Tutorial 'Introduction to Semantic Theory' (No. 4)

Lecture 2&3

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Session 2

November 8, 2019

Our agenda today

- We will finish what we left unexplained last time:

  Excersise 3: Set relations, sets and their characteristic functions, excersise 4 for functions

- Assignment 1&2

- Also something new:

  Rules for compositional interpretation

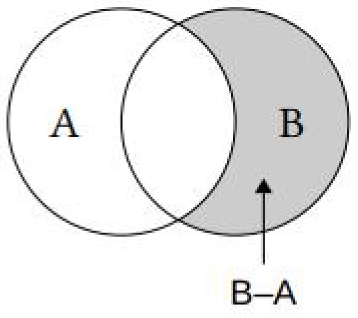
  Intransitive and transitive verbs

  Schonfinkelization

Basic concepts in set theory: Check list

Set relations:

- <u>Equivalence</u> A = B, iff A and B have exactly the same elements.
- <u>Subset</u> A $\subseteq$ B, iff all members of A are members of B.
- <u>Proper Subset</u> A $\subset$ B, iff A is a subset of B but not equivalent to B
- <u>Intersection</u> A $\cap$ B, is the set C with exactly those elements which are shared by A and B.
- <u>Overlap and disjointness</u> A and B overlap iff A $\cap$ B $\neq$ Ø A and B are disjoint if A $\cap$ B = Ø
- <u>Union</u> A $\cup$ B, the C with all the members of A and B and nothing else.
- <u>Complement</u> The complement of A relative to B, B–A



Excersise 3: Set relations

(13) True or false?

a. {a} = {b}

b. {x: x=a} = {a}

c. {x: x is green}={y: y is green}

d. {x: x likes a} = {y: y likes b}

e. $\{x : x \in \{y : y \in B\}\} = B$

f. $\{x : \{y : y \text{ likes } x\} = \emptyset\} = \{x : \{x : x \text{ likes } x\} = \emptyset\}$

(14) True or false?

a. $\{a,b,c\} \subseteq \{a,b,c\}$

b. $A \subset A$

c. If $A \subseteq B$ and $B \subseteq A$, then $A=B$

d. For every set S, $\emptyset \subseteq S$

e. i. $\{a,b\} \in \{\{a,b\}, \{c,d\}\}$ ii. $\{a,b\} \subseteq \{\{a,b\}, \{c,d\}\}$ iii. $\{a,b\} \in \{a,b,c,d\}$

Solutions for (14)

(14) a. True. Every set is a subset of itself.

b. False. Every set can't be a proper subset of itself.

c. True. $A \subseteq B$ means, all members of A are members of B.

$B \subseteq A$ means, all members of B are members of A.

Hence, A=B.

d. True.

e. True, false, false

Tips for questions such as (14e)

<u>When in doubt, go back to the basic definitions of set and set realtions.</u>

<u>Subset</u> $A \subseteq B$, iff all members of A are members of B.

Trick: When you see “$\subseteq$”, list all members of set A and B and compare them.

<u>Members/elements:</u> The formula “$x \in B$” can be read as: “x is a member (or element) of set B”.

Trick: When you see “$\in$”, list all the members of B and check if x is one of them.

Let's look at (14e) again with these definitions in mind.

Solutions for (14e)

i. $\underline{\{a,b\}} \in \{\underline{\{a,b\}}, \{c,d\}\}$

True. $\{\underline{\{a,b\}}, \{c,d\}\}$ has two elements $\{a,b\}$ and $\{c,d\}$.

ii. $\{a,b\} \subseteq \{\{a,b\}, \{c,d\}\}$

False. $\{a,b\}$ has two elements a and b. $\{\underline{\{a,b\}}, \{c,d\}\}$ has two elements $\{a,b\}$ and $\{c,d\}$.

iii. $\underline{\{a,b\}} \in \{a,b,c,d\}$

False. $\{a,b,c,d\}$ has four elements a,b,c and d. $\{a,b\}$ is not one of them.

Assignment 1&2

Any questions?

- x, y, z are variables ranging over objects
- a, b, c are names of objects
- X, Y, Z are variables ranging over sets
- A, B, C are names of sets

When a=1, b=1, a=b.   When A={1}, B={1}, A=B

But x,y are variables/place holders, not names. ~~x=y~~

Variables are called variables because they vary, i.e. they can have a variety of values.

A:= {x: x is green}={y: y is green}= {z: z is green}= {$: $ is green}={abc: abc is green}

f. $\{x : \{y : y \text{ loves } x\} = \{\text{John, Bill}\}\} = \{x : \{\text{Bill, John}\} = \{y : x \text{ loves } y\}\}$

Assignment 1&2

$$\text{g.} \quad \{x : x \text{ loves } b\} = \{x : \{y : x \text{ loves } y\} = \{b\}\}$$

Assume a *situation S1:* Mary loves John and Jane. John loves Jane. b= Jane

{x: x loves b}= {x: x loves Jane}= {Mary, John}

{x: {y: x loves y}={b}}= {x: x loves only b and loves nothing else}

= {x: x loves only Jane and loves nothing else}

= {John}

Therefore, g is false.

Assignment 1&2

g. $\{x : x \text{ loves } b\} = \{x : \{y : x \text{ loves } y\} = \{b\}\}$

Assume a *situation S2:* Mary loves Jane. John loves Jane. b= Jane

{x: x loves b}= {x: x loves Jane}= {Mary, John}

{x: {y: x loves y}={b}}= {x: x loves only b and loves nothing else}

= {x: x loves only Jane and loves nothing else}

= {Mary, John}

g is true because {x: x loves Jane} = {x: x loves only Jane and loves nothing else}.

Why functions?

Up to this point, we only looked at sets of individuals.

What about sets of couples (or triples, quadruples,etc.) of individuals?

Example: The set of all married couples who live in New York.

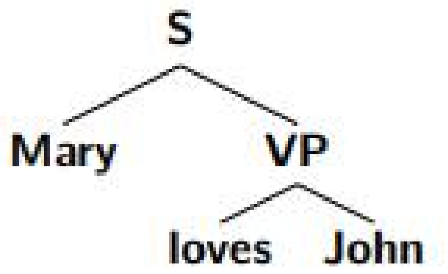
{William & Dorothy Bradford, William & Mary Brewster, Myles & Rose Standish, Edward & Elizabeth Winslow, …}

In this example, the set is defined as a set of pairs, but the order doesn't matter.

Ordered pairs

In certain cases, specifying the order in which the members of each pair occur is important. Consider sentence (15) with the transitive verb “love”:

(15)



Note: <Mary, John> ≠ <John, Mary>

Relations

A set of ordered pairs is called a **relation**.

Relations are defined by a condition on two variables to the right of ":"

(16) *Situation: Mary loves John. Mary loves Jane. John loves Jane. Jane loves Sue.*

R:= {<x, y>: x loves y}

<Mary, John> ∈ R

<Mary, Jane> ∈ R

<John, Jane> ∈ R

<Jane, Sue> ∈ R <Sue, Jane> ∉ R

R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}

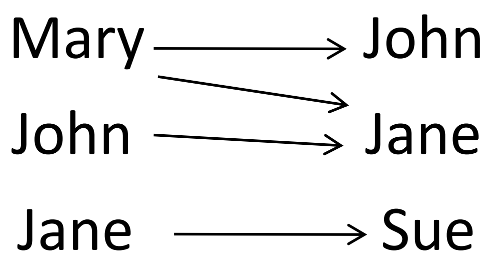
The **domain** of R is the set {Mary, John, Jane}.

The **range** of R is the set {John, Jane, Sue} .

Relations define mappings

A set of ordered pairs defines a **mapping,** or correspondence from the domain onto the range.

(16) R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}

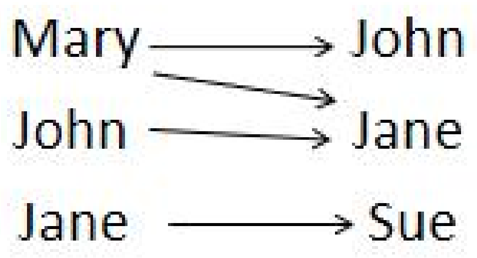


Functions and its uniqueness

First of all, a **Function is a relation.**

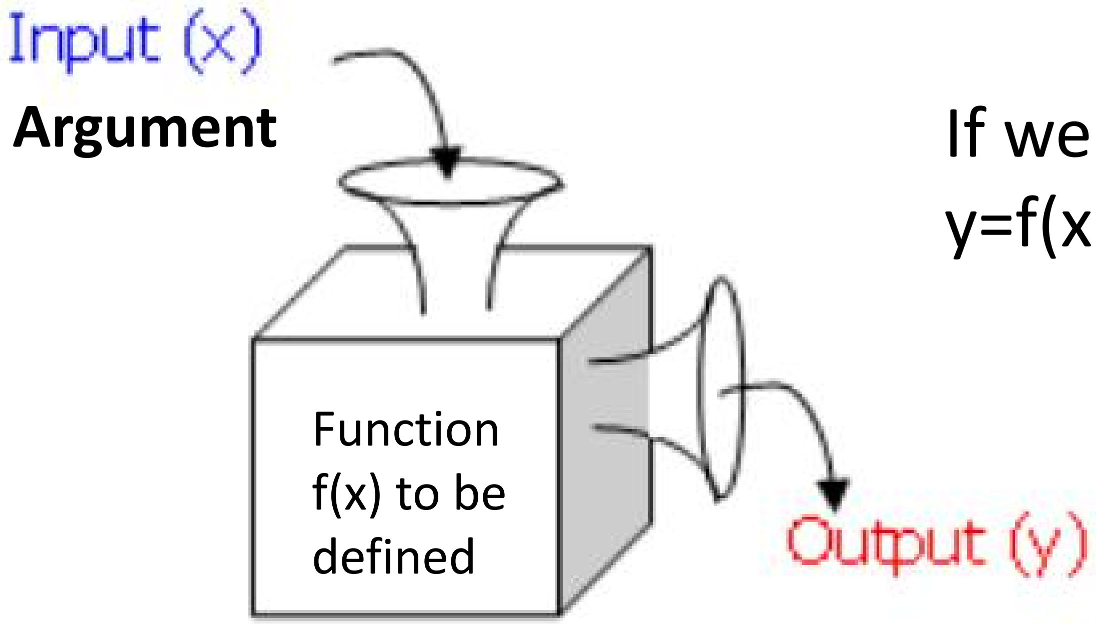
But, unlike relations, for a function, each element of the domain is mapped to a **single, unique value** in the range.

(16) R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}



The relation in (16) is not a function, because the domain contains two distinct ordered pairs which have the same first element <Mary, John>, <Mary, Jane>.

A tip for understanding functions



If we set our machine to $y=f(x)=x+1$ ...

Value $f(x)$ = 'the value of f for the argument x'.

$f(x) :=$ the unique y such that $\langle x, y \rangle \in f$.

Defining functions

- via listing:

$$F := \{\langle a, b\rangle, \langle b, c\rangle, \langle c, a\rangle\}$$

$$=$$

$$F := \begin{bmatrix} a \to b \\ b \to c \\ c \to a \end{bmatrix}$$

- with conditions (3 types):

$F := \{\langle x, y\rangle : x$ is a French president and y is x's year of birth$\}$

Let F be that function f such that
$f : \{x : x$ is a French president$\} \to \{x : x$ is a year$\}$, and for every $x \in \{x : x$ is a French president$\}$, $f(x) =$ the year in which x was born.

$$F := \begin{array}{l} f : \{x : x \text{ is a French president}\} \to \{x : x \text{ is a year}\} \\ \text{For every } x \in \{x : x \text{ is a French president}\}, f(x) = \text{the year } x \text{ was born.} \end{array}$$

Excersise 4: Functions/relations

(17) Function or not?

a. $R_1 := \{\langle x,y \rangle : y=2x+1\}$ (where x and y are integers)

b. $R_2 := \{\langle x,y \rangle :$ x is a human being and y is x's birth mother$\}$

c. $R_3 := \{\langle x,y \rangle :$ x is a human being and y is x's son$\}$

d. $R_4 := \{\langle y,x \rangle :$ x is a human being and y is x's son$\}$

e. Assume we have two sets. $A = \{a,b,c\}$ and $B = \{1,2,3,4\}$. The following relations from A to B:

$R_5 := \{\langle a,1 \rangle, \langle b,2 \rangle, \langle c,3 \rangle\}$

$R_6 := \{\langle a,3 \rangle, \langle b,4 \rangle, \langle c,1 \rangle, \langle a,2 \rangle\}$

$R_7 := \{\langle a,1 \rangle, \langle b,2 \rangle\}$

Solutions: Excersise 4

(17) a. R_1 is a function.

b. R_2 is a function. Human beings can only one birth mother.

c. R_3 is a not function. Human beings can more than one sons.

d. R_3 is a not function. One son can be mapped to more than one parent.

e. R_5 is a function.

R_6 is not a function. The argument *a* can be mapped to more than one value *2* and *3*.

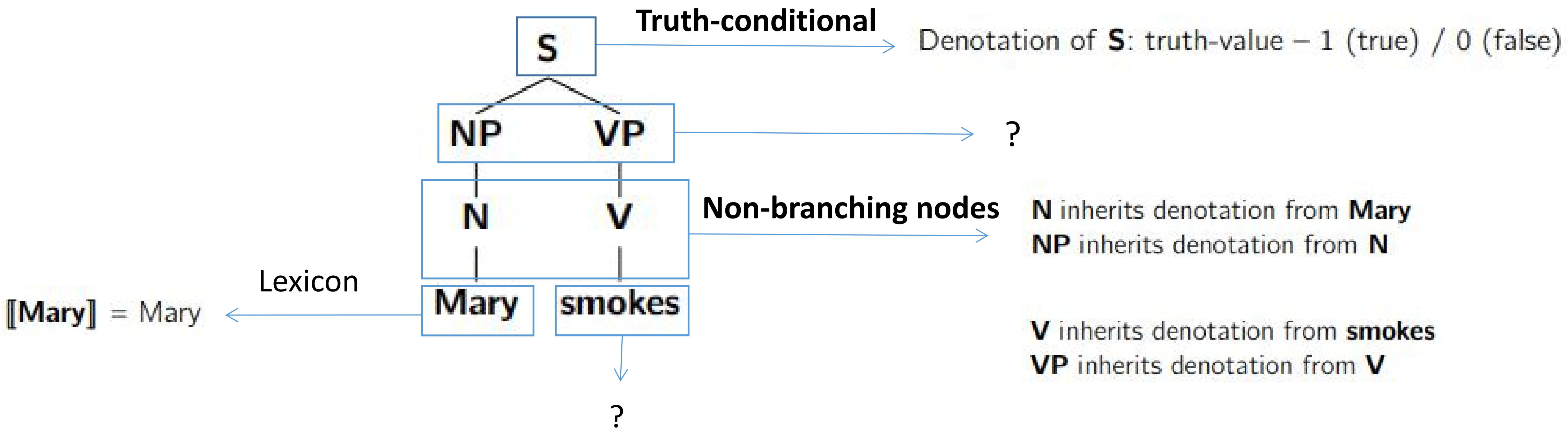
R_7 is not a function. All arguments within the domain D should be mapped to a value.

How does compositional interpretation work?

Truth-conditional semantics: Knowing the meaning of an expression consists in knowing the conditions under which it is true.

Mary smokes.

(18)

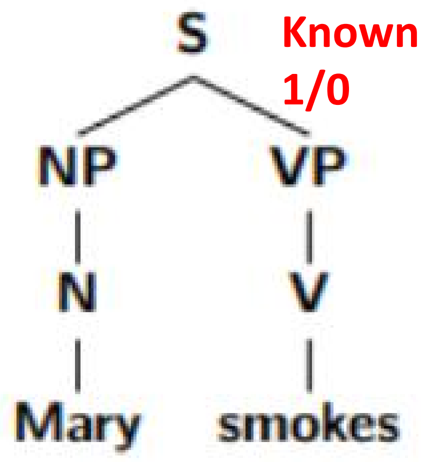


What remains unclear

[[S]] = [[VP]]([[NP]]) How do we combine two braches?

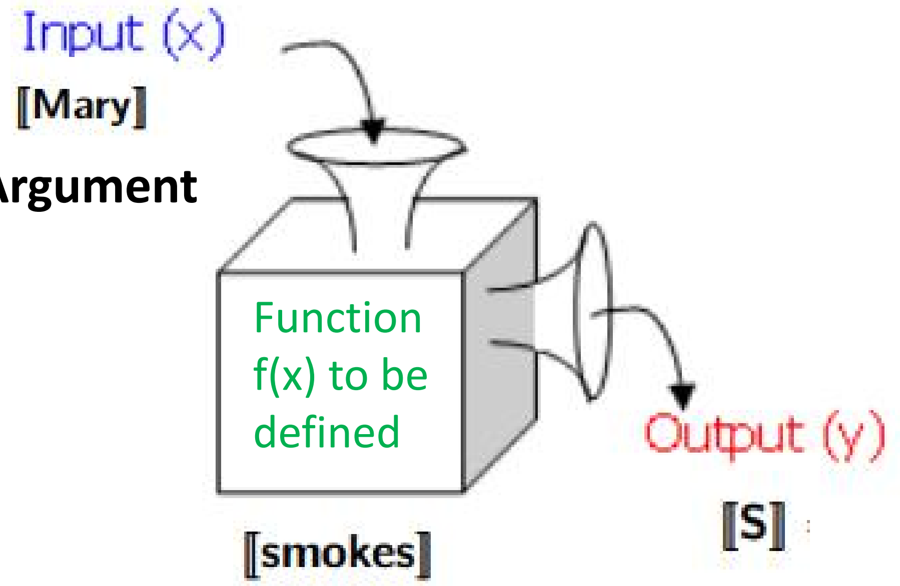
[[VP]]= ? How do we understand verbs?

Output:
Known
1/0



Known:
Individual
(D)

Unknown
To be defined



Value

Function Application (FA) as a model

Our solution: Using **function** as a tool to interpret any syntactic structure with two branches.

If $\boldsymbol{\alpha}$ has the form $\begin{array}{c} \textsf{S} \\ \diagup\ \diagdown \\ \beta \quad \gamma \end{array}$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

One branch is interpreted as a **function**, and the otherbranch is interpreted as a possible **argument** of the function.

$$
\begin{aligned}
[\![\text{S}]\!] &= [\![\text{VP}]\!]([\![\text{NP}]\!]) && \text{(S1)} \\
&= [\![\text{smokes}]\!]([\![\text{Mary}]\!]) && ([\![\text{VP}]\!], [\![\text{NP}]\!]) \\
&= \boxed{\left[\begin{array}{l} f : D \rightarrow \{0,1\} \\ \text{For all } x \in D,\ f(x) = 1 \text{ iff } x \text{ smokes} \end{array}\right]}\boxed{(\text{Mary})} && ([\![\text{smokes}]\!], [\![\text{Mary}]\!])
\end{aligned}
$$

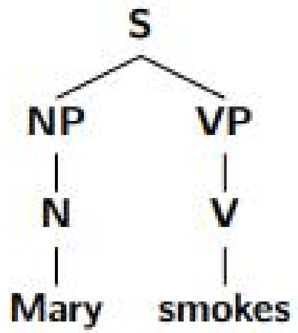
Intransitive verbs as functions *and* sets?

Intransitive verbs are functions from individuals to truth values:

$$[\![\textbf{dances}]\!] = f : D \to \{0, 1\}$$
$$\text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ dances}$$

$$[\![\textbf{smokes}]\!] = f : D \to \{0, 1\}$$
$$\text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes}$$

According to our intuition, there is another way to combine the two branches: **Set membeship**.



S= 1 iff [[Mary]] $\in$ $[\![\textbf{smokes}]\!] = \{x : x \in D \text{ and } x \text{ smokes}\}$

abbreviated as: $[\![\textbf{smokes}]\!] = \{x \in D : x \text{ smokes}\}$

The individual Mary

Intransitive verbs can been seen as both functions and sets. There must be some kind of relationship between sets and functions.

Sets and their characteristic functions

We use **characteristic function** to express the membership of the elements of any set S.

Mapping from Domain: The elements of S $\longrightarrow$ Range: The set of truth-values {1,0}

(18) Assume domain D = {John Lennon, Paul McCartney, George Harrison, Ringo Starr, Jay-Z}

A:= {x: x is a member of the Beatles}

The characteristic function of A is

$$\begin{bmatrix} \text{John Lennon} \rightarrow 1 \\ \text{Paul McCartney} \rightarrow 1 \\ \text{George Harrison} \rightarrow 1 \\ \text{Ringo Starr} \rightarrow 1 \\ \text{Jay-Z} \rightarrow 0 \end{bmatrix}$$

The "table notation"

Relation between sets and their characteristic functions

Mapping of **characteristic functions**:

The elements of S $\longrightarrow$ The set of truth-values {1,0}

This means:

Each subset S of the domain D defines such a function uniquely.

We write this as $\boldsymbol{char_{S,}}$ the characteristic function of S.

Any such function f (whose range is {0, 1}) corresponds to a unique subset S of the domain D.

We write this as $\boldsymbol{char_{f,}}$ is the set characterized by f.

$$\boldsymbol{char_S \longleftrightarrow char_f}$$

$char_S \longleftrightarrow char_f$: *One-on-one correspondence*

(19) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

There are some subsets of D:

A:= [[is a member of the Beatles]]={x: x is a member of the Beatles}

B: =[[raps]]={x: x raps}

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguist]]={x: x is a linguist}

$$char_A = \begin{bmatrix} \text{John Lennon} \longrightarrow 1 \\ \text{Paul McCartney} \longrightarrow 1 \\ \text{George Harrison} \longrightarrow 1 \\ \text{Ringo Starr} \longrightarrow 1 \\ \text{Jay-Z} \longrightarrow 0 \end{bmatrix}$$

$$char_B = \begin{bmatrix} \text{John Lennon} \longrightarrow 0 \\ \text{Ringo Starr} \longrightarrow 0 \\ \text{Jay-Z} \longrightarrow 1 \\ \text{Chomsky} \longrightarrow 0 \\ \text{Spongebob} \longrightarrow 0 \end{bmatrix}$$

Excersise 5: Characteristic function

What is the characteristic function of set C and D? Use table notation.

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguist]]={x: x is a linguist}

Solutions: Excersise 5

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguist]]={x: x is a linguist}

$$char_C = \begin{bmatrix} \text{John Lennon} \longrightarrow 0 \\ \text{Ringo Starr} \longrightarrow 0 \\ \text{Jay-Z} \longrightarrow 0 \\ \text{Chomsky} \longrightarrow 0 \\ \text{Spongebob} \longrightarrow 1 \end{bmatrix} \qquad char_D = \begin{bmatrix} \text{John Lennon} \longrightarrow 0 \\ \text{Ringo Starr} \longrightarrow 0 \\ \text{Jay-Z} \longrightarrow 0 \\ \text{Chomsky} \longrightarrow 1 \\ \text{Spongebob} \longrightarrow 0 \end{bmatrix}$$

$char_S \longleftrightarrow char_f$: One-on-one correspondence

(19) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

There are some fuctions whose range is {0, 1}:

[[is a member of the Beatles]]= $f_1 : D \longrightarrow \{0, 1\}$

For all x $\in$D, f(x)=1 iff x is a member of the Beatles.

[[raps]]=f_2...

[[lives in a pineapple under the sea]]=f_3...

[[is a professional musician]]=f_4...

$char_{f1}$= {x: x$\in$D and x is a member of the Beatles} = {John Lennon, Ringo Starr}

$char_{f1}$= {x: x$\in$D and x raps} = {Jay-Z}

Excersise 6: Characteristic function

What are the sets characterized by f_3 and f_4?

[[lives in a pineapple under the sea]]=$f_{3…}$

[[is a professional musician]]=$f_{4…}$

Solutions: Excersise 6

[[lives in a pineapple under the sea]]=$f_{3...}$

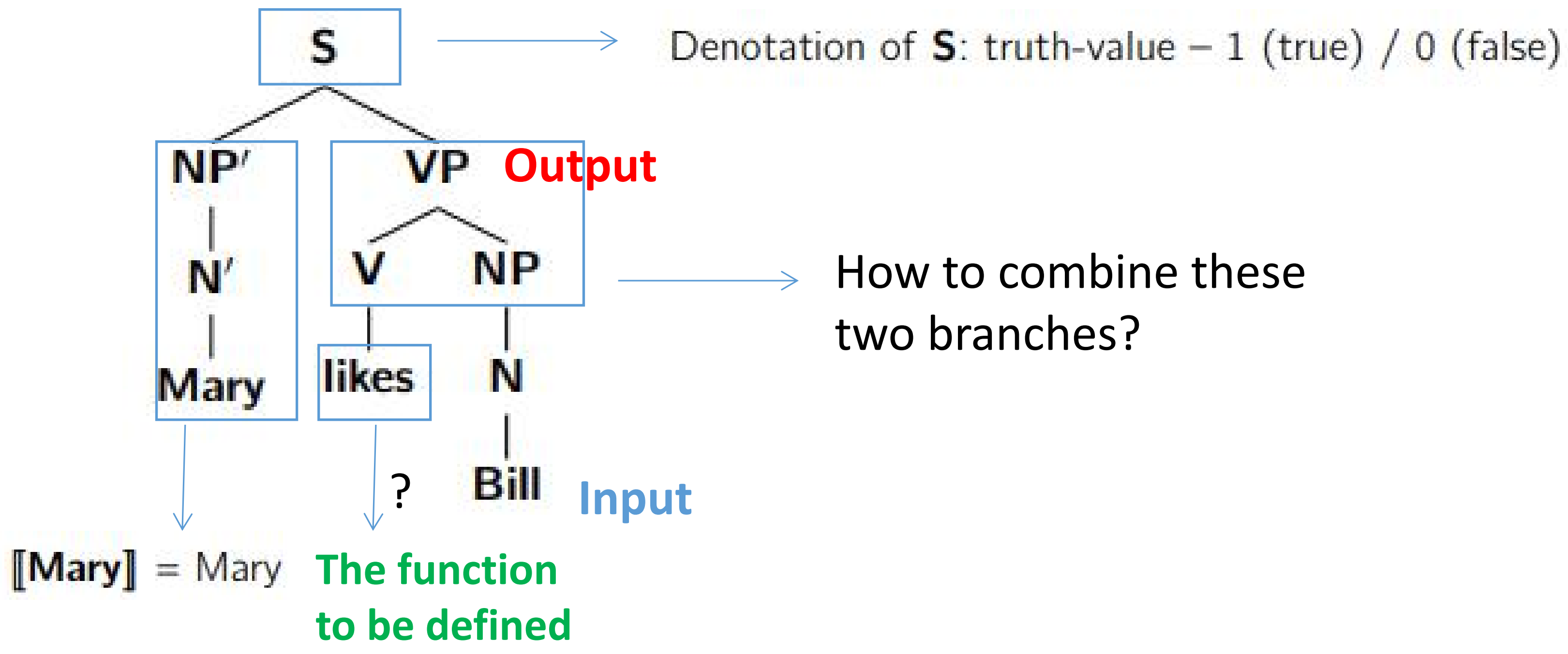
[[is a professional musician]]=$f_{4...}$

The sets characterized by f_3 and f_4 :

$char_{f3}$= {Spongebob}

$char_{f4}$= {John Lennon, Ringo Starr, Jay-Z}

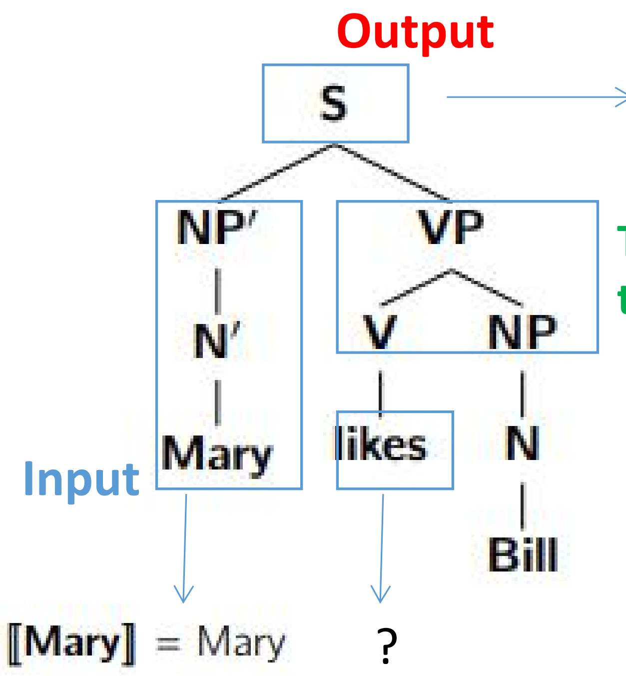
What about transitive Verbs?



We need new rules to combine [[V]] and [[NP]] to the node [[VP]].   Bill likes? likes Bill?

Unlike intransitive verbs, transitive verb cannot denote a function from individuals to truth-values.

Branching VP-nodes



Denotation of **S**: truth-value – 1 (true) / 0 (false)

The function
to be defined

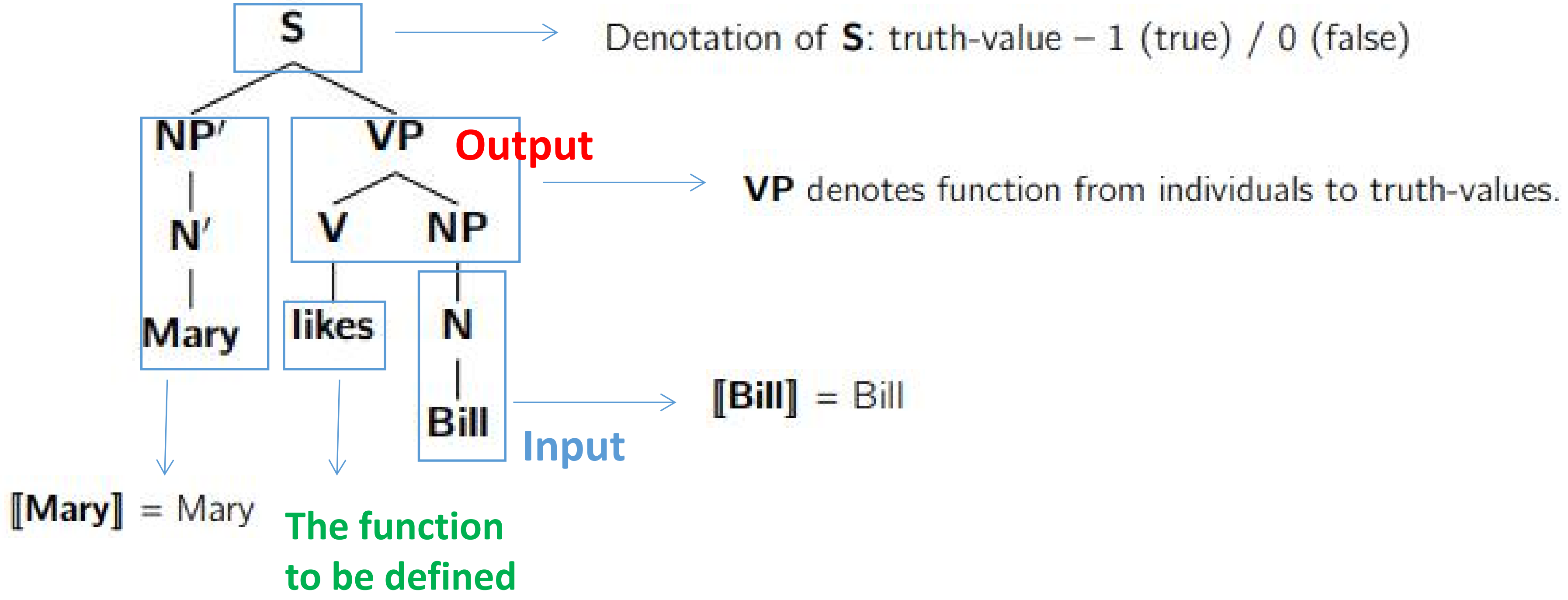
How to combine these
two branches?

Same kind of function

char[[VP]]=char[[likes Bill]]= {x: x likes Bill} ⟷ *char[[smokes]]*= {x: x smokes}

VP denotes **a (characteristic) function from individuals to truth-values**. This is something we already know from intransitive verbs.

Filling in the blanks...



Transitives verbs “likes” denote a function from individuals to functions from individuals to truth-values.

$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{1,0\}\} \\ \text{For all } x \in D,\ f(x) = \begin{array}{l} g_x : D \to \{1,0\} \\ \text{For all } y \in D,\ g_x(y) = 1 \text{ iff } y \text{ likes } x. \end{array} \end{array}$$

Function-valued functions as n-ary function.

(20) Assume D = {Fiona, Patsy, Jenny}

$R_{like} = \{\langle \text{Fiona, Patsy} \rangle, \langle \text{Patsy, Jenny} \rangle, \langle \text{Jenny, Jenny} \rangle\}$

Characteristic function of R_{like}=

$$\begin{bmatrix} <\text{Fiona, Fiona} > \rightarrow 0 \\ <\text{Fiona, Patsy} > \rightarrow 1 \\ <\text{Fiona, Jenny} > \rightarrow 0 \\ <\text{Patsy, Fiona} > \rightarrow 0 \\ <\text{Patsy, Patsy} > \rightarrow 0 \\ <\text{Patsy, Jenny} > \rightarrow 1 \\ <\text{Jenny, Fiona} > \rightarrow 0 \\ <\text{Jenny, Patsy} > \rightarrow 0 \\ <\text{Jenny, Jenny} > \rightarrow 1 \end{bmatrix}$$

This is a **binary function** that takes two arguments. But our assumption before tells us that “likes” denote a function takes exactly one individual as argument and maps it to a function.

Reducing n-ary functions to unary ones

Schonfinkelization/currying: Turning n-ary functions into unary functions.

Left-to-right

$$\begin{bmatrix} \text{Fiona} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 1 \\ \text{Jenny} \rightarrow 0 \end{bmatrix} \\ \text{Patsy} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \\ \text{Jenny} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \end{bmatrix}$$

Right-to-left

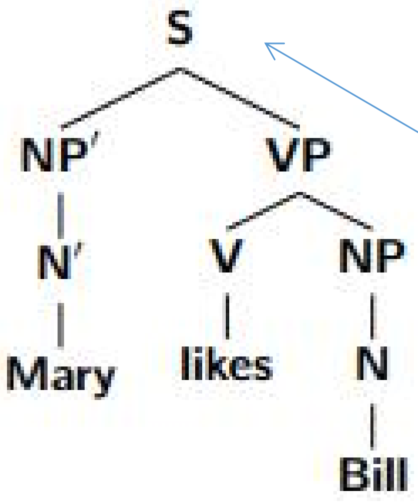
$$\begin{bmatrix} \text{Fiona} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 0 \end{bmatrix} \\ \text{Patsy} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 1 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 0 \end{bmatrix} \\ \text{Jenny} \rightarrow & \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 1 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \end{bmatrix}$$

The "likers" The ones being liked The ones being liked The "likers"

Schonfinkelization and grammar

From the ontological point of view, it does not matter which of the two ways we choose to model them.

But, does it make a difference from the point of view of the grammar of English?



Left-branching

In hierarchical terms, the object “Bill” is the closest to the predicate “likes”. Therefore, it should provide the argument for the function denoted by the predicate.

The **right-to-left Schonfinkelization** reflects the **left-branching** syntactic structure of English sentences.

Excersise 7: Ditransitive verbs (homework)

(21) Assume D = {Mary, John, Jane, Sue}

$R_{introduce}$= {<Mary, John, Sue>, <Mary, Jane, John>, <John, Jane, Sue>}

To be clear, <Mary, John, Sue> means “Mary introduces John to Sue”.

i. What is the characteristic function of $R_{introduce}$? Write this first as a ternary relation.

ii. There are two ways of using the **ditransitive** verb “introduce” in English:

a. Mary introduces John to Sue.

b. Mary introduces Sue John.

Which one appears more natural or acceptable to you?

Which schönfinkelization of the characteristic function of $R_{introduce}$ does each of these two forms correspond to? Specify this as a unary function using the table notation

Excersise 7: Hint 1

i. We did some excersise to write the characteristic function of R_{like} as binary relation as shown in (20a). What would the table notation look like for ternary relation with ordered triples like <Mary, John, Sue>?

(20a)

$$\begin{bmatrix} <\text{Fiona, Fiona}> \rightarrow 0 \\ <\text{Fiona, Patsy}> \rightarrow 1 \\ <\text{Fiona, Jenny}> \rightarrow 0 \\ <\text{Patsy, Fiona}> \rightarrow 0 \\ <\text{Patsy, Patsy}> \rightarrow 0 \\ <\text{Patsy, Jenny}> \rightarrow 1 \\ <\text{Jenny, Fiona}> \rightarrow 0 \\ <\text{Jenny, Patsy}> \rightarrow 0 \\ <\text{Jenny, Jenny}> \rightarrow 1 \end{bmatrix}$$

Excersise 7: Hint 2

ii. There are two ways of using the ditransitive verb “introduce” in English:

a. Mary introduces John to Sue.

b. Mary introduces Sue John.

As we all agreed in class, sentence (iia) *Mary introduces John to Sue* appears more natural or acceptable than (iib). Is it possible that this is due to the fact that the schönfinkelization of $R_{\text{introduce in}}$ sentence (iia) matchs the **right-to-left Schonfinkelization** of English in general?

Specify the schönfinkelization of both (iia) and (iib) as a unary function as in (20b). Which kind of schönfinkelization

(20b)

$$\begin{bmatrix} \text{Fiona} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 1 \\ \text{Jenny} \rightarrow 0 \end{bmatrix} \\ \text{Patsy} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \\ \text{Jenny} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \end{bmatrix}$$

Next time...

Textbooks (relevant chapters posted on StudIP)

H&K 2.5, 3.1

C&MG appendices 3, 4

Assignment 3

Negation, semantic types, Coordination, λ-notation, conversion, type-driven interpretation

Thanks and see you next week!