Assignment - (V)

RMC

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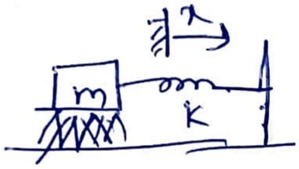
(9.2) $m=2,\ b=6,\ k=4$

given initial position $x(0)=1$



char eq.^{n} is,

$$ms^2+bs+k=0$$

$$2s^2+6s+4=0$$

$$(s+1)(s+2)=0$$

$$\boxed{s=-1,-2}$$

Response has the form,

$$x(t)=c_1e^{-t}+c_2e^{-2t}$$

at $t=0$,

$$x(0)=c_1+c_2=1$$

$$c_1=1-c_2 \quad \text{—(1)}$$

WKT, $\dot{x}(0)=0$; $0=-c_1-2c_2$

sub in (1) we get, $-1+c_2-2c_2=0$

$$\boxed{c_2=-1}$$

we get, $\boxed{c_1=2}$

$$\boxed{x(t)=2e^{-t}-e^{-2t}}$$

(9.3) $m=1,\ b=2,\ k=1$; initial position $x(0)=4$.

char eq.^{n} is $\quad ms^2+bs+k=0$

$$s^2+2s+1=0.$$

$$(s+1)(s+1)=0$$

$$s_1=s_2=-1$$

PS ②

$x(t) = c_1 e^{s_1 t} + c_2 t e^{s_2 t}$

at $t=0$,

$\boxed{4 = c_1}$

$\dot{x}(t) = s_1 c_1 e^{s_1 t} + c_2 [s_2 t e^{s_2 t} + e^{s_2 t}]$

$0 = -c_1 + c_2$

$c_2 = c_1$

$\boxed{c_2 = 4}$

$\boxed{x(t) = 4e^{-t}(1+t)}$

9.8

$m=1,\ b=4,\ k=5$

$w_{res} = 6.0$ rad/sec

control law, $ms^2 + (b+k_v)s + (k+k_p) = 0$ — ①

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ — ②

comparing, $b + k_v = 2\zeta\omega_n$

$4 + k_v = \zeta\omega_{res}$.

$4 + k_v = 6 \quad (\zeta = 1)$

$\boxed{k_v = 2}$

$k + k_p = \omega_n^2 \Rightarrow 5 + k_p = 9$

$\boxed{k_p = 4}$

$2\omega_n = \omega_{res}$

$\boxed{\omega_{res} = 3}$

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9.21 $m=3,\ b=5,\ k=2$

initial position $x(0)=3$

char. eq.^{n} is, $ms^2+bs+k=0$

$$3s^2+5s+2=0$$

$$s=\frac{-5\pm\sqrt{25-24}}{6} \Rightarrow s=\frac{-4}{6};-1$$

$$s_1=-4/6;\quad s_2=-1$$

response :- $x(t)=c_1e^{-s_1t}+c_2e^{-s_2t}$

$$x(0)=c_1+c_2=3 \Rightarrow c_1=3-c_2$$

$$x(t)=0$$

$$\frac{2}{3}(3-c_2)+c_2=0.$$

$$\boxed{c_2=-6}\ ;\ \boxed{c_1=9}$$

$$\boxed{x(t)=9e^{-2/3t}-6e^{-t}}$$

9.22 $m=2,\ b=3,\ k=8,\ \omega_{res}=6$ rad/sec.

Control: $ms^2+(b+kv)s+(k+kp)=0$ —(1)

also: $s^2+2Z\omega_n+\omega_n^2=0.$ —(2)

comparing $\frac{b+kv}{m}=2Z\omega_n$

$$\frac{3+kv}{2}=6 \Rightarrow \boxed{kv=9}$$

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$$\frac{k+k_p}{m} = \omega_n^2$$

$$8 + k_p = 18$$

$$\boxed{k_p = 10}$$

$$2\omega_n = 6$$

$$\boxed{\omega_n = 3}$$

10.1

$$z = \alpha t' + \beta$$

comparing: $\alpha = 2\sqrt{\theta} + 1$

$\beta = 3\theta^2 - \sin\theta$.

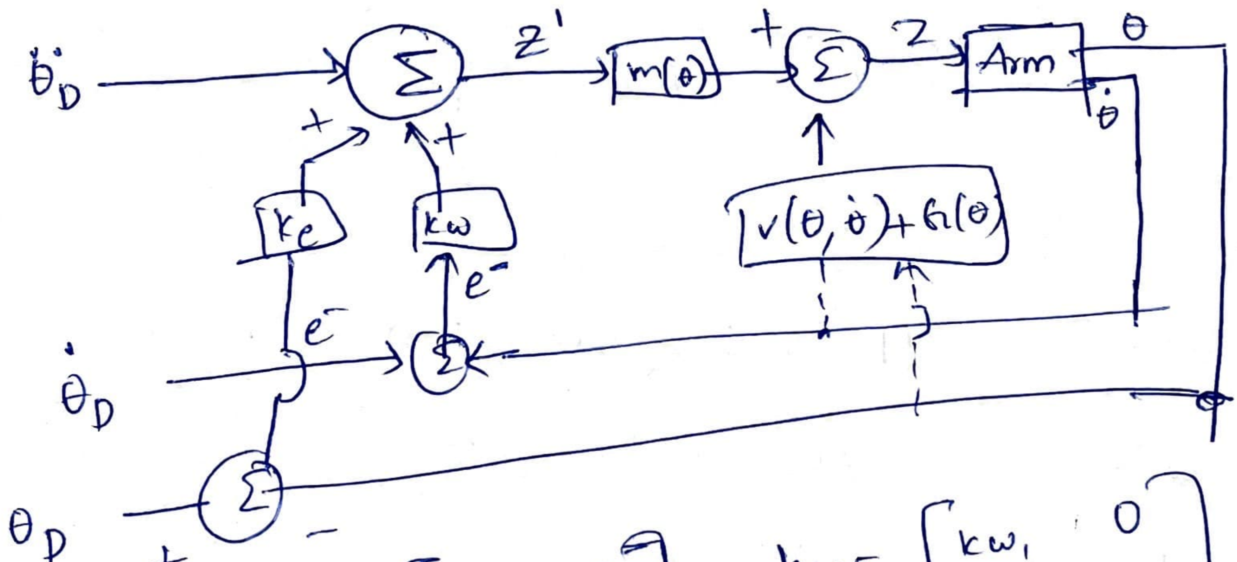
$$z' = \theta_D + k_\omega e + k_\theta e$$

& $e = e_D - \theta$.

$$K_{CL} = k_\theta = 10, \quad k_\omega = 2\sqrt{k_{CL}} = 2\sqrt{10}$$

$$\boxed{k_\omega = 2\sqrt{10}}$$

10.3



$$k_\theta = \begin{bmatrix} k\theta_1 & 0 \\ 0 & k\theta_2 \end{bmatrix}, \quad k_\omega = \begin{bmatrix} k\omega_1 & 0 \\ 0 & k\omega_2 \end{bmatrix}$$