

# Artificial Intelligence

## Exercises week 8 - Knowledge representation and uncertainty

COMP3411/9814

### Question 1: Logic

Consider the following predicate logic expressions. Which one are true?

1. The predicate logic of “There exists a student who has taken both calculus and linear algebra” is  $\exists x(S(x) \wedge C(x) \wedge L(x))$  where:  
  - $S(x)$  represents “ $x$  is a student.”
  - $C(x)$  represents “ $x$  has taken calculus.”
  - $L(x)$  represents “ $x$  has taken linear algebra.”T
2. The predicate logic of “Every person who owns a car has a driver’s license” is  $\forall x(P(x) \wedge O(x) \wedge D(x))$  where:  
  - $P(x)$  represents “ $x$  is a person.”
  - $O(x)$  represents “ $x$  owns a car.”
  - $D(x)$  represents “ $x$  has a driver’s license.”F
3. The predicate logic of “Some employees who work more than 40 hours a week do not receive overtime pay” is  $E(x) \wedge W(x) \wedge \neg O(x)$  where:  
  - $E(x)$  represents “ $x$  is an employee.”F

- $W(x)$  represents “ $x$  works more than 40 hours a week.”
  - $O(x)$  represents “ $x$  receives overtime pay.”
4. The predicate logic of “If any customer buys a product and the product is defective, then the customer will return the product” is  $\forall x(C(x) \wedge P(x) \wedge D(x) \rightarrow R(x))$  where:
- $C(x)$  represents “ $x$  is a customer.”
  - $P(x)$  represents “ $x$  buys a product.”
  - $D(x)$  represents “The product bought by  $x$  is defective.”
  - $R(x)$  represents “ $x$  will return the product.”

## Question 2: Bayes nets

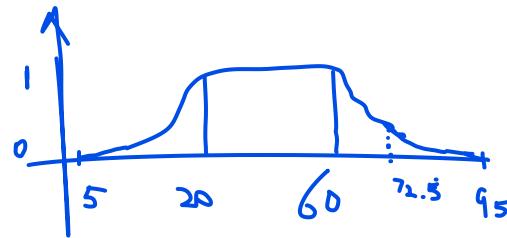
Suppose a certain medical test for a disease is known to have a false positive rate of 5% and a false negative rate of 10%. The prevalence of the disease in the population is 1%. If a person tests positive, what is the probability  $P_1$  that they actually have the disease? If the false negative rate increases to 15%, recalculate the probability  $P_2$  that a person actually has the disease given a positive test result. If the false positive rate decreases to 1%, while keeping the true positive rate at 90%, what is the new probability  $P_3$  that a person has the disease given a positive test result?

$$\begin{aligned} PP|D=0.05 \\ PN|D=0.1 \\ PD=0.01 \\ PD|P=PP|D*PD/PP \\ =0.9*0.01/0.0585 \end{aligned}$$

## Question 3: Fuzzy logic

In a streaming platform, the analysis team is performing a study on how a “normal customer” uses the platform. Based on the number of hours each customer uses the platform, they have ordered the customers and defined a fuzzy set for normal customers using a trapezoid membership function. The analysis indicate the following:

- Customers in the lowest 5% of use have a membership equal to zero.
- Customers between 5% and 20% of use have an increasing membership from zero to one.



- Customers between 20% and 60% of use have a membership equal to one.
- Customers between 60% and 95% of use have a decreasing membership from one to zero.
- Customers in the highest 5% of use (between 95% and 100%) have a membership equal to zero.

For a customer  $X$  located at position 72.5% of use compared with other customers, what would be the membership value of  $X$ ?