B. Shafai, Robotics

Solution to Practice Homework

1.

Method 1 (Geometry)

The end-effector coordinates (x, y) can be expressed as

$$\begin{cases} x = a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) + a_3 \cos(\theta_1+\theta_2+\theta_3) \\ y = a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2) + a_3 \sin(\theta_1+\theta_2+\theta_3) \end{cases}$$

$$dx = \frac{\partial x}{\partial\theta_1} d\theta_1 + \frac{\partial x}{\partial\theta_2} d\theta_2 + \frac{\partial x}{\partial\theta_3} d\theta_3 \qquad \frac{dx}{dt} = \dot{x} = V_x$$

$$dy = \frac{\partial y}{\partial\theta_1} d\theta_1 + \frac{\partial y}{\partial\theta_2} d\theta_2 + \frac{\partial y}{\partial\theta_3} d\theta_3 \qquad \frac{dy}{dt} = \dot{y} = V_y$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} - a_3 S_{123} & -a_2 S_{12} - a_3 S_{123} & -a_3 S_{123} \\ a_1 C_1 + a_2 C_{12} + a_3 C_{123} & a_2 C_{12} + a_3 C_{123} & a_3 C_{123} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Method 2 (Homogenous Transformation "Denavit_Hartenberg")

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

Since all joints are revolute the homogeneous transformations for all joints are the same

$$A_i^{i-1} = \begin{bmatrix} C_i & -S_i & 0 & a_i C_i \\ S_i & C_i & 0 & a_i S_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad i = 1, 2, 3$$

Jacobian

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \hline \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\begin{array}{c|c|c} R_0^0 k \times (d_3^0 - d_0^0) & R_1^0 k \times (d_3^0 - d_1^0) & R_2^0 k \times (d_3^0 - d_2^0) \\ \hline R_0^0 k & R_1^0 k & R_2^0 k \end{array}\right] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

If we multiply $A_1^0 A_2^1 A_3^2 = A_3^0$, then the last column of A_3^0 specifies d_3^0. So, we have

$$d_3^0 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}, \quad d_2^0 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

obtained from $A_1^0 A_2^1 = A_2^0$ $\begin{bmatrix} R_2^0 & d_2^0 \\ 0 & 1 \end{bmatrix}$

$$d_1^0 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad d_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Note that $R_0^0 = I_3$, and R_1^0, R_2^1, R_3^2 can be extracted from A_i^{i-1} for $i = 1, 2, 3$.

After substitution of all parameters in Jacobian expression and evaluating all entries we get the following result

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} - a_3 s_{123} & -a_2 s_{12} - a_3 s_{123} & -a_3 s_{123} \\ a_1 c_1 + a_2 c_{12} + a_3 c_{123} & a_2 c_{12} + a_3 c_{123} & a_3 c_{123} \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Note that the first two rows of J matches with the geometry solution, which correspond to $v_x = \dot{x}$, $v_y = \dot{y}$. Obviously, $v_z = 0$ and the rest of terms correspond to

$$\omega_x = \omega_y = \omega_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. The Denavit-Hartenberg link parameters for the manipulator can be written as

Link	a_i	α_i	d_i	θ_i
1	a_1	90	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

Rotation Matrices :

$$R_0^0 = I_3 \qquad R_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} X_1 & Y_1 & Z_1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} X_0 \\ Y_0 \\ Z_0 \end{array} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 \\ s\theta_1 & 0 & -c\theta_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 = R_1^0 R_2^1$$

$$\downarrow$$

$$\begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 \\ s\theta_2 & c\theta_2 & 0 \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Displacement vectors :

$$d_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_1^0 = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix}, \quad d_2^1 = \begin{bmatrix} a_2 c\theta_2 \\ a_2 s\theta_2 \\ 0 \end{bmatrix}, \quad d_3^2 = \begin{bmatrix} a_3 c\theta_3 \\ a_3 s\theta_3 \\ 0 \end{bmatrix}$$

Using these vectors and rotation matrices, we can write homogeneous transformation matrices

$$A_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}, \quad A_2^1 = \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix}, \quad A_3^2 = \begin{bmatrix} R_3^2 & d_3^2 \\ 0 & 1 \end{bmatrix}$$

d_3^0 can be computed from $A_3^0 = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{bmatrix}$

which is obtained as

$$d_3^0 = \begin{bmatrix} c_1(a_2 c_2 + a_3 c_{23}) \\ s_1(a_2 c_2 + a_3 c_{23}) \\ a_2 s_2 + a_3 s_{23} + a_1 \end{bmatrix} \Leftarrow \left\{ \begin{array}{l} c\theta_1 = c_1, \ s\theta_1 = s_1 \\ c(\theta_1+\theta_2) = c_{12} \\ s(\theta_1+\theta_2) = s_{12} \\ c(\theta_1+\theta_2+\theta_3) = c_{123} \\ s(\theta_1+\theta_2+\theta_3) = s_{123} \end{array} \right\}$$

Similarly

$$d_2^0 = \begin{bmatrix} a_2 c_1 c_2 \\ a_2 c_2 s_1 \\ a_2 s_2 + a_1 \end{bmatrix}$$

from $A_1^0 A_2^1 = \begin{bmatrix} R_2^0 & d_2^0 \\ 0 & 1 \end{bmatrix}$

Now, Jacobian can be constructed by Using the above vectors and matrices:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \hline \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\begin{array}{c|c|c} R_0^0 k \times (d_3^0 - d_0^0) & R_1^0 k \times (d_3^0 - d_1^0) & R_2^0 k \times (d_3^0 - d_2^0) \\ \hline R_0^0 k & R_1^0 k & R_2^0 k \end{array}\right] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$J = \left[\begin{array}{ccc} -s_1(a_2c_2 + a_3c_{23}) & -c_1(a_2s_2 + a_3s_{23} + a_1) & -a_3c_1s_{23} \\ c_1(a_2c_2 + a_3c_{23}) & -s_1(a_2s_2 + a_3s_{23} + a_1) & -a_3s_1s_{23} \\ 0 & a_2c_2 + a_3c_{23} & a_3c_{23} \\ \hline 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{array}\right]$$

3. From Exercise 3.3 of the text-book we have:

$$ {}^{0}_{3}T = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & L_1 C_1 + L_2 C_1 C_2 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & L_1 S_1 + L_2 S_1 C_2 \\ S_{23} & C_{23} & 0 & L_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

and

$$ {}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{0}_{4}T = {}^{0}_{3}T\,{}^{3}_{4}T \ \checkmark $$

Applying the method of your text-book (see also my lecture notes) we can get Jacobian as follows:

$$ {}^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix}, \quad {}^{1}V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^{2}\omega_2 = {}^{2}_{1}R\,{}^{1}\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} C_2 & 0 & S_2 \\ -S_2 & 0 & C_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} S_2\dot\theta_1 \\ C_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} $$

Note: $ {}^{1}P_2 = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}, $

$$ {}^{2}V_2 = {}^{2}_{1}R\,({}^{1}V_1 + {}^{1}\omega_1 \times {}^{1}P_2) = \begin{bmatrix} C_2 & 0 & S_2 \\ -S_2 & 0 & C_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ L_1\dot\theta_1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} $$

$$ {}^{3}\omega_3 = {}^{3}_{2}R\,{}^{2}\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} C_3 & S_3 & 0 \\ -S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_2\dot\theta_1 \\ C_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} S_{23}\dot\theta_1 \\ C_{23}\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix} $$

Note: $ {}^{2}P_3 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}, $

$$ {}^{3}V_3 = {}^{3}_{2}R\,({}^{2}V_2 + {}^{2}\omega_2 \times {}^{2}P_3) = \begin{bmatrix} C_3 & S_3 & 0 \\ -S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2\dot\theta_2 \\ -L_2 C_2\dot\theta_1 \end{bmatrix} \right) $$

$$ = \begin{bmatrix} S_3 L_2 \dot\theta_2 \\ C_3 L_2 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix}, \quad {}^{4}\omega_4 = {}^{3}\omega_3 $$

Note: $ {}^{3}P_4 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} $

<u>Jacobian in Frame 4</u>

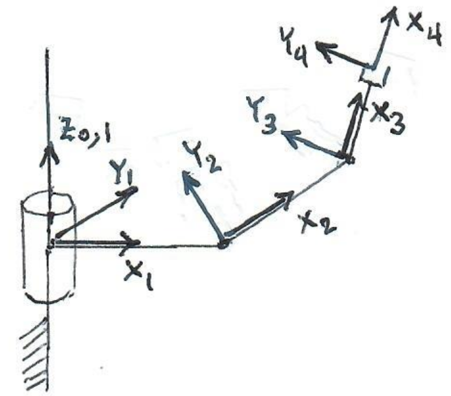
$$ {}^{4}V_4 = {}^{4}_{3}R\,({}^{3}V_3 + {}^{3}\omega_3 \times {}^{3}P_4) = \begin{bmatrix} S_3 L_2 \dot\theta_2 \\ C_3 L_2 \dot\theta_2 - L_3(\dot\theta_2 + \dot\theta_3) \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 - L_3 C_{23}\dot\theta_1 \end{bmatrix} \Rightarrow {}^{4}J = \begin{bmatrix} 0 & S_3 L_2 & 0 \\ 0 & C_3 L_2 + L_3 & L_3 \\ -L_1 - L_2 C_2 - L_3 C_{23} & 0 & 0 \end{bmatrix} $$

<u>For Jacobian w.r.t. frame 0</u>: Compute ${}^{0}_{4}R = {}^{0}_{1}R\,{}^{1}_{2}R\,{}^{2}_{3}R\,{}^{3}_{4}R$, ${}^{0}V_4 = {}^{0}_{4}R\,{}^{4}V_4 \Rightarrow {}^{0}J = \cdots$

" See my alternative Solution in the next page "

Alteranhve Solution to Problem 3

Consider again Exercize 3.3 of the text-book, which was one of the problems assigned in Homework 3. With the additional frame located at the tip of the hand we have the following configuration



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	L_1	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	θ_4

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{i-1}_{i}T = A_i^{i-1}$$

$${}^0_2T = {}^0_1T\,{}^1_2T = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & C_1L_1 \\ S_1C_2 & -S_1S_2 & -C_1 & S_1L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T = {}^0_2T\,{}^2_3T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & -S_1 & L_1C_1 + L_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & L_1S_1 + L_2S_1C_2 \\ S_{23} & C_{23} & 0 & L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4T = {}^0_3T\,{}^3_4T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & -S_1 & C_1C_{23}L_3 + L_1C_1 + L_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1C_{23}L_3 + L_1S_1 + L_2S_1C_2 \\ S_{23} & C_{23} & 0 & S_{23}L_3 + L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that all Rotation Matrices and Displacement Vectors can be identified from the above Homogeneous Transformation Matrices : $R_0^0, R_1^0, R_2^0, R_3^0, R_4^0$ and $d_0^0, d_1^0, d_2^0, d_3^0, d_4^0$.

Consequently, we can construct the Jacobian as follows :

$$J = \begin{bmatrix} R_0^0 k \times (d_4^0 - d_0^0) & R_1^0 k \times (d_4^0 - d_1^0) & R_2^0 k \times (d_4^0 - d_2^0) & R_3^0 k \times (d_4^0 - d_3^0) \\ R_0^0 k & R_1^0 k & R_2^0 k & R_3^0 k \end{bmatrix}$$

4. **Note** : The Problem is associated with Example 5.3 of your textbook

$$\tau = {}^3J^T \, {}^3F \qquad \text{"Textbook Notation"}$$

$${}^3F = \left[{}^3J^T\right]^{-1} \tau \qquad \text{Since} \quad {}^3J = \begin{bmatrix} L_1 S_2 & 0 \\ L_1 C_2 + L_2 & L_2 \end{bmatrix}$$

we have

$${}^3F = \frac{1}{L_1 L_2 S_2} \begin{bmatrix} L_2 & -L_1 C_2 - L_2 \\ 0 & L_1 S_2 \end{bmatrix} \tau$$

Solution is repeated

4. $\tau = {}^3J^T\,{}^3F \qquad {}^3J = \begin{bmatrix} L_1S_2 & 0 \\ L_1C_2+L_2 & L_2 \end{bmatrix}$

$${}^3F = \left[{}^3J^T\right]^{-1}\tau = \frac{1}{L_1L_2S_2}\begin{bmatrix} L_2 & -L_1C_2-L_2 \\ 0 & L_1S_2 \end{bmatrix}\tau$$

5. From velocity transformation section of your textbook and my lecture notes we have

$${}^B\nu_B = {}^B_AT\,{}^A\nu_A \quad \text{or Simple Notation} \quad \underline{\underline{{}^B\nu = {}^B_AT\,{}^A\nu}}$$

Since ${}^A_BT$ is given, we need ${}^B_AT = \left({}^A_BT\right)^{-1}$

$$\underline{\underline{{}^B\nu = \begin{bmatrix} {}^B_AR & \boxed{-{}^B_AR\,{}^Ap^*} \\ 0 & {}^B_AR \end{bmatrix}{}^A\nu}}$$

$\underbrace{{}^Ap^*}_{\text{My Note}}$ or $\underbrace{{}^Ap^{x}}_{\text{Textbook}}$

$$\begin{bmatrix} 0 & -P_x & P_y \\ P_x & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix}$$

$$-{}^B_AR\,{}^Ap^* = -\begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2.5 & 4.3 & 5 \\ -4.3 & -2.5 & 8.6 \\ 0 & -10 & 0 \end{bmatrix}$$

$${}^B\nu = \begin{bmatrix} 0.86 & 0.5 & 0 & -2.5 & 4.3 & 5 \\ -0.5 & 0.86 & 0 & -4.3 & -2.5 & 8.6 \\ 0 & 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.86 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.41 \\ 1.41 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.52 \\ -7.8 \\ -17.1 \\ 1.91 \\ 0.51 \\ 0 \end{bmatrix}$$

6.
$$\tau = {}^0J^T(\theta)\,{}^0F = \begin{bmatrix} -L_1S_1 - L_2S_{12} & L_1C_1 + L_2C_{12} \\ -L_2S_{12} & L_2C_{12} \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\tau_1 = -10L_1S_1 - 10L_2S_{12}$$
$$\tau_2 = -10L_2S_{12}$$

7. The last column of ${}^0_3T$ gives the displacement vector of frame $\{3\}$ relative to frame $\{0\}$ i.e. (x, y, z) components of the end-effector. So, it is sufficient to take the derivative of the last column of ${}^0_3T$ with respect to $\theta_1, \theta_2, \theta_3$, which can easily determine ${}^0J(\theta)$ as

$${}^0J(\theta) = \begin{bmatrix} -L_1S_1 - L_2S_1C_2 & -L_2C_1S_2 & 0 \\ L_1C_1 + L_2C_1C_2 & -L_2S_1S_2 & 0 \\ 0 & L_2C_2 & 0 \end{bmatrix}$$

8. Similar to problem 7, we need to take the partial derivative of ${}^0P_{2ORG}$

$${}^0P_{2ORG} = \begin{array}{cc} \begin{bmatrix} a_1C_1 - d_2S_1 \\ a_1S_1 + d_2C_1 \\ 0 \end{bmatrix} & \begin{array}{c} \leftarrow x \\ \leftarrow y \\ \leftarrow z \end{array} \end{array}$$

$${}^0\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}_{2ORG} = \begin{bmatrix} -a_1S_1 - d_2C_1 & -S_1 \\ a_1C_1 - d_2S_1 & C_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$