RSS HW1

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(1) (a) $${}^{A}T_{B} = \left(\begin{array}{c|c} R & t \\ \hline 0 & 1 \end{array}\right) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos\theta & -\sin\theta & 2 \\ 0 & \sin\theta & \cos\theta & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) $${}^{B}T_{A} = {}^{A}T_{B}^{-1} = \left(\begin{array}{c|c} R^T & -R^T t \\ \hline 0 & 1 \end{array}\right) = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & \cos\theta & \sin\theta & -2\cos\theta - 3\sin\theta \\ 0 & -\sin\theta & \cos\theta & 2\sin\theta - 3\cos\theta \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

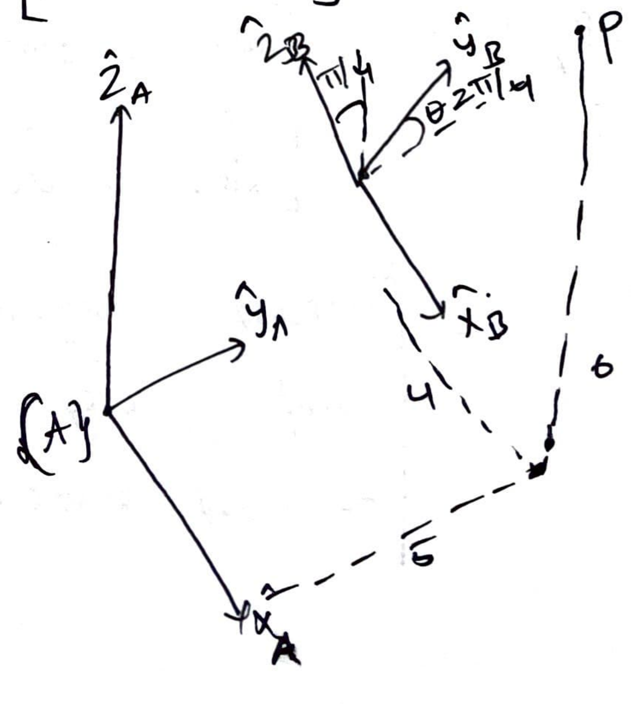
(c) Given, $\theta = \pi/4$ & ${}^{A}p = [4\ 5\ 6]^T$

$${}^{B}p = {}^{B}T_{A}\, {}^{A}p$$

$${}^{B}p = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & -2\cos\frac{\pi}{4} - 3\sin\frac{\pi}{4} \\ 0 & -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 2\sin\frac{\pi}{4} - 3\cos\frac{\pi}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 1 \end{bmatrix}$$

$${}^{B}p = [5\ \ 4.2426\ \ 0]^T$$

(d)



② ⓐ $^{A}R_{C} = {}^{A}R_{B} \cdot {}^{B}R_{C}$

where,

$$^{A}R_{B} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$^{B}R_{C} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{A}R_{C} = \begin{bmatrix} \cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix}$$

ⓑ Given, rotation by angle ϕ about Z_A & then rotation by angle θ about Y_A

This is rotation about a fixed frame $\{A\}$

$$R = R_{y_A}(\theta) \cdot R_{z_A}(\phi) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix}$$

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As you can see,

the rotation matrix obtained by using fixed frame rotation is same as the one obtained in 2(a) using current frame rotation method. Here the order of rotation is y_A & z_B in 2(a) & it is z_A & y_A in 2(b).

3 (a)

$$^{0}T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^{0}T_2 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $^{3}T_2 = {}^{3}T_0 \cdot {}^{0}T_2 = {}^{0}T_3^{-1} \cdot {}^{0}T_2$

$$^{3}T_2 = \begin{bmatrix} 0 & 1 & 0 & -1.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) (e) $^3P = \begin{bmatrix} 0.2 & -0.1 & 2 \end{bmatrix}^T$

$^0P = {}^0T_3 \cdot {}^3P$

$$= \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ -0.1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 1.7 \\ 1 \\ 1 \end{bmatrix}$$

(4) (a) The forward kinematic map of RRR manipulator

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & +\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, $c\theta_1 = \cos\theta_1$ & $s\theta_1 = \sin\theta_1$

$$^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$${}^0T_3 = {}^0T_1\, {}^1T_2\, {}^2T_3$$

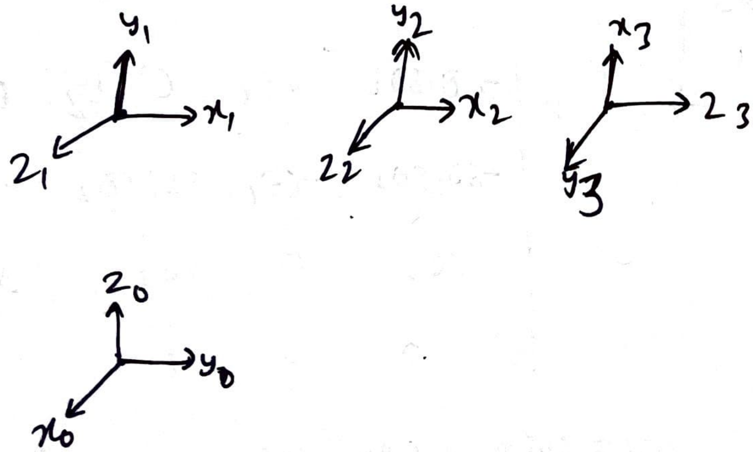
$$= \begin{bmatrix} c\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_2 s\theta_3 & -c\theta_1 c\theta_2 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 & s\theta_1 & L_2 c\theta_1 c\theta_2 + L_3 c\theta_1 c\theta_2 c\theta_3 - L_3 c\theta_1 s\theta_2 s\theta_3 \\ c\theta_2 c\theta_3 s\theta_1 - s\theta_1 s\theta_2 s\theta_3 & -c\theta_2 s\theta_1 s\theta_3 - c\theta_3 s\theta_1 s\theta_2 & -c\theta_1 & L_2 c\theta_2 s\theta_1 + L_3 c\theta_2 c\theta_3 s\theta_1 - L_3 s\theta_1 s\theta_2 s\theta_3 \\ c\theta_2 s\theta_3 + c\theta_3 s\theta_2 & c\theta_2 c\theta_3 - s\theta_2 s\theta_3 & 0 & L_1 + L_2 s\theta_2 + L_3 c\theta_2 s\theta_3 + L_3 c\theta_3 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, $c\theta_1 = \cos\theta_1$; $s\theta_1 = \sin\theta_1$

$c\theta_2 = \cos\theta_2$; $s\theta_2 = \sin\theta_2$

$c\theta_3 = \cos\theta_3$; $s\theta_3 = \sin\theta_3$

4 (b) RRP manipulator:



Using intermediate co-ordinates as shown:

$$\cancel{{}^0T_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 \end{bmatrix}}$$

$${}^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad {}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3$$

$$= \begin{bmatrix} -c\theta_1 s\theta_2 & s\theta_1 & c\theta_1 c\theta_2 & \theta_3 s\theta_1 + l_2 c\theta_1 c\theta_2 \\ -s\theta_1 s\theta_2 & -c\theta_1 & s\theta_1 c\theta_2 & -\theta_3 c\theta_1 + l_2 s\theta_1 s\theta_2 \\ c\theta_2 & 0 & s\theta_2 & l_1 + l_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, $c\theta_1 = \cos\theta_1$; $c\theta_2 = \cos\theta_2$

$s\theta_1 = \sin\theta_1$; $s\theta_2 = \sin\theta_2$

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⑤ ⓐ By law of cosines,

$$\cos\beta = \frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}$$

ⓑ β & θ_2 are on the same line.

so, $\beta + \theta_2 = \pi$

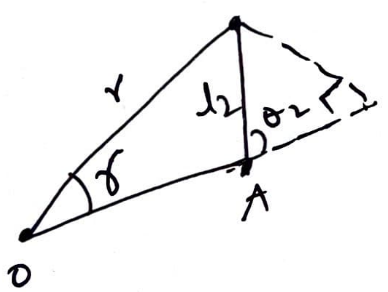
$\theta_2 = \pi - \beta$

then by law of cosines,

$$\beta = \cos^{-1}\left(\frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}\right)$$

$$\theta_2 = \pi - \cos^{-1}\left(\frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}\right)$$

ⓒ



from diagram,

$r \sin\gamma = l_2 \sin\theta_2$

$$\gamma = \sin^{-1}\left(\frac{l_2}{r}\sin\theta_2\right)$$

ⓓ from diagram, $\tan\alpha = \frac{y_w}{x_w}$

$\alpha = \theta_1 + \gamma$

$$\theta_1 = \alpha - \gamma = \tan^{-1}\left(\frac{y_w}{x_w}\right) - \sin^{-1}\left(\frac{l_2}{r}\sin\theta_2\right)$$

ⓔ → The 2-DOF planar arm will have 0 solutions, if the goal position's distance is more than (l_1+l_2) from the Origin O.

→ It has 1 solution, if the goal position's distance is (l_1+l_2) from the Origin O.

→ It has 2 solutions, if the goal position's distance is less than (l_1+l_2) from the origin O.

for other sol.^{n}

$\theta_1' = \alpha + \gamma$

$\theta_2' = -\theta_2$.