Introduction to Gravitational Waves

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Outline

- ⋆ Gravitational waves from the Einstein’s equations,
- ⋆ Detection principles (what is actually measured by interferometers?)
- ⋆ Newtonian intuitions from inspiralling binary system,
- ⋆ Application: standard sirens.


ME
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GRAVITATIONAL-WAVE MEMES
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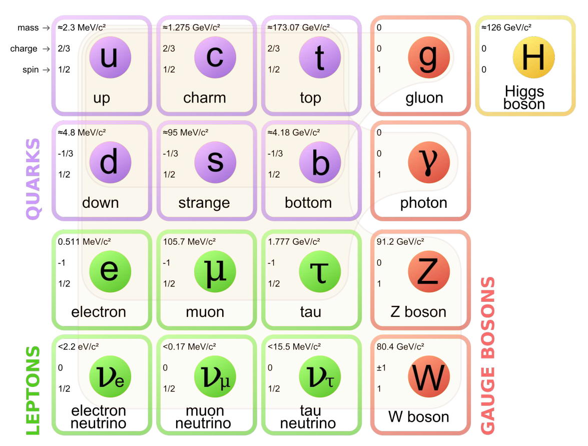
Fundamental interactions

Standard Model:



- ⋆ **Interaction (coupling strength):**
- ⋆ Strong nuclear interactions (1),
- ⋆ Electromagnetism $(1/137)$,
- ⋆ Weak nuclear interactions (10^{-9}),
- ⋆ Gravitation (10^{-38})

Gravitation: Newton vs Einstein

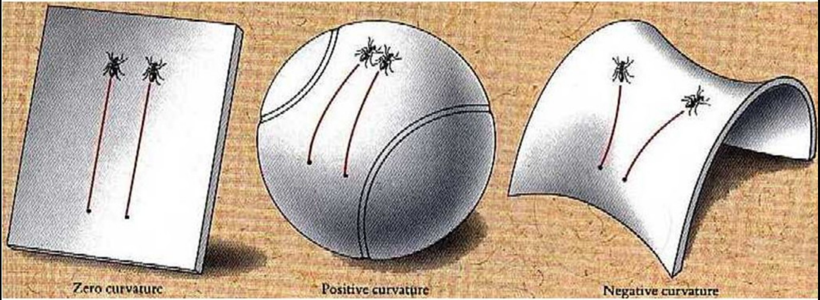
Newton:

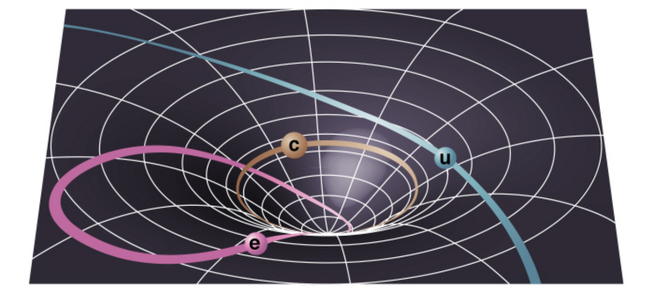
- ⋆ Space is euclidean, time is absolute, there is no relation between them
- ⋆ Gravitation is a force acting between masses ($F = -Gm_1 m_2/r^2$)
- ⋆ Force of gravitation acts immediately at any distance

Einstein:

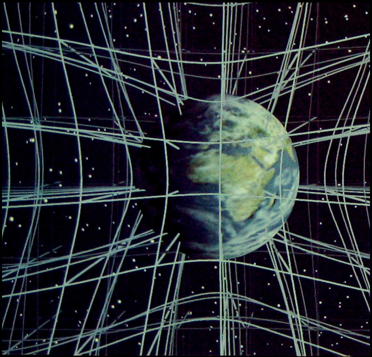
- ⋆ Space and time are related
- ⋆ 4-dimensional space-time is curved by masses, and gravitation is an effect of this curvature
- ⋆ Effects of gravitation travel with the speed of light

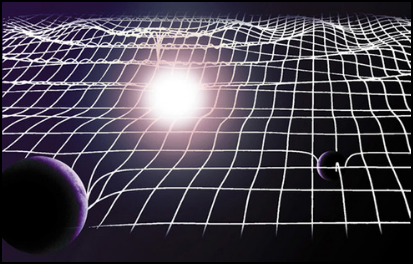
Gravity $\equiv$ geometry



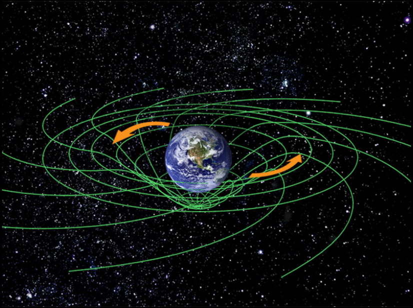


Einstein (1915): gravitation = geometry of spacetime





"Mass tells spacetime how to curve, and spacetime tells mass how to move."
(John A. Wheeler)

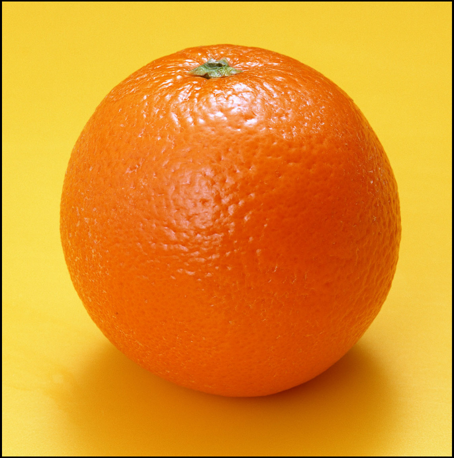


Gravitational waves

Einstein (1916) - wave-like solutions to GR equations *(time-varying distortions of the curvature propagating with the speed of light)*:

- ⋆ In realistic astrophysical situations, length-scale of the wave λ is much smaller than other important curvatures $\mathcal{L}$,
- ⋆ Split of the Riemann curvature tensor

$$R_{\alpha\beta\gamma\delta} = R^{GW}_{\alpha\beta\gamma\delta} + R^{B}_{\alpha\beta\gamma\delta}$$



"Kip Thorne's orange": B - large-scale background ($\mathcal{L} \simeq$ 10 cm), GW - fine-scale distortions/waves ($\lambda \simeq$ few mm).

Gravitational waves: indirect evidence

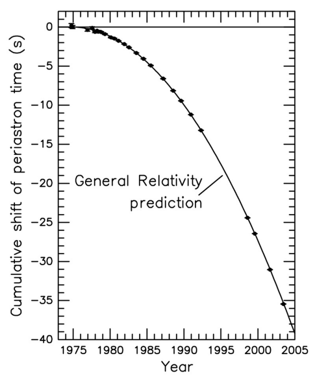
The 50s - breakthrough in theoretical understanding of the nature of the waves:

- ⋆ Herman Bondi, Felix Pirani, Andrzej Trautman (gravitational waves carry energy!)

The 60s - early insight of Bohdan Paczyński:

- ⋆ *"Gravitational Waves and the Evolution of Close Binaries", AcA 1967* - orbital period evolution of WZ Sge and HZ29 driven by the GW emission.

70s - observations of pulsars in relativistic binary systems (e.g. Hulse-Taylor pulsar):



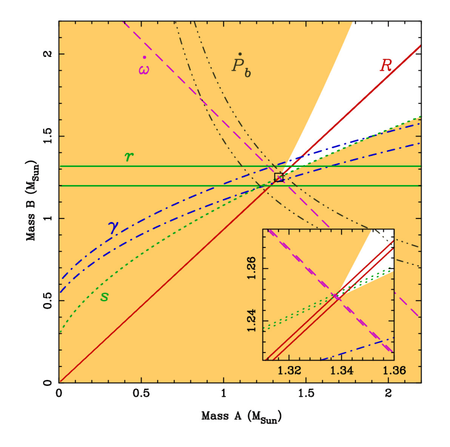
System is losing energy as if by emittion of gravitational waves in concordance with GR.

Neutron stars in relativistic binaries: PSR J0737-3039

- ⋆ Periastron advance:
  $$\dot{\omega} = 3\left(\frac{P_b}{2\pi}\right)^{-5/3}(T_\odot M)^{2/3}(1-e^2)^{-1}$$
- ⋆ Orbit decay:
  $$\dot{P}_b = -\frac{192\pi m_p m_c}{5M^{1/3}}\left(\frac{P_b}{2\pi}\right)^{-5/3}\times$$
  $$\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}T_\odot^{5/3}$$
- ⋆ Shapiro effect:
  $$r = T_\odot m_c,$$
  $$s = \frac{a_p \sin i}{c m_c}\left(\frac{P_b}{2\pi}\right)^{-2/3}T_\odot^{-1/3}M^{2/3}$$
- ⋆ Gravitational redshift:
  $$\gamma = e\left(\frac{P_b}{2\pi}\right)^{1/3}T_\odot^{2/3}M^{-4/3}m_c(M+m_c)$$

where $T_\odot = GM_\odot/c^3$, $M = m_p + m_c$.

Relativistic binaries show a number of effects compatible with GR!

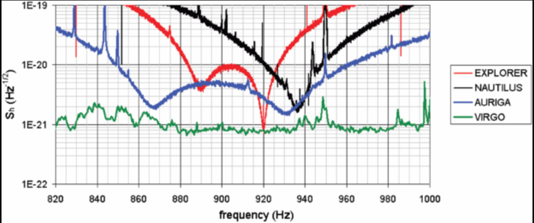


- ⋆ Pulsar A: $P = 22.7\ ms$, pulsar B: $P = 2.77\ s$,
- ⋆ Orbital period $\simeq$ 2.4 h,
- ⋆ eccentricity $\simeq$ 0.08,
- ⋆ Orbit decay $\simeq$ 7 mm/day.

Detection principle: resonant bars

Pioneered by Joseph Weber in the 1960s:

- ⋆ Passing gravitational wave carries energy → induces mechanical vibrations
- ⋆ A narrow-band detector (sensitive near characteristic frequencies of the bar)



Gravitational waves: weak field wave zone

"Ripples" in the "nearly flat" spacetime metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where e.g., $\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, 1, 1)$, and $|h_{\mu\nu}| \ll 1$ for all μ, ν.

In the weak-field limit h is small, 1st order (linear) sufficient: $h_{\mu\nu} = \eta_{\mu\alpha}\eta_{\beta\nu}h^{\alpha\beta}$

Coordinate transformations that preserve "nearly flat" (nearly Lorentz) spacetime:

- $\star$ background Lorentz transformations (boosts with $v \ll 1$),

$$g'_{\mu\nu} = \eta'_{\mu\nu} + \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}h_{\alpha\beta} = \eta'_{\mu\nu} + h'_{\mu\nu}$$

- $\star$ Gauge transformations (ξ^μ, $|\xi^\mu_{,\nu}|$, $|\xi_{,\mu\nu}| \ll 1$):

$$x'^\mu = x^\mu + \xi^\mu\left(x^\nu\right), \quad \text{so that}$$

$$g'_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \ll 1.$$

Gravitational waves: wave equation

In linear regime, weak field the Riemann tensor is

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\alpha\gamma,\beta\delta} - h_{\beta\delta,\alpha\gamma}\right).$$

Ricci tensor: $R_{\mu\nu} = \frac{1}{2}\left(h^{\alpha}_{\mu,\nu\alpha} + h^{\alpha}_{\nu,\mu\alpha} - h_{\mu\nu,\alpha}^{,\alpha} - h_{,\mu\nu}\right)$,

where $h \equiv h^{\mu}_{\mu} = \eta^{\mu\nu} h_{\mu\nu}$, $\quad h_{\mu\nu,\alpha}^{,\alpha} = \eta^{\alpha\gamma} h_{\mu\nu,\alpha\gamma}$.

And so... Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}\left(h^{\alpha}_{\mu,\nu\alpha} + h^{\alpha}_{\nu,\mu\alpha} - h_{\mu\nu,\alpha}^{,\alpha} - h_{,\mu\nu} - \eta_{\mu\nu}\left(h_{\alpha\beta}^{,\alpha\beta} - h_{,\beta}^{,\beta}\right)\right).$$

Using trace-reversed form, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{1}{2}\left(\bar{h}_{\mu\nu,\alpha}^{,\alpha} + \eta_{\mu\nu} \bar{h}_{\alpha\beta}^{,\alpha\beta} - \bar{h}_{\mu\alpha,\nu}^{,\alpha} - \bar{h}_{\nu\alpha,\mu}^{,\alpha}\right) \overset{vacuum}{=} 0.$$

'Good choice' of gauge (Lorentz gauge $\bar{h}^{\mu\alpha}_{,\alpha} = 0$) reduces it to

$$\bar{h}_{\mu\nu,\alpha}^{,\alpha} \equiv \eta^{\alpha\alpha} \bar{h}_{\mu\nu,\alpha\alpha} = \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}_{\mu\nu} = 0.$$

Plane gravitational waves

$$\bar{h}_{\mu\nu} = Re\left(A_{\mu\nu}\exp\left(ik_\alpha x^\alpha\right)\right),$$

$$\text{with} \quad k_\alpha k^\alpha = 0 \quad \rightarrow \quad \omega = k^t = \sqrt{k_x^2 + k_y^2 + k_z^2}.$$

From the choice of Lorentz gauge: $A_{\mu\alpha}k^\alpha = 0$.

Using remaining freedom, apply the transverse-traceless gauge for a wave traveling in the z direction:

- ⋆ $k^t = k^z = \omega, k^x = k^y = 0, \quad A_{\alpha z} = 0,$
- ⋆ $A^\mu_\mu = \eta^{\mu\nu}A_{\mu\nu} = 0, \quad A_{\alpha t} = 0.$

In the TT gauge, $\bar{h}^{(TT)}_{\mu\nu} = A^{(TT)}_{\mu\nu}\cos\left(\omega\left(t - z\right)\right)$, with

$$A^{(TT)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A^{(TT)}_{xx} & A^{(TT)}_{xy} & 0 \\ 0 & A^{(TT)}_{xy} & -A^{(TT)}_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{Also,} \quad \bar{h}^{(TT)}_{\mu\nu} = h^{(TT)}_{\mu\nu}.$$

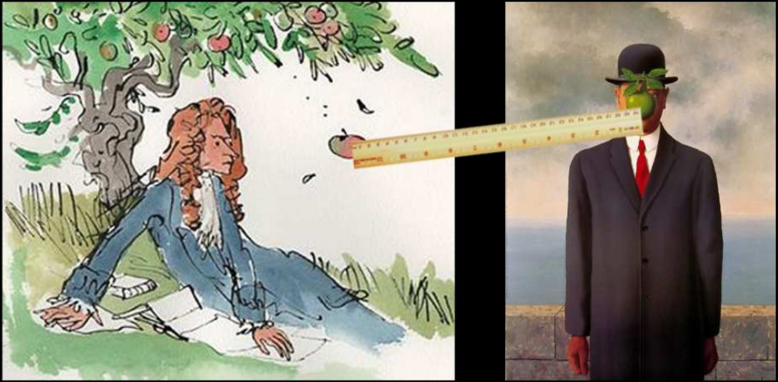
Gravitational waves: TT gauge

For a free test particle initially at rest, in the coordinate system corresponding to the TT gauge, it stays at rest: coordinates do not change, particles remain attached to initial positions.

TT gauge represents a coordinate system comoving with freely-falling particles.

What about the **proper distance** between neighbouring particles?

Detection principle: spacetime distance measurement



(Quentin Blake "Izaak Newton")

(Rene Magritte "The Son of Man")

"How to measure distance when the ruler also changes length?"

Proper distance between test particles

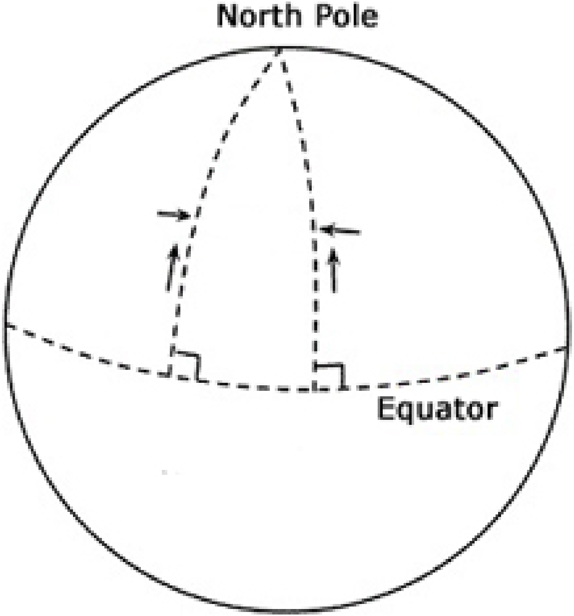
Consider two test particles, both initially at rest, one at $x = 0$ and the other at $x = \epsilon$. The proper distance is

$$\Delta s = \int |g_{\mu\nu} dx^{\mu} dx^{\nu}|^{1/2} \to \int_0^{\epsilon} |g_{xx}|^{1/2} \approx \epsilon \sqrt{g_{xx}\,(x = 0)}.$$

If $g_{xx}\,(x = 0) = \eta_{xx} + h_{xx}^{(TT)}\,(x = 0)$, then

$$\Delta s \approx \epsilon \left(1 + \frac{1}{2} h_{xx}^{(TT)}(x = 0) \right),$$

which, in general, is time-varying $\ddot\smile$


North Pole
Equator

Geodesic deviation - effect of tidal forces

Consider two test particles, both initially at rest ($u^\alpha = (1,0,0,0)$) one at $x = 0$ and the other at $x = \epsilon$ (distance between particles $\xi^\alpha = (0,\epsilon,0,0)$). Geodesic deviation equation in the weak field (proper time $\tau \approx$ coordinate time t),

$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = R^\alpha_{\beta\gamma\delta} u^\beta u^\gamma \xi^\delta$$

simplifies further to

$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = \epsilon R^\alpha_{ttx} = -\epsilon R^\alpha_{txt},$$

with $R^x_{txt} = \eta^{xx} R_{xtxt} = -\frac{1}{2} h^{(TT)}_{xx,tt}$, $R^y_{txt} = \eta^{yy} R_{ytxt} = -\frac{1}{2} h^{(TT)}_{xy,tt}$,

$$\frac{\partial^2 \xi^x}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2 h^{(TT)}_{xx}}{\partial t^2}, \quad \frac{\partial^2 \xi^y}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2 h^{(TT)}_{xy}}{\partial t^2}.$$

Geodesic deviation - effect of tidal forces

More general case; $x = \epsilon \cos\theta,\ y = \epsilon \sin\theta,\ z = 0$:

$$
\begin{aligned}
\frac{\partial^2 \xi^x}{\partial t^2} &= \frac{1}{2}\epsilon \cos\theta \frac{\partial^2 h_{xx}^{(TT)}}{\partial t^2} + \frac{1}{2}\epsilon \sin\theta \frac{\partial^2 h_{xy}^{(TT)}}{\partial t^2}, \\
\frac{\partial^2 \xi^y}{\partial t^2} &= \frac{1}{2}\epsilon \cos\theta \frac{\partial^2 h_{xy}^{(TT)}}{\partial t^2} - \frac{1}{2}\epsilon \sin\theta \frac{\partial^2 h_{xx}^{(TT)}}{\partial t^2}.
\end{aligned}
$$

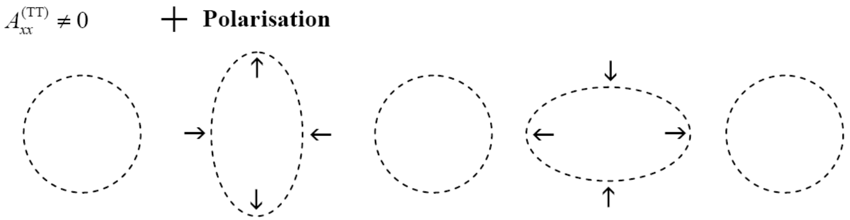
with solutions, for the plane wave in the z direction,

$$
\begin{aligned}
\xi^x &= \epsilon \cos\theta + \frac{1}{2}\epsilon \cos\theta A_{xx}^{(TT)} \cos\omega t + \frac{1}{2}\epsilon \sin\theta A_{xy}^{(TT)} \cos\omega t, \\
\xi^y &= \epsilon \sin\theta + \frac{1}{2}\epsilon \cos\theta A_{xy}^{(TT)} \cos\omega t - \frac{1}{2}\epsilon \sin\theta A_{xx}^{(TT)} \cos\omega t.
\end{aligned}
$$

The $+$ polarisation

$A_{xx}^{(TT)} \neq 0$, $A_{xy}^{(TT)} = 0$

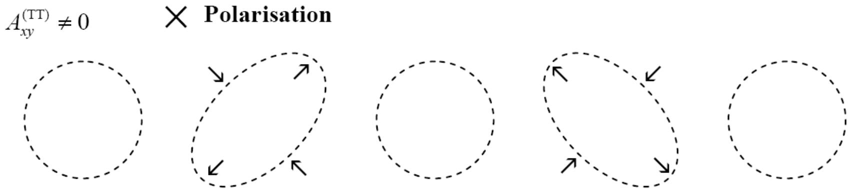
$$\begin{aligned} \xi^x &= \epsilon \cos\theta \left(1 + \frac{1}{2} A_{xx}^{(TT)} \cos\omega t\right), \\ \xi^y &= \epsilon \sin\theta \left(1 - \frac{1}{2} A_{xx}^{(TT)} \cos\omega t\right). \end{aligned}$$



The $\times$ polarisation

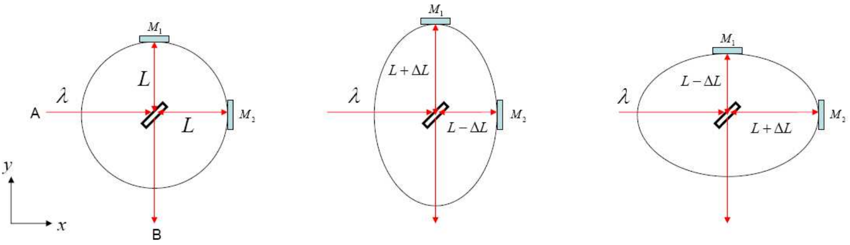
$A^{(TT)}_{xy} \neq 0$, $A^{(TT)}_{xx} = 0$

$$
\begin{aligned}
\xi^x &= \epsilon \cos\theta + \frac{1}{2}\epsilon \sin\theta A^{(TT)}_{xy} \cos\omega t, \\
\xi^y &= \epsilon \sin\theta - \frac{1}{2}\epsilon \cos\theta A^{(TT)}_{xy} \cos\omega t.
\end{aligned}
$$



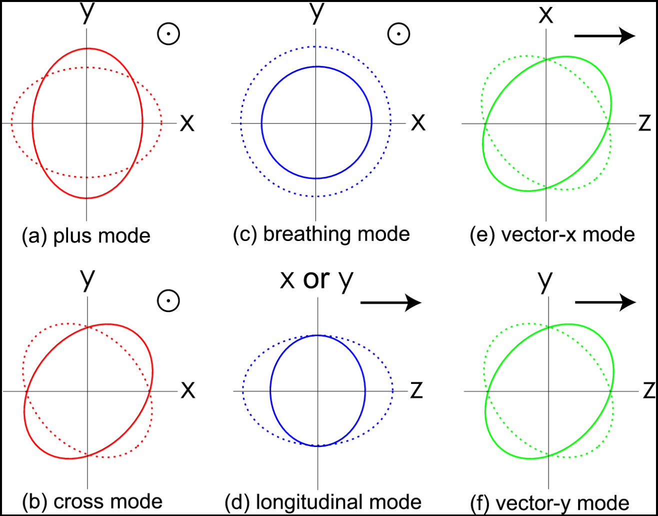
For purely $+$ mode wave ($\mathbf{h} = h\mathbf{e}_+$), fractional change in proper distance is

$$\frac{\Delta L}{L} = \frac{h}{2}$$



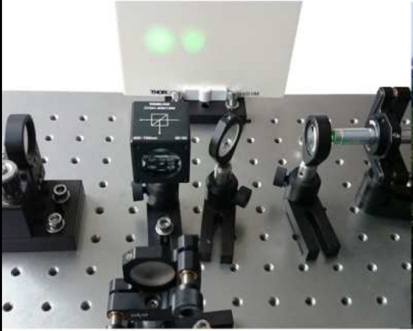
Gertsenshtein & Pustovit (1962) were first to suggest an interferometer to detect GWs. In the 70s Rainer Weiss had the same idea $\rightarrow$ LIGO

In general metric theory: 6 independent polarizations



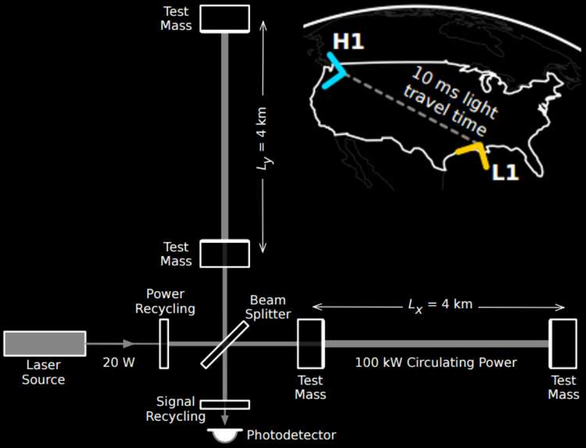
In GR: $h = h_+ F_+ + h_x F_x$, with F_+, $F_\times$ detector antenna patterns.

You vs. the interferometer she tells you not to worry about



Detection principle: laser interferometry

"How to measure distance when the ruler also changes length?"



Changes in arms length are **very** small: $\delta L_x - \delta L_y = \Delta L < 10^{-18}$ m (smaller than the size of the proton). Wave amplitude $h = \Delta L / L \leq 10^{-21}$.

Change of arms' length $\leftrightarrow$ variation in light travel time

Change of the x-arm: $ds^2 = -c^2 dt^2 + (1 + h_{xx})\, dx^2 = 0.$

Assume $h(t)$ is constant during light's travel through interferometer, replace $\sqrt{1 + h_{xx}}$ with $1 + h_{xx}/2$, integrate from $x = 0$ to $x = L$:

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{xx}\right) dx \quad \rightarrow \quad t_x = h_{xx} L/2c.$$

Round-trip time in the x-arm: $t_x = h_{xx} L/c.$

Round-trip time in the y-arm: $t_y = -hL/c \qquad (h_{yy} = -h_{xx} = -h)$

Round-trip times difference: $\boxed{\Delta\tau = 2hL/c}$

Phase difference (dividing $\Delta\tau$ by the radian period of light $2\pi/\lambda$):

$$\boxed{\Delta\phi = \frac{4\pi}{\lambda} hL = \frac{2\pi c}{\lambda} h\tau}.$$

- Do test masses move in response to a gravitational wave?
  - No, in the TT gauge (free-falling masses define the coordinates),
  - Yes, in the laboratory coordinates (masses move affected by tidal forces).
- Do light wavelength change in response to a gravitational wave?
  - No (see above),
  - Yes, stretch by h as the masses move (as in the cosmological redshift).
- If light waves are stretched by gravitational waves, how can light be used as a ruler?
  - Indeed, the instantaneous response of an interferometer to a gravitational wave is *null*.
  - But the light travels through the arms for some finite time allowing for the phase shift to build up.

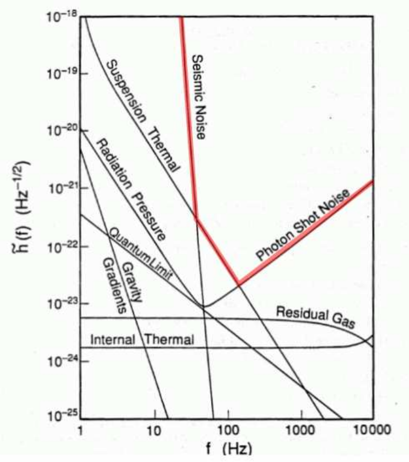
See also Saulson, P.R. (1997), *Am. J. Phys*. 65, 501

Orders of magnitude comparison

- ⋆ GW150914: $h = \Delta L/L \simeq 10^{-21}$
- ⋆ Two neutron stars merging near Sgr A*: $\sim 10^{-19}$
- ⋆ Io orbiting Jupiter: $\sim 3 \times 10^{-25}$
- ⋆ Hulse-Taylor pulsar: $\sim 10^{-26}$
- ⋆ Dumbbell 1 tonnes masses, 1 m arm from 300 m: $\sim 10^{-35}$
- ⋆ Collision of two aircraft carriers: 5×10^{-46}
- ⋆ Angry protester shaking her fist: $\sim 7 \times 10^{-52}$
- ⋆ Tennis ball rotating on 1 m string, from 10 m: $\sim 10^{-54}$.

- ⋆ The amplitude $h = \Delta L/L \leq 10^{-21}$ corresponds to the distance measurement between Earth and Sun with the accuracy of the size of the atom (10^{-10} m)
- ⋆ Ground motion amplitude near the detector: $\Delta L \sim 10^{-6}$ m ($10^{12} \times h$)
- ⋆ Laser wavelength: 10^{-6} m ($10^{12} \times h$)

How does the sensitivity curve look like?



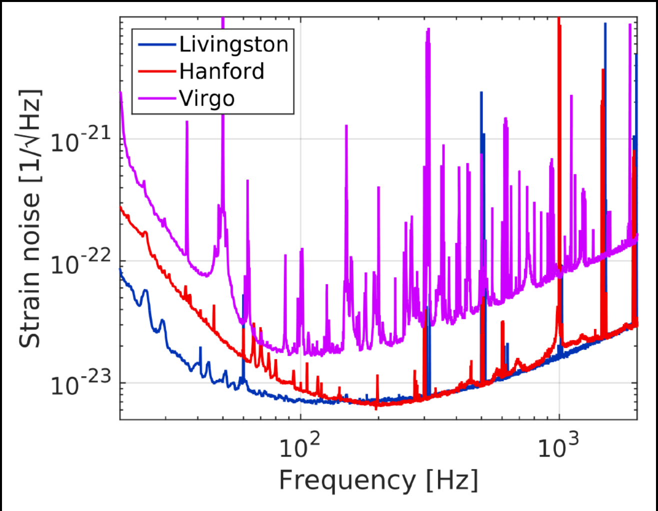
Initial LIGO proposal (1989)

- ⋆ Range of frequencies similar to human ears:

From 20 Hz (H0) to a few thousands Hz (3960 Hz, H7) - 8 octaves.

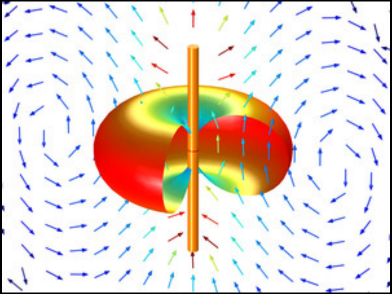
- ⋆ Poor, like for an ear, angular resolution.

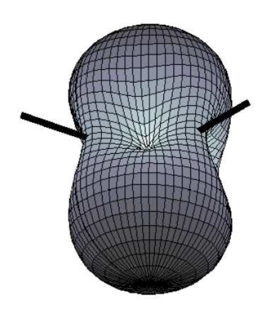
Noise-limited detector: amplitude spectral density

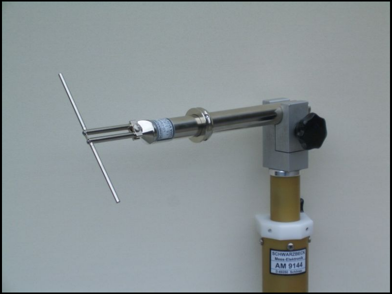


Lines here are all instrumental (*plot is dominated by instrumental noise*): mirror suspension resonances at 500 Hz and harmonics, calibration lines and power lines (60 Hz and harmonics) etc.

A word about antenna patterns







Antenna patterns

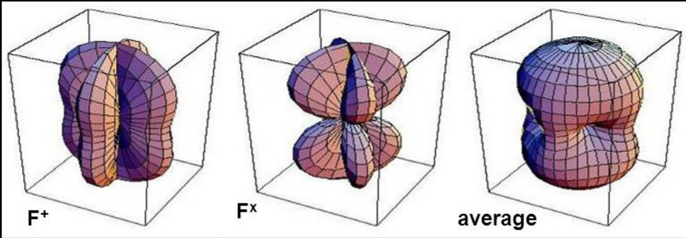
Response of x and y arms to a GW from arbitrary direction

$$h_{xx} = -\cos(\theta)\sin(2\phi)h_\times + (\cos^2(\theta)\cos\phi^2 - \sin\phi^2)h_+$$

$$h_{yy} = \cos\theta\sin 2\phi h_\times + (\cos\theta^2\sin\phi^2 - \cos\phi^2)h_+$$

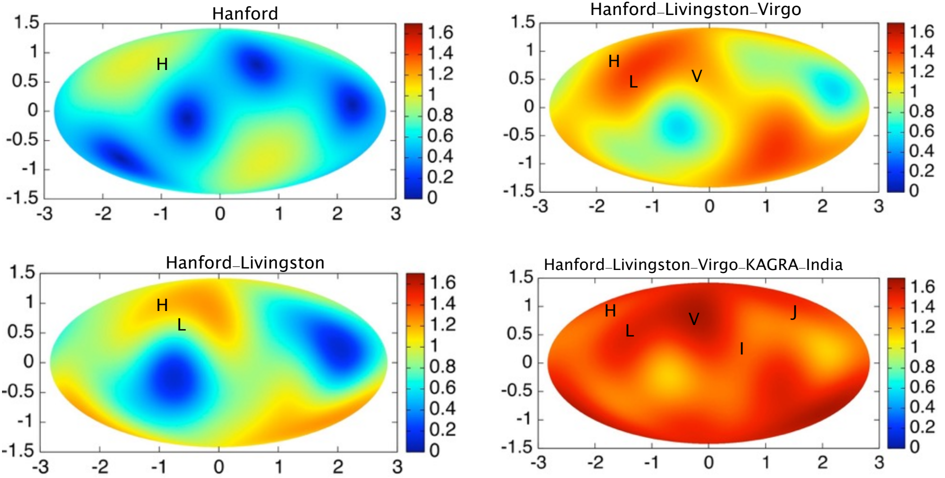
$$|h_{yy} - h_{xx}|$$

$$\frac{\delta L(t)}{L} = h(t) = F^+ h_+(t) + F^\times h_\times(t)$$

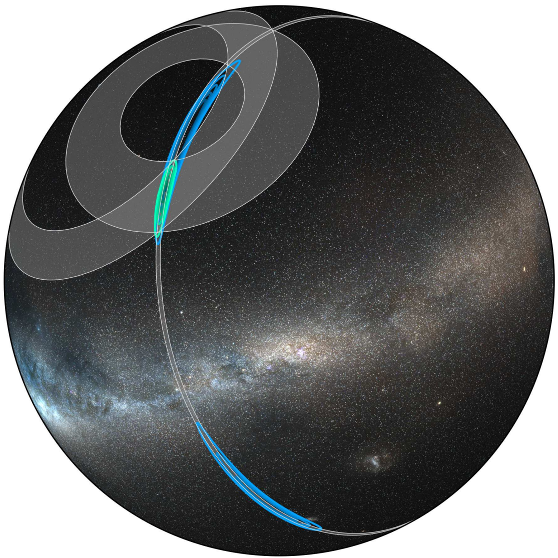


- Interferometers have a broad antenna pattern
  - Cannot locate direction of the source with a single detector
  - Can scan large portions of the sky simultaneously

Antenna pattern of the network of GW detectors

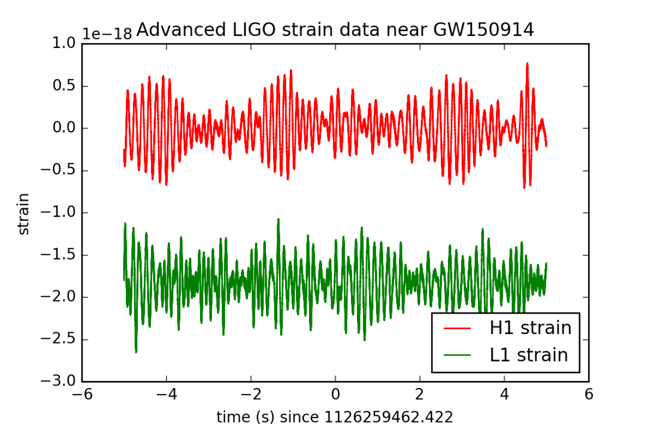


GW170817: LIGO-Virgo triangulation



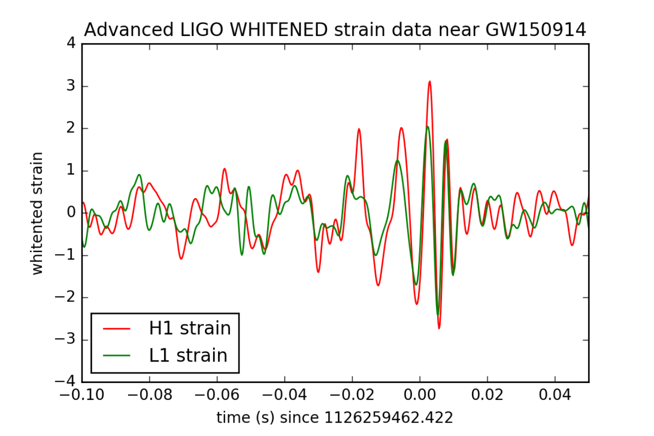
Credit: LIGO/Virgo/NASA/Leo Singer (Milky Way image: Axel Mellinger)

How the raw data looks like



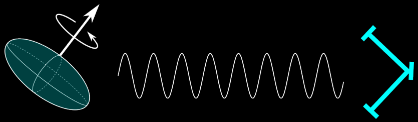
The data are dominated by the **low frequency noise** (L1 offset by -2×10^{-18} due to very low frequency oscillations).

Digging up the hidden signal (GW150914)



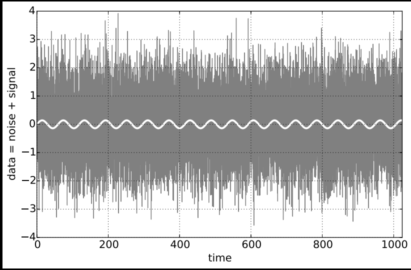
Whitening: dividing the data by the noise amplitude spectrum in the Fourier domain, filtering the frequencies outside the desired band with bandpass filter, suppressing the instrumental lines.

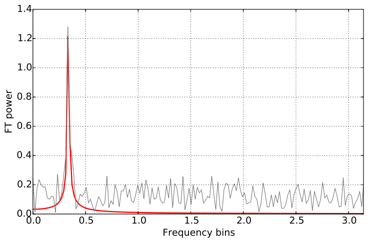
Matched filtering: a monochromatic signal



In this case a Fourier transform is sufficient to detect the signal (simplest matched filter method):

$$F = \left| \int_0^{T_0} x(t) \exp(-i\omega t) dt \right|^2$$





T_0 - time series duration, S_0 - spectral density of the data.

Signal-to-noise $SNR = h_0 \sqrt{\frac{T_0}{S_0}}$

Matched filtering

Assuming a signal model h, looking for the "best match" correlation $C(t)$ in data stream x, for a given time offset t

$$C(t) = \int_{-\infty}^{\infty} \underbrace{x(t')}_{\text{Data}} \times \underbrace{h(t'-t)}_{\text{Template with time offset } t} dt'$$

Rewrite correlation using Fourier transforms

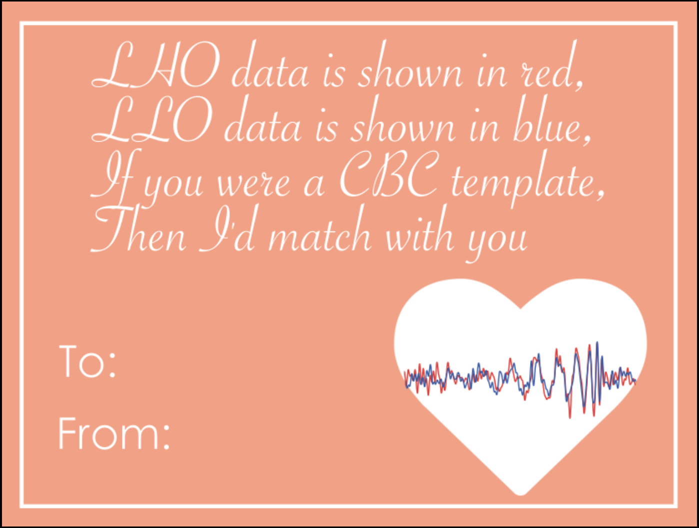
$$C(t) = 4 \int_{0}^{\infty} \tilde{x}(f)\tilde{h}^*(f) e^{2\pi i f t} df$$

(an inverse FT of $\tilde{x}(f)\tilde{h}^*(f)$). In practice, optimal matched filtering with the frequency weighting

$$C(t) = 4 \int_{0}^{\infty} \frac{\tilde{x}(f)\tilde{h}^*(f)}{S_n(f)} e^{2\pi i f t} df$$

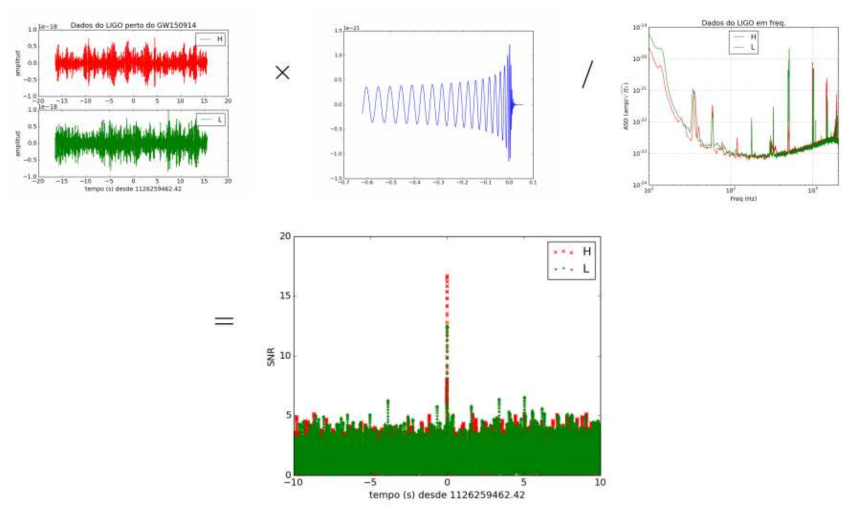
$S_n(f)$ - noise power spectral density

Choosing the best match



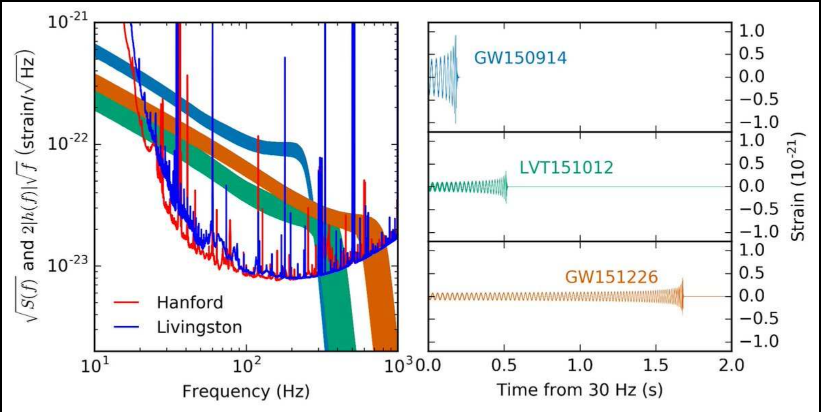
(CBC = Compact Binary Coalescence)

Matched filter in pictures



(from Riccardo Sturani's presentation)

LIGO O1 events



Optimal signal-to-noise ρ : $$\rho^2 = \int_0^\infty \left(\frac{2|\tilde{h}(f)|\sqrt{f}}{\sqrt{S_n(f)}} \right)^2 d\ln(f)$$

(GW150914: $\rho \simeq 24$, GW151226: $\rho \simeq 13$, LVT151012: $\rho \simeq 10$)

Astrophysical sources

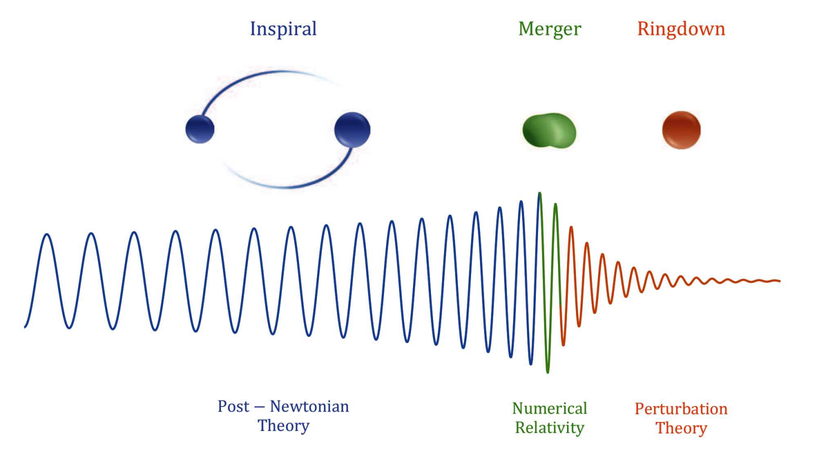
Non-axisymmetric (rapid) movemement of (large) masses
$\Longrightarrow$ GW emission

- ⋆ Cataclysmic:
  - ⋆ **binary systems** (black holes, neutron stars),
  - ⋆ **bursts** (e.g. supernovæ).
- ⋆ Long-lasting phenomena:
  - ⋆ **continuous waves** (rotating, deformed neutron stars = ‘gravitational pulsars’),
  - ⋆ **stochastic background** from a population of many objects.

Binary systems: Newton vs Einstein

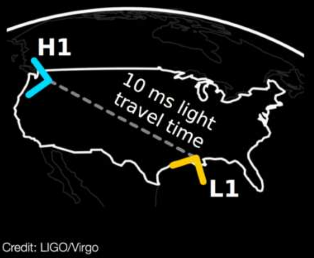
Spacetime, a third "body" in the binary system, is actively participating in the evolution!

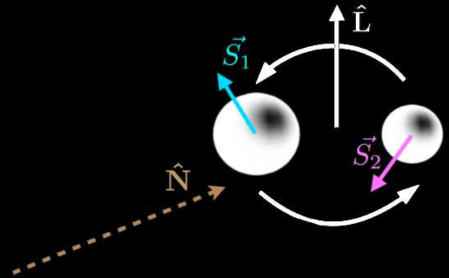
Evolution of a binary system



Binary system: 15+ parameters

- Intrinsic:
  - masses
  - spins
  - tidal deformability

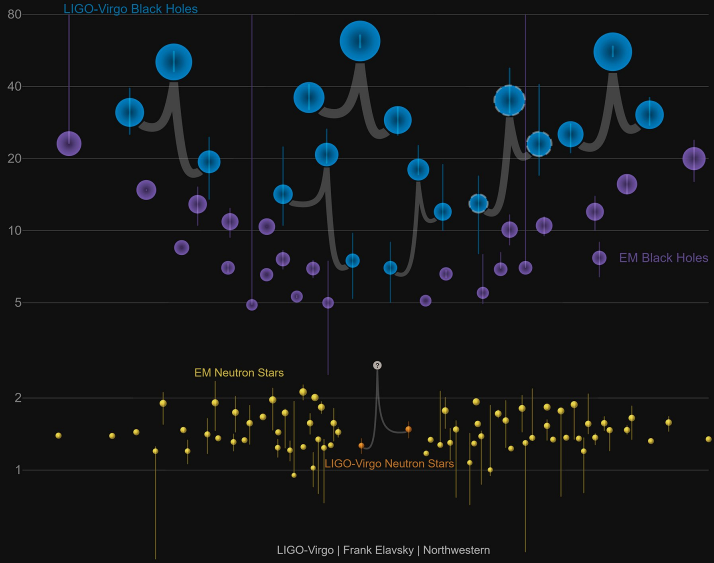




- Extrinsic:
  - Inclination, distance, polarisation
  - Sky location
  - Time, reference phase

Masses in the Stellar Graveyard

in Solar Masses



LIGO-Virgo | Frank Elavsky | Northwestern

Strain amplitude *h* and radiation modes

For a spherical wave of amplitude $h(r)$,
flux of energy is $F(r) \propto h^2(r)$
and the luminosity $L(r) \propto 4\pi r^2 h^2(r)$.

Hence, because of the conservation of energy:

$$h(r) \propto 1/r.$$

Radiating modes: quadrupole and higher

For a mass distribution $\rho(r)$, conserved moments:

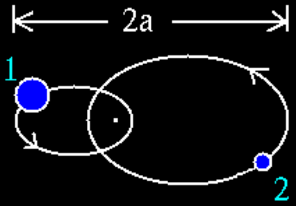
- ⋆ monopole $\int \rho(r) d^3 r$ - total mass-energy (energy conservation),
- ⋆ dipole $\int \rho(r) r d^3 r$ - center of mass-energy (momentum conservation).

Gravitational waves: some estimates

GWs correspond to accelerated movement of masses.

Consider a binary system of m_1 and m_2, semiaxis a with

- ⋆ total mass $M = m_1 + m_2$,
- ⋆ reduced mass $\mu = m_1 m_2 / M$,
- ⋆ mass quadrupole moment $Q \propto Ma^2$,
- ⋆ Kepler's third law $GM = a^3\omega^2$.



$$h(r) \propto \frac{1}{r}\frac{\partial^2 (Ma^2)}{\partial t^2} = \boxed{\frac{G^2}{c^4}\frac{1}{r}\frac{M\mu}{a} = \frac{G^{5/3}}{c^4}\frac{1}{r}M^{2/3}\mu\,\omega^{2/3}.}$$

Gravitational waves: quadrupole approximation

The quadrupole approximation (slowly-moving sources, Einstein 1918), wave amplitude is

$$h^{\mu\nu} = \frac{2}{r}\frac{G}{c^4}\ddot{Q}^{\mu\nu}, \quad \text{or, in terms of kinetic energy,} \quad h \sim \frac{E_{kin.}^{nsph.}}{r}.$$

Resulting GW luminosity is

$$L_{GW} \equiv \frac{dE_{GW}}{dt} \approx \frac{1}{5}\frac{G}{c^5}\langle \dddot{Q}^{\mu\nu}\dddot{Q}_{\mu\nu}\rangle$$

$$\propto \frac{G}{c^5}Q^2\omega^6 \propto \frac{G^4}{c^5}\left(\frac{M}{a}\right)^5 \propto \frac{c^5}{G}\left(\frac{R_s}{a}\right)^2\left(\frac{v}{c}\right)^6.$$

($R_s = 2GM/c^2$, $c^5/G \simeq 3.6 \times 10^{52}$ Joule/s)

Binary system: evolution of the orbit

Waves are emitted at the expense of the orbital energy:

$$E_{orb} = -\frac{Gm_1 m_2}{2a}, \qquad \frac{dE_{orb}}{dt} \equiv \frac{Gm_1 m_2}{2a^2}\dot{a} = -\frac{dE_{GW}}{dt}.$$

Evolution of the semi-major axis:

$$\frac{da}{dt} = -\frac{dE_{GW}}{dt}\frac{2a^2}{G\underbrace{m_1 m_2}_{\mu M}} \rightarrow \frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^4}{a^3}.$$

The system will coalesce after a time τ,

$$\tau = \frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^4},$$

where a_0 is the initial separation.

Binary system: chirp mass

Waves are emitted at the expense of the orbital energy:

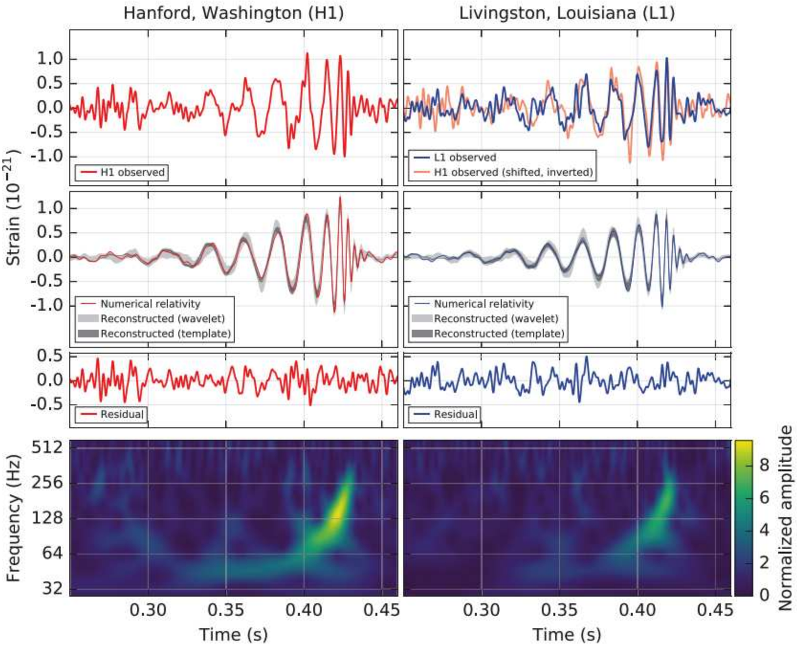
$$E_{orb} = -\frac{Gm_1 m_2}{2a}, \qquad \frac{dE_{orb}}{dt} \equiv \frac{Gm_1 m_2}{2a^2}\dot{a} = -\frac{dE_{GW}}{dt}.$$

Resulting evolution of the orbital frequency ω:

$$\dot{\omega}^3 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mu^3 M^2 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mathcal{M}^5,$$

where $\mathcal{M} = \left(\mu^3 M^2\right)^{1/5} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$ is the chirp mass. GWs frequency from a binary system is primarily twice the orbital frequency ($2\pi f_{GW} = 2\omega$). Hence $\mathcal{M}$ is a directly measured quantity:

$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3} f_{GW}^{-11/3} \dot{f}_{GW}\right)^{3/5}.$$


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Normalized amplitude
Time (s)
1.0
0.5
0.0
-0.5
-1.0
512
256
128
64
32
0.30
0.35
0.40
0.45
0
2
4
6
8

Binary system: energy emitted in GWs

End of the chirp f_{GW}^c is related to critical distance between masses a_{fin}:

$$a_{fin} = R_{s1} + R_{s2} = \frac{2G}{c^2}\,(m_1 + m_2)\,.$$

It can be used to estimate the total mass M:

$$M = m_1 + m_2 \approx \frac{c^3}{2\sqrt{2}G\pi}\frac{1}{f_{GW}^c}.$$

Energy emitted during the life of the binary system:

$$E = E_{ms} + E_{orb} = (m_1 + m_2)\,c^2 - \frac{Gm_1 m_2}{2a}.$$

(for $m_1 = m_2$, $a_{fin} = 2R_s = 4Gm_1/c^2$, $\Delta E \approx 6\%$).

Parameter estimation basics (GW510914)

GW amplitude dependence for a binary system

$$h \propto \mathcal{M}^{5/3} \times f_{GW}^{2/3} \times r^{-1}$$

where $\mathcal{M}$ is the chirp mass, $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$, known from the observations:

$$\mathcal{M} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f_{GW}^{-11/3}\dot{f}_{GW}\right]^{3/5}$$

From higher-order post-Newtonian corrections: $q = m_2/m_1$, spin components parallel to the orbital angular momentum...

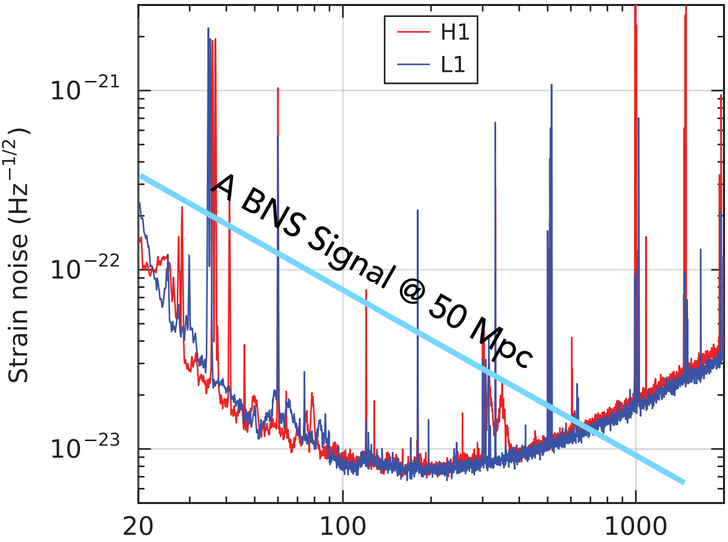
$$\mathcal{M} \simeq 30 M_{\odot} \Longrightarrow M = m_1 + m_2 \simeq 70 M_{\odot} \quad (\text{if } m_1 = m_2,\ M = 2^{6/5}\mathcal{M})$$

8 orbits observed until 150 Hz (orbital frequency 75 Hz):

- ⋆ Double neutron star system compact enough, but too light,
- ⋆ Neutron star-black hole system - black hole too big, would merge at lower frequency.

→ Double black hole binary.

LIGO SENSITIVITY DURING FIRST OBSERVING RUN (O1)



Binary inspiral vs the sensitivity curve

The so-called *Newtonian* signal at instantaneous frequency f_{GW} is

$$h = Q(\text{angles}) \times \mathcal{M}^{5/3} \times f_{GW}^{2/3} \times r^{-1} \times e^{-i\Phi}.$$

where the signal's phase is

$$\Phi(t) = \int 2\pi f_{GW}(t')dt'.$$

The relation between f_{GW} and t

$$\pi\mathcal{M} f_{GW}(t) = \left(\frac{5\mathcal{M}}{256(t_c - t)}\right)^{3/8}$$

The orbital velocity

$$v \propto (\pi\mathcal{M} f_{GW})^{1/3}$$

Binary inspiral vs the sensitivity curve

Matched filtering means that the signal is integrated with a proper phase as it sweeps through the range of frequencies.

Sensitivity curves most often show the effective (match-filtered) h_{eff}, and not the instantaneous h.

Dimensonal estimation of the frequency slope:

$$N_{cycles} \approx f_{GW}^2 \times \left(\frac{df_{GW}}{dt} \right)^{-1}$$

$$h_{eff} \propto \sqrt{N_{cycles}}\, h \propto \sqrt{f_{GW} t}\, h \propto \sqrt{f_{GW} \times f_{GW}^{-8/3}} \times f_{GW}^{2/3} \propto \boxed{f_{GW}^{-1/6}}.$$

Binary inspiral vs the sensitivity curve

Actually used in estimating the SNR is the frequency-domain match-filtering signal model $\tilde{h}(f)$ ($\simeq h_{eff}$, Fourier transform of $h(t)$),

$$\tilde{h}(f) = Q(angles)\sqrt{\frac{5}{24}}\pi^{-2/3}\frac{\mathcal{M}^{5/6}}{r}f_{GW}^{-7/6}e^{-i\Psi(f)},$$

where the frequency domain phase Ψ is (in point-particle approximation):

$$\Psi(f) \equiv \Psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu v^{5/2}}\sum_{k=0}^{N}\alpha_k v^{k/2}.$$

Binary system: source distance estimate

- ⋆ At cosmological distances, the observed frequency f_{GW} is redshifted by $(1+z)$:

$$f \to f/(1+z)$$

- ⋆ There is no mass scale in vacuum GR, so redshifting of f_{GW} cannot be distinguished from rescaling the masses

  because of the expansion in powers of $v \propto (\pi \mathcal{M} f_{GW})^{1/3}$

$\Longrightarrow$ inferred masses are $m = (1+z) m^{source}$

$\rightarrow$ Direct, independent **luminosity distance** measurement (but not z) from GW with f_{GW} and the strain h:

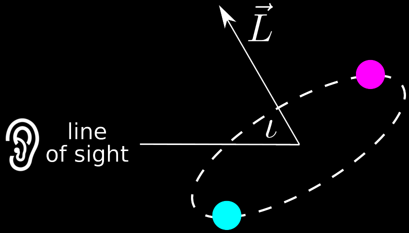
$$r = \frac{5}{96\pi^2} \frac{c}{h} \frac{\dot{f}_{GW}}{f_{GW}^3}.$$

Binary system: distance-inclination degeneracy

Luminosity distance $\sim 1/h$, and

$$h = h_+ F_+ + h_\times F_\times$$

depends on the inclination of the binary with repect to the ”line of sight”.



Two independent polarizations h_+ and $h_\times$:

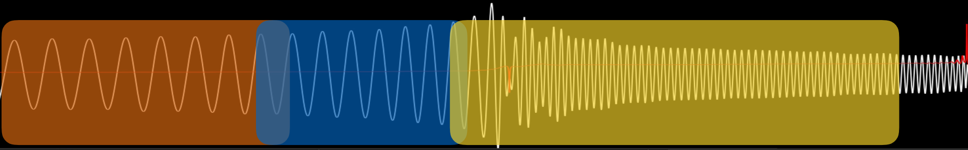
$$\begin{aligned} h_+ &= \frac{2\mu}{r} (\pi M f_{GW})^{2/3} \left(1 + \cos^2 \iota\right) \cos(2\phi(t)), \\ h_\times &= \frac{4\mu}{r} (\pi M f_{GW})^{2/3} \cos\iota \sin(2\phi(t)) \end{aligned}$$

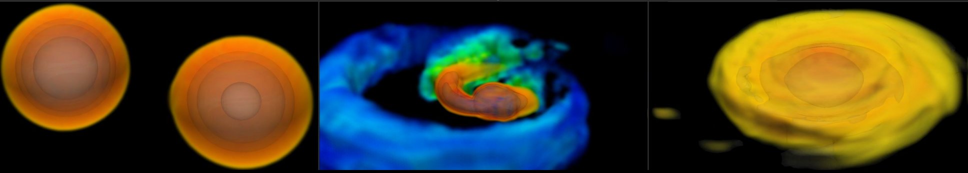
PHYSICAL EFFECTS IN BINARY NEUTRON STAR COALESCENCE WAVEFORMS

dominated by gravitational radiation back reaction - masses and spins

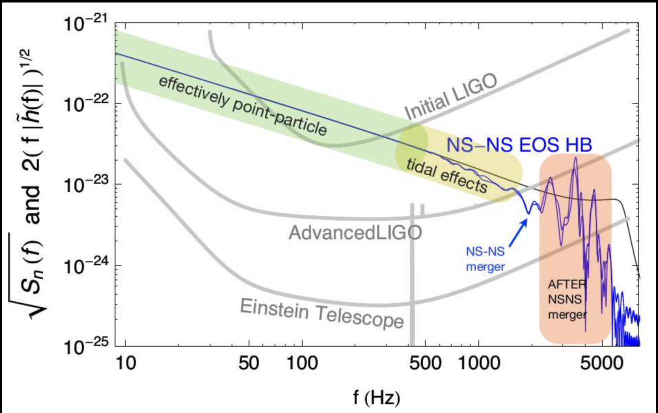
tidal effects appear at high PN order, dynamical tides might be important

complex physics of the merger remnant, multi-messenger source, signature of neutron star EoS





GW spectrum of 'material' binaries (e.g., BNSs)



Phase evolution differs from PP because of extended-body interactions:

$$\Psi(f) = \Psi_{PP}(f) + \Psi_{tidal}(f)$$

Ψ_{tidal} breaks the v expansion degeneracy.

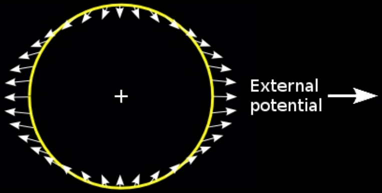
Signature of matter in binary NS waveforms

Tidal tensor $\mathcal{E}_{ij}$ of one of the components induces quadrupole moment Q_{ij} in the other:

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \quad \rightarrow \quad \lambda = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

In lowest-order approximation: $\boxed{\lambda = \frac{2}{3} k_2 R^5}$

λ - tidal deformability,
$k_2 \in (0.05, 0.15)$ - Love number
(dependent on M and EOS).



- $\star$ From the scaling this is a 5PN effect $(v/c)^{10}$
- $\star$ Convenient redefinition: $\boxed{\Lambda = G\lambda \left(\frac{GM}{c^2} \right)^{-5} \in (500, 3000)}$

Astrophysically-interesting parameters

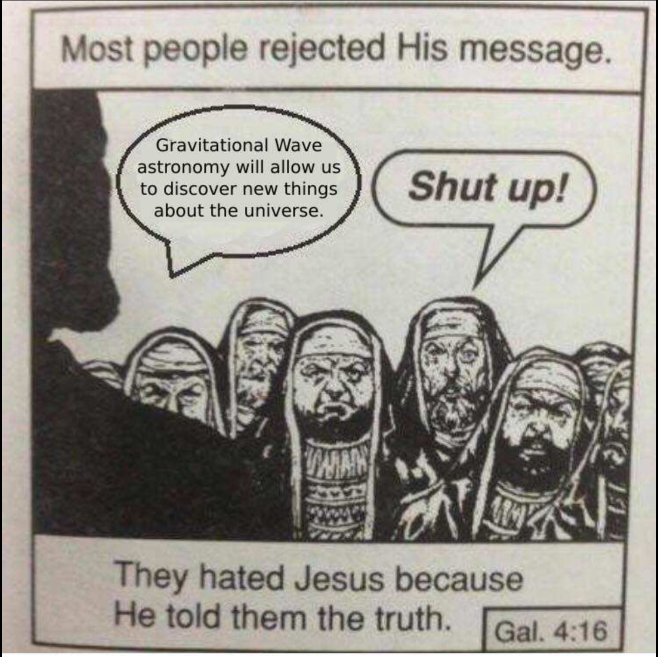
- ⋆ Chirp mass $\mathcal{M} = \left(\mu^3 M^2\right)^{1/5} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$,
- ⋆ Mass ratio $q = m_2/m_1$ (at 1PN), alternatively $\nu = m_1 m_2/(m_1 + m_2)^2$,
- ⋆ Spin-orbit and spin-spin coupling (at 2PN and 3PN, resp.) $\rightarrow$

$$\chi_{eff} = (m_1\chi_{1z} + m_2\chi_{2z})/(m_1 + m_2)$$

  where χ_{iz} are spin components along system's total angular momentum,
- ⋆ Tidal deformability Λ (at 5PN) $\rightarrow$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1}{(m_1 + m_2)^5} + (1 \leftrightarrow 2)$$

- ⋆ Direct "luminosity" ("loudness") distance: binary systems are "standard sirens".


Most people rejected His message.
Gravitational Wave astronomy will allow us to discover new things about the universe.
Shut up!
They hated Jesus because He told them the truth.
Gal. 4:16

EM vs GW

EM:

- ⋆ Created in microscopic processes by accelerated charges,
- ⋆ lowest multipole: dipole radiation,
- ⋆ scatters & is processed by matter.

Timing, spectrum, redshift, particle acceleration and thermal signatures $\rightarrow$ standard candles, outflows, last scattering surface …

GW:

- ⋆ Created in macroscopic processes by accelerated masses,
- ⋆ lowest multipole: quadrupole radiation (in GR),
- ⋆ once emitted interacts very weakly with matter.

Timing, mass & spin parameters $\rightarrow$ standard sirens (direct luminosity distance), core engine, cosmology, gravity theory tests …

Standard sirens in action

Binaries are standard sirens

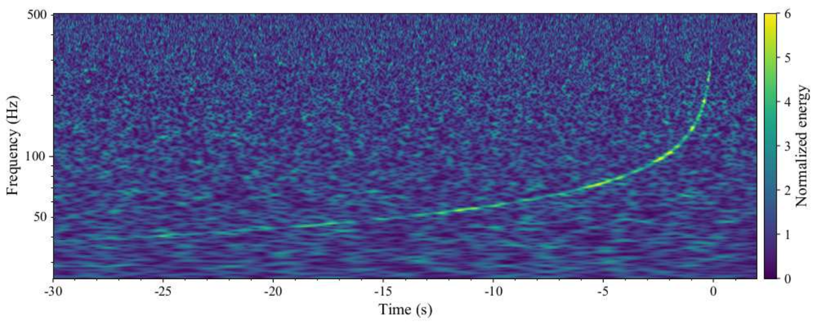
Binaries are *clean* systems: we have accurate models even in full general relativity.

Loss of energy to GWs causes orbit to decay, orbital frequency to go up. So the GWs will chirp up in frequency. Chirp time $t_{\rm chirp} \sim f\,/\,[{\rm d}f\,/\,{\rm d}t\,]$.

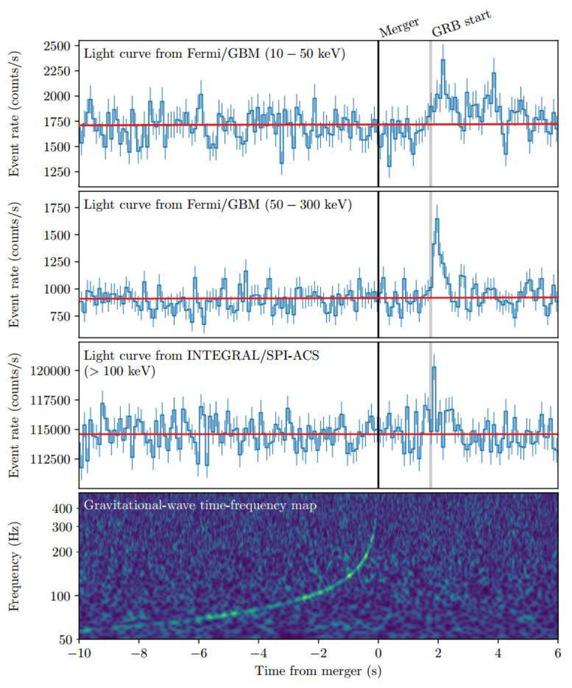
Signal contains both apparent brightness (from h and f) and intrinsic luminosity (from $t_{\rm chirp}$), from which we can compute the distance to the source:

$$\text{Distance} \propto c\frac{}{\text{frequency}^2 \times t_{\rm chirp}}$$

GW170817: 17 August 2017, 14:41:04 CEST

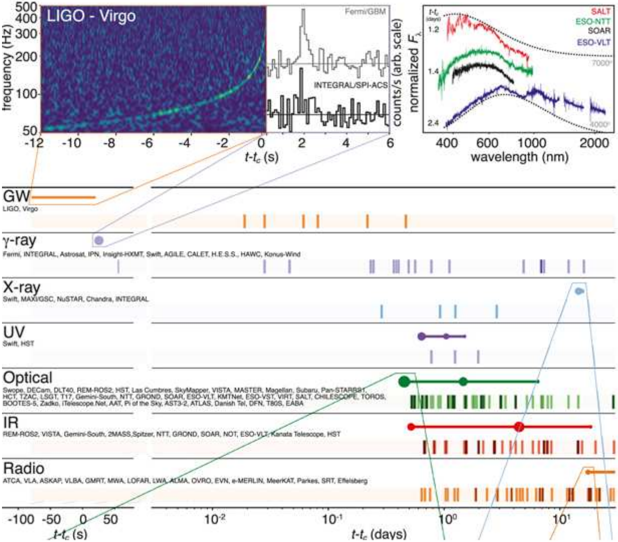


- ⋆ Combined LIGO-Virgo signal-to-noise ratio: SNR=32.4 (strongest signal so far!),
- ⋆ False alarm rate: less than one in 80000 years,
- ⋆ Chirp mass $\mathcal{M} = 1.188^{+0.004}_{-0.002}\ M_{\odot} \rightarrow$ a very light system!
- ⋆ Distance $d = 40^{+8}_{-14}$ Mpc (90% credible intervals)

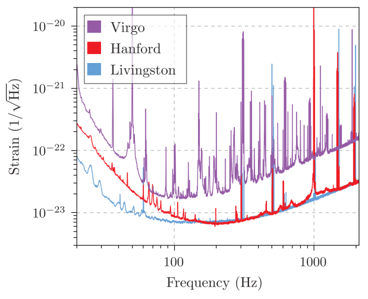


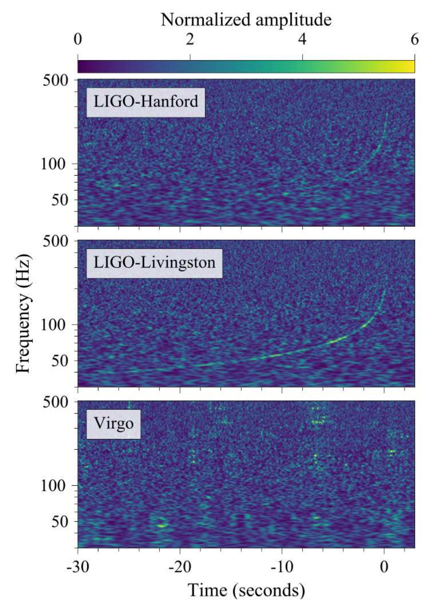
- ⋆ GW170817: **very long inspiral "chirp" (>100 s!)** firmly detected by the LIGO-Virgo network,
- ⋆ GRB 170817A: 1.74 $\pm$ 0.05 s later, weak **short gamma-ray burst** observed by Fermi (also detected by INTEGRAL),
- ⋆ First LIGO-Virgo alert 27 minutes later.

Marvel: “Infinity war is the most ambitious crossover event in history”
Me:



GW170817: localization of the source

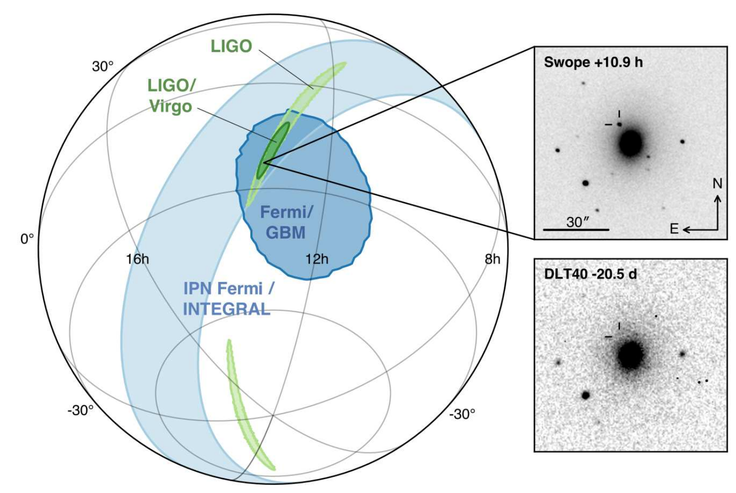




Average ranges for a BNS detection:

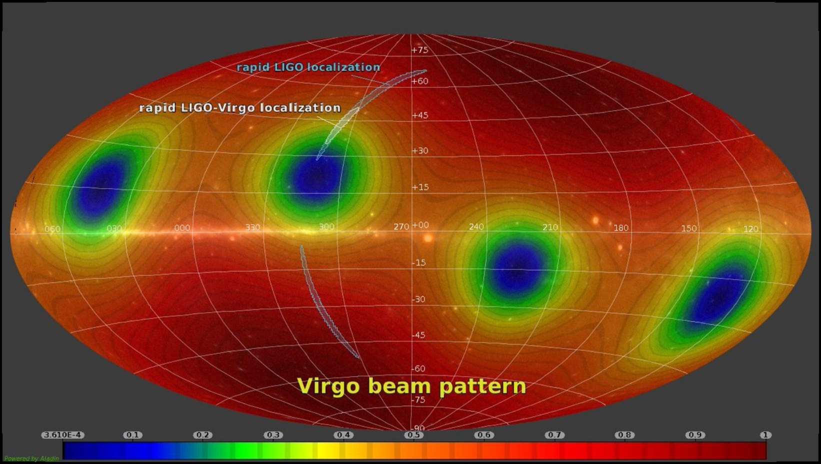
- ⋆ Livingston: $\approx$ 100 Mpc,
- ⋆ Hanford: $\simeq$ 60 Mpc,
- ⋆ Virgo: $\simeq$ 30 Mpc.

GW170817: LIGO-Virgo triangulation



- ⋆ Fortunate orientation of Virgo w.r.t. signal $\rightarrow$ small sky patch: $28^{\circ 2}$,
- ⋆ New EM source in NGC 4993, consistent with GW distance 40^{+8}_{-14} Mpc,
- ⋆ Chance of temporal-spatial coincidence $< 5 \times 10^{-8}$.

GW170817: Virgo beam patterns



GW170817: speed of gravitation

Relative speed difference between GWs and photons:

$$\frac{v_{GW} - c}{c} = \frac{\Delta v}{c} \approx \frac{c\Delta t}{d}.$$

Assuming very conservative values:

- ⋆ Distance $d = 26$ Mpc (lower bound from 90% credible interval on luminosity distance derived from the GW signal),
- ⋆ Time delay $\Delta t = 10$ s

$$-3 \times 10^{-15} \leq \frac{\Delta v}{c} \leq 7 \times 10^{-16}$$

$$\boxed{v_{GW} = 299792458^{+0.000001}_{-0.000006}\ \mathrm{m/s} = c^{+0.000001}_{-0.000006}\ \mathrm{m/s}}$$

Hubble plot ($v_H = H_0 d$)

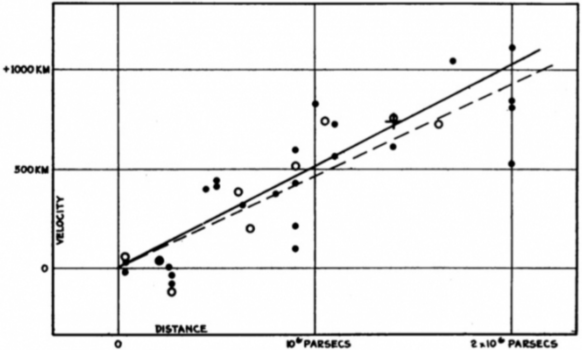
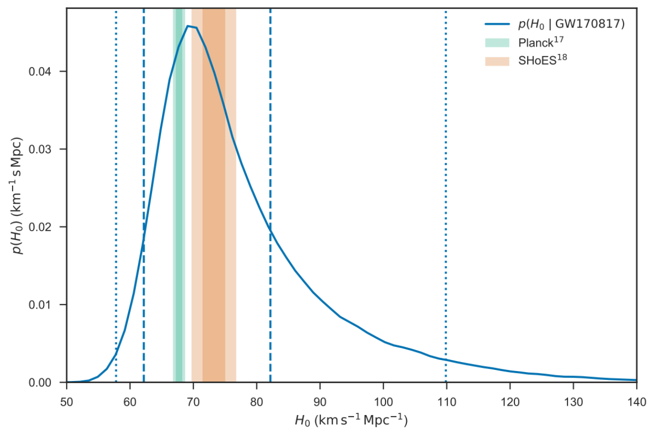


FIGURE 1

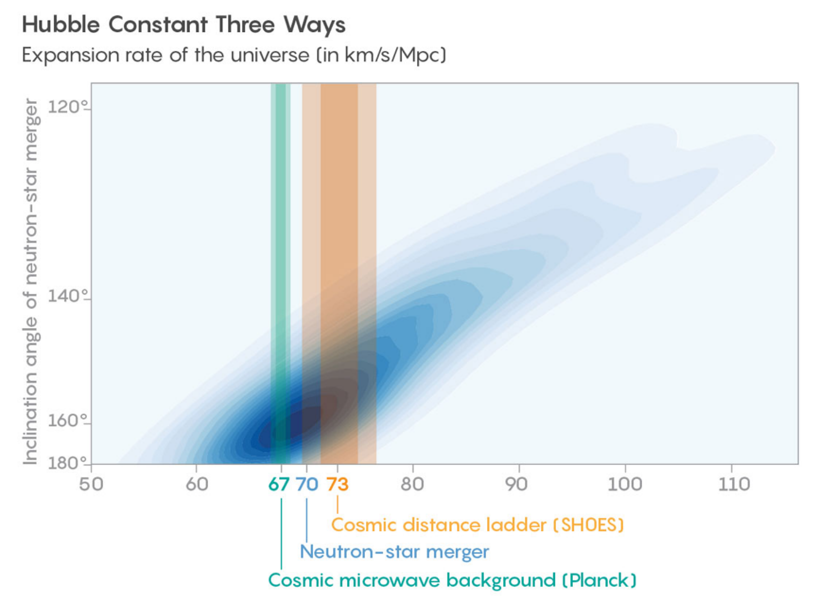
Velocity-Distance Relation among Extra-Galactic Nebulae.

GW170817: First "standard siren" H_0 measurement



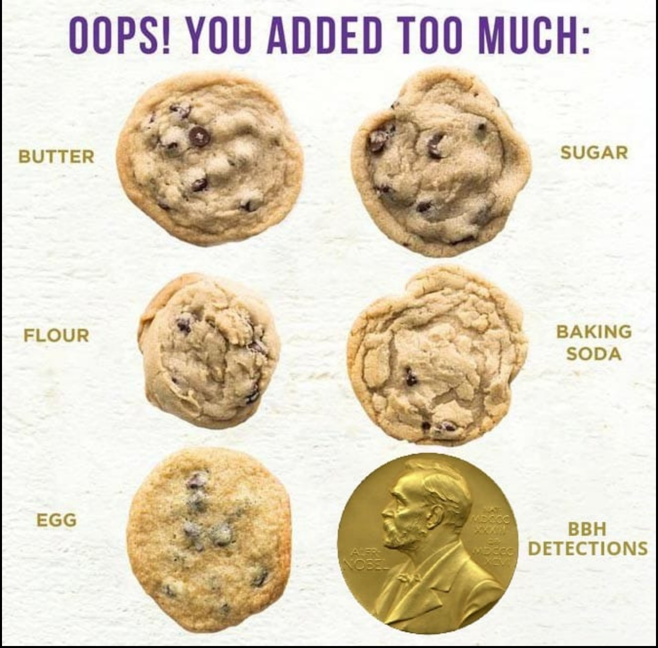
- $70.0^{+12.0}_{-8.0}$ km s^{-1} Mpc^{-1} (maximum a posteriori and 68% credible interval) = $\sim$14% at 1σ:
  - $\sim$11% because of GW luminosity distance,
  - The rest from the peculiar velocity of the galaxy.
- Planck: 67.74 ± 0.46, SHoES: 73.24 ± 1.74 km s^{-1} Mpc^{-1}

GW170817: Distance-binary inclination study



GWTC-1 - O1 & O2 detections (Phys. Rev. X 9, 031040 2019)

Nobel prize in physics, 2017



An unexpected shortage of neutron-star mergers?

BBH merger rate $\sim$10 – 100 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$, BNS $\sim$1000 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$

- ⋆ Salpeter initial mass function, $\xi(M) \propto M^{-2.35}$, for BHs and NSs progenitor stars:

$$\frac{N(M > 80M_\odot)}{N(M > 10M_\odot)} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06$$

- ⋆ If one assumes the same merger rates

$$\frac{\mathcal{R}_{BH}}{\mathcal{R}_{NS}} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06$$

- ⋆ Signal-to-noise $\propto \mathcal{M}^{5/6}$, detection volume $\propto SNR^3 \propto r^3$

$$\frac{\mathcal{D}_{BH}}{\mathcal{D}_{NS}} = \frac{\mathcal{R}_{BH}}{\mathcal{R}_{NS}} \left(\frac{\mathcal{M}_{BH}}{\mathcal{M}_{NS}}\right)^{5/2} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \left(\frac{8.7M_\odot}{1.4M_\odot}\right)^{5/2} \simeq 5.8$$

(Phys. Usp. 44 1 2001 `[astro-ph/0008481]`)


VIRGO
LIGO
What about BNS announcement?
Scientific community
We already had it.
LIGO
We've had one, yes.
LIGO
What about second BNS announcement?

Soon (feat. ML!)

Neutron stars: dense matter equation of state, mass-tidal deformability relationship, maximum mass $\leftrightarrow$ mass gap, phase transitions to quark-gluon plasma, additional fields and particles / exotic matter, elastic properties matter and phases, viscosity, magnetic field, oscillations and stability of stars, glitches, superfluidity and superconductivity, ...

Multi-messenger astronomy: gamma-ray bursts, production of elements, mass function of black holes and neutron stars, environment in host galaxies, various channels for creating binary systems, ...

Cosmology: Hubble parameter, dark energy equation of state, interaction with dark matter particles, large-scale structure, stochastic gravitational-wave background, lensing, ...

Tests of gravity theories: black hole spectroscopy (ringdown / ”podzwonne”), no-hair theorem, echoes, scalar & vector polarizations, strong field tests outside the quadrupole approximation, spin precession, Lorentz invariance, massive gravity, gravitational-wave memory, ...