = h_i x + ni$$

Where y represents the received signal, x represents the transmitted signal, n represents noise, and h represents a channel. The energy to noise ratio for a bit at $i^{th}$ receive antenna is given.

$$\gamma_i = \frac{|h_i|^2 Eb}{N_o}$$

After equalizing the channel h with a number of antennas L = 1, 2, 3, 4 receiving antennas, the bit energy to noise ratio is given [5].

$$\gamma = \sum_{i=1}^{L} \frac{|h_i|^2 Eb}{N_o} = L_{\gamma_i}$$

The above equation shows that the bit energy to noise ratio for L receiving antennas is equal to the L times bit energy to noise ratio in the signal antenna. $h_i$ is given from the chi-squared distribution [3].

$$\rho(\gamma_i) = \frac{1}{\frac{E_b}{N_o}} \exp\left(\frac{-\gamma_i}{\frac{E_b}{N_o}}\right)$$

The L random variables sum provides the bit energy to noise ratio. The $\gamma_i$ PDF is a chi-squared random variable with 2N degrees of freedom that is given.

$$\rho(\gamma_i) = \frac{1}{(L-1)!\left(\frac{E_b}{N_o}\right)^L} \gamma^{L-1} \exp\left(\frac{\gamma_i}{\frac{E_b}{N_o}}\right), \qquad \gamma \geq 0$$

The BER for MRC is given.

$$BER_{MRC} = K^L \sum_{i=0}^{L-1} \binom{L-1+i}{i} (1 - \mathrm{K})$$

Where

$$K = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{\frac{E_b}{N_o}}\right)^{-\frac{1}{2}}$$

The bit error rate for QPSK in AWGN is given with the bit energy to noise ratio.

$$P_e = Q(\sqrt{\frac{2E_b}{N_o}})$$

### 2.5 Phase Estimation

The phase estimation is investigated in this section for Rayleigh fading and AWGN channels. The QPSK modulated is represented by x(k), where 0 ≤ k ≤ $L_o - 1$ and $L_o$ is the observed length for the interval period of symbols. The received signal faced phase error due to Rayleigh fading and AWGN channel and generated phase is represented by $\theta_k$ where $-\frac{\pi}{4} \leq \theta_k \leq \frac{\pi}{4}$. According to this, frequency and timing are considered to be accurate. The fourth power equation used for estimating the phase is given [7].

$$\theta = \frac{1}{4} angle\ (E\{x^4(k)\})$$

## 3. Problem Formulation

The digital communication system given in this project is based on the Bit source, transmitter, channel, receiver, and bit sink. There is a need to investigate all parts of the given communication system. The whole digital communication system is shown in figure (2). Firstly, a random signal with the sequence number generates a bit source. Then using the bit source, the generated number of the sequence is transferred to the transmitter. The QPSK modulator is used at the transmitter side, which converts the sequence number into symbols. After the QPSK modulator, the Root Raised Cosine filter is used with the roll of a factor of 0.35 for pulse shaping. The resulted output from the transmitter will be transferred to the channel. This project investigated the two types of channels, such as Additive White Gaussian Noise (AWGN) and Rayleigh Fading. The Additive White Gaussian Noise (AWGN) is also considered a Gaussian noise process. For modeling the AWGN, the complex Gaussian random variable is added to the channel. There is a need to add the noise vector into a signal to get a specific SNR value in dB. But to make the Rayleigh Fading channel, the signal is multiplied by the Gaussian random variable. The output generated by the channel is transferred now to the transmitter. The transmitter has single and multiple antennas to receive the signal transmitted by the transmitter. The Maximum Ratio Combiner (MRC), also called the diversity combiner method, is used to combine the received signal at multiple antennas. The Root Raised Cosine filter is used at the receiver side to maximize the Signal to Noise ratio (SNR). The transmitted pulse shape is matched with the receiver filter. For the system operation, synchronization is crucial. That's why it is assumed that this project has perfect synchronization.

The channel with Rayleigh fading and AWGN reduces the communication system's performance and introduces the phase error. The antenna diversity is helpful to minimize system performance-related issues by multiple antennas at the receiver and to combine the received output from all antennas; the MRC is used. After that, the blind phase estimation is used to reduce the phase error. For making a decision, demodulation and detection are used. The performance evaluation of communication is done by calculating the BER vs. SNR in the decision process.

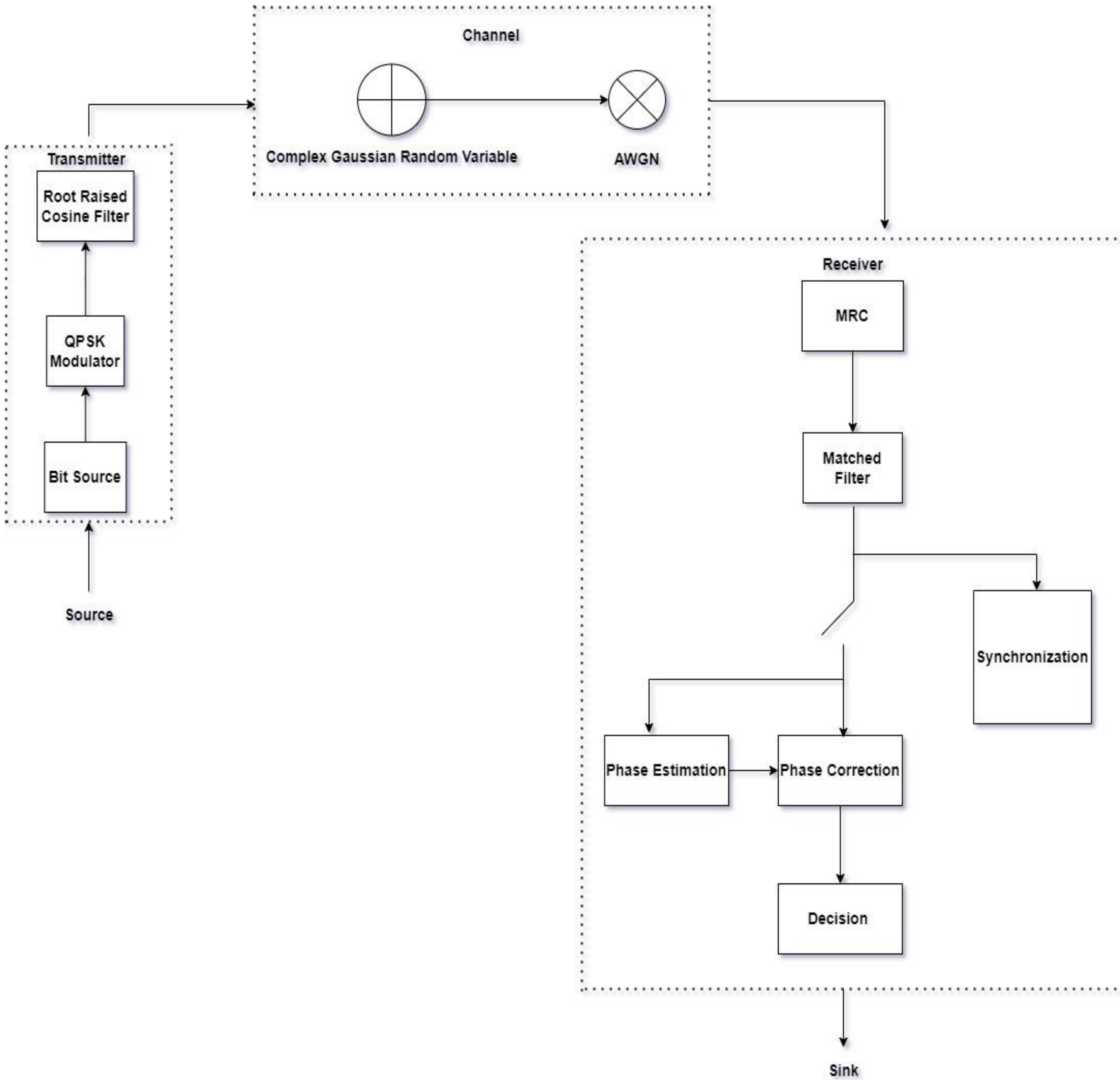


*Figure 3 Digital Communication System*

## 4. Methodology

In this project, we have assumed perfect synchronization. MATLAB software is used to simulate the entire given communication system. The most common technique Quadrature Phase Shift Keying (QPSK), is used for modulation and demodulation. This report includes a detailed discussion about all components of a given communication system, such as root-raised cosine filter, QPSK modulation, transmission channels, Rayleigh fading, phase estimation, and antenna diversity. The MATLAB function "pwelch" is used to plot the power spectral density of modulated signal for the different values of a roll of a factor of root raised cosine filter. The signal constellation points are studied in the AWGN channel for the different SNR values at the receiver side. The bit error rate (BER) as a function of SNR is investigated, and theoretical results are compared with the simulation results. Finally, phase estimation is done for the observation of different length intervals. All simulation results with the comments are given below.

## 5. Results and Comments

This section includes the simulation results for a given digital communication system and the problems mentioned in the report. The baseband QPSK modulation is adopted for this project, and a root-raised cosine filter with the roll of factor-alpha = 0.35 is used. Two channels are used for communication, such as Rayleigh fading and AWGN. For comparison between both channels, BER is calculated as a function of SNR. The receiver can have only one antenna or multiple antennas. The whole designed system is studied with phase estimation, phase offset, and correction.

### 5.1 Problem I

This problem provides the study of the power spectral density of transmitted signals for a different roll of factor values. Figure (4) shows the transmitted signal power spectral density for a roll of a factor of alpha = 0.35. Figure (5) illustrates the PSD of the transmitted signal for different values of a roll of factor. By using a roll of factor-alpha, it is easy to calculate the bandwidth of the signal. Figure (5) shows that when a roll of factor increases, then the bandwidth of the filter also increases. The roll-off factor varies between 0 and 1, and it has an effect on the impulse response. When the roll of factor decreases, then the ripple level in the time domain increases, and due to this filter, excess bandwidth is also reduced.

By comparing raised cosine filter response to our results, we can see that the PSD of the QPSK signal after raised cosine filter agrees with the filter response. Concretely, as depicts in figure (5) the smaller the roll-off factor indicates the narrower PSD.

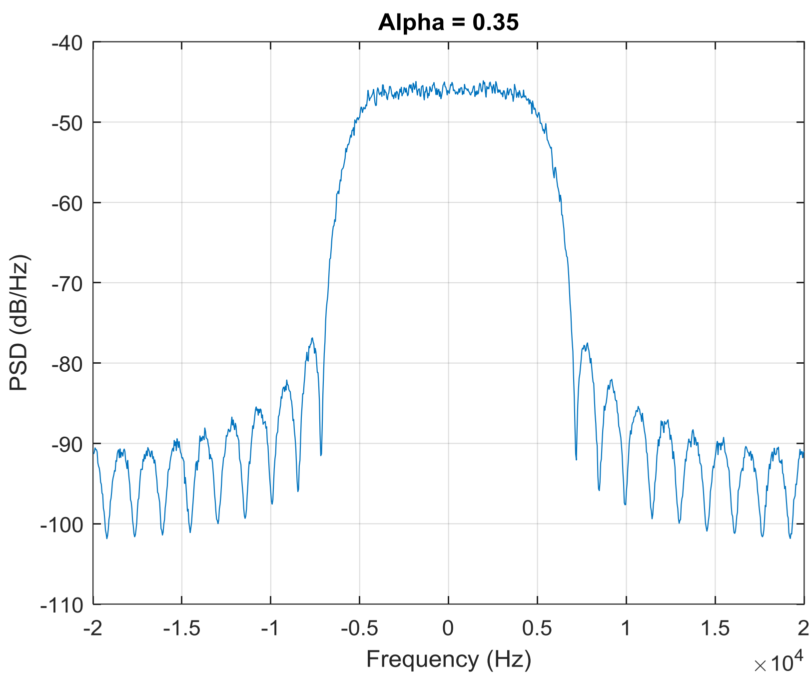


*Figure 4 PSD of Transmitted Signal for a roll of factor 0.35*

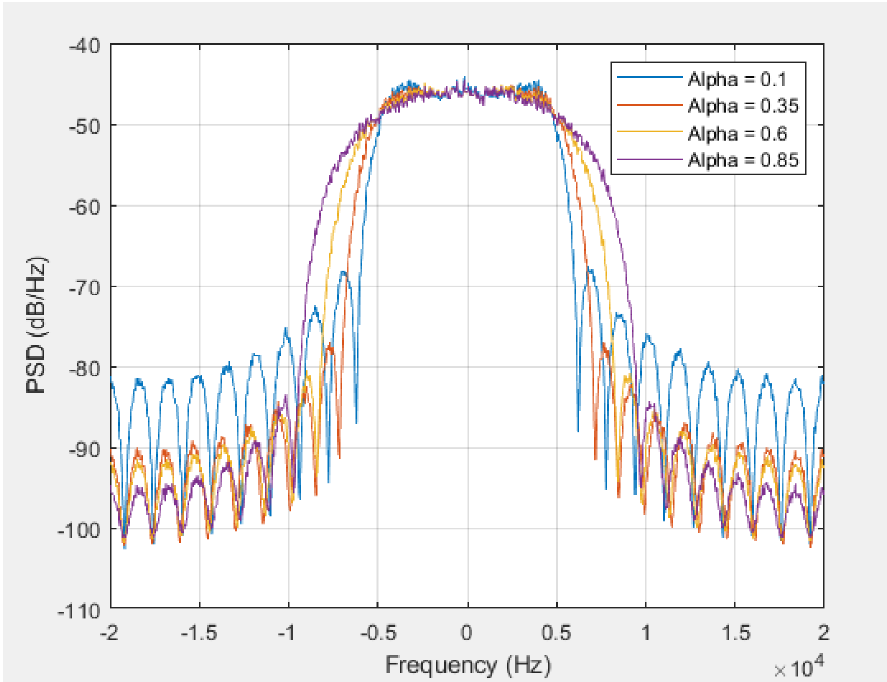


*Figure 5 PSD of Transmitted Signal for different values of a roll of factor*

## 5.2 Problem II

A constellation diagram is used to represent the signal modulated by any time of modulation technique. This project uses a modulation type of Quadrature Phase Shift Keying (QPSK). The constellation diagram displays the signal in a two-dimensional XY plane scatters diagram in the complex plane at symbol sampling instants. Figure (6) shows the signal points modulation by QPSK with AWGN channel for different values of SNR. It can be noticed that by increasing SNR, the constellation points become more concentrated. The distance between each point decreases by reducing the value of SNR. Due to this reason, there could be overlapping between symbols, and demodulation can cause phase errors.

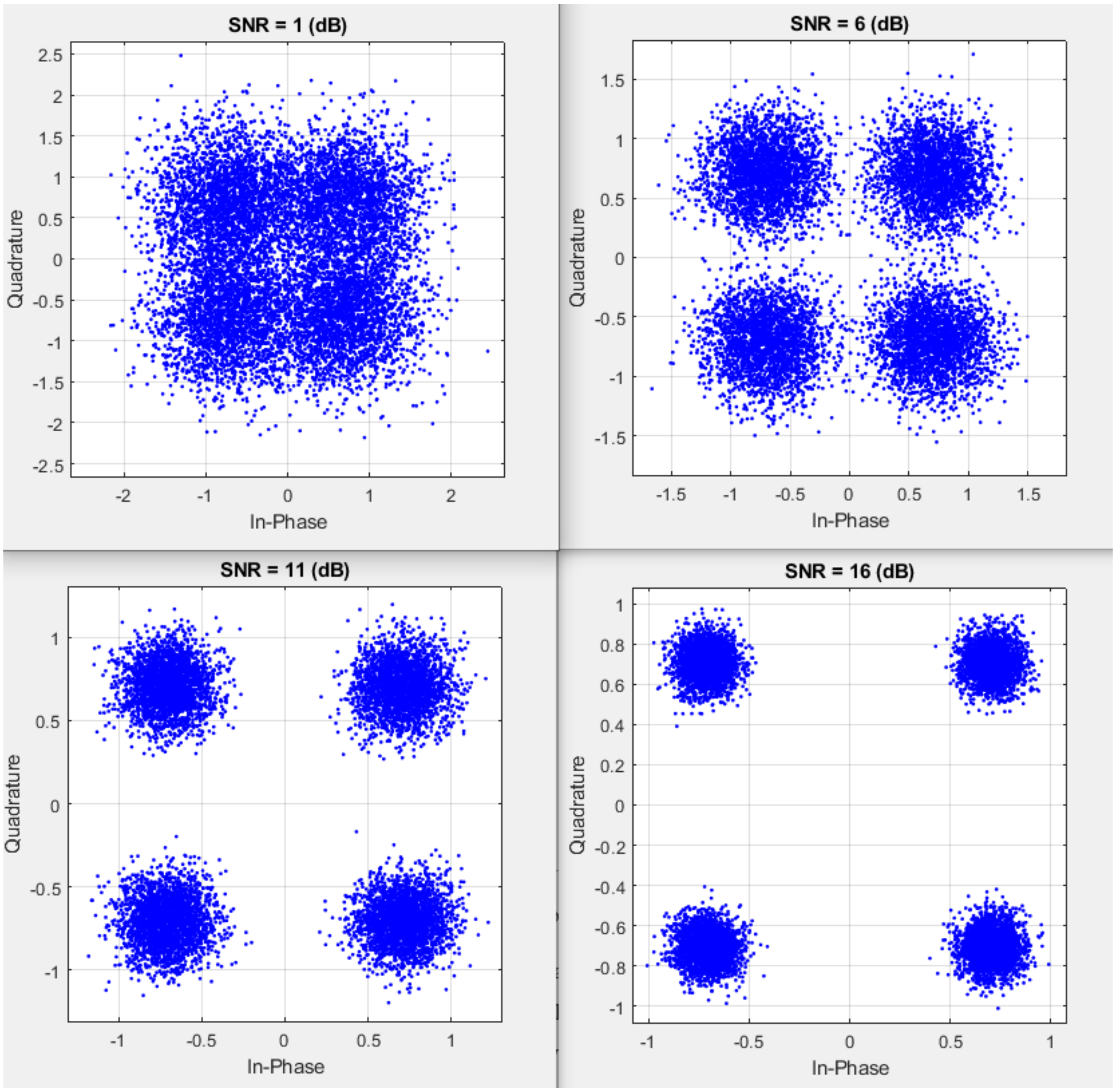


*Figure 6 Constellation points with different SNR*

The phase estimation plays an important role, so if it is not calculated accurately, then constellation points can shift into another region. By decreasing the value of SNR, the distance between lines of detection and edges of possible symbol cloud also decreases. Hence, a small error can lead to detecting another symbol. The system can also become more sensitive while estimating phase error when there are more symbols. Figure (7) shows that when phase error is introduced, it shifts the constellation points.

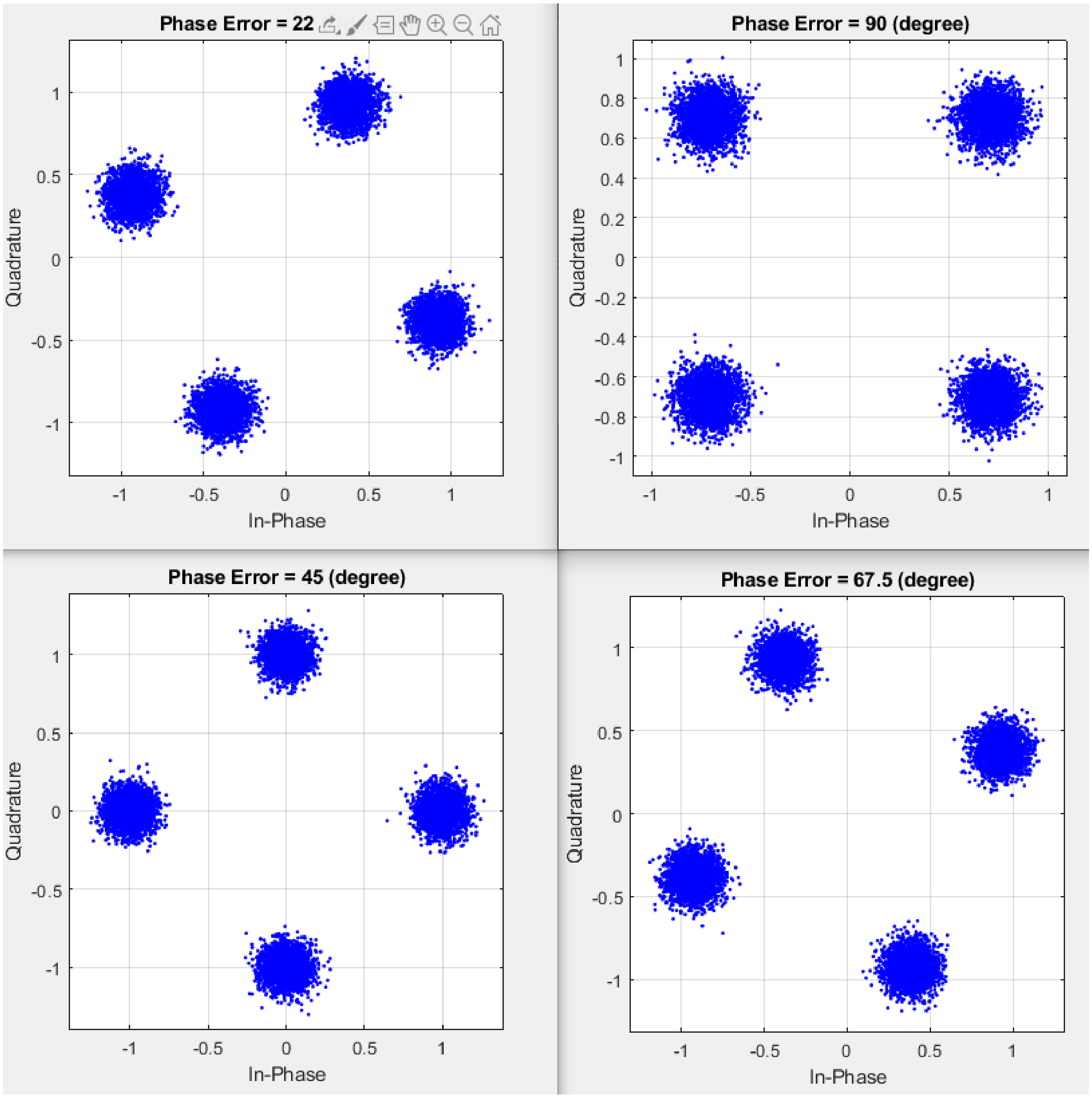


*Figure 7 Constellation with different values of phase error*

## 5.3 Problem III

In this section, the pulse shape generated from the root raised cosine filter is passed through AWGN and Rayleigh Fading channels. The equalization must be done for signals when the Rayleigh fading channels will be divided by the Rayleigh fading factor H.

From figure (8), we can see that the BER got smaller as the SNR increased. Overall, the BER, as a function of SNR, has a decreasing tendency with the SNR increases. For the AWGN channel, the value of BER is lower than the Rayleigh channel. For example, when SNR is 5, the BER of AWGN and Rayleigh channel are 0.595% and 6.418%, respectively.

From figure (9), we can see that the BER decreased more from one antenna to two antennas in comparison with others. Furthermore, with the increase of antenna's number, the BER decreases obviously. For example, when SNR is 16, the BER of a single antenna and 4 antennas are 0.62% and 0.0009%, respectively.

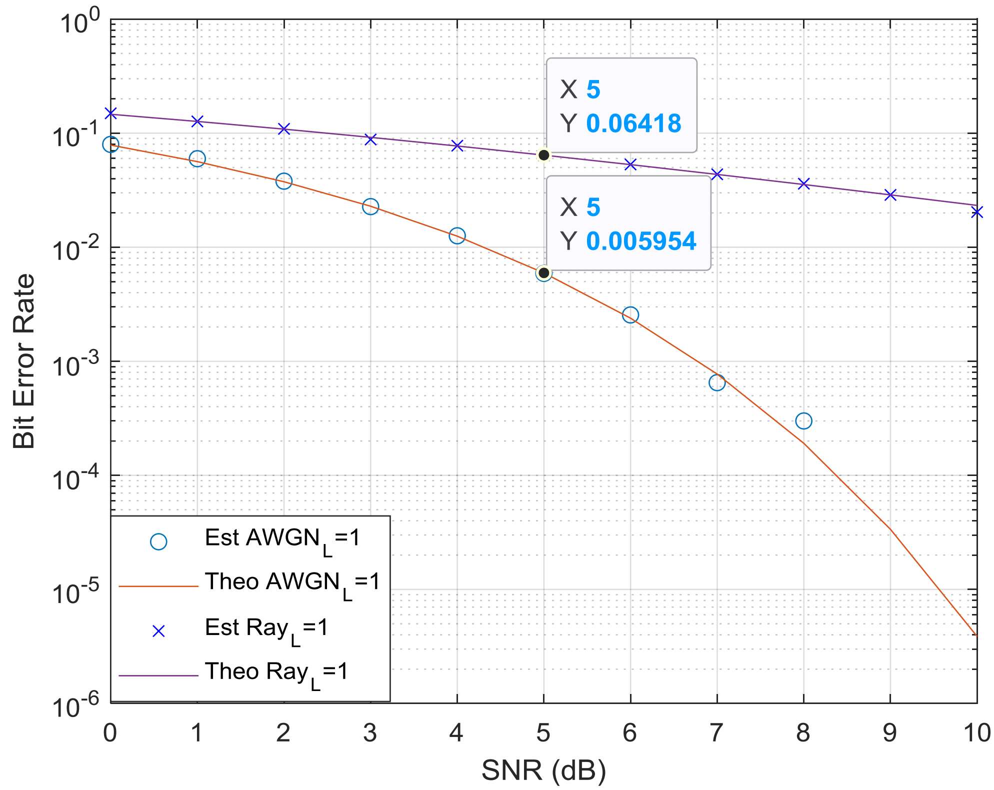


*Figure 8 BER with a single antenna*

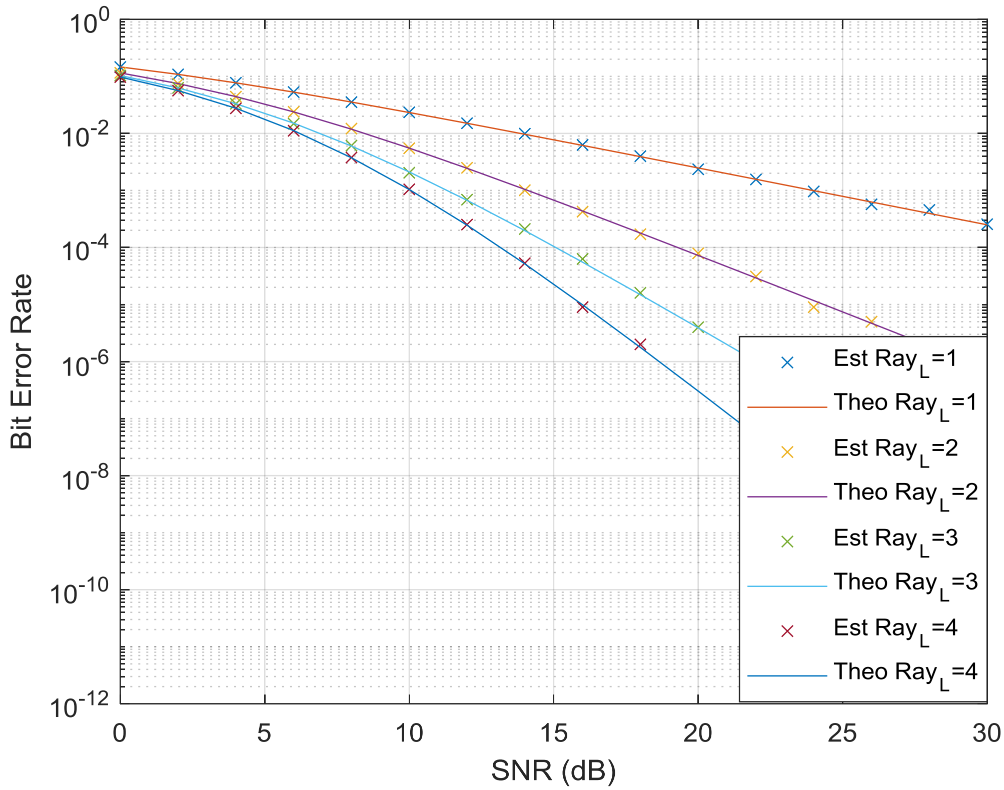


*Figure 9 BER with multiple antennas*

## 5.4 Problem IV

The difference between this question and previous one is that we use the blind estimation to estimate the phase error. The formulate is shown as follows,

$$\emptyset = \frac{1}{4} \text{ angle } E\{Y^4(k)\}$$

Where Y is the received signal vector. The MATLAB code example is as follows,

Phase_est = angle (mean (channelOutput. ^4))/4;

From figure (10), we can see that there are some mismatches, but totally due to good phase estimation, the results still match well.

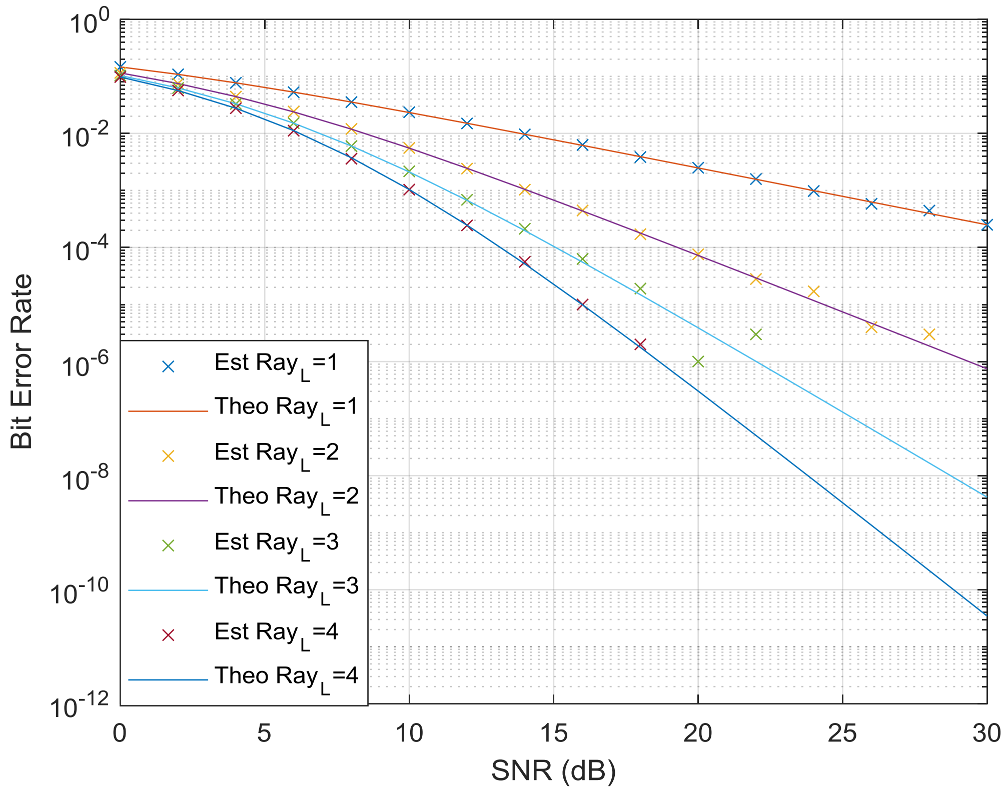


*Figure 10 BER with blind estimation*

Data length is another factor to affect the performance of phase estimation. In this project, we generate various length of signals to test the phase estimation. To prevent accidental, we repeat 10 times to get a smooth curve shown in Figure 11.

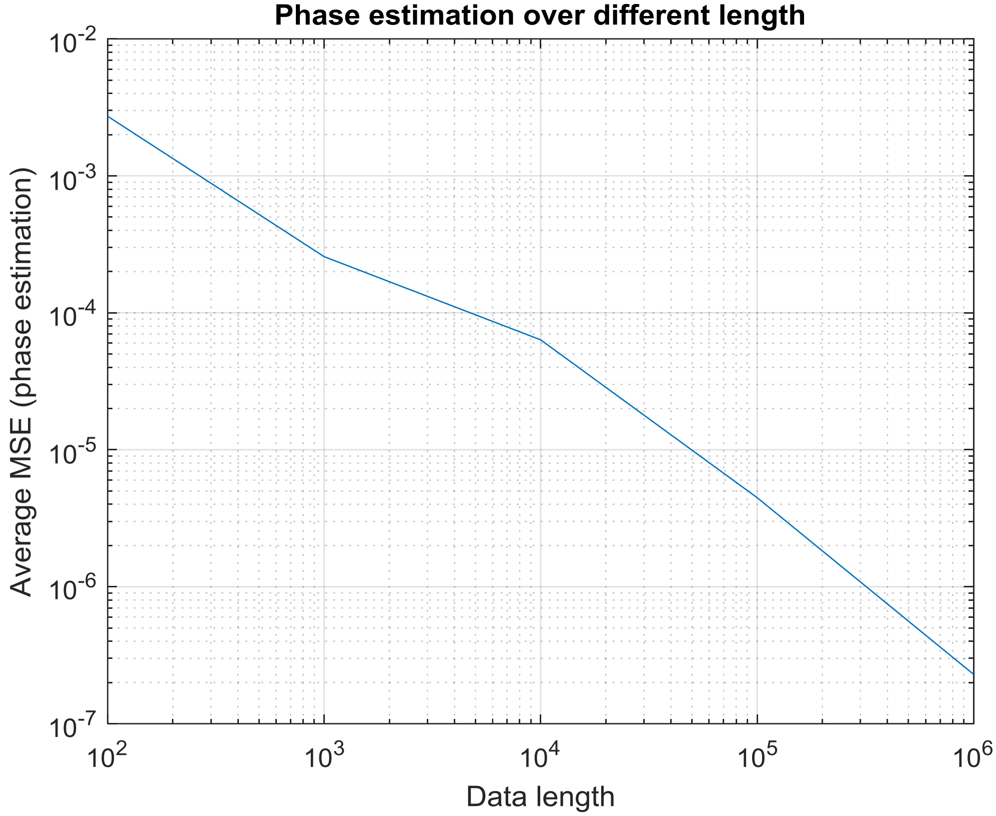


*Figure 11 BER with different signal length*

From figure (11), we can see that the average MSE (phase estimation) decreases obviously from $10^{-2}$ to $10^{-7}$ with the length of signal increases, which means the performance of phase estimation works better.

## 6. Conclusion

A digital communication system based on QPSK is simulated in this project using MATLAB software. Different components used to design the system are a bit generator for generating information, QPSK modulation, pulse shaping root raised cosine filter, the technique of MRC diversity, and phase estimation. The functions of all given components are explained in this report. To overcome the effect of ISI, root raised cosine filter at the transmitter side is used as pulse shaping. The root raised cosine filter is also tested in terms of power spectral density of transmitted values for different values of a roll of factor. The bandwidth is increased by the larger value of a roll of factor, and it reduces the efficiency of the spectrum. However, impulse response has sharp decay when the roll of factor has the smallest value.

A diversity method is used to reduce the flat fading and noise effects. For this purpose, the MRC technique is used, and it improves the output in terms of BER. The BER is decreased when the receiver has multiple antennas. The system shows more improvement in terms of BER as a function of SNR when more antennas are used at the receiver in comparison with a single antenna. Thus, more antennas are required for the better performance of the system.

## 7. References


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[8] Constantine A. Balanis, Antenna Theory analysis and design, 3rd ed., 2015.

# 8. Appendix (MATLAB Code)

## 8.1. Problem I

```
clear all
close all
clc


data_number = 2e4;                  % Data size
rolloff_factor = 0.1:0.25:0.9;       % Rolloff factor
Rs = data_number/2;                 % symbol Rate
span = 6;                           % Filter span in symbols (Number of Symbols)
sps = 4;                            % Samples per symbol


%          Data Generation
data_bit = randi([0 1], data_number, 1);


for i=1:length(rolloff_factor)

%          QPSK Modulation
QPSK_modulator = comm.QPSKModulator('BitInput',true);
modulated_transmiter_signal = QPSK_modulator(data_bit);

%        root raised cosine filter and upconversion
Root_raised_cosine_filter = rcosdesign(rolloff_factor(i), span, sps, 'sqrt');
upconverted_Transmitted_signal = upfirdn(modulated_transmiter_signal,
Root_raised_cosine_filter, sps);



%          PSD Calculator
figure(i)
[pxx frequency] =
pwelch(upconverted_Transmitted_signal,hamming(1024),[],[],Rs*sps,'centered');
plot(frequency,10*log10(pxx))
title(['Alpha = ' num2str(rolloff_factor(i))]);
ylabel('PSD (dB/Hz)')
xlabel('Frequency (Hz)')
hold on
grid on
end
```

### 8.2. Problem II

```
clear all
close all
clc


data_number = 2e4;          % Data size
rolloff_factor = 0.35;      % Rolloff factor
Rs = data_number/2;          % symbol Rate
M = 4;
span = 6;                   % Filter span in symbols (Number of Symbols)
sps = 4;                    % Samples per symbol
phase_err = -pi/4;          % Phase Error



%                       Data Generation
data_bit = randi([0 1], data_number, 1);

%-------------------------QPSK Modulation-------------------------------
QPSK_modulator = comm.QPSKModulator('PhaseOffset',phase_err,'BitInput',true);
modulated_transmiter_signal = QPSK_modulator(data_bit);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%

%                   Channel and Noise

%               generating constallations

SNR_dB = 1:5:16;                                %SNR DB
SNR = (10.^(SNR_dB./10));                          %SNR linear
for i=1:length(SNR_dB)

Noise = sqrt((0.5)/(log2(M)*SNR(i)))*(randn(data_number/log2(M),1) +
1i*randn(data_number/log2(M),1));

channelOutput = modulated_transmiter_signal + Noise;
%%%%%%%------------------------ Plot constellations-----------------------
    scatterplot(channelOutput);
    title(['SNR = ' num2str(SNR_dB(i)) ' (dB)']);
    grid on
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                    Phase Error

for Phase_err=-pi/4:pi/8:pi/4
%-------------------------QPSK Modulation-------------------------------
QPSK_modulator = comm.QPSKModulator('PhaseOffset',Phase_err,'BitInput',true);
modulated_transmiter_signal = QPSK_modulator(data_bit);

Noise = sqrt((0.5)/(log2(M)*SNR(i)))*(randn(data_number/log2(M),1) +
1i*randn(data_number/log2(M),1));

channelOutput = modulated_transmiter_signal + Noise;

scatterplot(channelOutput);
title(['Phase Error = ' num2str(rad2deg(Phase_err+pi/4)) ' (degree)']);
grid on

end
```

## 8.3. Problem III

### 8.3.1. Part 1

```
clear all
close all
clc


data_number = 2e4;          % Data size
rolloff_factor = 0.35;      % Rolloff factor
Rs = data_number/2;         %symbol Rate
M = 4;
span = 6;                   % Filter span in symbols (Number of Symbols)
sps = 4;                    % Samples per symbol
SNR_dB = 0:1:10;            %SNR dB
SNR = (10.^(SNR_dB./10));   %SNR linear
phase_err = 0;
L = 1;

%                    Data Generation

data_bit = randi([0 1], data_number, 1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%AWGN%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for j = 1:length(SNR_dB)
    %                        QPSK Modulation
```

```
    QPSK_modulator = comm.QPSKModulator('BitInput',true);
    modulated_transmiter_signal = QPSK_modulator(data_bit);

    %                    AWGN
    Noise = sqrt((0.5)/(log2(M)*SNR(j)))*(randn(data_number/log2(M),L) +
1i*randn(data_number/log2(M),L));
    channelOutput = modulated_transmiter_signal + Noise;


    %                  QPSK Demodulation
    QPSK_demodulator = comm.QPSKDemodulator('BitOutput',true);
    demodulated_received_signal = QPSK_demodulator(channelOutput);


    %                  Calculating BER with AWGN
    num_err(j) = biterr(data_bit,demodulated_received_signal);
    BER_AWGN(j) = num_err(j)/(length(data_bit));

    Theoretical_AWGN = berawgn(SNR_dB,'psk',M,'nondiff');
end
figure(1)
semilogy(SNR_dB,BER_AWGN,'O')
hold on
semilogy(SNR_dB,Theoretical_AWGN)
grid on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%RAY &
AWGN%%%%%%%%%%%%%%%%%%%%%%%%
for j = 1:length(SNR_dB)
        %                   QPSK Modulation
        QPSK_modulator = comm.QPSKModulator('BitInput',true);
        modulated_transmiter_signal = QPSK_modulator(data_bit);


        H = (randn(Rs,L)+1i*randn(Rs,L))/sqrt(2); % Generating fading gain

        Rep_sig = repmat(modulated_transmiter_signal, 1, L)./sqrt(L);
        Noise = sqrt((0.5)/(log2(M)*SNR(j)))*(randn(data_number/log2(M),L) +
1i*randn(data_number/log2(M),L));


        chanOut12 = H .* Rep_sig + Noise;
        x = conj(H).*chanOut12;
        channelOutput = sum(x, 2);
```

```
    %                       QPSK Demodulation
    QPSK_demodulator = comm.QPSKDemodulator('BitOutput',true);
    demodulated_received_signal = QPSK_demodulator(channelOutput);


    %                    Calculating BER with Ray & AWGN
    num_err(j) = biterr(data_bit,demodulated_received_signal);
    BER_RAY(j) = num_err(j)/(length(data_bit));                     %BER of Rayliegh Fading


    berTheory = berfading(SNR_dB,'psk',M,L);
end
semilogy(SNR_dB,BER_RAY,'Xbl')
hold on
semilogy(SNR_dB,berTheory)
grid on
legend('Est AWGN_L=1','Theo AWGN_L=1','Est Ray_L=1','Theo Ray_L=1')
hold on
xlabel('SNR (dB)')
ylabel('Bit Error Rate')
```

### 8.3.2. Part 2

```
clear all
close all
clc


data_number = 1e6;           % Data size
rolloff_factor = 0.35;       % Rolloff factor
Rs = data_number/2;          %symbol Rate
M = 4;
span = 6;                    % Filter span in symbols (Number of Symbols)
sps = 4;                     % Samples per symbol
SNR_dB = 0:2:30;             % SNR dB
SNR = (10.^(SNR_dB./10));    % SNR linear
Phase_err = rand*(-2*pi/4) + pi/4;  %Phase Error [-pi/4 , pi/4]
Phase_est = Phase_err;

%                 Data Generation
data_bit = randi([0 1], data_number, 1);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Rayleigh Fading & AWGN
%%%%%%%%%%%%%%%%%%%%%%%
```

```
for L = 1:4

    for j = 1:length(SNR_dB)
        %                    QPSK Modulation
        QPSK_modulator = comm.QPSKModulator('BitInput',true);
        modulated_transmiter_signal = QPSK_modulator(data_bit);


        H = (randn(Rs,L)+1i*randn(Rs,L))/sqrt(2); % Generating fading gain

        Rep_sig = repmat(modulated_transmiter_signal, 1, L)./sqrt(L);
        Noise = sqrt((0.5)/(log2(M)*SNR(j)))*(randn(data_number/log2(M),L) +
1i*randn(data_number/log2(M),L));


        chanOut12 = H .* Rep_sig + Noise;
        x = conj(H).*chanOut12;
        channelOutput = sum(x, 2);


        %                     QPSK Demodulation
        QPSK_demodulator = comm.QPSKDemodulator('BitOutput',true);
        demodulated_received_signal = QPSK_demodulator(channelOutput);


        %                   Calculating BER with AWGN
        num_err(j) = biterr(data_bit,demodulated_received_signal);
        BER_RAY(j) = num_err(j)/(length(data_bit));                    %BER of Rayliegh Fading


        berTheory = berfading(SNR_dB,'psk',M,L);
    end
    figure(2)
    semilogy(SNR_dB,BER_RAY,'X')
    hold on
    semilogy(SNR_dB,berTheory)
    grid on
    legend('Est Ray_L=1','Theo Ray_L=1','Est Ray_L=2','Theo Ray_L=2','Est Ray_L=3','Theo
Ray_L=3','Est Ray_L=4','Theo Ray_L=4')
    hold on
    xlabel('SNR (dB)')
    ylabel('Bit Error Rate')
end
```

### 8.4. Problem IV

#### 8.4.1. Part 1

```
 clear all
close all
clc


data_number = 1e6;          % Data size
rolloff_factor = 0.35;      % Rolloff factor
Rs = data_number/2;         %symbol Rate
M = 4;
span = 6;                   % Filter span in symbols (Number of Symbols)
sps = 4;                    % Samples per symbol
SNR_dB = 0:2:30;            % SNR dB
SNR = (10.^(SNR_dB./10));   % SNR linear
Phase_err = rand*(-2*pi/4) + pi/4;  %Phase Error [-pi/4 , pi/4]


%                  Data Generation
data_bit = randi([0 1], data_number, 1);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Rayleigh Fading & AWGN
%%%%%%%%%%%%%%%%%%%%%

for L = 1:4

   for j = 1:length(SNR_dB)
      %                     QPSK Modulation
      QPSK_modulator = comm.QPSKModulator('PhaseOffset',Phase_err,'BitInput',true);
      modulated_transmiter_signal = QPSK_modulator(data_bit);


      H = (randn(Rs,L)+1i*randn(Rs,L))/sqrt(2); % Generating fading gain

      Rep_sig = repmat(modulated_transmiter_signal, 1, L)./sqrt(L);
      Noise = sqrt((0.5)/(log2(M)*SNR(j)))*(randn(data_number/log2(M),L) +
1i*randn(data_number/log2(M),L));


      chanOut12 = H .* Rep_sig + Noise;
      x = conj(H).*chanOut12;
      channelOutput = sum(x, 2);
      Phase_est = angle(mean(channelOutput.^4))/4;

      %                          QPSK Demodulation
```

```
    QPSK_demodulator = comm.QPSKDemodulator('PhaseOffset',Phase_est,'BitOutput',true);
    demodulated_received_signal = QPSK_demodulator(channelOutput);


    %                     Calculating BER with AWGN
    num_err(j) = biterr(data_bit,demodulated_received_signal);
    BER_RAY(j) = num_err(j)/(length(data_bit));                        %BER of Rayliegh Fading


    berTheory = berfading(SNR_dB,'psk',M,L);
  end

  semilogy(SNR_dB,BER_RAY,'X')
  hold on
  semilogy(SNR_dB,berTheory)
  grid on
  legend('Est Ray_L=1','Theo Ray_L=1','Est Ray_L=2','Theo Ray_L=2','Est Ray_L=3','Theo
Ray_L=3','Est Ray_L=4','Theo Ray_L=4')
  hold on
  xlabel('SNR (dB)')
  ylabel('Bit Error Rate')
end
```

### 8.4.2. Part 2

```
clear all
close all
clc


data_number = [1e2,1e3,1e4,1e5,1e6];          % Data size
rolloff_factor = 0.35;        % Rolloff factor
Rs = data_number/2;          %symbol Rate
M = 4;
span = 6;                    % Filter span in symbols (Number of Symbols)
sps = 4;                     % Samples per symbol
SNR_dB = 10;             % SNR dB
SNR = (10.^(SNR_dB./10));      % SNR linear
Phase_err = rand*(-2*pi/4) + pi/4;  %Phase Error [-pi/4 , pi/4]
L = 1;




%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Rayleigh Fading & AWGN
%%%%%%%%%%%%%%%%%%%%%%%%
```

```
Phase_error_MSE = 0;

   for i = 1:length(data_number)
      for j=1:10

      %                  Data Generation
      data_bit = randi([0 1], data_number(i), 1);

      %                      QPSK Modulation
      QPSK_modulator = comm.QPSKModulator('PhaseOffset',Phase_err,'BitInput',true);
      modulated_transmiter_signal = QPSK_modulator(data_bit);


      H = (randn(Rs(i),L)+1i*randn(Rs(i),L))/sqrt(2); % Generating fading gain

      Rep_sig = repmat(modulated_transmiter_signal, 1, L)./sqrt(L);
      Noise = sqrt((0.5)/(log2(M)*SNR))*(randn(data_number(i)/log2(M),L) +
1i*randn(data_number(i)/log2(M),L));


      chanOut12 = H .* Rep_sig + Noise;
      x = conj(H).*chanOut12;
      channelOutput = sum(x, 2);


      Phase_est = angle(mean(channelOutput.^4))/4;

      %                        QPSK Demodulation
      QPSK_demodulator = comm.QPSKDemodulator('PhaseOffset',Phase_est,'BitOutput',true);
      demodulated_received_signal = QPSK_demodulator(channelOutput);


      %                     Calculating BER

      Phase_error_MSE(j) = immse(Phase_err,Phase_est);
      end
      MSE(i) = sum (Phase_error_MSE);
   end

   loglog(data_number,MSE./10)
   hold on
   grid on
   title ('Phase estimation over different length')
   xlabel('Data length')
   ylabel('Average MSE (phase estimation)')
```