Assignment

Incremental differentiation

1. Let $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, if $R_1 = 300$ ohms with maximum error of 2% and $R_2 = 500$ ohms with maximum error of 3%. Find maximum % of error in R.

2. The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm and 6cm respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible approximate error in the value computed for the volume where the volume of the cylinder is given by $\pi r^2 h$.

3. The diameter and height of a right circular cylinder are found by measurement to be 8cm and 12.5cm respectively with the possible errors of 0.05cm in each measurement. Find the maximum possible approximate error in the computed volume where the volume of the cylinder is given by $\pi r^2 h$.

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4. If $w = f\left(\frac{r-s}{s}\right)$. Show $r\frac{dw}{dr} + s\frac{dw}{ds} = 0$.

5. If $w = f(x, y)$. Where $x = e^u \cos v$, $y = e^u \sin v$. Show $y\frac{dw}{du} + x\frac{dw}{dv} = e^{2u}\frac{dw}{dy}$.

6. If $z = f(u, v^2)$. Show $2u\frac{dz}{du} - v\frac{dz}{dv} = 0$.

7. If $z = \log(x^2 + y^2 + xy)$. Show $x\frac{dz}{dx} + y\frac{dz}{dy} = 2$.

8. If $x = f(2a - 3b, 3b - 4c, 4c - 2a)$. Prove that $\frac{1}{2}\frac{\partial x}{\partial a} + \frac{1}{3}\frac{\partial x}{\partial b} + \frac{1}{4}\frac{\partial x}{\partial c} = 0$.

9. Examine whether the function $f(x, y) = \begin{cases} x^2 + 4y^2 & (x, y) \neq (1,2) \\ 0 & (x, y) = (1,2) \end{cases}$ is continuous at (1,2).

Jacobian

10. If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$. Show that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$.

11. If $x = r\cos\theta$, $y = \sin\theta$. Prove that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$. Also evaluate $\frac{\partial(r,\theta)}{\partial(x,y)}$.

12. Verify chain rule for Jacobian if $x = a$, $y = a\tan b$, $z = c$ where x, y and z are function of a, b and c and a,b and c are functions of x, y and z.

13. Compute Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ when $u = x\sin y\cos z$, $v = x\sin y\sin z$ and $w = x\cos y$.

Hessian

14. Find the relative maxima/minima using Hessian matrix for function $f(x, y, z) = 6x^2 - 9x - 3xy - 7y + 5y^2$.

15. Find the relative maxima/minima using Hessian matrix for $f(x, y, z) = x^2 + y^2 + z^2 - 4x + 8y - 12z + 56$.

Vector Calculus

16. Find an equation for the tangent plane to the surface $xz^2 + x^2y = z - 1$ at the point (1, -3, 2).

17. Find an equation for the tangent plane to the surface $2x^2 - y^2 + 5y$ at the point (-2, 2, 14).

18. Find an equation for the tangent plane to the surface $4x^3y - 7xy^2$ at the point (1,1,-3).

19. Find the directional derivative of function $\varphi = x^2yz + 4xz^2$ at point (1, -2, 1) in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

20. Find the directional derivative of function $\varphi = xy + yz + zx$ at point (1, 2, 0) in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

21. Find the directional derivative of function $\varphi = (x^2 + y^2 + z^2)^{-1/2}$at point (3,1,2) in the direction of $yz\,\mathbf{i} + zx\,\mathbf{j} + xy\,\mathbf{k}$.

22. Find the directional derivative of the function $\varphi = x^2 - y^2 + 2z^2$ at the point P (1,2,3) in the direction of the line PQ where Q is the point (5,0,4).

23. If $A(x, y, z) = x^2z\,\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\,\mathbf{k}$. Find $\nabla . \boldsymbol{A}$ at the point $(1, -1, 1)$.

24. If $A(x, y, z) = xz^3\,\mathbf{i} - 2x^2z\,\mathbf{j} + 2y^2z^4\,\mathbf{k}$. Find $\nabla \times \boldsymbol{A}$ at the point $(-1, -1, -2)$.

25. Find div F where $F = grad(x^3 + y^3 + z^3 - 3xyz)$

26. Find divergence of the vector function $A(x, y, z) = e^{xyz}(xy^2\,\mathbf{i} + yz^2\,\mathbf{j} + zx^2\,\mathbf{k})$ at the point P(1,2,3)

Maxima Minima

27. Use second derivative test to determine whether each critical number of function $f(x) = 12 + 2x^2 - x^4$ corresponds to a relative maximum, a relative minimum, or neither.

28. Use second derivative test to determine whether each critical number of function $f(x) = 12 + 2x^2 - x^4$ corresponds to a relative maximum, a relative minimum, or neither.

29. Find the maxima and minima for the following function $f(x, y) = 3x^2 - y^2 + x^3$

30. Find the maxima and minima for the following function $f(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$

31. Determine the critical points for the function $f(x) = x^3 + x^2 - 5x - 5$.

32. Determine the critical points for the function $f(x) = x^3 + 6x^2 - 15x + 2$.

33. If (a, b) are the critical points of a function $z = f(x, y)$ and $r = f_{xx}, s = f_{xy}, t = f_{yy}$ then write down the conditions for z having a maximum or minimum value at point (a, b).

Lagrange Multiplier

34. Fine the extrema of the function $f(x, y, z) = x^2 + y^2 + z^2$ such that constraint $g_1(x) = x^2 + y^2 - 1 = 0$ and $g_2(x) = x + y + z - 1 = 0$ using the method of Lagrange multiplier.

35. Write the expression for Lagrange's function. Use Lagrange multiplier method to determine the dimension of a rectangular box, open at the top having a volume of 32 ft^3 and requiring the least amount of material for its construction.

36. Find the point on the circle $x^2 + y^2 = 4$ that is closest to the point (3, 4) using method of Lagrange multiplier.

Taylor Series

37. Expand $f(x, y) = x^2 + xy + y^2$ in powers of $(x - 2)$ and $(y - 3)$ using Taylor's theorem.

38. Expand $f(x, y) = x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem.

39. Expand the function $f(x, y) = e^x \sin y$ in powers of x and y using Taylor's theorem, at (0, 0) as far as terms of third degree i.e., for $n = 3$.

40. Expand the function $f(x, y) = e^{x+y}$ using Taylor's theorem, at (0, 0) as far as terms of third degree i.e., for $n = 3$.

41. Expand $log\, x$ in ascending power of $(x - 1)$ by using Taylor's theorem.

42. Expand $\log(a + h)$ by using Taylor's theorem.