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The ludic and powerful Mayan mathematics for teaching

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Abstract

Apparently, the Mayas discovered and used the abstraction of zero many centuries before any other culture, about 400 years before our era. Some of the remarkable cultural achievements of the Maya people show the need for a mathematical tool, powerful, and accurate. In this work, we describe the numbering system that was positional, similar to that currently used, but using base 20, and with only three signs: the point, the bar, and the zero. The latter was represented in different ways. The most common was the snail shell. Here we show, how with these three signs, all operations were performed. We exhibit the advantages of using dots, bars, and snail shells, to perform arithmetic operations. We show how to use the Mayan system in base 10, for a fast understanding. We display on this basis, the operations of addition, subtraction, multiplication, division, and square root without using tables. It is an elegant, dynamic, ludic, and intuitive process. We also show some initial results obtained from the application of this method for teaching mathematics in elementary schools in Yucatán, México.

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Mayan mathematics; base 20; base10.

1. Introduction

The Mayas wrote their numbers vertically and using dots, bars and seashells. They used base 20. In figure 1, we show the first 21 Mayan numerals. Notice the blocks of powers of 20 in the vertical direction and notice that 5 points are equivalent to a bar. In figure 2, we show the representation of the number 4002 using the Mayan system. The methodology that, more likely, used the Maya to conduct their operations included the following rules [Morley, 1968;Thompson, 1941].

I. Five points are equivalent to one bar in the same level.
II. One bar is equivalent to five points in that level.
III. Four bars in a level are equivalent to one point in the superior immediate level.

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IV. One point in a level is equivalent to four bars in the inferior immediate level.

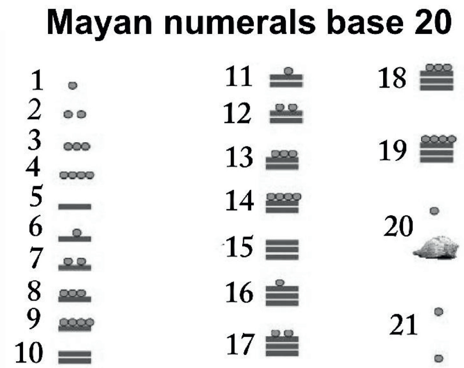


Figure 1

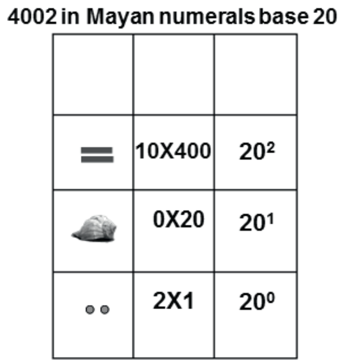


Figure 2

Here, we will use the same methodology to perform arithmetic operations, but we will do it in base 10 for reasons of clarity. We show, in figures 3 and 4, Mayan numerals in base10. Notice now the blocks of powers of 10 in the vertical direction. To perform arithmetic operations in base 10, we will use the following rules [Magaña, 2003, 2010, 2012], which are the same as the Mayan rules, with a small change only.

I. Five points are equivalent to one bar in the same level.

II. One bar is equivalent to five points in that level.

III. Two bars in a level are equivalent to one point in the superior immediate level.

IV. One point in a level is equivalent to two bars in the inferior immediate level.

It is not necessary the use of tables of any kind to carry out the arithmetic operations. We can perform the addition, subtraction, multiplication, division and square root. The mathematical proofs that support this methodology are suggested by Calderón (1966) and given by Magaña (1990).

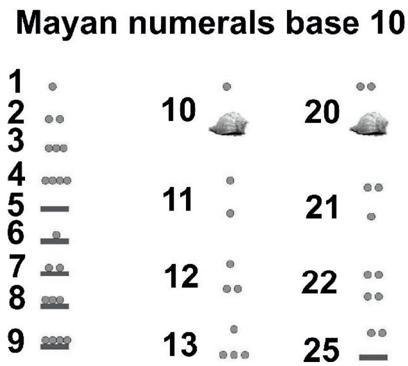


Figure 3

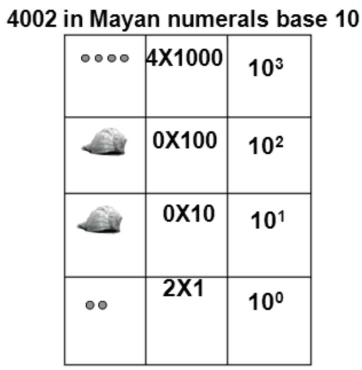


Figure 4

2. Addition

We make the operation 161 + 671, as we indicate in figure 5. We make the addition in figure 6, grouping points and bars on each level and transforming bars into points, as shown figure 7. For every two bars, we put a point in the superior immediate level, like in figures 7, 8 and 9. We show the result in figure 9, and is 832. For adding several numbers, the procedure is the same. The addition of many numbers can be done very quickly.

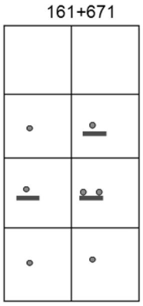
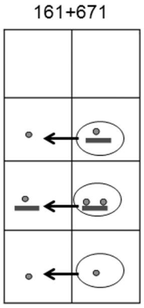


Figure 5



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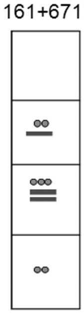

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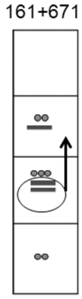

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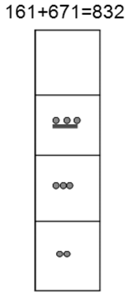


Figure 9

3. Subtraction

The rule for the subtraction is that one point annihilates one point, and one bar annihilates one bar. We will perform the subtraction 752-644. Notice that in figure 10, the number 752 is in the column on the left, and the number 644 is on the right. We work on the minuend without altering the subtrahend. We proceed along for each level, starting from the lowest one. In figures 10 to 16, we describe the subtraction procedure. In figure 12, we change one bar by five dots. Then, one dot goes one level down, becoming two bars, as shown in figure 13. Afterwards, we convert one bar into five points again, as we show in figure 14. Then, we culminate the subtraction as we show in figures 15 and 16. Notice that we can make the test of the subtraction just by summing the final numbers on the left column, and in the column on the right in figure 16. This means 108+644=752.

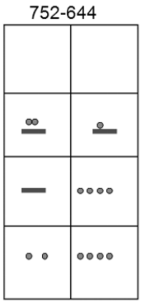
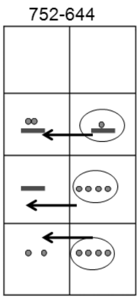


Figure 10



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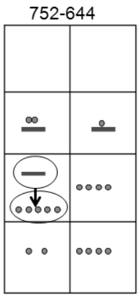

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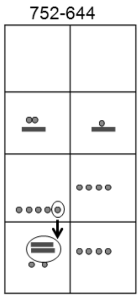

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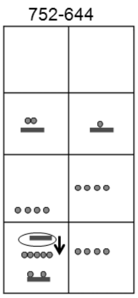

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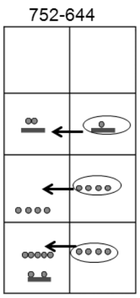

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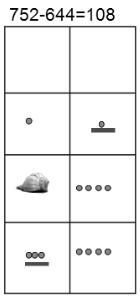
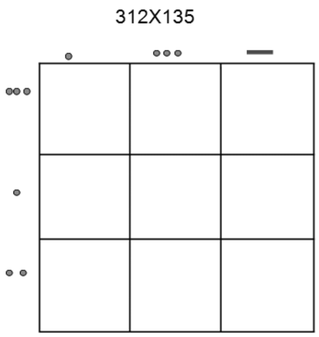


Figure 16

4. Multiplication

We do not need tables to perform a multiplication. We show this in the following example. Let us make the product 312X135. We put the factors outside the board; one (312), vertically and the other horizontally, like we show in figure 18. Then, we reproduce in each square, the figure that we have to the left by outside the board, so many times, as the number of the superior part indicates. Clearly, we can perform the reciprocal operation too. We do the easier of these two possibilities. We put a group of three dots, or thrice a dot in the first square of the left column. In this way, we solve the squares of the initial column on the left, as we show in figure 19. We solve the squares of the following columns in a similar fashion, as we show in figures 20 and 21. We have almost finished the multiplication.

Then, we start the final part to obtain the product. We group along the main diagonal, as we show on the figure 22. Each transversal diagonal corresponds to a power of 10. Afterwards, we use the rules we have given above. Each group of five dots is transformed into a bar, and every two bars become a dot in the superior immediate level, leaving a zero (this is a seashell) in its place. After doing this, we read the result directly along the main diagonal. The square of the right inferior corner corresponds to the units. We show the sequence in figures 23 to 25. Notice that in the figure 26 we have left a seashell in the square corresponding to the units. Now we read in the figure 25 the result: 42120.



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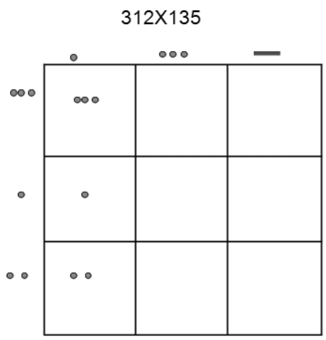

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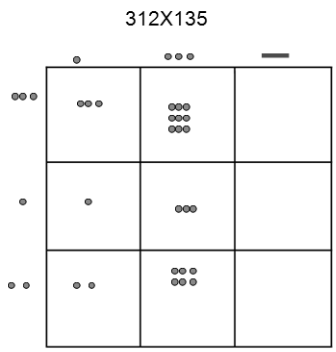

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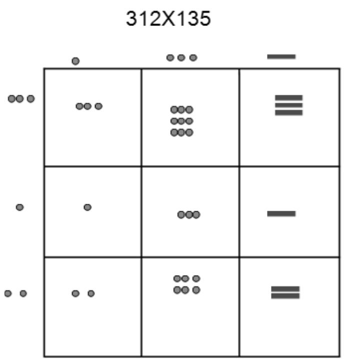

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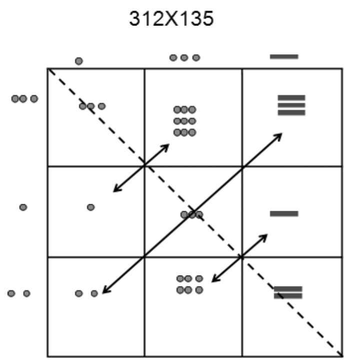

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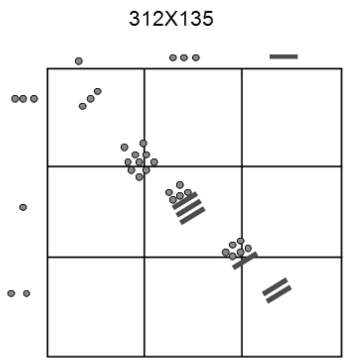

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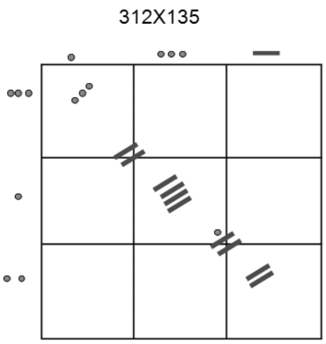

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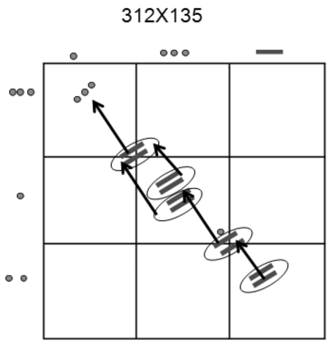


Figure 24

Figure 25

4.1 Non square matrix products

Now let us make the product of two numbers, which does not lead to a square matrix. We perform the product of 12 times 251. We show the sequence in figures 27 to 35. The dashed line indicates the main diagonal. We group along this diagonal, as we show in the figure 29. Then, we transform each group of five dots into a bar, and every two bars become a dot in the superior immediate level, leaving a zero in its place. The result is in the figure 32, along the main diagonal: 12X251=3012.

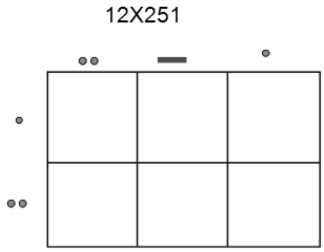
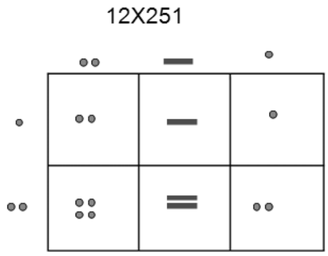


Figure 26



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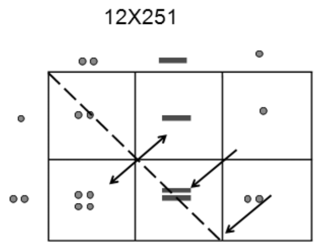

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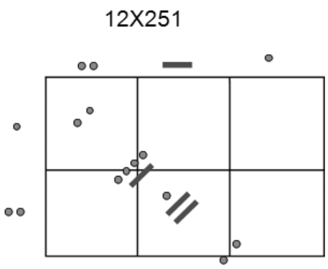

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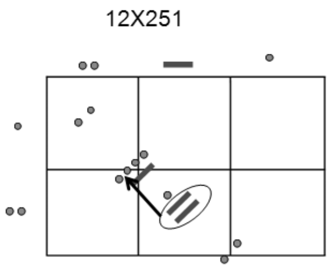

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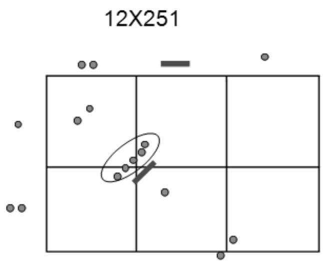

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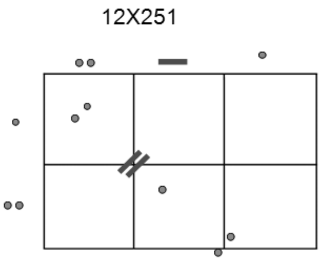
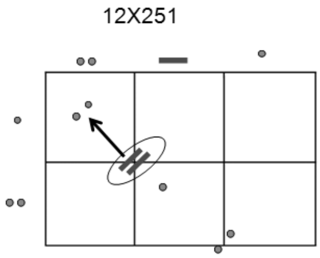


Figure 32



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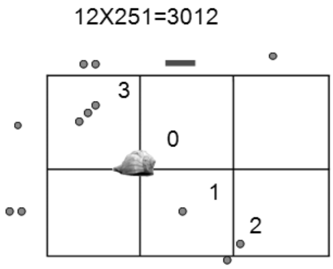


Figure 34

4.2 Constructing the multiplication tables

Let us now consider the product of two digits. We show how to obtain the product of six times seven. This is a typical case when students memorize without understanding the meaning of what they are doing. With this method, it is not necessary to memorize the product. They can reproduce it as many times as they need. However, what is more likely is that at large, they will memorize the result after going through the procedure enough number of times. For this case, our board is made of only one square. We place the factors outside of our board, as we show in figure 35. In the only square of our board, we reproduce six times one bar and two dots, as we show in the figure 36.

Then, we exchange every five dots with one bar. This means that we have to put two additional bars; see the figure 37. Afterwards, we substitute every pair of bars for one dot on the immediate superior level. We have four pairs of bars, which we show encircled in the figure 30. These four pairs correspond to four dots in the immediate superior level; see the figure 30. Finally, we can read the result: 42, in the figure 39.

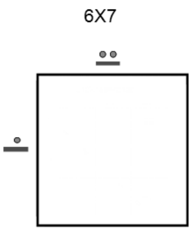
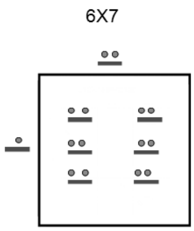


Figure 35



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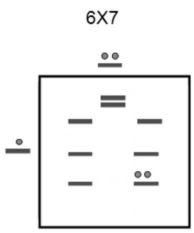

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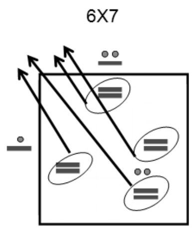

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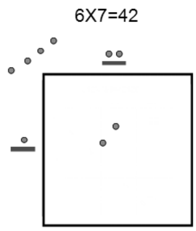


Figure 39

5. Division

This is the inverse operation of the multiplication, and thus we will solve it. The dividend is conceived like the product of two numbers. One of them is the divisor and the other, unknown, is the quotient. Therefore, the dividend is placed on the main diagonal of the board. We use our knowledge on the multiplication to go backwards, in order to find the missing factor, which is the quotient.

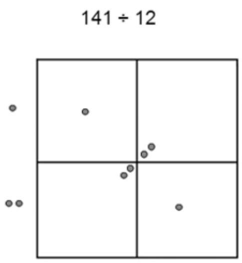
Let us consider 141 ÷ 12, see figure 40. We will put the divisor in a vertical form and by outside of the board. The quotient will be in a horizontal form and by the exterior of the board too. These locations can be exchanged with no problem. We begin the division proposing a number to be in the external part, by above of the square of the left corner. This choice is such that, reproducing the first figure external to the left (one dot) as many times as the number that we are looking for, we obtain the dot of the left superior square on the board. Clearly, we are following an inverse way from the multiplication.

We find that we have to put one dot, as we show in the figure 32. With this, we satisfy the multiplication rules for the first square of the initial column. In order to complete the inferior immediate square of the same column, we see that we need two points. We take those two points from the number that is in the diagonal, as we show in figures 41 and 42. In this way, we have completed satisfactorily the first column.

Now, we work with the second column, see figure 43. We must find the following number of the quotient that goes in the external part of the board by above of the second column. If we use a number two (i. e, two dots) we complete the first square of this column, but we exceed what we have in the inferior square of this column. In this way, we try a solution by using one dot, as we show in the figure 44. In the first square of the second column, one dot is in excess, thus we lower it to the inferior immediate square like two bars. We show this in figure 44. Then, we convert one of bars into five points, as we show in figure 45. We take the required two dots to leave in that square, and we displace the rest (one bar and four dots) to the first square of the third column. Clearly, 141÷12 results in 11 plus a residue of 9. We show this in figures 46 and 47, and we use a star to begin with decimal fractions.

Afterwards, we work with the third column; see figures 47, and 48. We must find the following number of the quotient that goes in the external part of the board by above of the third column. We try with a bar and two dots, as we show in the figure 48. A pair of dots is in excess in the first square on the third column. We lower the two dots to the immediate inferior level, as four bars; see figure 48. Then, we convert one bar into five dots. We leave the required two bars and four dots in that square; see figures 49 and 50. We displace the remaining bar and dot, to the upper square of the fourth column; see figures 50 and 51.

Then, we work with the fourth column; see figures 51, and 52. We must find the following number of the quotient that goes in the external part of the board by above of the fourth column. We try with a bar, as we show in the figure 52. A dot is in excess in the upper square of the fourth column, as we show in the figure 52. We lower that dot, as two bars, to the immediate inferior level. See figures 52 and 53. In the inferior square of the fourth column, we must have two bars, and we have exactly that. Thus the division has concluded, and 141÷12= 11.75, as we show in the figure 53.



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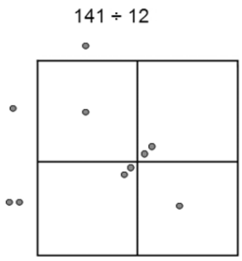

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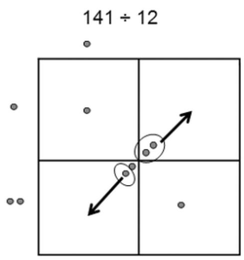

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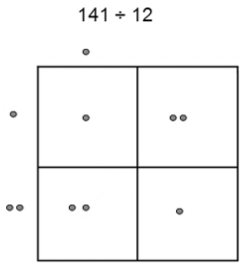

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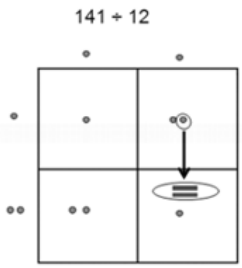

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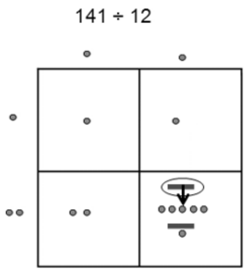

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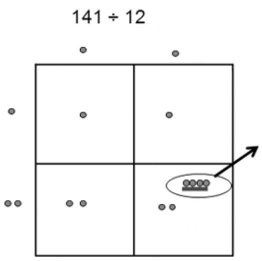

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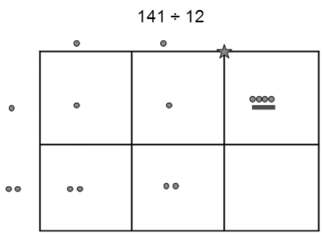

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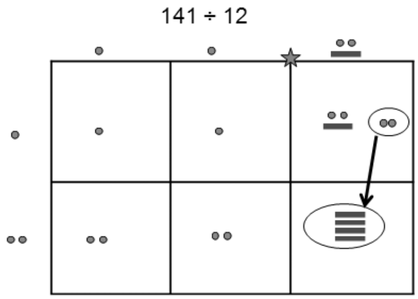

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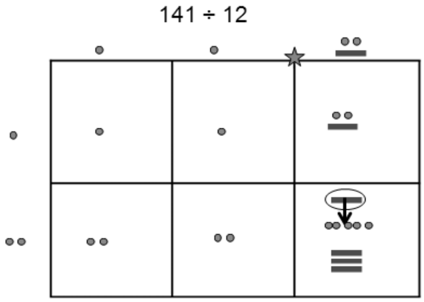

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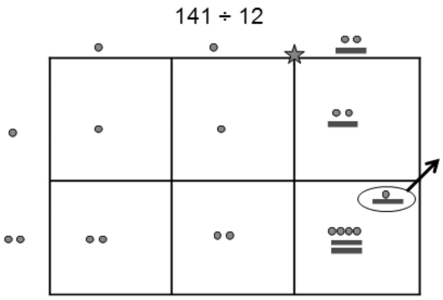

Figure 50

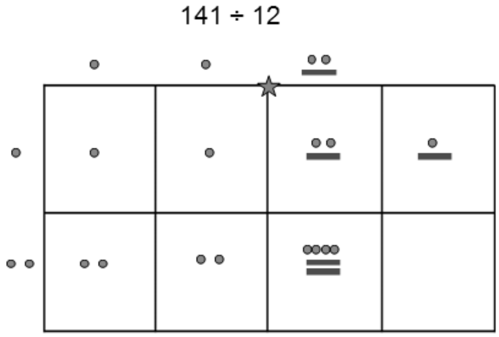

Figure 51

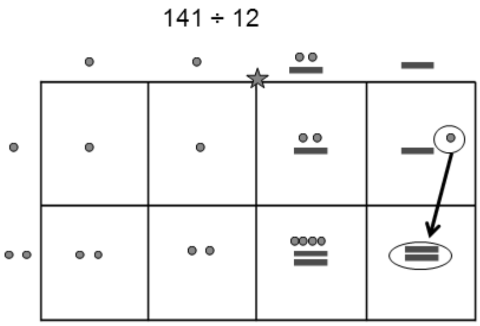

Figure 52

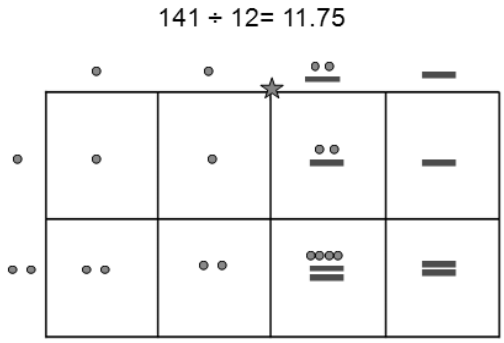

Figure 53

6. Square root

We solve the square root in the same manner that we solve the division. The radicand is the dividend. Of course that we do not know the divisor or the quotient, but we know that they are the same, and we use this fact to find the solution.

Let us find the square root of 225. As we are considering it as a division, we place the radicand along the main diagonal of the board; see the figure 54. We proceed as we did in the division, but knowing that the quotient must be equal to the divisor. We have two dots within the square of the left corner of the board. We try one dot like the first digit of the divisor and of the quotient, as we show in the figure 55. With this trial, one dot is in excess in the upper square belonging to the column on the left. We lower this dot to the immediate inferior level, along the main diagonal, as two bars. See figures 55 and 56. After finding the solution for the first square, we follow an additional rule for the square root. We have to distribute symmetrically as possible, the number of the radicand located in the immediate inferior level. We distribute among the squares located on the diagonal that corresponds to the same power of 10. We show this distribution in figures 57 and 58.

Now we proceed to find the following number of the solution. We may try a bar in the external superior part of the second column, and simultaneously, in the external part of the second row on the board, see figure 59. We see that with this trial, we require having only one bar in the first square of the second column, and we have one dot in excess. We lower that dot to the immediate inferior level, as two bars. We also require having only one bar on the first square of the second row. Again, we have one dot in excess. We lower it to the immediate inferior level too. See the figure 60. Finally, we need to solve the second square of the second column. In this square, we already have five bars. These five bars correspond to the number of bars that we need, according to our trial of a bar. In this manner, the square root is solved, see figure 61. The result is exact and is 15.

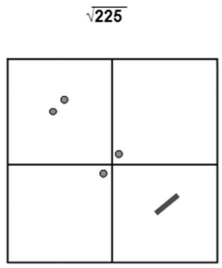

Figure 54

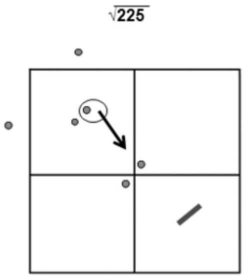

Figure 55

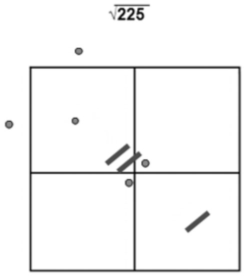

Figure 56

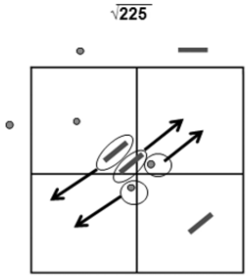

Figure 57

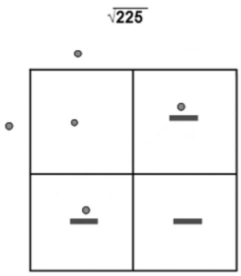

Figure 58

Figure 59

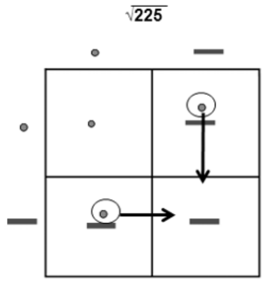

Figure 60

Figure 61

7. Preliminary results from the application of this method

This method has been used as an auxiliary tool for teaching mathematics in some of the elementary schools in Yucatán, Mexico. These schools are located in the rural zones.

It is convenient to mention that the resulting algorithms from this method, when using Arabic notation, are simpler than the ones which are currently taught [Magaña, 2012]. However, it is necessary to master Mayan mathematics previously. Besides,

The students at those schools are traditionally very weak in mathematics learning. In 2010, the Yucatan Public Education Secretary asked me to start a training pilot program on Mayan mathematics. This was for teachers of the elementary school level. Then, since 2011, those teachers taught Mayan mathematics during half an hour at the end of the day, after the regular program was satisfied. The ludic nature of Mayan mathematics was very useful to attract children´s attention. The national tests on mathematics for the elementary schools in 2011 and 2012 were very satisfactory for those children. It is convenient to mention that the elementary school in Peto in a distant rural zone in Yucatan was the second place in the last two years. This suggests the convenience of changing the traditional way of teaching mathematics to this manner.

As a final comment, we should mention that the nature of Mayan mathematics induces to mathematical reasoning. It is not necessary at all, to memorize tables of any kind. This occurs with each of the arithmetic operations. With Mayan mathematics, children understand the concept behind each arithmetic operation.

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