Artificial Intelligence

Exercises week 8 - Knowledge representation and uncertainty - **Solutions**

COMP3411/9814

Question 1: Logic

Consider the following predicate logic expressions. Which one are true?

1. The predicate logic of "There exists a student who has taken both calculus and linear algebra" is $\exists x(S(x) \wedge C(x) \wedge L(x))$ where:

   - S(x) represents "x is a student."
   - C(x) represents "x has taken calculus."
   - L(x) represents "x has taken linear algebra."

2. The predicate logic of "Every person who owns a car has a driver's license" is $\forall x(P(x) \wedge O(x) \wedge D(x))$ where:

   - P(x) represents "x is a person."
   - O(x) represents "x owns a car."
   - D(x) represents "x has a driver's license."

3. The predicate logic of "Some employees who work more than 40 hours a week do not receive overtime pay" is $E(x) \wedge W(x) \wedge \neg O(x)$ where:

   - E(x) represents "x is an employee."

- W(x) represents "x works more than 40 hours a week."
- O(x) represents "x receives overtime pay."

4. The predicate logic of "If any customer buys a product and the product is defective, then the customer will return the product" is $\forall x(C(x) \wedge P(x) \wedge D(x) \rightarrow R(x))$ where:

   - C(x) represents "x is a customer."
   - P(x) represents "x buys a product."
   - D(x) represents "The product bought by x is defective."
   - R(x) represents "x will return the product."

Answer: Both 1 and 4 are true.

Question 2: Bayes nets

Suppose a certain medical test for a disease is known to have a false positive rate of 5% and a false negative rate of 10%. The prevalence of the disease in the population is 1%. If a person tests positive, what is the probability P1 that they actually have the disease? If the false negative rate increases to 15%, recalculate the probability P2 that a person actually has the disease given a positive test result. If the false positive rate decreases to 1%, while keeping the true positive rate at 90%, what is the new probability P3 that a person has the disease given a positive test result?

Answer:

P(disease) = 0.01 (given)
P(positive | No disease) = 0.05 (given)
P(No positive | disease) = 0.10 (given)
P(disease | positive) = ??

P(disease | positive) = P(positive | disease)*P(disease) / P(positive)

⇒ P(positive | disease) = 1 - P(No positive | disease) = 1 - 0.1 = 0.90
⇒ P(positive) = P(positive | disease) * P(disease) + P(positive | No disease) * P(No disease)
= (0.90 * 0.01) + (0.05 * 0.99) = 0.0585

Putting values in P(disease | positive) formula:
P1(disease | positive) = 0.90 * 0.01 / 0.0585 = **0.1538**

⇒ P(positive | disease) = 1 - P(No positive | disease) = 1 - 0.15 = 0.85
⇒ P(positive) = P(positive | disease) * P(disease) + P(positive | No disease) * P(No disease)
= (0.85 * 0.01) + (0.05 * 0.99) = 0.058
Putting values in P(disease | positive) formula:
P2(disease | positive) = 0.85 * 0.01 / 0.058 = **0.1466**

⇒ P(positive | disease) = 1 - P(No positive | disease) = 1 - 0.1 = 0.90
⇒ P(positive) = P(positive | disease) * P(disease) + P(positive | No disease) * P(No disease)
= (0.90 * 0.01) + (0.01 * 0.99) = 0.0189
Putting values in P(disease — positive) formula:
P3(disease | positive) = 0.90 * 0.01 / 0.0189 = **0.476**

Question 3: Fuzzy logic

In a streaming platform, the analysis team is performing a study on how a "normal customer" uses the platform. Based on the number of hours each customer uses the platform, they have ordered the customers and defined a fuzzy set for normal customers using a trapezoid membership function. The analysis indicate the following:

- Customers in the lowest 5% of use have a membership equal to zero.
- Customers between 5% and 20% of use have an increasing membership from zero to one.
- Customers between 20% and 60% of use have a membership equal to one.
- Customers between 60% and 95% of use have a decreasing membership from one to zero.
- Customers in the highest 5% of use (between 95% and 100%) have a membership equal to zero.

For a customer X located at position 72.5% of use compared with other customers, what would be the membership value of X?

Answer: The parameters for the membership function are [5, 20, 60, 95], based on that, the membership value for X is 0.6428