

New Foundations is consistent

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Abstract

We give a self-contained account of a version of Holmes’ proof [1] that Quine’s set theory *New Foundations* [2] is consistent. This is a ‘deformalisation’ of the formal proof written in Lean at [3].

Note: At the present time, the formal proof [3] is incomplete, and this paper reflects the unfinished state of that proof. We aim to keep this paper proof in line with the formal proof, although as the project is ongoing, some variance is to be expected.

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1 The theories at issue

2 Outline

References

- [1] M. Randall Holmes. *NF is Consistent*. 2023. arXiv: [1503.01406](https://arxiv.org/abs/1503.01406) [math.LO].
- [2] W. V. Quine. “New Foundations for Mathematical Logic”. In: *American Mathematical Monthly* 44 (1937), pp. 70–80. URL: <https://api.semanticscholar.org/CorpusID:123927264>.
- [3] Sky Wilshaw et al. *The consistency of New Foundations*. 2022–2023. URL: <https://leanprover-community.github.io/con-nf/>.