EXPONENTIAL SMOOTHING

- Exponential smoothing is the most widely used class of procedures for smoothing discrete time series to forecast the immediate future.
- In exponential smoothing, however, we want to allow the more recent values of the series to have greater influence on the forecast of future values than the more distant observations.
- Exponential smoothing is a simple and pragmatic approach to forecasting, whereby the forecast is constructed from an **exponentially weighted average** of past observations.

SIMPLE EXPONENTIAL SMOOTHING

- This forecasting method is most widely used of all forecasting techniques. It requires little computation.
- This method is used when data pattern is approximately horizontal (i.e., there is neither cyclic variation nor trend in the historical data).
- Let an observed time series be y1, y2, …. yn. Formally, the simple exponential smoothing equation takes the form of

$$S_{t+1} = \alpha y_t + (1-\alpha)\, S_t$$

Where $S_i \rightarrow$ The smoothed value of time series at time i

$y_i \rightarrow$ Actual value of time series at time i

$\alpha \rightarrow$ Smoothing constant

In case of simple exponential smoothing, the smoothed statistic is the forecasted value.

$$F_{t+1} = \alpha y_t + (1-\alpha) F_t$$

Where F_{t+1} → Forecasted value of time series at time t+1

Ft → Forecasted value of time series at time t

This means:

$F_t = \alpha y_{t-1} + (1-\alpha) F_{t-1}$

$F_{t-1} = \alpha y_{t-1} + (1-\alpha) F_{t-2}$

$F_{t-2} = \alpha y_{t-2} + (1-\alpha) F_{t-3}$

$F_{t-3} = \alpha y_{t-3} + (1-\alpha) F_{t-4}$

Substituting, $F_{t+1} = \alpha y_t + (1-\alpha) F_t = \alpha y_t + (1-\alpha)(\alpha y_{t-1} + (1-\alpha)F_{t-1}) =$

$= \alpha y_t + \alpha (1-\alpha) y_{t-1} + (1-\alpha)^2 F_{t-1} =$

$= \alpha y_t + \alpha (1-\alpha) y_{t-1} + \alpha (1-\alpha)^2 y_{t-2} + (1-\alpha)^3 F_{t-2}$

$= \alpha y_t + \alpha (1-\alpha) y_{t-1} + \alpha (1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 y_{t-3} + (1-\alpha)^4 F_{t-3}$

Generalizing,

$$F_{t+1} = \sum_{i=0}^{t-1} \alpha(1-\alpha)^i y_{t-i} + (1-\alpha)^t F_1$$

The series of weights used in producing the forecast F_t are α , $\alpha (1-\alpha)$, $\alpha(1-\alpha)^2$, $\alpha(1-\alpha)^3$…. These weights decline toward zero in an exponential fashion; thus, as we go back in the series, each value has a smaller weight in terms of its effect on the forecast. The exponential decline of the weights towards zero is evident.

<u>Choosing α:</u>

After the model is specified, its performance characteristics should be verified or validated by comparison of its forecast with historical data for the process it was designed to forecast.

We can use the error measures such as *MAPE* (Mean absolute percentage error), *MSE* (Mean square error) or *RMSE* (Root mean square error) and α is chosen such that the error is minimum.

Usually the *MSE* or *RMSE* can be used as the criterion for selecting an appropriate smoothing constant. For instance, by assigning a values from 0.1 to 0.99, we select the value that produces the smallest *MSE* or *RMSE*

Since F1 is not known, we can:

- Set the first estimate equal to the first observation. Thus we can use 1
- Use the average of the first five or six observations for the initial smoothed value.

DOUBLE EXPONENTIAL SMOOTHIG-HOLT'S TREND METHOD

Under the assumption of no trend in the data, simple exponential smoothing yields goods results but it fails in case of existence of trend. Double exponential smoothing is used when there is a linear trend in the data.

The basic idea behind double exponential smoothing is to introduce a term to take into account the possibility of a series exhibiting some form of trend. This slope component is itself updated via exponential smoothing.

Suppose the data exhibits a linear trend as:

$y_t = b_0 + b_1 t + e_t$

where, b_0 and b_1 may change slowly with time.

The basic equations for Holt's Method are:

1. $\mu_t = \alpha y_t + (1 - \alpha)(\mu_{t-1} + T_{t-1})$
2. $T_t = \beta(\mu_t - \mu_{t-1}) + (1 - \beta)T_{t-1}$
3. $F_{t+m} = \mu_t + mT_t$

where

$\mu_t \rightarrow$ *Exponentially smoothed value of the series at time t*

$y_t \rightarrow$Actual observation of time series at time t

Tt $\rightarrow$Trend Estimate

$\alpha\rightarrow$Exponential Smoothing Constant for the data

$\beta\rightarrow$Smoothing constant for trend

$F_{t+m}\rightarrow$m period ahead forecasted value

The difference between 2 successive exponential smoothing values is $(\mu_t - \mu_{t-1})$ used as an estimate of the trend. The estimate of the trend is smoothed by multiplying it by β and then multiplying the old estimate of the trend by $(1 – \beta)$. To forecast, the trend is multiplied by the number of periods ahead that one desires to forecast and then the product is added to μ_t.

Choice of α and β

Choose one that minimize MSE or MAPE.

Initialization

Level : $\mu_1 = y_1$

Trend : $T_1 = (y_2 - y_1)/(T_2 - T_1)$ or $(y_4 - y_1)/(T_4 - T_1)$

TRIPLE EXPONENTIAL SMOOTHING HOLT'S WINTERS TREND AND SEASONALITY METHOD:

Under the assumption of presence of only linear trend in the data, double exponential smoothing yields goods results but it fails in case of existence of trend and seasonality. Triple exponential smoothing is used when there is trend in the data along with seasonal variations.

Two Holt-Winters methods are designed for time series that exhibit linear trend

1. **Additive Holt-Winters method**: used for time series with constant (additive) seasonal variations
2. **Multiplicative Holt-Winters method**: used for time series with increasing (multiplicative) seasonal variations

Holt- Winter's Trend and Seasonality Method for Multiplicative Model:

It is generally considered to be best suited to forecasting time series that can be described by the equation:

$$y_t = (T_t * S_t * I_t)$$

This method is appropriate when a time series has a linear trend with a multiplicative seasonal pattern.

- Smoothing equation for the series
  $\mu_t = \alpha \frac{Yt}{St-p} + (1-\alpha)(\mu_{t-1} + b_{t-1}) \qquad 0 \leq \alpha \leq 1$
- Trend estimating equation
  $b_t = \beta(\mu_t - \mu_{t-1}) + (1-\beta) b_{t-1}$
- Seasonality updating equation
  $S_t = \gamma \frac{Yt}{\mu t} + (1-\gamma) S_{t-p}$
- Forecast equation
  $F_{t+m} = (\mu_t + m\, b_t) S_{t+m-p}$

where

μ_t →Exponentially smoothed value of the series at time t

y_t →Actual observation of time series at time t

T_t →Trend Estimate

α→Exponential Smoothing Constant for the data

β→Smoothing constant for trend

γ→ Smoothing constant for seasonality

F_{t+m}→m period ahead forecasted value

p →the period of seasonality (p=4 for quarterly data & p=12 for monthly data

<u>Initialising:</u>

$\mu_p = (y_1 + y_2 + \ldots .. y_p) / p$

$b_p = ((y_{p+1} + y_{p+2} + \ldots y_{p+p}) - (y_1 + y_2 + \ldots . y_p)) / p^2$

$S_i = y_i / \mu_p \qquad i=1,2,3\ldots .p$

<u>Choice of α,β,γ</u>

α is used to smooth randomness, β to smooth trend and γ to smooth seasonality. Choose α,β,γ which minimize MSE or MAPE.

Holt- Winter's Trend and Seasonality Method for Additive Model:

It is generally considered to be best suited to forecasting time series that can be described by the equation:

$$y_t=(T_t + S_t + I_t)$$

- Exponentially smoothed series equation

  $\mu_t=\alpha\ (y_t-S_{t-p}) + (1-\alpha)\ (\mu_{t-1}+b_{t-1})$ $\qquad 0\leq \alpha \leq 1$
- Trend estimating equation

  $b_t = \beta(\mu_t - \mu_{t-1}) + (1-\beta)\ b_{t-1}$
- Seasonality updating equation

  $S_t = \gamma\ (y_t-\mu_t)+ (1-\gamma)\ S_{t-p}$
- Forecast equation

  $F_{t+m} = \mu_t + m\ b_t+S_{t+m-p}$

where

μ_t → Exponentially smoothed value of the series at time t

y_t → Actual observation of time series at time t

T_t → Trend Estimate

α → Exponential Smoothing Constant for the data

β → Smoothing constant for trend

γ → Smoothing constant for seasonality

F_{t+m} → m period ahead forecasted value

p → the period of seasonality (p=4 for quarterly data & p=12 for monthly data)

Initialising:

$\mu_p = (y_1 + y_2 + \ldots\ldots y_p) / p$

$b_p = ((y_{p+1} + y_{p+2} + \ldots y_{p+p})-(y_1+y_2+\ldots\ldots y_p)) / p^2$

$S_i = y_i - \mu_p \qquad i=1,2,3\ldots\ldots p$

Choice of α,β,γ

α is used to smooth randomness, β to smooth trend and γ to smooth seasonality. Choose α,β,γ that minimize MSE or MAPE.

- **Time series**- A time series consists of data which are arranged chronologically. It establishes a relationship between two variables in which one of the variable is independent variable i.e. the time and other variable y is the dependent variable whose value changes with regard to time variable e.g. total agricultural production in different years.

  Mathematically, a time series is defined by the values Y_1, Y_2, Y_3… Y_n of the variable Y at times t_1, t_2, t_3… t_n.

  Symbolically, Y = f (t), i.e., Y is a function of time t.

  Components of time series

  A time series consists of the following four components-

  - Trend
  - Seasonal variations
  - Cyclical variations
  - Irregular variations

- **Trend**-Trend refers to long term movement in the time series, i.e. Trend refers to the ability of the time series to increase or to decrease or to remain constant over a long period of time. If the values of the variable are scattered around a straight line, then we have a linear trend. Otherwise, the trend is non-linear e.g. long- term changes in productivity.

- **Seasonal variations**- Seasonal variations involve patterns of change within a year that tend to be repeated from year to year. They are short- term periodic movements. The time interval of occurrence of seasonal variations may vary from a few hours to a few weeks or a few months. To note the seasonal variations, the data must be recorded at least quarterly, monthly, weekly, or daily depending on the nature of the variable under consideration.

- **Cyclical variations**- Cyclical variations are oscillatory variations in the time series that oscillate around the trend line with period of oscillation as more than one year. These variations do not follow any regular pattern and move in somewhat unpredictable manner. These are upswings and downswings in the time series that are observable over extended periods of time.

- **Irregular variations**- The irregular component of the time series is the residual factor that accounts for the deviations of the actual time series values from what we would expect from the trend, seasonal, and cyclical components. It accounts for the random availability in the time series. The irregular component is caused by the short-term, unanticipated, and non-recurring factors that affect the time series, viz. earthquakes, floods etc.

MODELS OF DECOMPOSITION

There are two models of decomposition of time series:

- **The additive model**- This model is used when it is assumed that the four components are independent of one another, i.e., when the pattern of occurrence and the magnitude of movement in any particular component are not affected by other components under this assumption, the magnitude of time series (Y(t)), at any time t is the sum of the separate influences of its four components, i.e.

$$Y(t) = T(t) + S(t) + C(t) + I(t)$$

where T(t)= Trend variations
S(t)= seasonal variations
C(t)= cyclical variations
I(t)= irregular variations

- **Multiplicative model**-This model is used when it is assumed that the components may depend on each other.

$$Y(t) = T(t)*S(t)*C(t)*I(t)$$

- The key smoothing hyperparameters are α, β, γ