Sets

Y = Set of residents' year y, $y \in \{1, \ldots, 5\}$
T = Set of residents' tier t, $t \in \{1, \ldots, 4\}$
T'_y = Set of type for residents' year y, $y \in Y$
T'_1 = {1c, 1p, 1v, 1prs, 1u, 1ent, 1o, 1a, 1ed}
T'_2 = {2c, 2prs , 2v}
T'_3 = {3c, 3prs, 3v, 3up}
T'_4 = {4}
T'_5 = {5}
I = Set of residents i, $i \in \{1, \ldots,89\}$
I_y^{year} = Set of residents i of residents' year y, $y \in Y$
I_1^{year} = {1, …, 56}
I_2^{year} = {57, …, 64}
I_3^{year} = {65, …, 78}
I_4^{year} = {79, …, 85}
I_4^{year} = {86, …, 89}
I_t^{tier} = Set of residents i of tier t, $t \in T$
I_1^{tier} = {1, …, 64}
I_2^{tier} = {57, …, 78}
I_3^{tier} = {65, …, 85}
I_4^{tier} = {79, …, 89}
$I_{t'}$ = Set of residents i of type $t', t' \in T'_y, y \in Y$
I_{1c} = {1, …, 5}
I_{1p} = {6, …, 15}
I_{1v} = {16}
I_{1prs} = {17, 18}
I_{1u} = {19, 20,21}
I_{1ent} = {22, 23}
I_{1o} = {24, …, 28}
I_{1a} = {29, …, 46}
I_{1ed} = {47, …, 56}
I_{2c} = {57, …, 61}
I_{2prs} = {62, 63}
I_{2v} = {64}
I_{3c} = {65, …, 68}
I_{3prs} = {69, 70}
I_{3v} = {71}
I_{3up} = {72, …, 78}
I_4 = {79, …, 85}
I_5 = {86, …, 89}
J = Set of services j, $j \in$ {Trauma, EGS, SICU, CRS, MIS, SO, Brendo, Peds, Thor, Txp, Holy, NFA, NFB,NFC, NFS, Lancaster, Elective, CGOH, Vasc, Wound}
$J_{t'}$ = Set of services j that must be picked from residents' type $t', t' \in T'_y, y \in Y$
J_{1p} = {NFA, NFB}
J_{1v} = {NFA, NFB}
J_{1prs} = {NFA, NFB, NFC}
D = Set of number of days in a month, d ∈ {1, …,31}
M = Set of months $m, m \in \{1, \ldots, 12\}$

K = Set of vacation weeks $k, k \in \{1, ..., 3\}$
Q = Set of quarters $q, q \in \{1, \ldots, 4\}$
M_q = Set of months m for quarter q in set Q
$M_1 = \{1, 2, 3\}$
$M_2 = \{4, 5, 6\}$
$M_3 = \{7, 8, 9\}$
$M_4 = \{10, 11, 12\}$

Parameters

V_{idm}^{k}	=	One if resident i take choice A vacation on day d on month m for week k and zero otherwise
H_{idm}^{k}	=	One if resident i take choice B vacation on day d on month m for week k and zero otherwise
R_{jy}^{min}	=	Minimum number of required residents in year y for service j
R_{jy}^{fix}	=	Fix number of required residents in year y for service j
C_{jt}^{min}	=	Minimum number of required residents in tier t for service j
C_{jt}^{fix}	=	Fix number of required residents in tier t for service j
N_{jy}^{min}	=	Minimum number of months required for service j in year y
N_{jy}^{max}	=	Maximum number of months required for service j in year y
G_{y}^{min}	=	Minimum number of months required for group of service J_y in year y
G_{y}^{max}	=	Maximum number of months required for group of service J_y in year y
c	=	Penalty constant for choice B vacation
a	=	Penalty constant for violate fix number of required residents for service j in year y
b	=	Penalty constant for violate fix number of required residents for service j in tier t

Decision Variables

x_{ijm}	=	One if resident i works on service j on month m and zero otherwise
z_{ijq}	=	One if resident i works on service j on quarter q and zero otherwise
w_{ij}	=	One if resident i had worked on service j and zero otherwise
e_{ik}	=	One if resident i take vacation choice A for week k and zero otherwise
v_{idm}	=	One if resident i actually takes vacation on day d on month m and zero otherwise
o_{jdm}	=	Number of residents who take vacation on day d on month m of service j
o_{max}	=	Maximum number of residents that allow to take vacation on day d on month m of service j
s_{jm}	=	Additional residents to satisfy min required residents from year y for service j on month m
r_{jm}	=	Additional residents to satisfy min required residents from tier t for service j on month m

Objective

$$\min\ o_{max} + c \sum_{i \in I} \sum_{k \in K} (1 - e_{ik}) + a \sum_{j \in J} \sum_{m \in M} s_{jm} + b \sum_{j \in J} \sum_{m \in M} r_{jm}$$

Constraints

Only one service is assigned to one resident on the same month

$$\sum_{j \in J} x_{ijm} = 1 \qquad \forall i \in I, \forall m \in M$$

The actual vacation date is either from vacation plan A or plan B for week k

$$v_{idm} \quad \geq \quad e_{ik}V_{idm}^{k} + (1 - e_{ik})H_{idm}^{k} \qquad \forall i \in I, \forall d \in D, \forall m \in M, \forall k \in K$$

Ensure that at least one week will have vacation from A

$$\sum_{k \in K} e_{ik} \quad \geq \quad 1 \qquad \forall i \in I$$

Find the maximum number of residents that allow to take vacation on day d on month m of service j

$$\sum_{\mathrm{i} \in I} v_{idm}x_{ijm} \quad = \quad o_{jdm} \qquad \forall j \in J, \forall d \in D, \forall m \in M$$

$$o_{max} \quad \geq \quad o_{jdm} \qquad \forall j \in J, \forall d \in D, \forall m \in M$$

$$o_{jdm} \quad \leq \quad 2 \qquad \forall j \in J, \forall d \in D, \forall m \in M$$

Residents of type t' for all year y must do service j for at least $N_{jt'}^{min}$ months

$$\sum_{m \in M} x_{ijm} \quad \geq \quad N_{jt'}^{min} \qquad \forall i \in I_{t'}, \forall j \in J, \forall t' \in T_y', \forall y \in Y$$

Residents of type t' for all year y must do service j for at most $N_{jt'}^{max}$ months

$$\sum_{m \in M} x_{ijm} \quad \leq \quad N_{jt'}^{max} \qquad \forall i \in I_{t'}, \forall j \in J, \forall t' \in T_y', \forall y \in Y$$

Number of residents in year y for service j must be at least R_{jy}^{min} for every month in M

$$\sum_{i \in I_y^{year}} x_{ijm} \quad \geq \quad R_{jy}^{min} \qquad \forall j \in J, \forall m \in M, \forall y \in Y$$

$$\sum_{i \in I_y^{year}} x_{ijm} + s_{jm} \quad \geq \quad R_{jy}^{min} \qquad \forall j \in J, \forall m \in M, \forall y \in Y$$

Number of residents in year y for service j must be equal to R_{jy}^{fix} for every month in M

$$\sum_{i \in I_y^{year}} x_{ijm} \quad = \quad R_{jy}^{fix} \qquad \forall j \in J, \forall m \in M, \forall y \in Y$$

Number of residents in tier t for service j must be at least C_{jt}^{min} for every month in M

$$\sum_{i \in I_t^{tier}} x_{ijm} \quad \geq \quad C_{jt}^{min} \quad \forall j \in J, \forall m \in M, \forall t \in T$$

$$\sum_{i \in I_t^{tier}} x_{ijm} + r_{jm} \quad \geq \quad C_{jt}^{min} \quad \forall j \in J, \forall m \in M, \forall t \in T$$

Number of residents in tier t for service j must be equal to C_{jt}^{fix} for every month in M

$$\sum_{i \in I_t^{tier}} x_{ijm} \quad = \quad C_{jt}^{fix} \quad \forall j \in J, \forall m \in M, \forall t \in T$$

Residents of type t' must choose one of services from set $J_{t'}$ for at least $G_{t'}^{min}$ months

$$\sum_{j \in J_{t'}} \sum_{m \in M} x_{ijm} \quad \geq \quad G_{t'}^{min} \quad \forall i \in I_{t'}, \forall t' \in T'_y, \forall y \in Y$$

Residents of type t' must choose one of services from set $J_{t'}$ for at most $G_{t'}^{max}$ months

$$\sum_{j \in J_{t'}} \sum_{m \in M} x_{ijm} \quad \leq \quad G_{t'}^{max} \quad \forall i \in I_{t'}, \forall t' \in T'_y, \forall y \in Y$$

Constraint to guarantee that residents type t' will pick only one of services from set $J_{t'}$ services

$$\sum_{j \in J_{t'}} w_{ij} \quad = \quad 1 \quad \forall i \in I_{t'}, \forall t' \in T'_y, \forall y \in Y$$

$$\sum_{m \in M} x_{ijm} \quad \geq \quad w_{ij} \quad \forall i \in I_{t'}, \forall j \in J_{t'}, \forall t' \in T'_y, \forall y \in Y$$

$$\sum_{m \in M} x_{ijm} \quad \leq \quad w_{ij}\, G_{t'}^{max} \quad \forall i \in I_{t'}, \forall j \in J_{t'}, \forall t' \in T'_y, \forall y \in Y$$

Residents from year 1p who got chosen to work in Elective service should be only from Jul to Oct

$$\sum_{m \in M} x_{ijm} \quad \geq \quad 0 \quad \forall i \in I_{1p}, j = \{Elective\}$$

$$\sum_{m \in M} x_{ijm} \quad \leq \quad 1 \quad \forall i \in I_{1p}, j = \{Elective\}$$

$$x_{ijm} \quad = \quad 0 \quad \forall i \in I_{1p}, j = \{Elective\}, m \in \{Jul, Aug, Sep, Oct\}$$

Residents from year 2c must do services CRS and EGS for consecutive 2 months

$$x_{ijm-1} + x_{ijm+1} \leq 2 - x_{ijm} \qquad \forall i \in I_{2c}, j \in \{CRS, EGS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm-1} + x_{ijm+1} \geq x_{ijm} \qquad \forall i \in I_{2c}, j \in \{CRS, EGS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm} \leq x_{ijm+1} \qquad \forall i \in I_{2c}, j \in \{CRS, EGS\}, m \in \{Jan\}$$

$$x_{ijm} \leq x_{ijm-1} \qquad \forall i \in I_{2c}, j \in \{CRS, EGS\}, m \in \{Dec\}$$

Residents from year 3c must do services Holy, Peds, and MIS for consecutive 2 months

$$x_{ijm-1} + x_{ijm+1} \leq 2 - x_{ijm} \qquad \forall i \in I_{3c}, j \in \{Holy, Peds, MIS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm-1} + x_{ijm+1} \geq x_{ijm} \qquad \forall i \in I_{3c}, j \in \{Holy, Peds, MIS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm} \leq x_{ijm+1} \qquad \forall i \in I_{3c}, j \in \{Holy, Peds, MIS\}, m \in \{Jan\}$$

$$x_{ijm} \leq x_{ijm-1} \qquad \forall i \in I_{3c}, j \in \{Holy, Peds, MIS\}, m \in \{Dec\}$$

Residents from year 4 must do services Lancaster, Peds, SO, and EGS for consecutive 2 months

$$x_{ijm-1} + x_{ijm+1} \leq 2 - x_{ijm} \qquad \forall i \in I_{4}, j \in \{Lancast, Peds, SO, EGS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm-1} + x_{ijm+1} \geq x_{ijm} \qquad \forall i \in I_{4}, j \in \{Lancast, Peds, SO, EGS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm} \leq x_{ijm+1} \qquad \forall i \in I_{4}, j \in \{Lancast, Peds, SO, EGS\}, m \in \{Jan\}$$

$$x_{ijm} \leq x_{ijm-1} \qquad \forall i \in I_{4}, j \in \{Lancast, Peds, SO, EGS\}, m \in \{Dec\}$$

Residents from year 5 must do services CRS, MIS, SO, and EGS for consecutive 3 months

$$\sum_{\forall q \in Q} z_{ijq} = 1 \qquad \forall i \in I_{5}, j \in \{CRS, MIS, SO, EGS\}$$

$$x_{ijm} = z_{ijq} \qquad \forall i \in I_{5}, j \in \{CRS, MIS, SO, EGS\}, \forall m \in M_q, \forall q \in Q$$

Residents from year 2c and 3c must not do Trauma for any consecutive month

$$x_{ijm-1} + x_{ijm+1} \leq 2(1 - x_{ijm}) \qquad \forall i \in I_{2c,3c}, j \in \{Trauma\}, m \notin \{Jan, Dec\}$$

$$x_{ijm-1} + x_{ijm+1} \geq 0 \qquad \forall i \in I_{2c,3c}, j \in \{Trauma\}, m \notin \{Jan, Dec\}$$

$$x_{ijm} + x_{ijm+1} \geq 1 \qquad \forall i \in I_{2c,3c}, j \in \{Trauma\}, m \in \{Jan, Dec\}$$

Residents from year 4 must not do NFS for any consecutive month

$$x_{ijm-1} + x_{ijm+1} \leq 2(1 - x_{ijm}) \qquad \forall i \in I_{4}, j \in \{NFS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm-1} + x_{ijm+1} \geq 0 \qquad \forall i \in I_4, j \in \{NFS\}, m \notin \{Jan, Dec\}$$

$$x_{ijm} + x_{ijm+1} \geq 1 \qquad \forall i \in I_4, j \in \{NFS\}, m \in \{Jan, Dec\}$$