Artificial Intelligence
Exercises week 5 - Optimisation - **Solutions**

COMP3411/9814

Question 1: Mutations

In a genetic algorithm with 10 chromosomes, each 20 genes long, and a mutation rate of 0.01 (per gene), how many gene mutations are expected per generation?

Answer: Total genes = $10 \times 20 = 200$. Expected mutations = $0.01 \times 200 = 2$

Question 2: Offspring fitness

Suppose a genetic algorithm encodes solutions as 4-bit chromosomes, and the fitness function is $f(b_1b_2b_3b_4) = b_1 + 2b_2 + 4b_3 + 8b_4$ (where $b_i \in \{0, 1\}$). Given parent chromosomes A = 1100 and B = 0011, after single-point crossover between the second and third bits, what is the fitness of the offspring?

Answer: $f_{AB}(1111) = 15$ and $f_{BA}(0000) = 0$.
Note that no selection method was indicated, therefore, we included the whole offspring.

Question 3: Tabu search

Given a small optimisation problem: Minimise the function $f(x) = x^2 - 4x + 4$ where x can take integer values between -10 and 10. Perform one iteration of the Tabu Search algorithm starting from $x = 0$. Assume two neighbours,

consisting of $x = -1$ and $x = 1$. What is the new value of x after the first iteration?

Answer:

Let's perform the Tabu Search algorithm step by step:

1. Initial Solution: $x_0 = 0$, $f(x_0) = 0^2 - 4(0) + 4 = 4$

2. Neighbourhood Solutions:

- $x_1 = 1$, $f(x_1) = 1^2 - 4(1) + 4 = 1$
- $x_{-1} = -1$, $f(x_{-1}) = (-1)^2 - 4(-1) + 4 = 9$

3. Selecting the Best Neighbour: The best neighbour solution is $x = 1$ with $f(x_1) = 1$.

4. Update Tabu List: Add $x = 1$ to the tabu list.

5. New Solution: Move to $x = 1$ for the next iteration.

Therefore, the correct answer is $x = 1$.

Question 4: Genetic operators

Consider two chromosomes in a genetic algorithm where symbol '|' marks the crossover point. The fitness function is defined as the number of '1' in a chromosome.

- Chromosome 1: 1100 | 1010
- Chromosome 2: 1010 | 1011

Questions:

- What are the fitness values of original chromosomes?
- What would be the two offspring or children after the crossover?
- What are the fitness scores of both offspring?
- Perform bit flip mutations at positions 2 and 5 on both offspring with 0 based index from left to right. What are the new offspring?
- An offspring is considered fit if its fitness score > 5. How many of the offspring after mutation are fit?

Answers:

- Original fitness values - 4, 5
- Offsprings after crossover - 11001011, 10101010
- Fitness score after crossover - 5, 4
- Mutated offspring - 11101111, 10001110
- Number of fit offspring - 1

Question 5: Genetic algorithm

A genetic algorithm is applied to optimise a binary chromosome problem. The initial population consists of the following chromosomes:

- Parent 1: 1101001011
- Parent 2: 1010110101

The algorithm proceeds with the following steps in sequence:

1. Perform single-point crossover at the 4th position (zero-based indexing) between Parent 1 and Parent 2.
2. Apply mutation to Offspring 1 by flipping the bits at positions 2, 6, and 9 (zero-based indexing).
3. Compute the fitness function for Offspring 1 after mutation using the equation:

$$\text{Fitness} = \sum_{i=0}^{n-1} b_i \cdot 2^{n-1-i}$$

Where:

- b_i is the bit value (0 or 1) at position i
- n is the total number of bits in the chromosome

After performing these operations, what is the fitness value of the final mutated chromosome?

Answer:

Step 1: Single-Point Crossover

We perform a single-point crossover at the 4th position (zero-based indexing) between Parent 1 and Parent 2.

- Parent 1: 1101|001011
- Parent 2: 1010|110101

After the crossover, the resulting offspring are:

- Offspring 1: 1101110101
- Offspring 2: 1010001011

Step 2: Mutation

Next, we apply a mutation operation to Offspring 1 by flipping the bits at positions 2, 6, and 9 (zero-based indexing).

Offspring 1 before mutation: 1101110101

Flipping the bits at positions 2, 6, and 9:

- Position 2: Flip the 3rd bit (0 to 1)
- Position 6: Flip the 7th bit (0 to 1)
- Position 9: Flip the 10th bit (1 to 0)

Offspring 1 after mutation: 1111111100

Step 3: Fitness Calculation

For the chromosome 1111111100, the fitness is calculated as follows:

$$\begin{aligned}
\text{Fitness} &= (1 \cdot 2^9) + (1 \cdot 2^8) + (1 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) && (1)\\
&\quad + (1 \cdot 2^4) + (1 \cdot 2^3) + (1 \cdot 2^2) + (0 \cdot 2^1) + (0 \cdot 2^0) && (2)\\
&= 512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 0 + 0 && (3)\\
&= 1020 && (4)
\end{aligned}$$

Therefore, the fitness value of the final mutated chromosome is 1020.