ELECTRIC CIRCUITS

TENTH EDITION

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Figure 14.18 summarizes the high-pass filter circuits we have examined. Look carefully at the expressions for $H(s)$. Notice how similar in form these expressions are—they differ only in the denominator, which includes the cutoff frequency. As we did with the low-pass filters in Eq. 14.13, we state a general form for the transfer function of these two high-pass filters:

Transfer function for a high-pass filter ▶

$$H(s) = \frac{s}{s + \omega_c}. \tag{14.20}$$

Any circuit with the transfer function in Eq. 14.20 would behave as a high-pass filter with a cutoff frequency of ω_c. The problems at the end of the chapter give you other examples of circuits with this voltage ratio.

We have drawn attention to another important relationship. We have discovered that a series RC circuit has the same cutoff frequency whether it is configured as a low-pass filter or as a high-pass filter. The same is true of a series RL circuit. Having previously noted the connection between the cutoff frequency of a filter circuit and the time constant of that same circuit, we should expect the cutoff frequency to be a characteristic parameter of the circuit whose value depends only on the circuit components, their values, and the way they are connected.

✓ ASSESSMENT PROBLEMS

Objective 2—Know the *RL* and *RC* circuit configurations that act as high-pass filters

14.3 A series RL high-pass filter has $R = 5\ \text{k}\Omega$ and $L = 3.5$ mH. What is ω_c for this filter?

Answer: 1.43 Mrad/s.

14.4 A series RC high-pass filter has $C = 1\ \mu$F. Compute the cutoff frequency for the following values of R: (a) 100 Ω; (b) 5 kΩ; and (c) 30 kΩ.

Answer: (a) 10 krad/s;
(b) 200 rad/s;
(c) 33.33 rad/s.

14.5 Compute the transfer function of a series RC low-pass filter that has a load resistor R_L in parallel with its capacitor.

Answer: $H(s) = \dfrac{\dfrac{1}{RC}}{s + \dfrac{1}{KRC}}$, where $K = \dfrac{R_L}{R + R_L}$.

NOTE: Also try ChapterProblems 14.13 and 14.17.

14.4 Bandpass Filters

The next filters we examine are those that pass voltages within a band of frequencies to the output while filtering out voltages at frequencies outside this band. These filters are somewhat more complicated than the low-pass and high-pass filters of the previous sections. As we have already seen in Fig. 14.3(c), ideal bandpass filters have two cutoff frequencies, ω_{c1} and ω_{c2}, which identify the passband. For realistic bandpass filters, these cutoff frequencies are again defined as the frequencies for which the magnitude of the transfer function equals $(1/\sqrt{2})H_{\text{max}}$.

Center Frequency, Bandwidth, and Quality Factor

There are three other important parameters that characterize a bandpass filter. The first is the **center frequency**, ω_o, defined as the frequency for which a circuit's transfer function is purely real. Another name for the center frequency is the **resonant frequency**. This is the same name given to the frequency that characterizes the natural response of the second-order circuits in Chapter 8, because they are the same frequencies! When a circuit is driven at the resonant frequency, we say that the circuit is *in resonance*, because the frequency of the forcing function is the same as the natural frequency of the circuit. The center frequency is the geometric center of the passband, that is, $\omega_o = \sqrt{\omega_{c1}\omega_{c2}}$. For bandpass filters, the magnitude of the transfer function is a maximum at the center frequency ($H_{\max} = |H(j\omega_o)|$).

The second parameter is the **bandwidth**, β, which is the width of the passband. The final parameter is the **quality factor**, which is the ratio of the center frequency to the bandwidth. The quality factor gives a measure of the width of the passband, independent of its location on the frequency axis. It also describes the shape of the magnitude plot, independent of frequency.

Although there are five different parameters that characterize the bandpass filter—ω_{c1}, ω_{c2}, ω_o, β, and Q—only two of the five can be specified independently. In other words, once we are able to solve for any two of these parameters, the other three can be calculated from the dependent relationships among them. We will define these quantities more specifically once we have analyzed a bandpass filter. In the next section, we examine two *RLC* circuits which act as bandpass filters, and then we derive expressions for all of their characteristic parameters.

The Series *RLC* Circuit—Qualitative Analysis

Figure 14.19(a) depicts a series *RLC* circuit. We want to consider the effect of changing the source frequency on the magnitude of the output voltage. As before, changes to the source frequency result in changes to the impedance of the capacitor and the inductor. This time, the qualitative analysis is somewhat more complicated, because the circuit has both an inductor and a capacitor.

At $\omega = 0$, the capacitor behaves like an open circuit, and the inductor behaves like a short circuit. The equivalent circuit is shown in Fig. 14.19(b). The open circuit representing the impedance of the capacitor prevents current from reaching the resistor, and the resulting output voltage is zero.

At $\omega = \infty$, the capacitor behaves like a short circuit, and the inductor behaves like an open circuit. The equivalent circuit is shown in Fig. 14.19(c). The inductor now prevents current from reaching the resistor, and again the output voltage is zero.

But what happens in the frequency region between $\omega = 0$ and $\omega = \infty$? Between these two extremes, both the capacitor and the inductor have finite impedances. In this region, voltage supplied by the source will drop across both the inductor and the capacitor, but some voltage will reach the resistor. Remember that the impedance of the capacitor is negative, whereas the impedance of the inductor is positive. Thus, at some frequency, the impedance of the capacitor and the impedance of the inductor have equal magnitudes and opposite signs; the two impedances cancel out, causing the output voltage to equal the source voltage. This special frequency is the center frequency, ω_o. On either side of ω_o, the output voltage is less than the source voltage. Note that at ω_o, the series combination of the inductor and capacitor appears as a short circuit.

The plot of the voltage magnitude ratio is shown in Fig. 14.20. Note that the ideal bandpass filter magnitude plot is overlaid on the plot of the series *RLC* transfer function magnitude.

Now consider what happens to the phase angle of the output voltage. At the frequency where the source and output voltage are the same, the phase angles are the same. As the frequency decreases, the phase angle contribution from the capacitor is larger than that from the inductor.

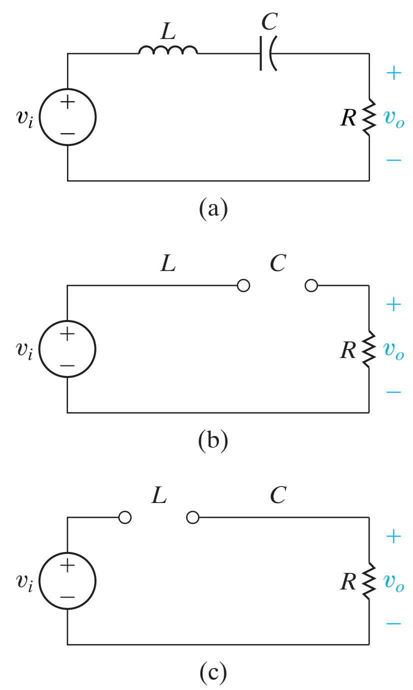

Figure 14.19 ▲ (a) A series *RLC* bandpass filter; (b) the equivalent circuit for $\omega = 0$; and (c) the equivalent circuit for $\omega = \infty$.

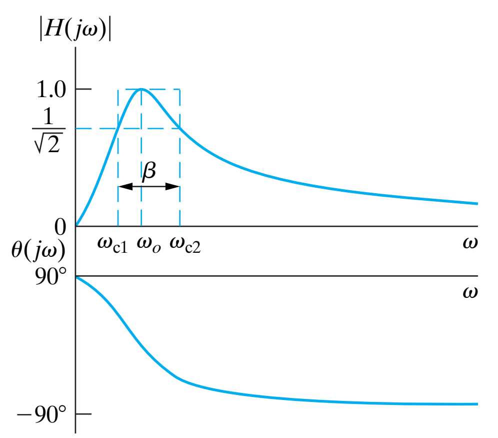

Figure 14.20 ▲ The frequency response plot for the series *RLC* bandpass filter circuit in Fig. 14.19.

Because the capacitor contributes positive phase shift, the net phase angle at the output is positive. At very low frequencies, the phase angle at the output maximizes at $+90^\circ$.

Conversely, if the frequency increases from the frequency at which the source and the output voltage are in phase, the phase angle contribution from the inductor is larger than that from the capacitor. The inductor contributes negative phase shift, so the net phase angle at the output is negative. At very high frequencies, the phase angle at the output reaches its negative maximum of -90°. The plot of the phase angle difference thus has the shape shown in Fig. 14.20.

The Series *RLC* Circuit—Quantitative Analysis

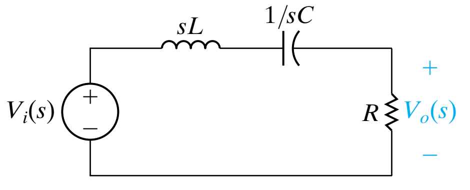

Figure 14.21 ▲ The s-domain equivalent for the circuit in Fig. 14.19(a).

We begin by drawing the s-domain equivalent for the series RLC circuit, as shown in Fig. 14.21. Use s-domain voltage division to write an equation for the transfer function:

$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}. \tag{14.21}$$

As before, we substitute $s = j\omega$ into Eq. 14.21 and produce the equations for the magnitude and the phase angle of the transfer function:

$$|H(j\omega)| = \frac{\omega(R/L)}{\sqrt{[(1/LC) - \omega^2]^2 + [\omega(R/L)]^2}}, \tag{14.22}$$

$$\theta(j\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega(R/L)}{(1/LC) - \omega^2}\right]. \tag{14.23}$$

We now calculate the five parameters that characterize this RLC bandpass filter. Recall that the center frequency, ω_o, is defined as the frequency for which the circuit's transfer function is purely real. The transfer function for the RLC circuit in Fig. 14.19(a) will be real when the frequency of the voltage source makes the sum of the capacitor and inductor impedances zero:

$$j\omega_o L + \frac{1}{j\omega_o C} = 0. \tag{14.24}$$

Solving Eq. 14.24 for ω_o, we get

Center frequency ▶

$$\boxed{\omega_o = \sqrt{\frac{1}{LC}}.} \tag{14.25}$$

Next, calculate the cutoff frequencies, ω_{c1} and ω_{c2}. Remember that at the cutoff frequencies, the magnitude of the transfer function is $(1/\sqrt{2})H_{\max}$. Because $H_{\max} = |H(j\omega_o)|$, we can calculate $H_{\max}$ by substituting Eq. 14.25 into Eq. 14.22:

$$\begin{aligned} H_{\max} &= |H(j\omega_o)| \\ &= \frac{\omega_o(R/L)}{\sqrt{[(1/LC) - \omega_o^2]^2 + (\omega_o R/L)^2}} \\ &= \frac{\sqrt{(1/LC)}(R/L)}{\sqrt{[(1/LC) - (1/LC)]^2 + \left[\sqrt{(1/LC)}(R/L)\right]^2}} = 1. \end{aligned}$$

Now set the left-hand side of Eq. 14.22 to $(1/\sqrt{2})H_{max}$ (which equals $1/\sqrt{2}$) and prepare to solve for ω_c:

$$\frac{1}{\sqrt{2}} = \frac{\omega_c(R/L)}{\sqrt{[(1/LC) - \omega_c^2]^2 + (\omega_c R/L)^2}}$$

$$= \frac{1}{\sqrt{[(\omega_c L/R) - (1/\omega_c RC)]^2 + 1}}. \quad (14.26)$$

We can equate the denominators of the two sides of Eq. 14.26 to get

$$\pm 1 = \omega_c \frac{L}{R} - \frac{1}{\omega_c RC}. \quad (14.27)$$

Rearranging Eq. 14.27 results in the following quadratic equation:

$$\omega_c^2 L \pm \omega_c R - 1/C = 0. \quad (14.28)$$

The solution of Eq. 14.28 yields four values for the cutoff frequency. Only two of these values are positive and have physical significance; they identify the passband of this filter:

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}, \quad (14.29)$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}. \quad (14.30)$$

◀ **Cutoff frequencies, series *RLC* filters**

We can use Eqs. 14.29 and 14.30 to confirm that the center frequency, ω_o, is the geometric mean of the two cutoff frequencies:

$$\omega_o = \sqrt{\omega_{c1} \cdot \omega_{c2}}$$

◀ **Relationship between center frequency and cutoff frequencies**

$$= \sqrt{\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]}$$

$$= \sqrt{\frac{1}{LC}}. \quad (14.31)$$

Recall that the bandwidth of a bandpass filter is defined as the difference between the two cutoff frequencies. Because $\omega_{c2} > \omega_{c1}$ we can compute the bandwidth by subtracting Eq. 14.29 from Eq. 14.30:

$$\beta = \omega_{c2} - \omega_{c1}$$

◀ **Relationship between bandwidth and cutoff frequencies**

$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right] - \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]$$

$$= \frac{R}{L}. \quad (14.32)$$

The quality factor, the last of the five characteristic parameters, is defined as the ratio of center frequency to bandwidth. Using Eqs. 14.25 and 14.32:

Quality factor ▶

$$Q = \omega_o/\beta = \frac{(1/LC)}{(R/L)} = \sqrt{\frac{L}{CR^2}}. \tag{14.33}$$

We now have five parameters that characterize the series RLC bandpass filter: two cutoff frequencies, ω_{c1} and ω_{c2}, which delimit the passband; the center frequency, ω_o, at which the magnitude of the transfer function is maximum; the bandwidth, β, a measure of the width of the passband; and the quality factor, Q, a second measure of passband width. As previously noted, only two of these parameters can be specified independently in a design. We have already observed that the quality factor is specified in terms of the center frequency and the bandwidth. We can also rewrite the equations for the cutoff frequencies in terms of the center frequency and the bandwidth:

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}, \tag{14.34}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}. \tag{14.35}$$

Alternative forms for these equations express the cutoff frequencies in terms of the quality factor and the center frequency:

$$\omega_{c1} = \omega_o \cdot \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right], \tag{14.36}$$

$$\omega_{c2} = \omega_o \cdot \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]. \tag{14.37}$$

Also see Problem 14.24 at the end of the chapter.

The examples that follow illustrate the design of bandpass filters, introduce another RLC circuit that behaves as a bandpass filter, and examine the effects of source resistance on the characteristic parameters of a series RLC bandpass filter.

Example 14.5 Designing a Bandpass Filter

A graphic equalizer is an audio amplifier that allows you to select different levels of amplification within different frequency regions. Using the series RLC circuit in Fig. 14.19(a), choose values for R, L, and C that yield a bandpass circuit able to select inputs within the 1–10 kHz frequency band. Such a circuit might be used in a graphic equalizer to select this frequency band from the larger audio band (generally 0–20 kHz) prior to amplification.

Solution

We need to compute values for R, L, and C that produce a bandpass filter with cutoff frequencies of 1 kHz and 10 kHz. There are many possible approaches to a solution. For instance, we could use Eqs. 14.29 and 14.30, which specify ω_{c1} and ω_{c2} in terms of R, L, and C. Because of the form of these equations, the algebraic manipulations might get complicated. Instead, we will use the fact that the center frequency is the geometric mean of the cutoff frequencies to compute ω_o, and we will then use Eq. 14.31 to compute L and C from ω_o. Next we will use the definition of quality factor to compute Q, and last we will use Eq. 14.33 to compute R. Even though this approach involves more individual computational steps, each calculation is fairly simple.

Any approach we choose will provide only two equations—insufficient to solve for the three unknowns—because of the dependencies among the bandpass filter characteristics. Thus, we need to select a value for either R, L, or C and use the two equations we've chosen to calculate the remaining component values. Here, we choose 1 μF as the capacitor value, because there are stricter limitations on commercially available capacitors than on inductors or resistors.

We compute the center frequency as the geometric mean of the cutoff frequencies:

$$f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{(1000)(10{,}000)} = 3162.28 \text{ Hz}.$$

Next, compute the value of L using the computed center frequency and the selected value for C. We must remember to convert the center frequency to radians per second before we can use Eq. 14.31:

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{[2\pi(3162.28)]^2(10^{-6})} = 2.533 \text{ mH}.$$

The quality factor, Q, is defined as the ratio of the center frequency to the bandwidth. The bandwidth is the difference between the two cutoff frequency values. Thus,

$$Q = \frac{f_o}{f_{c2} - f_{c1}} = \frac{3162.28}{10{,}000 - 1000} = 0.3514.$$

Now use Eq. 14.33 to calculate R:

$$R = \sqrt{\frac{L}{CQ^2}} = \sqrt{\frac{0.0025}{(10^{-6})(0.3514)^2}} = 143.24\ \Omega.$$

To check whether these component values produce the bandpass filter we want, substitute them into Eqs. 14.29 and 14.30. We find that

$$\omega_{c1} = 6283.19 \text{ rad/s (1000 Hz)},$$

$$\omega_{c2} = 62{,}831.85 \text{ rad/s (10,000 Hz)},$$

which are the cutoff frequencies specified for the filter.

This example reminds us that only two of the five bandpass filter parameters can be specified independently. The other three parameters can always be computed from the two that are specified. In turn, these five parameter values depend on the three component values, R, L, and C, of which only two can be specified independently.

Example 14.6 Designing a Parallel *RLC* Bandpass Filter

a) Show that the *RLC* circuit in Fig. 14.22 is also a bandpass filter by deriving an expression for the transfer function $H(s)$.
b) Compute the center frequency, ω_o.
c) Calculate the cutoff frequencies, ω_{c1} and ω_{c2}, the bandwidth, β, and the quality factor, Q.
d) Compute values for R and L to yield a bandpass filter with a center frequency of 5 kHz and a bandwidth of 200 Hz, using a 5 μF capacitor.

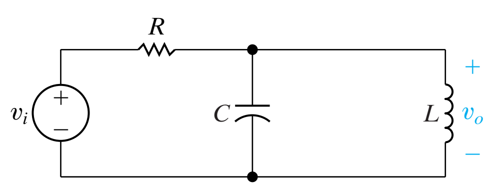

Figure 14.22 ▲ The circuit for Example 14.6.

Solution

a) Begin by drawing the s-domain equivalent of the circuit in Fig. 14.22, as shown in Fig. 14.23. Using voltage division, we can compute the transfer function for the equivalent circuit if we first compute the equivalent impedance of the parallel combination of L and C, identified as $Z_{eq}(s)$ in Fig. 14.23:

$$Z_{eq}(s) = \frac{\frac{L}{C}}{sL + \frac{1}{sC}}.$$

Now,

$$H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$

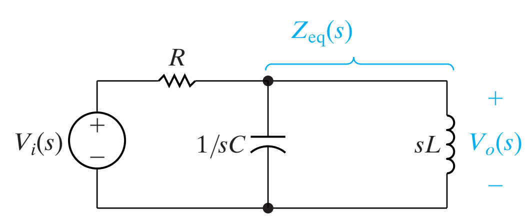

Figure 14.23 ▲ The s-domain equivalent of the circuit in Fig. 14.22.

b) To find the center frequency, , we need to calculate where the transfer function magnitude is maximum. Substituting $s = j\omega$ in $H(s)$,

$$|H(j\omega)| = \frac{\dfrac{\omega}{RC}}{\sqrt{\left(\dfrac{1}{LC} - \omega^2\right)^2 + \left(\dfrac{\omega}{RC}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\omega RC - \dfrac{1}{\omega \dfrac{L}{R}}\right)^2}}.$$

The magnitude of this transfer function is maximum when the term

$$\left(\frac{1}{LC} - \omega^2\right)^2$$

is zero. Thus,

$$\omega_o = \sqrt{\frac{1}{LC}}$$

and

$$H_{\max} = |H(j\omega_o)| = 1.$$

c) At the cutoff frequencies, the magnitude of the transfer function is $(1/\sqrt{2})H_{\max} = 1/\sqrt{2}$. Substituting this constant on the left-hand side of the magnitude equation and then simplifying, we get

$$\left[\omega_c RC - \frac{1}{\omega_c \dfrac{L}{R}}\right] = \pm 1.$$

Squaring the left-hand side of this equation once again produces two quadratic equations for the cutoff frequencies, with four solutions. Only two of them are positive and therefore have physical significance:

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}.$$

▲ **Cutoff frequencies for parallel *RLC* filters**

We compute the bandwidth from the cutoff frequencies:

$$\beta = \omega_{c2} - \omega_{c1}$$

$$= \frac{1}{RC}.$$

Finally, use the definition of quality factor to calculate Q:

$$Q = \omega_o/\beta$$

$$= \sqrt{\frac{R^2C}{L}}.$$

Notice that once again we can specify the cutoff frequencies for this bandpass filter in terms of its center frequency and bandwidth:

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2},$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}.$$

d) Use the equation for bandwidth in (c) to compute a value for R, given a capacitance of 5 μF. Remember to convert the bandwidth to the appropriate units:

$$R = \frac{1}{\beta C}$$

$$= \frac{1}{(2\pi)(200)(5 \times 10^{-6})}$$

$$= 159.15\ \Omega.$$

Using the value of capacitance and the equation for center frequency in (c), compute the inductor value:

$$L = \frac{1}{\omega_o^2 C}$$

$$= \frac{1}{[2\pi(5000)]^2(5 \times 10^{-6})}$$

$$= 202.64\ \mu\text{H}.$$

Example 14.7 Determining Effect of a Nonideal Voltage Source on a *RLC* Bandpass Filter

For each of the bandpass filters we have constructed, we have always assumed an ideal voltage source, that is, a voltage source with no series resistance. Even though this assumption is often valid, sometimes it is not, as in the case where the filter design can be achieved only with values of R, L, and C whose equivalent impedance has a magnitude close to the actual impedance of the voltage source. Examine the effect of assuming a nonzero source resistance, R_i, on the characteristics of a series RLC bandpass filter.

a) Determine the transfer function for the circuit in Fig. 14.24.
b) Sketch the magnitude plot for the circuit in Fig. 14.24, using the values for R, L, and C from Example 14.5 and setting $R_i = R$. On the same graph, sketch the magnitude plot for the circuit in Example 14.5, where $R_i = 0$.

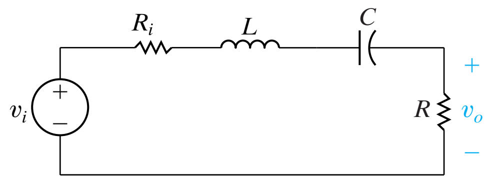

Figure 14.24 ▲ The circuit for Example 14.7.

Solution

a) Begin by transforming the circuit in Fig. 14.24 to its s-domain equivalent, as shown in Fig. 14.25. Now use voltage division to construct the transfer function:

$$H(s) = \frac{\frac{R}{L}s}{s^2 + \left(\frac{R + R_i}{L}\right)s + \frac{1}{LC}}.$$

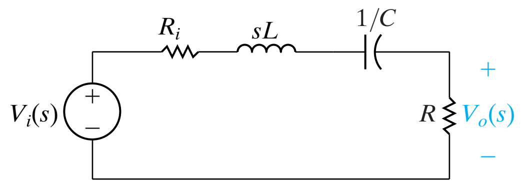

Figure 14.25 ▲ The s-domain equivalent of the circuit in Fig. 14.24.

Substitute $s = j\omega$ and calculate the transfer function magnitude:

$$|H(j\omega)| = \frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\frac{R + R_i}{L}\right)^2}}.$$

The center frequency, ω_o, is the frequency at which this transfer function magnitude is maximum, which is

$$\omega_o = \sqrt{\frac{1}{LC}}.$$

At the center frequency, the maximum magnitude is

$$H_{\max} = |H(j\omega_o)| = \frac{R}{R_i + R}.$$

The cutoff frequencies can be computed by setting the transfer function magnitude equal to $(1/\sqrt{2})H_{\max}$:

$$\omega_{c1} = -\frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c2} = \frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}}.$$

The bandwidth is calculated from the cutoff frequencies:

$$\beta = \frac{R + R_i}{L}.$$

Finally, the quality factor is computed from the center frequency and the bandwidth:

$$Q = \frac{\sqrt{L/C}}{R + R_i}.$$

From this analysis, note that we can write the transfer function of the series RLC bandpass filter with nonzero source resistance as

$$H(s) = \frac{K\beta s}{s^2 + \beta s + \omega_o^2},$$

where

$$K = \frac{R}{R + R_i}.$$

Note that when $R_i = 0$, $K = 1$ and the transfer function is

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$

b) The circuit in Example 14.5 has a center frequency of 3162.28 Hz and a bandwidth of 9 kHz, and $H_{\max} = 1$. If we use the same values for R, L, and C in the circuit in Fig. 14.24 and let $R_i = R$, then the center frequency remains at

3162.28 kHz, but $\beta = (R + R_i)/L = 18$ kHz, and $H_{max} = R/(R + R_i) = 1/2$. The transfer function magnitudes for these two bandpass filters are plotted on the same graph in Fig. 14.26.

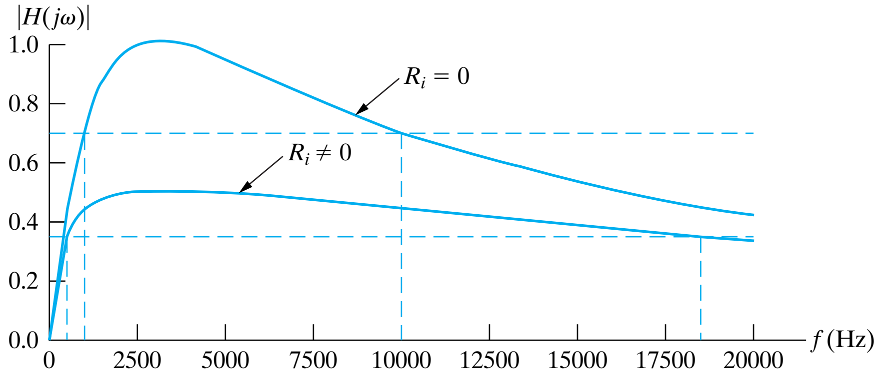

Figure 14.26 ▲ The magnitude plots for a series *RLC* bandpass filter with a zero source resistance and a nonzero source resistance.

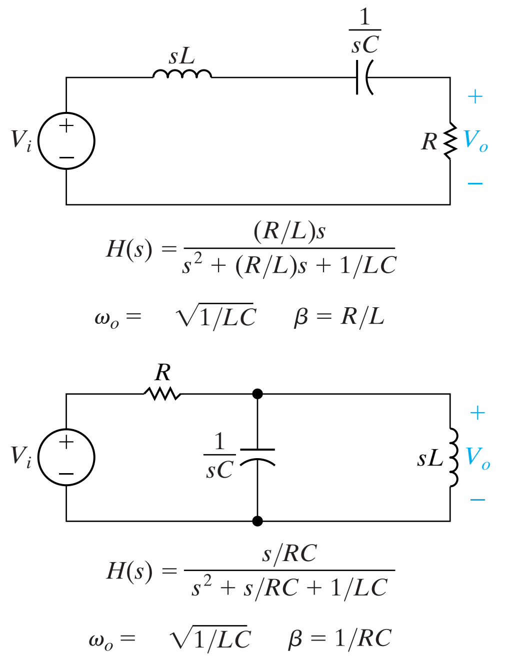

Figure 14.27 ▲ Two *RLC* bandpass filters, together with equations for the transfer function, center frequency, and bandwidth of each.

If we compare the characteristic parameter values for the filter with $R_i = 0$ to the values for the filter with $R_i \neq 0$, we see the following:

- The center frequencies are the same.
- The maximum transfer function magnitude for the filter with $R_i \neq 0$ is smaller than for the filter with $R_i = 0$.
- The bandwidth for the filter with $R_i \neq 0$ is larger than that for the filter with $R_i = 0$. Thus, the cutoff frequencies and the quality factors for the two circuits are also different.

The addition of a nonzero source resistance to a series *RLC* bandpass filter leaves the center frequency unchanged but widens the passband and reduces the passband magnitude.

Here we see the same design challenge we saw with the addition of a load resistor to the high-pass filter, that is, we would like to design a bandpass filter that will have the same filtering properties regardless of any internal resistance associated with the voltage source. Unfortunately, filters constructed from passive elements have their filtering action altered with the addition of source resistance. In Chapter 15, we will discover that active filters are insensitive to changes in source resistance and thus are better suited to designs in which this is an important issue.

Figure 14.27 summarizes the two *RLC* bandpass filters we have studied. Note that the expressions for the circuit transfer functions have the same form. As we have done previously, we can create a general form for the transfer functions of these two bandpass filters:

Transfer function for *RLC* bandpass filter ▶

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}. \tag{14.38}$$

Any circuit with the transfer function in Eq. 14.38 acts as a bandpass filter with a center frequency ω_o and a bandwidth β.

In Example 14.7, we saw that the transfer function can also be written in the form

$$H(s) = \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \tag{14.39}$$

where the values for K and β depend on whether the series resistance of the voltage source is zero or nonzero.

Relating the Frequency Domain to the Time Domain

We can identify a relationship between the parameters that characterize the frequency response of *RLC* bandpass filters and the parameters that characterize the time response of *RLC* circuits. Consider the series *RLC* circuit in Fig. 14.19(a). In Chapter 8 we discovered that the natural response of this circuit is characterized by the neper frequency (α) and the resonant frequency (ω_o). These parameters were expressed in terms of the circuit components in Eqs. 8.58 and 8.59, which are repeated here for convenience:

$$\alpha = \frac{R}{2L} \text{ rad/s,} \tag{14.40}$$

$$\omega_o = \sqrt{\frac{1}{LC}} \text{ rad/s.} \tag{14.41}$$

We see that the same parameter ω_o is used to characterize both the time response and the frequency response. That's why the center frequency is also called the resonant frequency. The bandwidth and the neper frequency are related by the equation

$$\beta = 2\alpha. \tag{14.42}$$

Recall that the natural response of a series *RLC* circuit may be underdamped, overdamped, or critically damped. The transition from overdamped to underdamped occurs when $\omega_o^2 = \alpha^2$. Consider the relationship between α and β from Eq. 14.42 and the definition of the quality factor Q. The transition from an overdamped to an underdamped response occurs when $Q = 1/2$. Thus, a circuit whose frequency response contains a sharp peak at ω_o, indicating a high Q and a narrow bandwidth, will have an underdamped natural response. Conversely, a circuit whose frequency response has a broad bandwidth and a low Q will have an overdamped natural response.

✓ ASSESSMENT PROBLEMS

Objective 3—Know the *RLC* circuit configurations that act as bandpass filters

14.6 Using the circuit in Fig. 14.19(a), compute the values of R and L to give a bandpass filter with a center frequency of 12 kHz and a quality factor of 6. Use a 0.1 μF capacitor.

Answer: $L = 1.76$ mH, $R = 22.10\ \Omega$.

14.7 Using the circuit in Fig. 14.22, compute the values of L and C to give a bandpass filter with a center frequency of 2 kHz and a bandwidth of 500 Hz. Use a 250 Ω resistor.

Answer: $L = 4.97$ mH, $C = 1.27\ \mu$F.

NOTE: Also try Chapter Problems 14.18 and 14.25.

14.8 Recalculate the component values for the circuit in Example 14.6(d) so that the frequency response of the resulting circuit is unchanged using a 0.2 μF capacitor.

Answer: $L = 5.07$ mH, $R = 3.98$ kΩ.

14.9 Recalculate the component values for the circuit in Example 14.6(d) so that the quality factor of the resulting circuit is unchanged but the center frequency has been moved to 2 kHz. Use a 0.2 μF capacitor.

Answer: $R = 9.95$ kΩ, $L = 31.66$ mH.

14.5 Bandreject Filters

We turn now to the last of the four filter categories—the bandreject filter. This filter passes source voltages outside the band between the two cutoff frequencies to the output (the passband), and attenuates source voltages