Additional Lecture Notes on

Denavit-Hartenberg Formulation of

Homogeneous Transformation

By using

John Craig Notation

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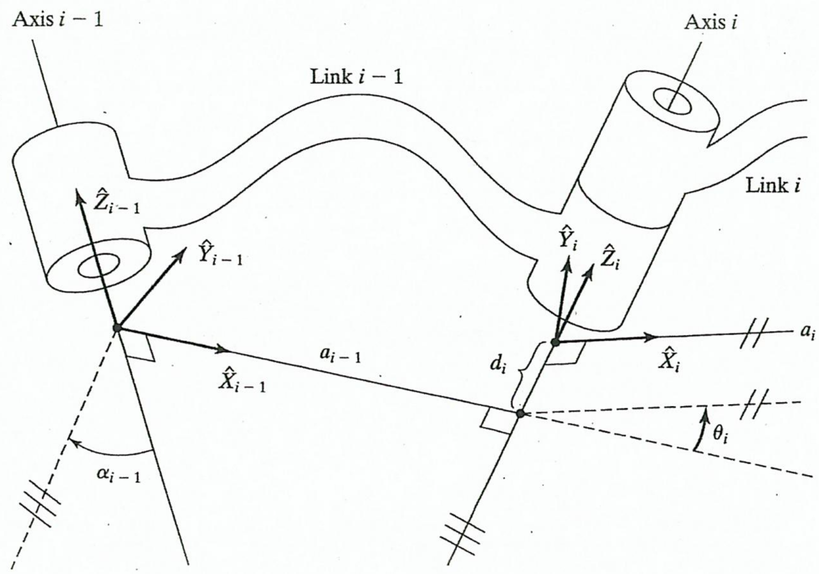
Denavit - Hartenberg Formulation of Homogeneous Transformation



Link frames are attached so that frame $\{i\}$ is attached rigidly to link i.

$a_i = the\ distance\ from\ \hat{Z}_i\ to\ \hat{Z}_{i+1}\ measured\ along\ \hat{X}_i$;

$\alpha_i = the\ angle\ from\ \hat{Z}_i\ to\ \hat{Z}_{i+1}\ measured\ about\ \hat{X}_i$;

$d_i = the\ distance\ from\ \hat{X}_{i-1}\ to\ \hat{X}_i\ measured\ along\ \hat{Z}_i$; and

$\theta_i = the\ angle\ from\ \hat{X}_{i-1}\ to\ \hat{X}_i\ measured\ about\ \hat{Z}_i$.

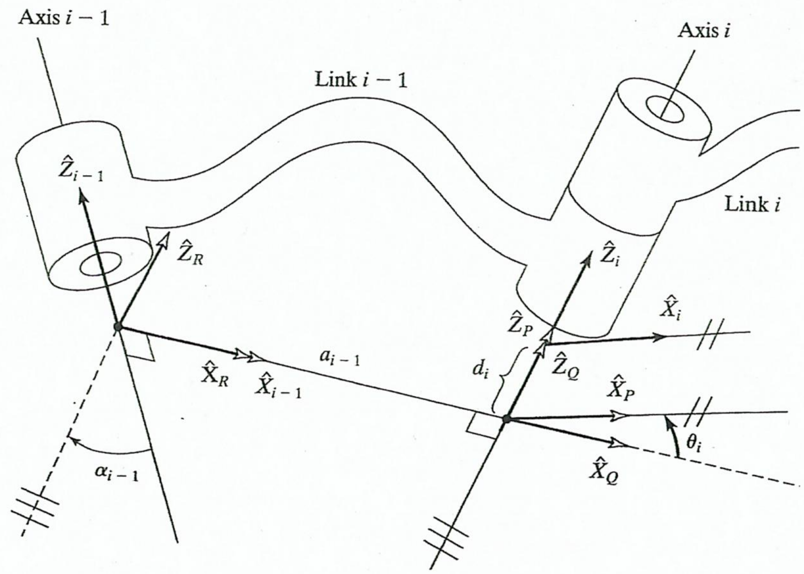


FIGURE 3.15: Location of intermediate frames $\{P\}$, $\{Q\}$, and $\{R\}$.

Frame $\{Q\}$ differs from $\{R\}$ by a translation a_{i-1}. Frame $\{P\}$ differs from $\{Q\}$ by a rotation θ_i, and frame $\{i\}$ differs from $\{P\}$ by a translation d_i. If we wish to write the transformation that transforms vectors defined in $\{i\}$ to their description in $\{i-1\}$, we may write

$$ {}^{i-1}P = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T \; {}^{i}P, $$

or

$$ {}^{i-1}P = {}^{i-1}_{i}T \; {}^{i}P, $$

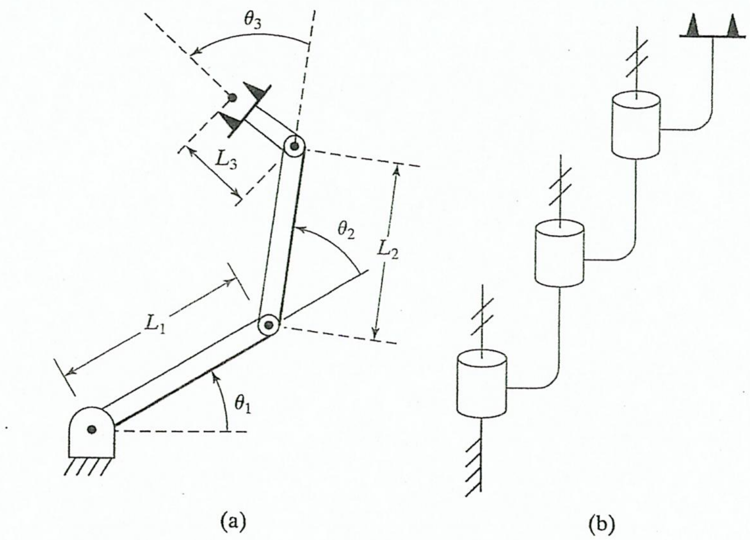
where

$$ {}^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T. $$

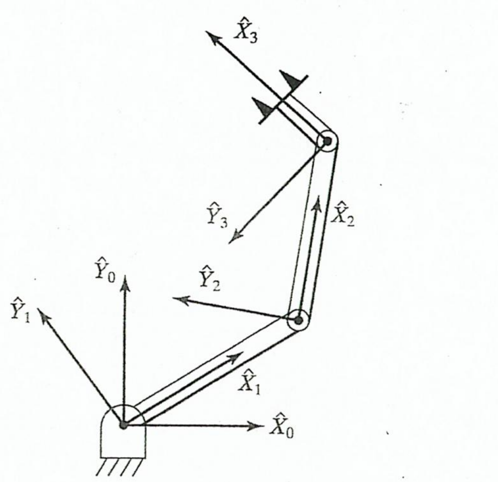
Considering each of these transformations, we see that

$$ {}^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i), $$

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$



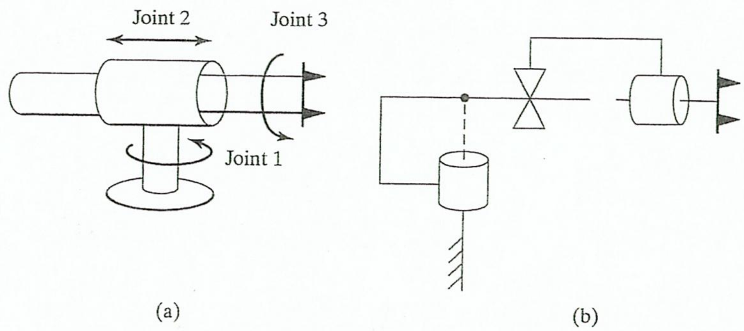
A three-link planar arm. On the right, we show the same manipulator by means of a simple schematic notation. Hash marks on the axes indicate that they are mutually parallel.



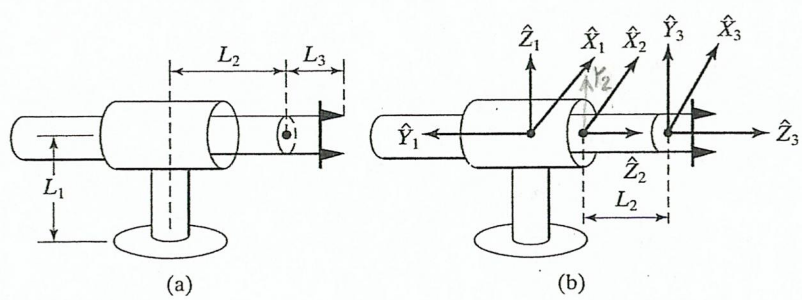
Link-frame assignments.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Link parameters of the three-link planar manipulator.



Manipulator having three degrees of freedom and one prismatic joint.



Link-frame assignments.

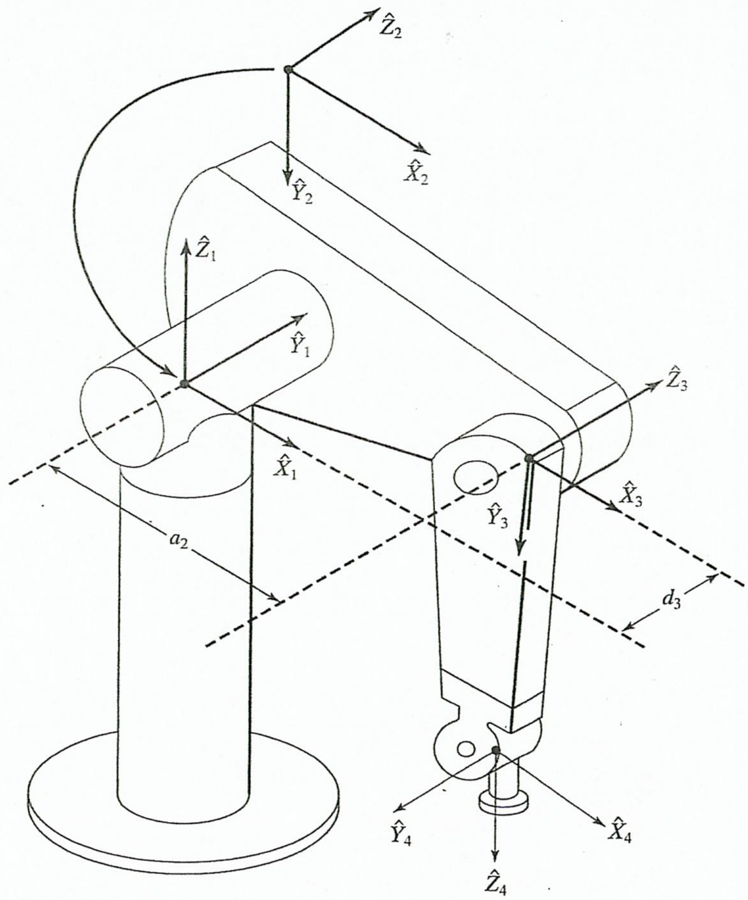
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

Link parameters for the *RPR* manipulator

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$



Some kinematic parameters and frame assignments for the PUMA 560

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	−90°	0	0	θ_2
3	0	a_2	d_3	θ_3
4	−90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	−90°	0	0	θ_6

Link parameters of the PUMA 560.

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$