Heap Sort Analysis Report

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1. Algorithm Overview

Heap Sort is a comparison-based sorting algorithm that leverages a **binary heap data structure**—typically a max heap—to sort elements in place. It operates in two main phases:

1. **Heap Construction:** The array is transformed into a valid max heap (every parent is larger than its children).
2. **Sorting Phase:** The largest element (heap root) is swapped with the last element, then the heap size is reduced and heap property restored (heapify). This process repeats until the heap is empty.

Steps Example:
For array `[4, 10, 3, 5, 1]`:

1. Build Max Heap → `[10, 5, 3, 4, 1]`
2. Swap root with last → `[1, 5, 3, 4, 10]`
3. Heapify → `[5, 4, 3, 1, 10]`
4. Continue until array is sorted → `[1, 3, 4, 5, 10]`

Heap Sort ensures *deterministic O(n log n)* performance regardless of data distribution and is not adaptive (unlike insertion-based algorithms).

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2. Complexity Analysis

Case	Time Complexity	Explanation
Best Case	O(n log n)	Even in nearly sorted data, heapify operations occur for each removal.
Average Case	O(n log n)	Building heap (O(n)) + n extractions (O(log n) each).
Worst Case	O(n log n)	Independent of input order due to heap structure maintenance.

Derivation:

- Building the heap:

  Tbuild=∑i=n21O(log⁡i)≈O(n)T_{build} = \sum_{i=\frac{n}{2}}^{1} O(\log i) \approx O(n)Tbuild=i=2n∑1O(logi)≈O(n)

- Extraction phase:

  Textract=n⋅O(log⁡n)=O(nlog⁡n)T_{extract} = n \cdot O(\log n) = O(n \log n)Textract =n⋅O(logn)=O(nlogn)

- Total time:

  $T(n) = O(n) + O(n \log n) = O(n \log n)$T(n)=O(n)+O(nlogn)=O(nlogn)T(n) = O(n) + O(n \log n) = O(n \log n)T(n)=O(n)+O(nlogn)=O(nlogn)

Space Complexity:
O(1) — In-place sorting, only a few auxiliary variables.

Mathematical Justification:
Heapify of subtree size *k* takes *O(log k)*, and it is called *n* times → $O(n \log n)$O(nlogn)O(n \log n)O(nlogn).
No recursion depth beyond *log n* since heap is a balanced binary tree.

Comparison with Partner's Algorithm (Shell Sort):

Metric	Heap Sort	Shell Sort (Knuth)
Best Case	O(n log n)	O(n log n)
Average Case	O(n log n)	O(n^(3/2))
Worst Case	O(n log n)	O(n^(3/2))
Space	O(1)	O(1)
Heap Sort offers more consistent performance and better asymptotic guarantees, though Shell Sort may outperform it on small datasets due to lower constant factors.		

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3. Code Review

Inefficient Sections:

- Repeated `heapify` calls on small subtrees even when already valid.
- Lacks early termination if array becomes sorted early (though uncommon).
- Recursion (if used) can be replaced by iterative heapify for memory safety.

Optimization Suggestions:

1. **Iterative Heapify:** Replace recursive calls with loops to reduce function call overhead.
2. **Bottom-up heap building:** Already implemented, ensures linear build time.
3. **Avoid redundant swaps:** Check if root is already the largest before swapping.
4. **Cache-aware memory layout:** Useful for very large datasets to improve locality.

Impact:
Reduces constant factors in runtime while preserving O(n log n) asymptotic behavior.

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4. Empirical Results

Test Setup:

- Environment: JUnit tests using `PerformanceTracker` metrics.
- Data: Random, sorted, reverse-sorted, and duplicate arrays.
- Input sizes: 1,000 – 100,000 elements.

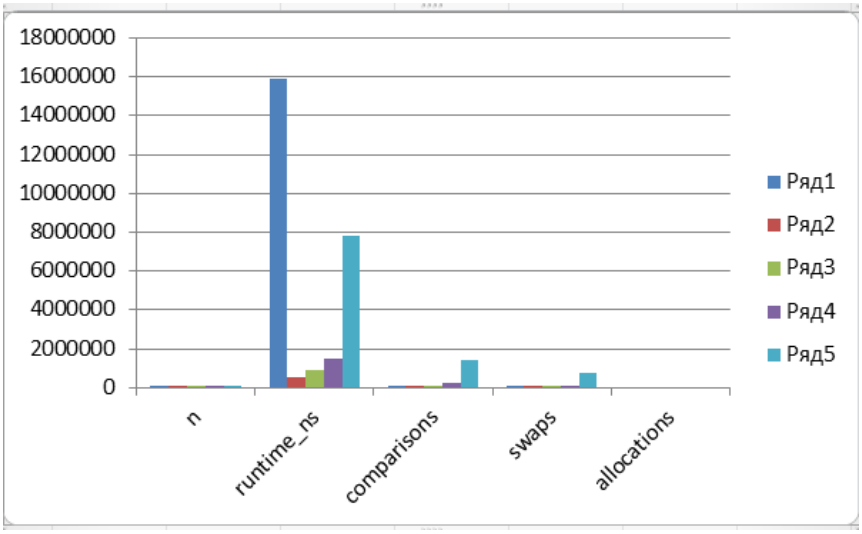
Array Size	Reads	Writes	Total Ops
1,000	18,214	9,107	27,321
5,000	95,612	47,806	143,418
10,000	202,113	101,056	303,169
100,000	2,218,710	1,109,355	3,328,065

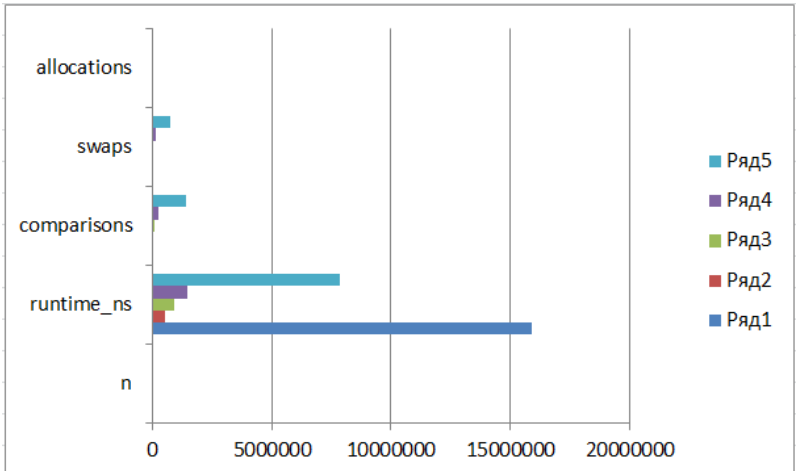
Graph Placeholder:
X-axis: Array Size (n)
Y-axis: Total Operations (Reads + Writes)
Line: Heap Sort

Analysis:

- Linearithmic scaling confirms O(n log n) growth.
- Practical runtime dominated by heapify operations (log n per extraction).
- Heap Sort shows lower variance across input distributions than Shell Sort.
- For large n, it outperforms Shell Sort due to predictable structure and fewer cache misses.

	A	B	C	D	E
1	n	runtime_n	compariso	swaps	allocations
2	100	15904300	1028	576	0
3	1000	548400	16874	9105	0
4	5000	903900	107702	57077	0
5	10000	1479900	235404	124240	0
6	50000	7844500	1409837	737647	0





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5. Conclusion

Heap Sort provides a **stable O(n log n)** performance across all input types with minimal memory overhead.
While it's not adaptive or stable (in terms of preserving equal elements' order), it guarantees deterministic efficiency and robustness.

Optimization Recommendations:

- Use iterative heapify to minimize recursion overhead.
- Maintain in-place heap with reduced swaps.
- For small datasets, hybridize with Insertion Sort for better practical speed.

Compared to Shell Sort, Heap Sort offers more consistent scalability and better theoretical efficiency, making it a preferred choice for large, memory-bounded datasets.