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A Dynamic Supplier Selection and Inventory Management Model for a Serial Supply Chain with a Novel Supplier Price Break Scheme and Flexible Time Periods

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A R T I C L E I N F O

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A B S T R A C T

A new supplier price break and discount scheme taking into account order frequency and lead time is introduced and incorporated into an integrated inventory planning model for a serial supply chain that minimizes the overall incurred cost including procurement, inventory holding, production, and transportation. A mixed-integer linear programming (MILP) formulation is presented addressing this multi-period, multi-supplier, and multi-stage problem with predetermined time-varying demand for the case of a single product. Then, the length of the time period is considered as a variable. A new MILP formulation is derived when each period of the model is split into multiple sub-periods, and under certain conditions, it is proved that the optimal solution and objective value of the original model form a feasible solution and an upper bound for the derived model. In a numerical example, three scenarios of the derived model are solved where the number of sub-period is set to 2, 3, and 4. The results further show the decrease of the optimal objective value as the length of the time period is shortened. Sufficient evidence demonstrates that the length of the time period has a significant influence on supplier selection, lot sizing allocation, and inventory planning decisions. This poses the necessity of the selection of appropriate length of a time period, considering the trade-off between model complexity and cost savings.

1. Introduction

Procurement, production and distribution are three fundamental processes involved in supply chain management (Thomas & Griffin, 1996). Procurement is one of the major strategies that a company faces as procurement incurred cost contributes in large percentage to the overall cost of the entire supply chain. In the process of procurement, the suppliers are first screened according to multiple criteria, mainly including price, delivery, quality, and reputation, to generate a reduced set of preferred suppliers. Then the firm makes the decision about how much to order from each preferred supplier according to its demand pattern for the purpose of increasing cost efficiency (Mendoza & Ventura, 2008). Extensive research has been conducted on the integrated model of supplier selection and order quantity allocation.

Inventory is recognized as one of the major drivers in a supply chain (Ravindran & Warsing, 2013). Maintaining appropriate inventory levels is an essential task for a firm since high inventory levels increase the responsiveness to customers while increasing the cost, whereas low inventory levels might cause shortages, which consequently impairs the firm's reputation.

Operational decisions planning regarding procurement or inventory management may not result in minimal cost across the entire supply chain. Recently, researchers have integrated supplier selection and inventory control to reach a global optimal solution aiming at minimizing purchasing cost and inventory cost simultaneously. Basnet and Leung (2005) presented a multi-product, multi-period inventory lot-sizing model considering supplier selection. An enumerative search algorithm and a heuristic algorithm were presented to solve the proposed mixed-integer programming formulation. Rezaei and Davoodi (2008) introduced a multi-period, multi-supplier, and multi-item inventory model with imperfect quality. Mendoza and Ventura (2010) studied a serial inventory system with supplier selection and order quantity allocation, considering the transfer of materials or products between consecutive stages of systems. Moqri et al. (2011) proposed a forward dynamic programming approach to solve the multi-period integrated supplier selection and inventory lot-sizing problem. Ventura, Valdebenito and Golany (2013) extended previous research on supply chain inventory and introduced a dynamic serial supply chain model to represent the multi-supplier, multi-stage, and multi-period operational planning problem

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coordinating purchasing, manufacturing, inventory, and transportation simultaneously. Recently, Cárdenas-Barrón, González-Velarde and Treviño-Garza (2015) improved the work of Basnet and Leung (2005) and proposed a new approach based on a "reduce and optimize" approach. Firouz, Keskin and Melouk (2017) developed a decomposition based heuristic algorithm with simulation to address an integrated supplier selection and inventory problem with multi-sourcing and lateral transshipments. As an effective practice for the supplier to promote their products, quantity discount is incorporated into the dynamic lot sizing and supplier selection problem. Parsa, Khiav, Mazdeh and Mehrani (2013) extended the result from Moqri et al. (2011) by considering quantity discount from suppliers. Choudhary and Shanker (2013) and Choudhary and Shanker (2014) developed integrated supplier selection, inventory lot-sizing, and carrier selection models incorporating economies of scale in purchasing and transportation costs. Lee, Kang, Lai and Hong (2013) and Mazdeh, Emadikhiav and Parsa (2015) presented a genetic algorithm and a heuristic respectively to solve the dynamic lot-sizing problem with supplier selection and quantity discount. Purohit, Choudhary and Shankar (2016) considered both time-varying stochastic demand and all-units quantity discounts in the supplier selection and inventory lot-sizing problem.

However, most of the foregoing papers do not take lead time into account or do not consider both lead time and quantity discounts simultaneously. Although Ventura et al. (2013) considered lead time between suppliers and the manufacturing site as well as between consecutive stages, lead time from a specified supplier or a particular stage was assumed to be a constant within the planning horizon, which is impractical in real world environments. According to Roy (2010), the next-generation supply chain responds to demand change in real time, requiring real time planning to make optimum decisions based on real demand and supply. That is to say, the build-to-forecast model is evolving into demand-driven supply chain in order to acquire a higher service level at lower operational cost. Given the dynamic environment in which a supply chain operates, an operational plan may become obsolete shortly after release or even before the plan is run. Therefore, the responsiveness of a supply chain becomes a vital measurement of supply chain performance and efficiency. In this context, it is necessary to build an integrated model, which not only coordinates decisions on procurement, production, distribution, and inventory management, but also responds quickly to real time demand change, to adapt to increasingly challenging environments.

Quantity discount pricing is one of the most popular coordinating mechanisms and has been studied by researchers under various contract conditions. For example, Lee and Rosenblatt (1986), Monahan (1984), Weng (1995), and Corbett and De Groote (2000) focused on the determination of discount schemes maximizing suppliers' benefits. Sarmah, Acharya and Goyal (2006) conducted a comprehensive review in the literature dealing with buyer vendor coordination models using quantity discounts as coordination mechanisms. They concluded that in the majority of the models suppliers provide all unit quantity discounts. Kim and Hwang (1988) have developed an algorithm to determine an incremental discount pricing scheme where the discount rate and the break point are both unknown or one of them is prescribed. Besides, Sadrian and Yoon (1992) first introduced a new approach called business volume discount where the amount of the discount is based on the total dollar volume of business awarded to a given supplier, not on the quantity of each individual product purchased. Later, Viswanathan and Wang (2003) examined a single-vendor, single-retailer distribution channel coordination mechanism, where the vendor offers quantity or volume discounts as coordination mechanisms. While according to Sarmah et al. (2006), most of the supply chain coordination models with quantity discount consider deterministic demand and zero lead time in a single period setting.

On the other hand, procurement auctions with piecewise linear cost curves have been proved to be profitable for the buyer (Hohner et al., 2003). Instead of competing only on the price, several aspects of the supplier performance should be addressed, for example, lead time, quality, delivery probability, etc., according to Kameshwaran, Narahari, Rosa, Kulkarni and Tew (2007). Therefore, multiple conflict objectives have to be considered in the optimization problems for real applications. Notice that lead time plays a vital role during a procurement process either from the supplier's or the buyer's point of view. Hence, it is necessary to include lead time to suppliers' price scheme.

This paper extends the dynamic inventory model proposed by Ventura et al. (2013), incorporating the concept of auctions, introduces a novel price break scheme considering order frequency and lead time from suppliers. The problem is then formulated as a mixed-integer linear programming (MILP) model aiming at minimizing the overall incurred cost across the entire supply chain with capacity constraints in production, inventory, and transportation. In the process of operational decisions planning, the length of a time period is constant. Under the motivation of exploring the impact of the length of the time period on the overall supply chain cost, we consider a flexible time period in the proposed MILP model. Our results show the fact that the length of time period influences significantly the selection of suppliers, the allocation of order quantity, and the planning of inventory.

The paper is organized as follows. Section 2 introduces a supplier price break scheme generalized from the incremental discount policy by means of taking into account multiple deliveries and lead time. In Section 3, with the presence of the above supplier price break scheme, the development of an integrated dynamic inventory planning model for a serial supply chain is presented. Section 4 discusses a reformulated MILP model where the length of a time period is shortened. The corresponding theorems showing the connection between the proposed model in Section 3 and the reformulated model are also given in this section. Section 5 provides a numerical example illustrating the mathematical model presented in Section 3. While in Section 6, sensitivity analysis regarding the length of a time period and the number of stages is provided. Finally, conclusions and future research are discussed in Section 7.

2. Supplier price break scheme

2.1. Price break scheme from the supplier

The two discount schemes, all-unit discount and incremental discount, have been widely utilized by sellers to promote their products in practice (Hadley & Whitin, 1963). The fundamental idea of the schemes is that sellers provide a price discount based on the quantity ordered, encouraging buyers to take advantage of economies of scale for the purpose of lowering their procurement cost. Both discount schemes offer lower price as the size of the order increases. Under the all-unit discount policy, the seller provides price discount in terms of every unit purchased; whereas for the incremental discount policy, the discounted price within each interval applies only to the quantities that lie between the two breakpoints.

As a result of these discount schemes, buyers tend to place large size orders, which accordingly lead to increasing holding cost as well as the potential loss in case the future price of raw materials decreases dramatically. Consequently placing orders of large size might not be a wise decision when considering the overall cost across the whole supply chain within a planning horizon. On the other hand, sellers may suffer higher production or acquisition

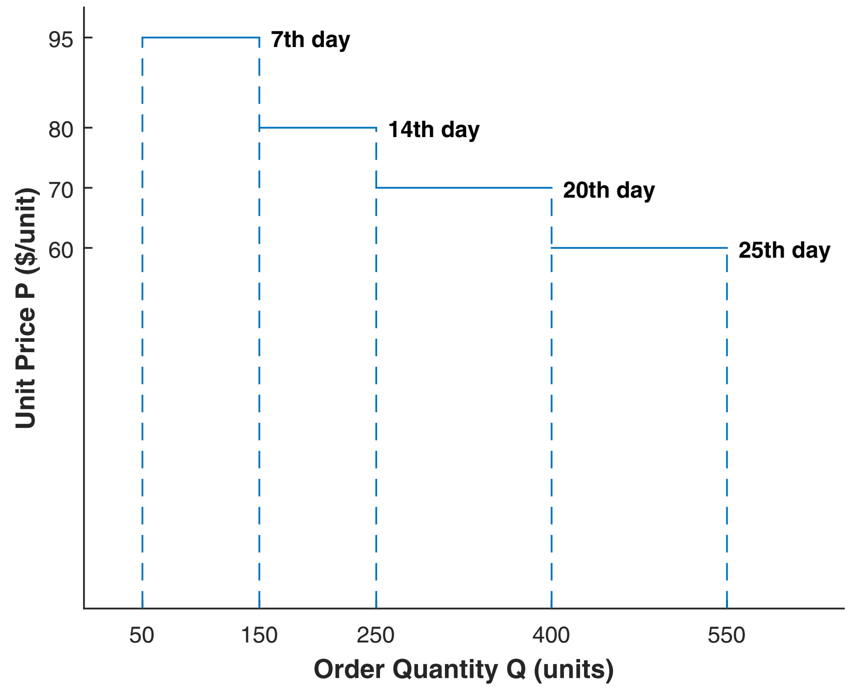

Fig. 1. Supplier's supply curve considering delivery time.

cost, such as extra labor involved in the production process or additional demand from upstream suppliers, as they promise to serve a large volume of products to downstream buyers. The profit of buyers within certain time horizon may not increase as expected. Hence, neither buyers nor sellers benefit from these discount schemes.

This causes the necessity to design an attractive supplier discount scheme taking into account order frequency and delivery time. First, the foregoing price discount policy only considers the one-time purchase. In the real world it is reasonable that better pricing should be given to customers who order more frequently within a given time horizon than customers with less order frequency. Secondly, either delivery time or lead time is one of the main factors affecting responsiveness and cost efficiency in the supply chain purchasing process. Instead of supplying a large quantity of raw materials at a time, suppliers rather provide a staggered supply pattern according to their own production planning process. For example, a small quantity from the supplier may be available with shorter lead time but relevantly higher price. Large volume of products with lower price will be subsequently available with longer lead time. This enables the buyer to place multiple orders within the planning horizon according to his own manufacturing schedule to satisfy the demand from final customers while maintaining minimal cost across the supply chain.

Notice that the purchasing process in supply chain management is similar to reverse auctions, referring to a single buyer and multiple sellers (Kalagnanam and Parkes, 2004). Sellers would provide volume discount for competing during an auction, this kind of mechanism is called volume discount auction (Davenport & Kalagnanam, 2001). Gautam et al. (2009) derive an optimal auction scheme for a single item, multi-unit procurement in terms of volume discount bids. In this work, the authors apply an incremental quantity-based discount policy. Verma, Hemachandra, Narahari and Tew (2014) introduce lead time constraints into single item, multi-unit procurement auctions with volume discount bids and formulate the problem as an integer programing model.

By incorporating lead time constraints into the volume discount auction, we apply this supplier price break scheme to the integrated supplier selection and inventory control problem. The scheme is a generalization of the incremental discount policy, considering delivery time and order frequency, aligning supplier deliveries with the buyer's production schedule. In this scheme, suppliers define several price break points along with the cumulative quantity available and the earliest delivery time. Then, buyers, according to their planning schedule as well as suppliers' availability, specify the order quantity, order frequency and delivery time within their planning horizon. Despite the size of each order, purchasing cost is calculated using incremental discount in terms of the cumulative order quantity from this supplier.

Suppose that a buyer has a pre-selected supplier set $S = \{1, 2, \ldots, n_S\}$. Each supplier s provides the following price break scheme:

$$\{m_s, Q_{0s}, L_s, (Q_{gs}, P_{gs}, L_{gs}),\ 1 \le g \le m_s\}. \tag{1}$$

where m_s is the number of break points from supplier s; Q_{0s} denotes minimal supply quantity; L_s represents the expiration date of this offer from supplier s; Q_{gs} denotes the cumulative quantity that supplier s is ready to provide on day L_{gs} or after L_{gs} with cost $Q_{0s}P_{1s} + \sum_{g'=1}^{g} P_{g'}(Q_{g's} - Q_{(g'-1)s})$; P_{gs} represents the unit discount price for the order quantity within interval $(Q_{(g-1)s}, Q_{gs})$; $Q_{0s} < Q_{1s} < Q_{2s} < \ldots < Q_{m_s s}$, $P_{1s} > P_{2s} > \ldots > P_{m_s s}$ and $L_{1s} \le L_{2s} \le \ldots \le L_{m_s s}$; the maximal supply quantity from supplier s is $Q_{m_s s}$. Therefore the purchasing cost to a buyer for an overall quantity of Q_s units from supplier s is determined as follows:

$$C(Q_s) = Q_{0s}P_{1s} + \sum_{g'=1}^{g} P_{g's}\left(Q_{g's} - Q_{g'-1s}\right) + P_{(g+1)s}(Q_s - Q_{gs}),\ Q_{gs} \le Q_s \le Q_{(g+1)s}.$$

For example, a supplier offers a price break scheme (see Fig. 1) defined as,

$$\{4, 50, 45, (150, 95, 7), (250, 80, 14), (400, 70, 20), (550, 60, 25)\}.$$

Fig. 1 shows that the supplier's minimum supply quantity is 50 units. The supplier is ready to serve up to 150 units at the price of \$95 per unit. The earliest delivery date is day 7. On day 14, 100

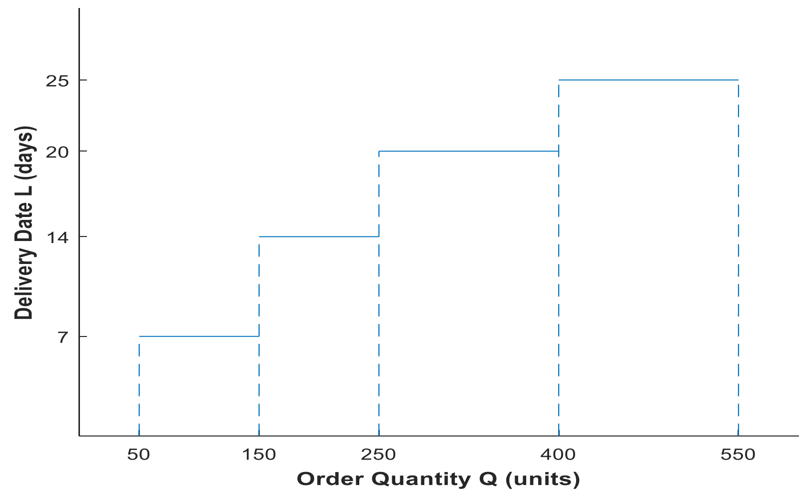

Fig. 2. Supplier's supply schedule.

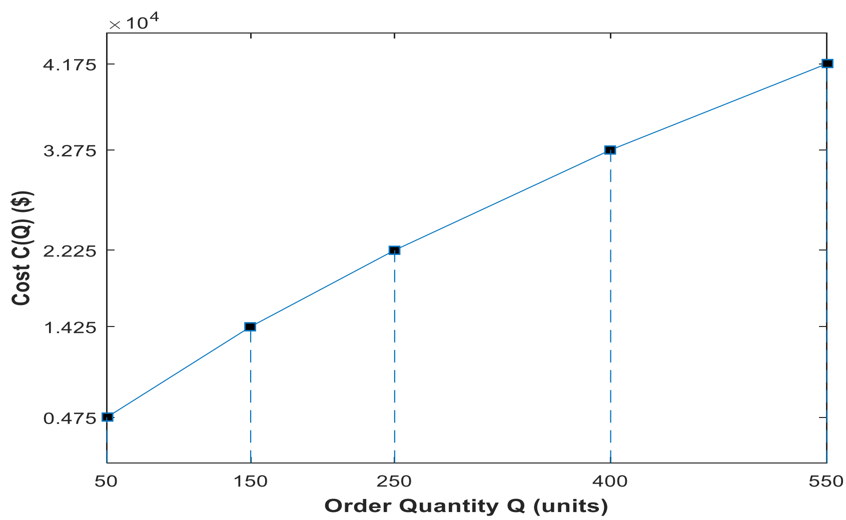

Fig. 3. Purchasing cost from the supplier.

additional units will be available at the unit price of \$80. On day 20, the supplier is ready to serve 150 extra units with a discount price of \$70 per unit. Finally, the supplier charges \$60 per unit for the remaining 150 units (from 400 to 550), which will be available as early as day 25. The maximal supply quantity of this supplier is 550 units.

In order to better understand the proposed price break scheme, let us first address the supplier's capacity (see Fig. 2). Note that at most 150 units can be served on day 7. The supplier can provide 100 additional units on day 14, while on day 20, the cumulative capacity from the supplier would be 400 units, and so on.

Assume a buyer has a purchasing plan of 200 units on day 15 and 200 units on day 30. According to Fig. 2, the supplier is capable of delivering the two orders of 200 units each on days 15 and 30. The cost to the buyer is only related to the cumulative order quantity of 400 units; that is, $C(400) = 150 \times 95 + 100 \times 80 + 150 \times 70 = 32,750$ (see Fig. 3).

This scheme allows multiple orders from the buyer to align with the buyer's production schedule, while the purchasing cost relies merely on the total order quantity during the planning horizon, taking advantage of economies of scale. As a generalization of incremental discount, such scheme is advantageous to both buyers and sellers. First, this scheme will decrease the inventory holding cost at the manufacturing site because multiple deliveries lead to lower the stock level of raw materials. Meanwhile the purchasing cost is calculated in terms of the cumulative order quantity based on the incremental discount. This will inevitably decreases the cost of the procurement process for the buyers. On the other hand, the sellers or suppliers applying this scheme do not need to produce large order quantities in short time periods as before, which could require overtime. Instead, they can design their manufacturing plan accordingly to meet the demand from buyers gradually, which turns out to be an efficient cost saving strategy for sellers.

2.2. Fitted price break scheme to a serial supply chain

In a serial supply chain, operational planning decisions such as purchasing, production, inventory, and transportation are usually made periodically, in most cases, at the beginning of each period. Let $T = \{1, 2, \ldots, n_T\}$ be the set of planning periods in a supply chain system. Let p be the length of each period. The earliest delivery date L_{gs} regarding each breakpoint $g, 1 \leq g \leq m_s$ defined in (1) might not be given exactly in multiples of p; On the other hand, the days an offer lasts L_s may not be the same as the planning horizon, which provides the possibility of multiple offers from the same supplier. Thus, in the process of making optimal planning, it is imperative to align the supplier's supply arrangement to the buyer's schedule. Simply stated, the supplier's price break scheme should accommodate the buyer's planning period. The following questions are to be answered:

- What quantity the supplier is able to provide at the beginning of each period?
- What price schedule should be taken to calculate the purchasing cost?

The following assumptions are considered:

- At the beginning of the planning horizon, an offer from a supplier may have already started.
- A supplier can only have an active offer in a particular period.
- A new offer can only start when the current offer ends.

Accordingly, the following parameters are defined given that an offer has already started prior to the beginning of the planning horizon and is still in effect within some planning periods:

ω_s: Number of periods since the beginning of the offer from supplier s to the beginning of the planning horizon, equal 0 when the offer from supplier s starts from period 1,

a_s: Overall supply quantity from supplier s within these ω_s periods.

Under these assumptions, at the beginning of the planning horizon, an offer may have already started with a_s units arrived at the manufacturing site prior to period 1. Some parts of a_s may have been processed to finished products within the prior planning horizon, while the remaining parts of a_s kept at the factory are considered as initial inventory. If this offer is expired before the end of the planning horizon, a new offer from the same supplier will start from the beginning of the succeeding period. Since a supplier is allowed to provide multiple offers within the planning horizon, without loss of generality, every offer can be treated as a different supplier for simplicity of the optimization process.

The following three-step procedures are applied to explicitly derive a fitted price break scheme to match the buyer's planning periods. First, parameters with respect to each offer from each supplier are determined such as the preliminary starting period, the completion period and the minimal order quantity. Then, the price break scheme regarding each offer is derived. Finally, each offer is treated as a different supplier and a set of suppliers is established for the convenience of optimization modeling.

Step 1. Calculate the starting period, the completion period, and the minimal order quantity with respect to each offer from suppliers.

Since L_s is the expiration date of each offer from supplier s, the number of periods each offer covers (τ_s) can be calculated as:

$$\tau_s = \left\lfloor \frac{L_s}{p} \right\rfloor. \tag{2}$$

Then the number of offers supplier s is able to provide (δ_s) within the planning horizon is:

$$\delta_s = \left\lceil \frac{n_T + \omega_s}{\tau_s + 1} \right\rceil. \tag{3}$$

According to these parameters, the preliminary starting period and the end period of each offer from supplier j are defined and derived as follows.

$s_{s,\theta}$: The first period of the θth offer from supplier s, $1 \le \theta \le \delta_s, s \in S$,

$c_{s,\theta}$: The completion period of the θth offer from supplier s, $1 \le \theta \le \delta_s, s \in S$.

Within the planning horizon, the starting time period of the first offer and the completion time period of the last offer are assumed to be period 1 and period n_T respectively. Hence, $s_{s\theta}$ and $c_{s\theta}$ can be written as follows:

$$s_{s,1} = 1, c_{s,1} = s_{s,1} + \tau_s - \omega_s,$$

$$s_{s,2} = s_{s,1} + \tau_s + 1 - \omega_s, c_{s,2} = s_{s,2} + \tau_s,$$

$$s_{s,\theta} = s_{s,\theta-1} + \tau_s + 1, c_{s,\theta} = s_{s,\theta} + \tau_s,\ \ \theta = 3, \ldots,\ \ \delta_s - 1,$$

$$s_{s,\delta_s} = c_{s,\delta_s-1} + 1, c_{s,\delta_s} = n_T.$$

Given that an offer from a supplier may have started prior to the beginning of period 1 and is still in effect within the planning horizon, the minimal order quantity with respect to offer 1 within the planning periods will be zero under the condition that a_s is positive. Hence, the minimal order quantity of the first offer from s^{th} supplier can be expressed as follows:

$$Q_{0s1} = \max\{0, Q_{0s} - a_s\}. \tag{4}$$

Step 2. Determine the price break scheme with respect to offers from supplier s within the planning horizon.

(i) The first offer from supplier s:

Considering the fact that the first offer may have already been in effect prior to the planning horizon, for simplicity, a new price break scheme regarding this first offer can be derived eliminating the beforehand order quantity (a_s) which has been shipped to the manufacturing site within the prior planning horizon.

According to (1), the parameter γ is determined where $Q_{(\gamma-1)s} \le a_s \le Q_{\gamma s}$.

Therefore, by ruling out a_s, the incremental discount policy for the first offer within the planning horizon can be represented as follows:

$$\{m_s - \gamma + 1, Q_{0s1}, (Q_{gs} - a_s, P_{gs}), \gamma \le g \le m_s\}, \tag{5}$$

where Q_{0s1} is determined in (4). The number of price break points is reduced to $m_s - \gamma + 1$.

Next, corresponding to planning periods of the buyer, we will combine the delivery date and the available quantity provided by the supplier to derive the cumulative supply capacity regarding every period. As a result, the delivery date included in the price break scheme will be eliminated and a straightforward representation of the price break scheme for the first offer will be provided. We define:

$Q_{ts\theta}$: Maximum supply quantity at the beginning of period t from offer θ of supplier s, $s_{s\theta} \le t \le c_{s\theta}, 1 \le \theta \le \delta_s, s \in S$.

According to (1), Q_{ts1} is derived from the following formula:

$$Q_{ts1} = \begin{cases} Q_{\upsilon s} - a_s, & \upsilon_{ts} > 0, \\ 0, & \upsilon_{ts} = 0, \end{cases} \qquad \forall\ s_{s1} \le t \le c_{s1},\ \ s \in S,$$

$$\upsilon_{ts} = \max\{0, \{g : L_{gs} \le (t + \omega_s - 1)p, 1 \le g \le m_s\}\}, \tag{6}$$

where υ_{ts} denotes the interval or the break point corresponding to the beginning of period t.

As a consequence, from (5) and (6), the price break scheme for offer 1 combining the delivery date is represented as:

$$\{(s_{s1}, c_{s1}); (m_s - \gamma + 1, Q_{0s1}); (t,\ Q_{ts1}), s_{s1} \le t \le c_{s1};$$
$$(Q_{gs} - a_s, P_{gs}), \gamma \le g \le m_s\},\ \ s \in S. \tag{7}$$

Note here the delivery date or lead time has been represented by the available cumulative supply quantity per period. Meanwhile, the incremental discount price break points are kept for purchasing cost calculation.

(ii) The θ^{th} offer from supplier s, $2 \le \theta \le \delta_s - 1$:

After the expiration of the preceding offer, a new offer from the same supplier can start at the beginning of the next period. The corresponding price break scheme with respect to the θ^{th} offer of supplier s can be derived:

$$Q_{ts\theta}=\begin{cases}Q_{vs}, & v_{ts}>0,\\ 0, & v_{ts}=0,\end{cases}\quad \forall\ s_{s\theta}\le t\le c_{s\theta},\quad s\in S,$$
$$v_{ts}=\max\{0,\{g:L_{gs}\le(t+\omega_s-1)p-(\tau_s+1)(\theta-1)p,1\le g\le m_s\}\}. \quad (8)$$

From (1) and (8), the price break scheme with respect to the intermediate offers can be expressed as follows:

$$\{(s_{s\theta},c_{s\theta});(m_s,Q_{0s});(t,\ Q_{ts\theta}),s_{s\theta}\le t\le c_{s\theta};(Q_{gs},P_{gs}),\ 1\le g\le m_s\},\ s\in S. \quad (9)$$

(iii) The last offer (δ_s) within the planning horizon:

Finally, the last offer from the same supplier is discussed separately because the offer may not expire at the beginning of last period of the planning horizon. Here, we manually set the completion period of the last offer $c_{s\delta_s}$ to n_{T}.

$$Q_{ts\delta_s}=\begin{cases}Q_{vs}, & v_{ts}>0,\\ 0, & v_{ts}=0,\end{cases}\qquad \forall\ s_{s\delta_s}\le t\le n_{\mathrm{T}},\ s\in S,$$
$$v_{ts}=\max\{0,\{g:L_{gs}\le(t+\omega_s-1)p-(\tau_s+1)(\delta_s-1)p,1\le g\le m_s\}. \quad (10)$$

Hence, combining (1) and (10), the following price break scheme is obtained.

$$\left\{\left(s_{s\delta_s},n_{\mathrm{T}}\right);(m_s,Q_{0s});\left(t,Q_{ts\delta_s}\right),s_{s\delta_s}\le t\le n_{\mathrm{T}};(Q_{gs},P_{gs}),\ 1\le g\le m_s\},\ s\in S. \quad (11)$$

Step 3. Construct a supplier pool by considering each offer as an individual supplier.

Once the price break scheme pertaining to each offer from suppliers is determined, considering that offers from the same supplier can be assumed to be independent, for simplicity purposes, it is reasonable to consider that each offer belongs to a different supplier. Consequently, a set of suppliers is created with the following general expression regarding the price break scheme for each supplier:

$$\left\{\left(s_j,c_j\right);(m_s,Q_{0j}),(t,\ Q'_{tj}),s_j\le t\le c_j;\left(Q_{gj},P_{gj}\right),\ 1\le g\le m_s\right\},\ j\in J \quad (12)$$

where

s_j: Starting period that supplier j is available,

c_j: The last period that supplier j is in use,

Q_{0j}: Minimal supply quantity for the first order from supplier j, $j\in J$,

m_j: Number of intervals for supplier j, $j\in J$,

Q_{gj}: Maximum supply quantity in interval g from supplier j, $j\in J$, $g=1,\ 2,\dots,m_j$,

P_{gj}: Price applied to the order quantity within interval g, $j\in J$, $g=1,\ 2,\dots,m_j$,

Q'_{tj}: Converted cumulative quantity that supplier j is able to provide at the beginning of period t, $t\in T_1$, $j\in J$.

The existence of lead time for a supplier indicates that the available supply quantity for the first few periods may be zero, as the supplier's lead time (s_j) may be longer than the length of a period (p). Thus, in order to simplify (12), we define:

$$s'_j=\min\{t:\ Q'_{tj}>0,s_j\le t\le c_j\}. \quad (13)$$

Then, (12) can be rewritten as:

$$\left\{\left(s'_j,c_j\right);(m_s,Q_{0j}),(t,\ Q'_{tj}),s'_j\le t\le c_j;\left(Q_{gj},P_{gj}\right),\ 1\le g\le m_s\right\},\ j\in J. \quad (14)$$

Recall the supplier (denoted by supplier 1) demonstrated in Fig. 1:

$$\{50,4,45,(150,95,7),\ (250,80,14),\ (400,70,20),\ (550,60,25)\}.$$

Let us consider an offer that started two periods prior to the beginning of the planning horizon with 100 units already served. Thus, in this case, we will have $L_1=45$, $a_1=100$, and $\omega_1=2$. In addition, let us consider that the length of the planning horizon is 60 days with 5 periods, which means that the length of each period (p) is 12 days. $n_T=5$.

Step 1:

$$\tau_1=\left\lfloor\frac{L_1}{p}\right\rfloor=3,$$

$$\delta_1=\left\lceil\frac{n_T+\omega_1}{\tau_1+1}\right\rceil=2,$$

$$s_{1,1}=1,c_{1,1}=s_{1,1}+\tau_1-\omega_1=2$$

$$s_{1,2}=s_{1,1}+\tau_1+1-\omega_1=3,c_{1,2}=5.$$

Therefore, each original offer from this supplier can supply 3 periods and a total of two offers are available from this supplier within the planning horizon. The first offer that has already been in effect before the beginning of period 1 will expire after 15th days, which is between periods 2 and 3. Consequently the starting time and completion time of the second offer are periods 3 and 5 respectively. The demands in both periods 1 and 2 are satisfied by offer 1. While starting from period 3, offer 2 becomes available. The minimal order quantity for offer 1 is calculated as:

$$Q_{011}=max\{0,Q_{01}-a_1\}=\max\{0,50-100\}=0.$$

Step 2:

(i) The first offer:

Since 100 is between 50 and 150, according to $Q_{\gamma-1j}\le a_j\le Q_{\gamma j}$, γ is 1 and the incremental discount for the first offer is updated to (see Fig. 4):

$$\{0,(50,95),\ (150,80),\ (300,70),(450,60)\}.$$

The supplier's capacity for each period can be derived as follows:

$$t=1:$$
$$v_{11}=\max\left\{0,\left\{g:L_{g1}\le 12\ \times(1+2-1),1\le g\le 4\right\}\right\}=3,$$
$$Q_{111}=Q_{31}-a_1=400-100=300.$$

$$t=2:$$
$$v_{21}=\max\left\{0,\ \left\{g:L_{g1}\le 12\ \times(2+2-1),1\le g\le 4\right\}\right\}=4,$$
$$Q_{211}=Q_{41}-a_1=550-100=450.$$

After conversion, the price break scheme with respect to the first offer can be written as:

$$\{(1,2);(4,0);(1,300),(2,\ 450);(50,95),\ (150,80),\ (300,70),(450,60)\}.$$

(ii) The second offer:

During the second offer, the supplier's capacity for each period can be calculated as:

$$t=3:$$
$$v_{31}=\max\left\{0,\ \left\{g:L_{g1}\le 12\ \times(3+2-1)-12\ \times\ (3+1)\ \times(2-1),1\le g\le 4\right\}\right\}=0,$$
$$Q_{312}=0.$$

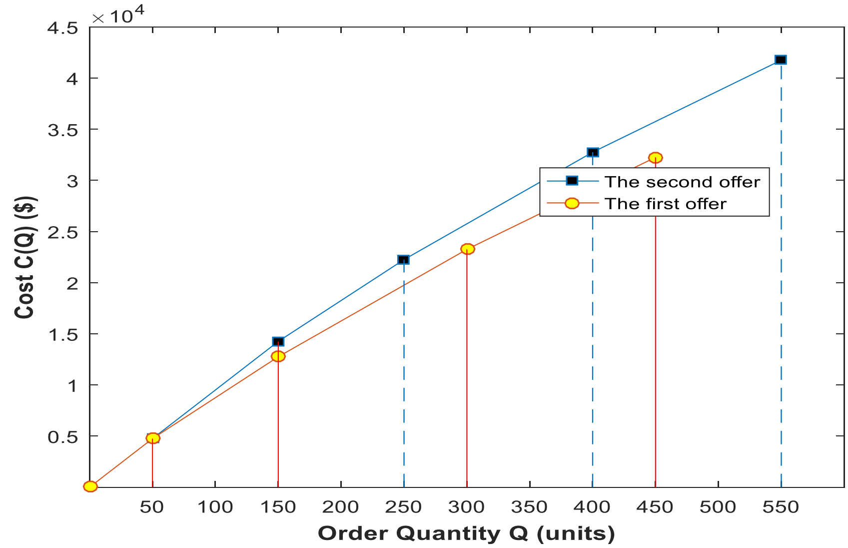

Fig. 4. Purchasing cost in terms of each offer.

$$t = 4:$$
$$v_{ts} = \max\left\{0, \left\{g : L_{g1} \le 12 \times (4 + 2 - 1) - 12 \times (3 + 1) \times (2 - 1), 1 \le g \le 4\right\}\right\} = 1,$$
$$Q_{412} = Q_{11} = 150.$$

$$t = 5:$$
$$v_{ts} = \max\left\{0, \left\{g : L_{g1} \le 12 \times (5 + 2 - 1) - 12 \times (3 + 1) \times (2 - 1), 1 \le g \le 4\right\}\right\} = 3,$$
$$Q_{512} = Q_{v1} = 400.$$

Although the incremental discount strategy of offer 2 is the original discount policy with 4 breakpoints, from the above calculation, the last breakpoint (550, 60, 25) is truncated due to the maximum cumulative supply capacity till the beginning of the last period Q_{512} is only 400 units.

Therefore, offer 2 from supplier 1 can be represented in this manner:

$$\{(3,5); (3,50); (4,300), (5, \ 450); (150,95), (250,80), \ (400,70)\}.$$

Step 3:

In general, two offers/suppliers will be in consideration for the purpose of seeking optimal operational decisions regarding procurement, production, distribution, and inventory planning. The price break scheme with respect to these two suppliers can be respectively expressed as follows:

Supplier 1:

$$\{(1,2); (4,0); (1,300), (2, \ 450); (50,95), (150,80), \ (300,70), (450,60)\}.$$

Supplier 2:

$$\{(4,5); (3,50); (4,150), (5, \ 400); (150,95), (250,80), \ (400,70)\}.$$

Define q_j^t as the replenishment quantity from supplier j arrived at the buyer's factory in period t, then on the one hand, the cumulative order quantity from supplier j till period t should not exceed the corresponding maximum supply quantity Q'_{tj}; on the other hand, raw material cost from supplier j is calculated based on the incremental discount in terms of total order quantity ($\sum_{t'=1}^{t} q_j^{t'}$ for all $t \in T$).

Now suppose a buyer has time-varying demand in five periods as indicated in Table 1. The following description demonstrates the quantity acquired from these two suppliers and the corresponding cost. 200 units will be delivered at the beginning of period 1. In period 2, the demand of 140 units can be satisfied on day 12. Then, on day 24, there is no available supply quantity. 140 units are also available to the buyer at the beginning of the fourth period, which is day 36. Finally, the demand in period 5 is also satisfied. As discussed, the purchasing cost only relates to the cumulative order quantity, regardless of the order quantity of each period. Since order quantity from each supplier are 340 and 370 respectively, then purchasing cost to the buyer can be calculated according to $C(Q_j) = Q_{0j}P_{1j} + \sum_{g'=1}^{g} P_{g'j}(Q_{g'j} - Q_{g'-1j}) + P_{(g+1)j}(Q_j - Q_{gj})$, $Q_{gj} \le Q_j \le Q_{(g+1)j}$.

Purchasing cost of offer 1: $50 \times 95 + 100 \times 80 + 150 \times 70 + 40 \times 60 = 25{,}650$.

Purchasing cost of offer 2: $50 \times 95 + 100 \times 95 + 100 \times 80 + 120 \times 70 = 30{,}650$.

In sum, totally 710 units have been acquired at the cost of \$56,300.

The discount policy presented ensures the supplier a stable supply pattern aligning his own production schedule. Moreover, the buyer has the flexibility to order multiple times while still acquiring price discount as a result of economies of scale.

As stated previously, supply chains have evolved from forecast-driven to demand-driven models, which requires quick response from all supply chain parties as the demand changes. That is to say, the operational plan coordinating the process of procurement, production, and distribution must adapt to real time demand. Especially, when suppliers submit offers using the foregoing time-sensitive price break scheme, what kind of centralized decisions should be made at every stage of the downstream entities to minimize the overall cost? To be specific, given the suppliers with their own scheme, within a planning horizon we aim to develop an operational plan on: when and what quantities should be placed to which supplier; when and what quantities should be manufactured and shipped from one stage to the next stage; what is the inventory level at each stage in each time period. In this context, an integrated supplier selec-

Table 1
The buyer's demand and order quantity.

Period	Delivery date (days)	Demand (units)	Cumulative demand (units)	Order quantity (units)	Cumulative order quantity (units)		Supply capacity (units)
					Offer 1	Offer 2	
1	0	200	200	200	200		300
2	12	140	340	140	340		450
3	24	150	490	0		0	0
4	36	140	630	140		140	150
5	48	230	850	230		370	400

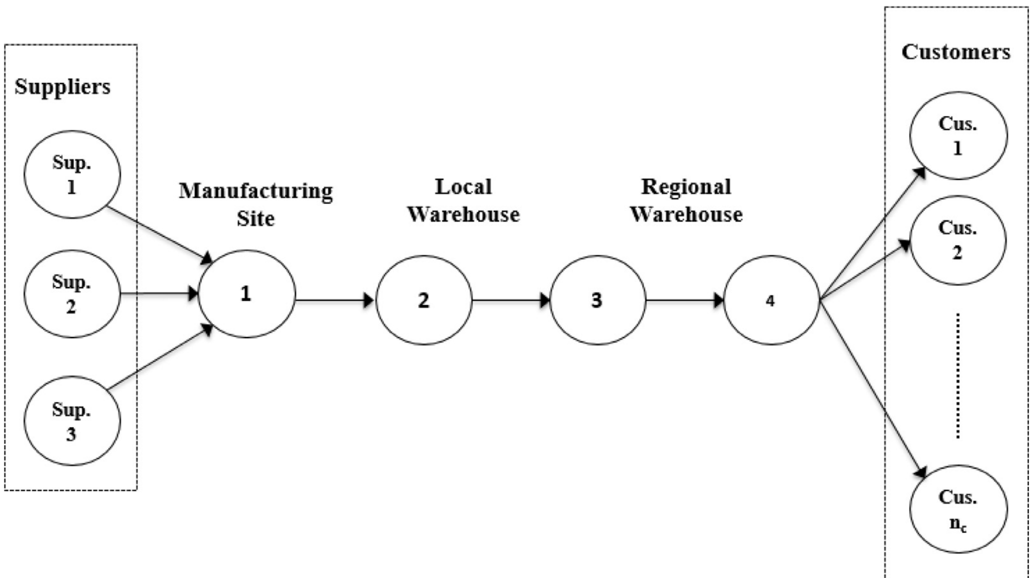

Fig. 5. A dynamic serial supply chain network with three suppliers, four stages, and five periods.

tion and inventory planning model for a serial supply chain considering this price break scheme will be developed in the next section.

3. The model

3.1. A supply chain network

Let us consider a serial supply chain system where the set of planning periods is denoted as $T = \{1,\ 2, \ldots, n_T\}$, with the set of stages $K = \{1,\ 2, \ldots, n_K\}$, among which stage 1 represents the manufacturing stage while stage 2 denotes the local warehouse. In this paper, one manufacturing site and one local house are in consideration. Raw materials arrive at stage 1 and are processed into finished products upon schedule. Then the finished products are stored at the local warehouse and are ready to be transited to stage 3. The succeeding stages $k = 3, \ldots, n_K$ are considered as regional warehouses or distribution centers. Finally customer demand is met with the inventory holding at the last stage n_K. Thus, $K_D = \{2,\ 3, \ldots, n_K - 1\}$ denotes the set of intermediate stages of the whole supply chain. Ventura et al. (2013) define this dynamic serial supply chain structure as a general transshipment network. An illustration of a dynamic serial supply chain network with four stages, five time periods and three suppliers is shown in Fig. 5.

Notice that Fig. 5 fails to represent the effect of changes in customer demand over time, Ventura et al. (2013) further define a time expanded static supply chain network including set of nodes $(t,\ k)$ and set of arcs connecting various nodes.

In the following theorem, accessible nodes $(t,\ k)$ and a set of time periods T_k for each stage $k,\ k \in K_D$ in the static serial supply chain network are defined and then the described problem is reduced to an equivalent problem with fewer variables but the same optimal solution.

Theorem 1. For any given stage $k \in K \backslash \{1\}$, node $(t,\ k),\ t \in T$ is feasible and reachable if and only if $t \in T_k$, where

$$T_k = \left\{ t : 1 + \sum_{k'=m_k}^{k-1} l_{k'} \le t \le n_T - \sum_{k'=k}^{m_k^*-1} l_{k'},\ t \in T \right\}.$$

m_k is 0 or the closest preceding stage from stage k that has initial inventory or pending inventory. m_k^* is n_K or the closest succeeding stage from stage k that has ending inventory on hand.

Proof. See Ventura et al. (2013).

The set of feasible time periods T_1 is derived in Corollary 1.1.

Corollary 1.1. Assume that neither initial inventory nor pending orders are available at stage 1 at the beginning of the first period. Let m_1^* is n_K or the closest succeeding stage from stage 1 that has ending inventory on hand. Define T_1 as the time periods that the supply to stage 1 is feasible to satisfy the demand in the planning horizon. Then node $(t,\ 1)$ is feasible if and only if $t \in T_1$, where

$$T_1 = \left\{ t : min\{s_j\} \le t \le n_T - \sum_{k=1}^{m_1^*-1} l_k,\ j \in J \right\}.$$

Proof. $min\{s_j\},\ j \in J$ denotes the earliest period that raw materials are available at stage 1. Apparently node $(t,\ 1)$ for any time period t less than $min\{s_j\}$ is infeasible. On the other hand, in order for final products flow sending from stage 1 arrives at stage m_1^*

the latest time period that is feasible at stage 1 should be at most $n_T - \sum_{k=1}^{m_1^*-1} l_k$. Otherwise, finished product sending from stage 1 at period t, $n_T - \sum_{k=1}^{m_1^*-1} l_k < t < n_T$ will not be received at stage m_1^*. □

Corollary 1.2. Define J_t as the preferred set of suppliers that are available to supply items at period t. Then J_t has to satisfy the following condition to ensure the predetermined amount of raw materials arrive at stage 1 as requested:

$$J_t = \left\{ j : s'_j \le t \le c_j,\ \ j \in J \right\},$$

where s'_j and c_j denote the first period and last period that supplier j is available. □

3.2. Problem formulation

Incorporating the price break scheme introduced in Section 2, the integrated multi-period, multi-stage, multi-supplier dynamic planning problem to determine the best procurement, production, distribution, and inventory strategies for a serial supply chain can be formulated as an MILP model. The lists of parameters, decisions variables, cost function components and constraints are presented below.

Parameters

- k: index for stages in consideration, $k \in K$,
- t: index for time period, $t \in T$,
- c_1^t: production cost per unit at stage 1 in period t, $t \in T_1$,
- f_1^t: production setup cost at stage 1 in period t, $t \in T_1$,
- h_k^t: unit inventory holding cost at stage k from period t to period $t+1$, $\in K$, $t \in T_k$,
- u_k^t: unit holding cost for in-transit inventory shipped from stage k to stage $k+1$ from period t to period $t+l_k$, $k \in K_D$, $t \in T_k$,
- l_k: lead time when products are shipped from stage k to $k+1$, $k \in K \setminus \{n_K\}$,
- d^t: customer demand in period t, $t \in T_{n_K}$,
- o_j: minimum order quantity per order except the first order from supplier j, $j \in J$,
- R_j: maximum quantity per order from supplier j, $j \in J$,
- i_k^0: initial inventory at each stage k, $k \in K$,
- $i_k^{n_T}$: ending inventory at each stage k, $k \in K$,
- b_1^t: production capacity at stage 1 in period t, $t \in T_1$,
- b_k^t: maximum delivery quantity at stage k in period t, $t \in T_k$, $k \in K_D$,
- e_k: inventory capacity at stage k, $k \in K$,
- M_j: primary order cost for supplier j, $j \in J$,
- N_j: secondary order cost for supplier j, $j \in J$,
- β_{k0}: fixed transportation cost from stage k to stage $k+1$, $k \in K_D$,
- α_{ke}: maximum shipping quantity in the interval at stage k, $e \in E$, $k \in K_D$, $t \in T_k$,
- β_{ke}: unit shipping cost when shipping quantity lies in interval e at stage k, $e \in E$, $k \in K_D$.

Decision variables

- z_j^t: binary variable, equals 1 if supplier j is selected in period t, equals 0 otherwise, $t \in T_1$, $j \in J_t$,
- q_j^t: replenishment quantity from supplier j arrived at stage 1 in period t, $t \in T_1$, $j \in J_t$,
- i_k^t: inventory level held at stage k from period t to period $t+1$, $k \in K$, $t \in T_k \backslash n_T$,
- x_1^t: production quantity at manufacturing stage 1 at period t, $t \in T_1$,
- y_k^t: shipping quantity at period t from stage k to stage $k+1$, $k \in K_D$, $t \in T_k$,
- w_1^t: binary variable, equals 1 if a production order is submitted in period t, equals 0 otherwise, $t \in T_1$,
- w_k^t: binary variable, equals 1 if a replenishment order is submitted at period t, equals 0 otherwise, $k \in K_D$, $t \in T_k$,
- s_{gj}: binary variable, equals 1 if the cumulative order quantity from supplier j till the end of the planning horizon lies in interval g, $j \in J$, $g = 1, 2, \dots, m_j$,
- r_{gj}: partial order quantity within interval g when the cumulative order quantity from supplier j lies in interval g, $j \in J$, $g = 1, 2, \dots, m_j$,
- y_{ke}^t: equals y_k^t when the quantity shipped from stage k to stage $k+1$ lies in the interval e, equals 0 if not, $e \in E$, $k \in K_D$, $t \in T_k$,
- f_{ke}^t: binary variable, equals 1 if order quantity shipped from stage k to stage $k+1$ in period t lies in interval e, equals 0 if not, $e \in E$, $k \in K_D$, $t \in T_k$.

According to defined parameters and decision variables, a time expanded static serial supply chain network considering one manufacturer (stage 1), one local warehouse (stage 2) and two regional warehouses or distribution centers (stages 3 and 4) is illustrated in Fig. 6. The lead time from stage 2 to stage 3 is considered to be 1 time unit, that is $l_2 = 1$, while the lead time between stages 1-2, and stages 3-4 are assumed to be zero. Notice that delivery time from suppliers are already taken into account and converted to the availability of each supplier at the beginning of each time period as we have introduced in Section 2.

Objective function

An MILP model can be formulated as:

$$\begin{aligned} Min\ Z = & \sum_{\boldsymbol{j \in J}} \sum_{\boldsymbol{g=1}}^{\boldsymbol{m_j}} \{ s_{\boldsymbol{gj}} C_{\boldsymbol{gj}} + P_{\boldsymbol{gj}} r_{gj} \} + \sum_{j \in J} \left(\sum_{g=1}^{m_j} M_j s_{gj} + N_j \sum_{t \in T_1} z_j^t \right) \\ & + \sum_{t \in T_1} (c_1^t x_1^t + f_1^t w_1^t) + \sum_{k \in K} \sum_{t \in T_k} h_k^t \left(i_k^t \right) + \sum_{k \in K_D} \sum_{t \in T_k} u_k^t \left(y_k^t \right) \\ & + \sum_{k \in K_D} \sum_{t \in T_k} \sum_{e \in E} \beta_{k0} f_{ke}^t + \sum_{k \in K_D} \sum_{t \in T_k} \sum_{e \in E} \beta_{ke} y_{ke}^t, \end{aligned} \tag{15}$$

Eq. (15) represents the total incurred cost function, which includes purchasing, manufacturing, transportation and inventory holding cost within the planning horizon.

Constraints

$$z_j^t Q_{0j} \le \sum_{t'=1}^{t} q_j^{t'}, \qquad j \in J_t, \quad t \in T_1, \tag{16}$$

$$\sum_{t'=1}^{t} q_j^{t'} \le Q'_{tj}, \qquad j \in J_t, \quad t \in T_1. \tag{17}$$

Constraint (16) and (17) represent supplier capacity constraints where cumulative order quantity from each supplier till period t cannot exceed the amount available by then.

$$q_j^t \le z_j^t R_j, \quad j \in J_t, \quad t \in T_1, \tag{18}$$

$$q_j^t \ge z_j^t o_j, \qquad j \in J_t, \quad t \in T_1. \tag{19}$$

The maximum quantity and minimum quantity for each are presented in constraints (18) and (19).

$$\sum_{t \in T_1} z_j^t \ge \sum_{g=1}^{m_j} s_{gj}, \qquad j \in J_t. \tag{20}$$

Constraint set (20) shows the order status from both the supplier's side and from the buyer's side. Once a supplier is selected, the right hand side of (20) should be equal to 1. Consequently, the left hand side, which denotes order frequency from this supplier must be greater than or equal to 1.

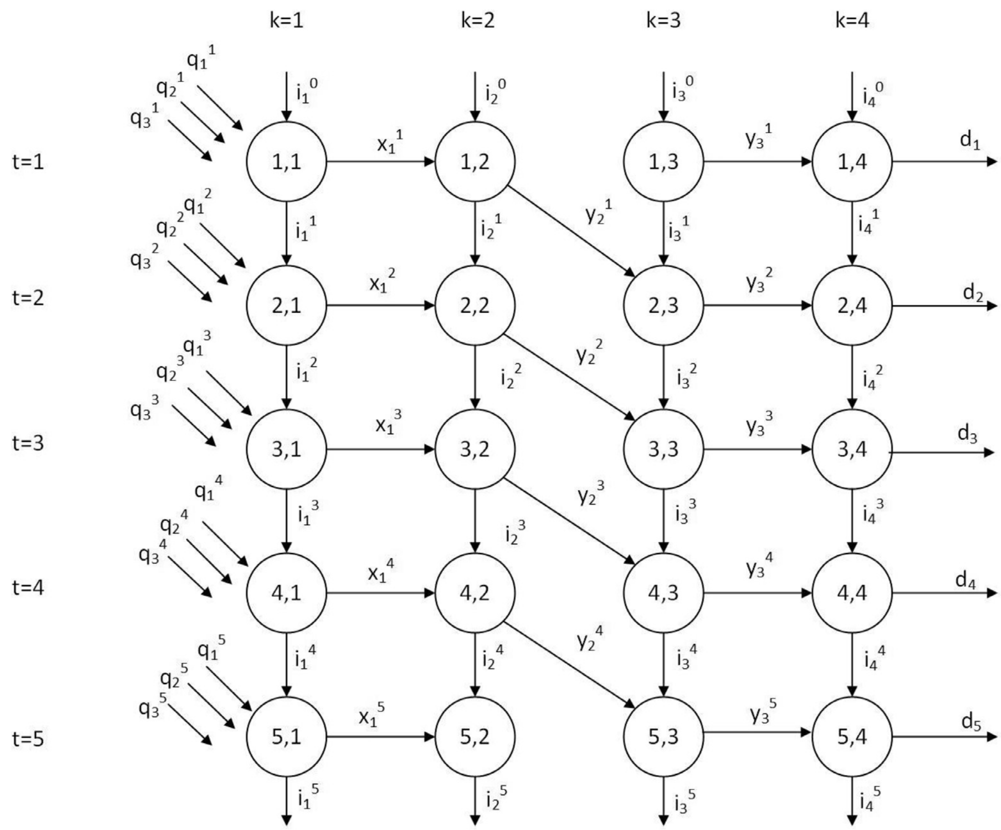

Fig. 6. A time-expanded static serial supply chain network with three suppliers, four stages and five periods.

Set of constraints (21)–(26) show cumulative order quantity and order cost constraint from given supplier j.

$$\sum_{g=1}^{m_j} s_{gj} Q_{g-1\ j} \le \sum_{t\in T_1} q_j^t,\ \ j \in J_t, \tag{21}$$

$$\sum_{t\in T_1} q_j^t \le \sum_{g=1}^{m_j} s_{gj} Q_{gj}, \quad j \in J_t, \tag{22}$$

$$\sum_{g=1}^{m_j} s_{gj} \le 1, \quad j \in J_t. \tag{23}$$

Constraint sets (21) and (22) guarantee that the total order quantity to each supplier is between any two consecutive breakpoints; while constraint set (23) ensures that only one interval is chosen for each supplier.

$$r_{gj} \le\ \ s_{gj}\left(Q_{gj} - Q_{g-1\ j}\right), \quad j \in J_t, \quad g = 1,\ 2, \ldots, m_j. \tag{24}$$

In constraint set (24), the partial order in any quantity interval must lie within the given interval for each supplier.

$$\sum_{t\in T_1} q_j^t = \sum_{g=1}^{m_j} (s_{gj} Q_{g-1\ j} +\ r_{gj}),\ \ j \in J_t, \tag{25}$$

$$C_{gj} = Q_{0j} P_{1j} + \sum_{g'=1}^{g-1} P_{g'j}\left(Q_{g'j} - Q_{g'-1j}\right), \quad j \in J_t,\ g = 1,\ 2, \ldots, m_j. \tag{26}$$

Eq. (25) demonstrates the cumulative order quantity from each supplier; In Eq. (26), C_{gj} denotes raw materials cost from supplier j where the order quantity is exactly $Q_{(g-1)j}$ which is the lower bound of the interval g, $j \in J_t$, $g = 1,\ 2, \ldots, m_j$.

$$\sum_{j\in S} q_j^t + i_1^{t-1} = x_1^t + i_1^t, \quad t \in T_1, \tag{27}$$

$$x_1^t + i_2^{t-1} = y_2^t + i_2^t, \quad t \in T_2, \tag{28}$$

$$y_{k-1}^{t-l_{k-1}} + i_k^{t-1} = y_k^t + i_k^t, \quad k \in K_D \backslash \{2\},\ t \in T_k, \tag{29}$$

$$y_{\ n_K-1}^{t-l_{n_K-1}} + i_{n_K}^{t-1} = d^t + i_{n_K}^t, \quad t \in T_{n_K}. \tag{30}$$

Constraint sets (27) and (28) represent flow balance at stage 1 and 2 respectively; while flow balance at all stages except stage 1, stage 2, and n_K is shown in constraint set (29); constraint set (30) ensures flow balance at stage n_K.

$$x_1^t \le b_1^t w_1^t, \quad t \in T_1, \tag{31}$$

$$y_k^t \le b_k^t w_k^t, \quad k \in K_D,\ t \in T_k, \tag{32}$$

$$i_k^t \le e_k, \quad k \in K,\ t \in T_k. \tag{33}$$

Constraint sets (31)–(33) show capacity restrictions of production, delivery within K_D, and inventory constraints, respectively.

$$y_{ke}^t \le f_{ke}^t \alpha_{ke}, \quad e \in E,\ k \in K_D,\ t \in T_k, \tag{34}$$

$$y_{ke}^t \ge f_{ke}^t \alpha_{ke-1},\ e \in E,\ k \in K_D,\ t \in T_k, \tag{35}$$

$$\sum_{e\in E} f_{ke}^t \le 1,\ k \in K_D,\ t \in T_k, \tag{36}$$

$$\sum_{e\in E} y_{ke}^t \ = y_k^t,\ k \in K_D,\ t \in T_k. \tag{37}$$

Constraint sets (34)–(37) guarantee the shipping quantities lie in the corresponding intervals.

$$r_{gj} \ge 0, s_{gj}\ biniary,\ j \in J_t,\ g = 1,\ 2, \ldots, m_j, \tag{38}$$

$$y_{ke}^t \geq 0,\ f_{ke}^t\ binary, \quad e \in E, k \in K_D,\ t \in T_k, \tag{39}$$

$$i_k^t \geq 0,\ k \in K,\ t \in T_k, \tag{40}$$

$$q_j^t \geq 0,\ z_j^t\ binary, \quad j \in J_t, t \in T_1, \tag{41}$$

$$x_1^t \geq 0,\ w_1^t\ binary, \quad t \in T_1, \tag{42}$$

$$y_k^t \geq 0, w_k^t\ binary, \quad k \in K_D,\ t \in T_k. \tag{43}$$

Finally constraint sets (38)–(43) define the types of decision variables.

Applying the price break scheme proposed in Section 2, the cost of raw materials across the planning horizon are calculated as follows:

$$\sum_{j \in J_t} \sum_{g=1}^{m_j} \{s_{gj} C_{gj} + P_{gj} r_{gj}\}. \tag{44}$$

In the formulation, ordering costs consist of two major components, primary ordering cost and secondary ordering cost. There is no order frequency limit from suppliers, but secondary ordering cost is considered when multiple orders are placed from the manufacturer. Specifically, there will be primary ordering cost for the selection of a given supplier. Afterwards, the secondary cost is added once an order is placed. This type of cost structure is applicable to the proposed price break scheme as the manufacturer is encouraged to maintain a long-term relationship with the supplier. Here, primary ordering cost refers to initial setup cost for selecting a supplier, while secondary cost occurs whenever an order is placed. For instance, a supplier with primary cost of $1000 and secondary cost of $500 would charge $1,500 for the first order, while additional $500 for the second orders placed.

In this model, transportation costs from suppliers to the manufacturer are assumed to be already included in the purchasing cost. Besides, a third-party provider, which offers an all-unit discount cost strategy, is selected for inter-stage transportation of products within the supply chain. According to Ventura et al. (2013), the concept of over-declaring is adopted for cost reduction by artificially increasing the shipment size to the next higher breakpoint. In practice, over-declaring has been applied frequently in a cost efficient manner.

Furthermore, the production variable costs and holding costs are defined as linear functions.

4. Analysis of the length of time period

Researches in supply chain management normally treat the length of the time period as a constant, which is given and fixed beforehand, operational decisions are then made based on a predetermined number of time periods to reach a global minimal cost across the entire supply chain. In this paper, the length of the time period (p) is considered as a variable and the shortening of p results in the increase in the time periods for the given planning horizon. Our objective is to explore the influence of the length of time period on the overall cost as well as operational decisions in supply chain planning problems. Suppose, for a certain serial supply chain with five periods, p is 12 days. If shortening the length of the time period to 6 days or 4 days, the total time periods for the corresponding problem increase to 10 and 15 respectively.

Let P be the MILP model discussed in Section 3. That is, model P has planning time period set T and set of stages K, where stage 1 denotes the production process and the customers' demands $d = [d_1, d_2, \ldots, d_{n_T}]$ are satisfied at the last stage (n_K). For any positive integer m, let P_m be the derived MILP model when evenly splitting every time period in P into m sub-periods, so the total number of time periods in P_m increases to mn_T. Time periods in P_m can be denoted as $(t-1)m+n,\ t \in T,\ n = 1, 2, \ldots,\ m$.

Table 2
Comparison of parameters in P and P_m.

Models	Periods		Production capacity
P	1	t	b_1^t
P_m	1	$(t-1)m+1$	b_1^t
	2	$(t-1)m+2$	$b_1^t - x_1^{(t-1)m+1}$
	…	…	…
	m	tm	$b_1^t - \sum_{n'=1}^{n} x_1^{(t-1)m+n'}$

In order to formulate P_m, the capacity constraints should reflect the corresponding capacity change regarding various facilities when shortening the length of the time period. Apparently, the capacity for suppliers, warehouse inventories, and inter-stage transportation in every period $(t-1)m+n$ at every stage in P_m are not affected and thus still equal to the corresponding parameters in period t at the same stage in P. However, after splitting every time period into m sub-periods, the total production capacity within these m sub-periods from period $(t-1)m+1$ to period tm equals the production capacity b_1^t in P. Define $x_1^{(t-1)m+1}$ as production units in period $(t-1)m+1$, then the available production capacity in period $(t-1)m+2$ would be the unused capacity from period $(t-1)m+1$, which is $b_1^t - x_1^{(t-1)m+1}$, and so forth (see Table 2). Similarly, the total production setup cost is still f_1^t as long as there is a production plan scheduled from period $(t-1)m+1$ to period tm.

Additional decision variables

$w_{1'}^t$: Binary variable, equals 1 if a production order is submitted within every m periods, equals 0 otherwise, $t \in T$. That is, if there is any production order submitted from period $m+1$ to period $2m$, then $w_{1'}^2 = 1$,

$w_1^{(t-1)m+n}$: Binary variable, equals 1 if a production order is submitted in period $(t-1)m+n$, equals 0 otherwise. $t \in T_1,\ n = 1, 2, \ldots, m$.

Accordingly, the following constraints are added to previously derived model and replace constraint (31):

$$x_1^{(t-1)m+n} \leq b_1^t w_1^{(t-1)m+n}, t \in T_1, n = 1, 2, \ldots, m, \tag{45}$$

$$x_1^{(t-1)m+n} \leq b_1^t - \sum_{n'=1}^{n} x_1^{(t-1)m+n'}, t \in T_1, n = 2, 3, \ldots, m, \tag{46}$$

$$\sum_{n=1}^{m} w_1^{(t-1)m+n} \leq m w_{1'}^t,\ t \in T_1. \tag{47}$$

Constraints (42) is substituted with:

$$x_1^{(t-1)m+n} \geq 0,\ \ w_1^{(t-1)m+n}, w_{1'}^t,\ \ binary, t \in T_1, n = 1, 2, \ldots, m. \tag{48}$$

To formulate P_m, we also need:

- Replace t with $(t-1)m+n,\ t \in T,\ n = 1, 2, \ldots,\ m$.
- Redefine converted quantity $Q'_{(t-1)m+n\ j}$ according to formula given in Section 2. The length of each period for P_m is the length of the time period in P divided by m.

The derived MIP model (P_m) is written as:

$$Min\ Z = \sum_{\boldsymbol{j} \in J_t} \sum_{\boldsymbol{g}=1}^{\boldsymbol{m_j}} \{s_{gj} C_{gj} + P_{gj} r_{gj}\} + \sum_{t \in T_1} \left\{ \sum_{n=1}^{m} c_1^{(t-1)m+n} x_1^{(t-1)m+n} + f_1^t w_{1'}^t \right\}$$

$$
\begin{aligned}
&+\sum_{k\in K}\sum_{t\in T_k}\sum_{n=1}^{m} h_k^{(t-1)m+n}\left(i_k^{(t-1)m+n}\right)\\
&+\sum_{k\in K_D}\sum_{t\in T_k}\sum_{n=1}^{m} u_k^{(t-1)m+n}\left(y_k^{(t-1)m+n}\right)\\
&+\sum_{k\in K_D}\sum_{t\in T_k}\sum_{e\in E}\sum_{n=1}^{m}\beta_{k0} f_{ke}^{(t-1)m+n}\\
&+\sum_{k\in K_D}\sum_{t\in T_k}\sum\nolimits_{e\in E}\sum\nolimits_{n=1}^{m}\beta_{ke} y_{ke}^{(t-1)m+n}\\
&+\sum_{j\in J_t}\left(\sum_{g=1}^{m_j} M_j s_{gj} + N_j\sum_{t\in T_1}\sum_{n=1}^{m} z_j^{(t-1)m+n}\right),
\end{aligned} \tag{49}
$$

Subject to
(16)–(30), (32)–(41), (43),

$$x_1^{(t-1)m+n} \le b_1^t w_1^{(t-1)m+n}, t\in T_1, n=1,2,\dots,m, \tag{50}$$

$$x_1^{(t-1)m+n} \le b_1^t - \sum_{n'=1}^{n} x_1^{(t-1)m+n'}, t\in T_1, n=2,\dots,m, \tag{51}$$

$$\sum_{n=1}^{m} w_1^{(t-1)m+n} \le m w_{1'}^t, t\in T_1, \tag{52}$$

$$x_1^{(t-1)m+n} \ge 0,\ \ w_1^{(t-1)m+n}, w_{1'}^t,\ \textit{binary},\quad t\in T_1, n=1,2,\dots,m. \tag{53}$$

Theorem 2. Consider MILP models P and P_m, define the optimal solution of P as $X^*=[q_j^{t*}, x_1^{t*},\ y_k^{t*}, i_k^{t*},\dots,\ t=1,2,\dots,\ n_T,\ k=1,2,\dots,n_K,\ \ j=1,2,\dots,n_J]$ with the corresponding optimal objective value Z^*. Then X^* can be transformed to $X_m=[q_j^{(t-1)m+n*}, x_1^{(t-1)m+n*},\ \ y_k^{(t-1)m+n*}, i_k^{(t-1)m+n*},\dots,\ t=1,2,\dots,\ \ n_T,\ k=1,2,\dots,n_K,\ j=1,2,\dots,n_J,\ n=1,2,\dots,m]$, where X_m and Z^* provide a feasible solution and an upper bound on the optimal objective for P_m, respectively, under the following conditions:

(i) In P_m, the unit holding cost $h_k^{(t-1)m+n}$ in the $\{(t-1)m+n\}$th period at a given stage k equals the corresponding holding cost h_k^t in period t at stage k divided by m. That is, for all $n=1,\ 2,\dots,\ m,\ t=1,2,\dots,n_T,\ k=1,2,\dots,n_K,\ h_k^{(t-1)m+n}=\frac{h_k^t}{m}$.
(ii) The production setup cost f_1^t for any time period t ($t=1,2,\dots,\ n_T$) in P is equal to the sum of production setup costs within the corresponding m periods in P_m.
(iii) The starting date and completion date of each offer from each supplier are fixed.

Proof. First we prove that X^* can be transformed to a feasible solution X_m of P_m.

Since one period in P is split into m sub-periods in P_m, any period t in P corresponds to period $(t-1)m+1$ in P_m. When converting the optimal solution X^* to X_m, all decision variables in $\{(t-1)m+n\}$th period take the corresponding values in X^*. To be specific, we have $q_j^{(t-1)m+1*}=q_j^{t*}$, $x_1^{(t-1)m+1*}=x_1^{t*}$, and so on. Furthermore, in X_m, except $i_k^{(t-1)m+n*}=i_k^{(t-1)m+1*}$, n = 2,…, m, all the remaining decision variables in period $(t-1)m+n$ are set to zero. The equation $i_k^{(t-1)m+n*}=i_k^{(t-1)m+1*}$ indicates the inventory held in every $\{(t-1)m+n\}$th period will be kept in stock for the following $m-1$ sub-periods. Note that the constraints in P_m are then simplified to be the same constraints as in P when X_m is substituted in the constraints of P_m. Therefore X_m is a feasible solution of P_m.

Then let us prove the optimal objective value Z^* of P is an upper bound on the objective value of P_m.

The procurement cost, $\sum_{j\in J_t}\sum_{g=1}^{m_j}\{s_{gj}C_{gj}+P_{gj}r_{gj}\}+\sum_{j\in J_t}(\sum_{g=1}^{m_j} M_j s_{gj}+N_j\sum_{t\in T_1}\sum_{n=1}^{m} z_j^{(t-1)m+n})$, depends merely on the total order quantity from each supplier as well as the order frequency. Given that the order quantity and order frequency with respect to X_m remain the same as in X^*, according to condition (iii), the procurement cost in terms of X_m will remain unchanged regardless of the length of the planning time period.

In P, holding cost at a given stage k ($k=1,2,\dots,\ n_K$) from period t to period $t+1$ is calculated as $i_k^t h_k^t$. According to condition (i), when shortening the length of time period, the unit holding cost per time period $h_k^{(t-1)m+n}, n=1,2,\dots,\ m$ is defined as $\frac{h_k^t}{m}$ accordingly. At the same time, in X_m, the property of $i_k^{(t-1)m+n*}=i_k^{(t-1)m+1*}=i_k^t,\ n=2,\dots,\ m$ represents the inventory in period $(t-1)m+1$ will be kept till period tm, therefore the corresponding inventory hold cost in these m periods is $\frac{h_k^t}{m}\times m\times i_k^t$, which is exactly $i_k^t h_k^t$. Furthermore, the in-transit inventory holding cost is related to the unit in-transit holding cost and the in-transit quantity. The unit in-transit holding cost $u_k^{(t-1)m+n}$ at the corresponding m periods from stage k to stage $k+1$ is still u_k^t because neither $u_k^{(t-1)m+n}$ nor u_k^t is relevant to the length of time period. Moreover, the in-transit shipping quantity in X_m is generated from X^*, therefore the incurred in-transit inventory holding cost in terms of the feasible solution X_m is equivalent to the in-transit hold cost in terms of X^* in P.

The variable component of the production cost $c_1^{(t-1)m+n}x_1^{(t-1)m+n}$ in P_m is exactly the same as that in P since the total production quantity and production schedule in terms of X_m remain at the same level. According to condition (ii), production setup cost is considered as f_1^t if there is a production plan from period $(t-1)m+1$ to period tm. So the feasible solution X_m leads to the equivalent fixed setup cost in P_m as that in P.

Notice that the transportation cost is related to the shipping quantity from one stage to the next stage. Since the same shipping strategy is adopted in X_m, there is also no difference in transportation cost between X_m and X^*. Thus the change of the length of the time period has no influence on transportation cost regarding the feasible solution X_m.

From above analysis, we conclude that the optimal objective value Z^* of P is the corresponding objective value of P_m with respect to the feasible solution X_m, hence an upper bound on the optimal objective value of P_m. □

Theorem 3. Let P_{md} be a derived model when splitting customer demand in period $(t-1)m+1,\ t=1,2,\dots,n_T$ of P_m into evenly distributed demand from period $(t-1)m+1$ to period $tm,\ t=1,2,\dots,\ n_T$. That is $\forall\ n=1,2,\dots,\ m,\ t=1,2,\dots,n_T,\ d_{(t-1)m+n}=\frac{d_t}{m}$ holds. Then, X_m and Z^* are a feasible solution and an upper bound of P_{md} respectively under the condition: $h_{n_K}^{(t-1)m+n}=0,\ n=1,2,\dots,m-1$.

Proof. Given the same number of time periods and same stages, X_m is apparently a feasible solution of P_{md}. Specifically, the explanation of X_m in P_{md} would be: from period $(t-1)m+2$ to period tm, there are no arrivals at the last stage; the demand from these $m-1$ sub-periods is satisfied with inventory. When the inventory holding cost at the last stage within every m periods is out of consideration, Z^* shows a feasible objective value of P_{md} with respect to the feasible solution X_m and hence an upper bound on the optimal objective of P_{md}. □

5. Numerical example

In this section, an illustrative example is presented as a demonstration of the serial supply chain network in Fig. 5.

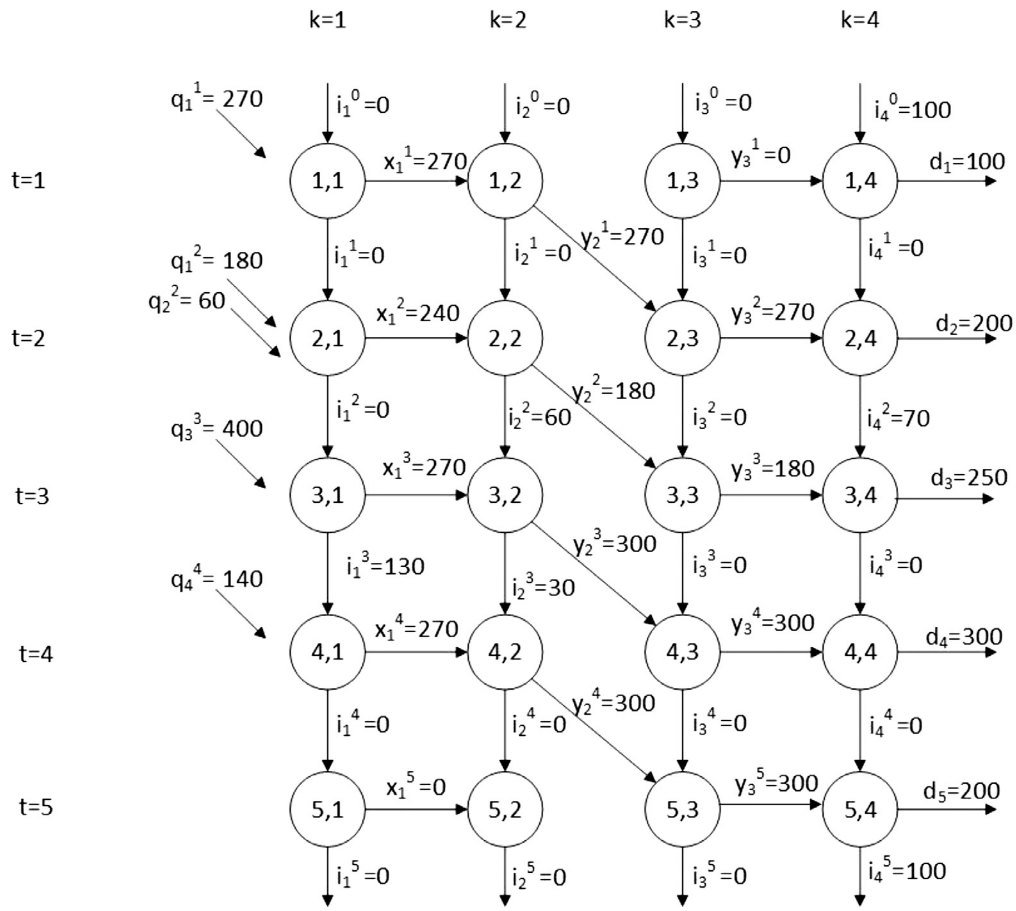

Fig. 7. Demonstration of the optimal solution of the serial supply chain network.

Table 3
Cumulative supply quantities from three suppliers.

Period	Supplier 1 (units)		Supplier 2 (units)	Supplier 3 (units)
	1st offer Offer	2nd offer		
1	300		200	100
2	450		400	100
3			650	400
4		150	900	400
5		400	1200	1000

Three suppliers are preselected by means of the Analytical Hierarchy Process (AHP) (Saaty, 1990; Mendoza & Ventura, 2008). Besides supplier 1, who was discussed in Section 2, the price break scheme of supplier 2 is $\{50, 5, 60; (200, 120, 0), (400, 100, 12), (650, 85, 17), (900, 70, 25), (1200, 60, 40)\}$, while the scheme of supplier 3, $\{50, 3, 50; (100, 110, 0), (400, 80, 15), (1000, 60, 37)\}$, provides fewer price breakpoints with larger capacity. After conversion, the cumulative quantity (Q'_{tj}) available for every period from each supplier is shown in Table 3.

Therefore, the fitted price break scheme with regard to suppliers 2 can be written as

$\{(1, 5); (50, 5); (1, 200), (2, 400), (3, 650), (4, 900), (5, 1200); (200, 120), (400, 100), (650, 85), (900, 70), (1200, 60)\}$. Similarly, the resulting price break scheme for suppliers 3 is

$$\{(1, 5); (50, 3); (1, 100), (2, 100), (3, 400), (4, 400), (5, 1000); (100, 110), (400, 80), (1000, 60)\}.$$

It is assumed that no pending orders exist at the beginning of period 1 and no ending inventories for the first three stages are considered. Deterministic and time varying demand in every time period is given in Table 5. The parameters related to supply, production, distribution, and inventory are also provided in Tables 4, 5, and 6. Initial inventory and ending inventory at stage 4 are 100 units.

Therefore, according to Theorem 1, $m_k = 0$ and $m_k^* = 4$ for $k \in K\backslash\{1\}$. Then, T_k, $k \in 2, 3, 4$ are calculated as follows: $T_2 = \{1, 2, 3, 4\}$, $T_3 = \{2, 3, 4, 5\}$, $T_4 = \{2, 3, 4, 5\}$. Similarly, applying Corollary 1.1, $m_1^* = 4$ and $T_1 = \{1, 2, 3, 4\}$.

Nominal freight rates denote the rates with the all-unit discount policy provided by the third-party carrier, while actual rates are the incurred transportation costs after adopting the concept of over-declaring. In this example, when the shipping size is between 49 and 62, the cost is greater than the corresponding cost of shipping 63 units under the all-unit discount policy. Therefore, in order to reduce the transportation cost, it is worthwhile for the decision maker to artificially increase the shipment size to the next higher breakpoint, which leads to the fixed freight rates as shown in Table 6.

A MILP with 250 constraints and 203 decision variables was formulated for this numerical example. The model was built using Matlab R2015a and solved by Matlab's MILP solver intlinprog on a PC with an Intel Core i7-4790S CPU at 3.2 GHZ and 16 GB RAM. Fig. 7 provides the optimal operational decisions on production, inventory and transportation planning within the given time horizon. As a result of high unit inventory holding cost for all stages, it can be seen that production and shipping are scheduled for all accessible nodes in Fig. 7. In addition, the fact that no inventory is held at the manufacturing site shows all raw materials arrive just in time for production.

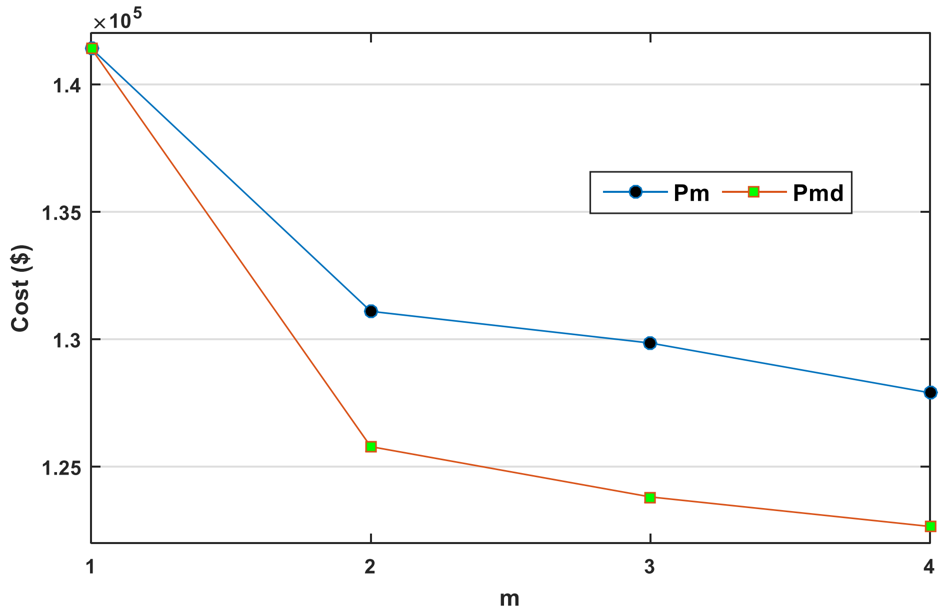

Fig. 8. Comparison of the incurred cost when splitting the length of period or the demand.

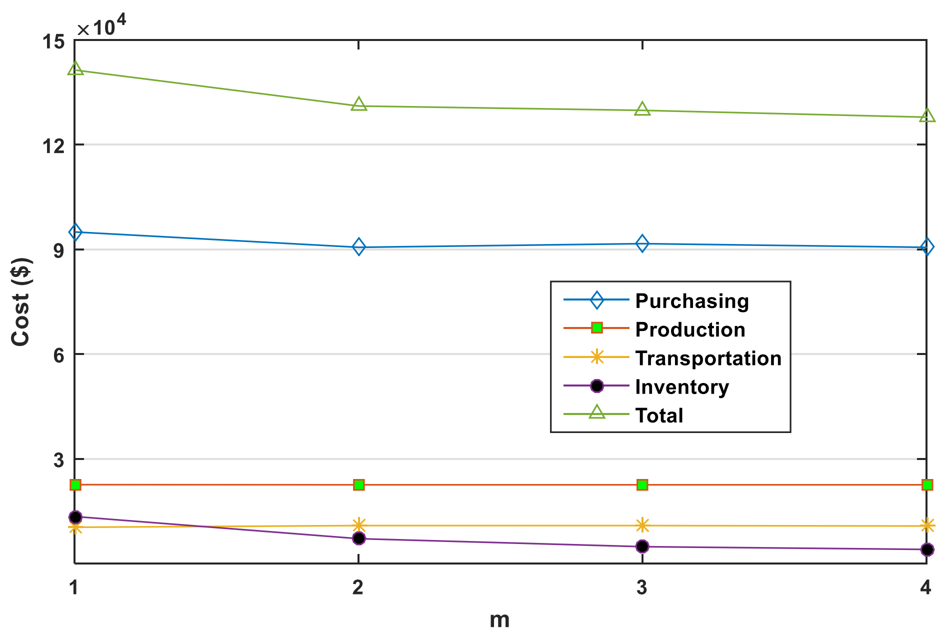

Fig. 9. Analysis of the incurred costs when increasing parameter m.

The best purchasing decisions are provided in Table 7. Similarly, it can be seen raw materials orders are placed every period in order to avoid holding cost of raw materials at the manufacturing facility. Furthermore, the overall order units from offer 1 of supplier 1 across the planning horizon reach its supply capacity, which indicates supplier 1 is the most preferred supplier, while other suppliers are considered only when supplier 1 reaches its capacity.

6. Sensitivity analysis

As discussed in Section 4, the length of the time period has an influence on decision-making and thus the overall cost. According to Theorem 2, as one time period is split into m sub-periods, the new model (P_m), where the total number of time periods increases to mn_T, is considered as an MILP with the optimal objective value lower than or equal to that of the original model (P) with only n_T periods. In this section, m is set to 2, 3 and 4 respectively. To be specific, in this example, the length of planning horizon is 60 days, five periods are considered in P and the length of each time period is 12 days. As m is set to 2, the derived model P_2 has 10 periods in all and the length of each time period is 6 days. Then the corresponding model P_3 is formulated as an MILP with 15 periods when m is 3, and so forth. Therefore the original model is reformulated as three separate MILPs when a period is split into 2, 3 and 4 sub-periods. P_2, P_3 and P_4 are implemented in Matlab R2015a.

Table 8 and Fig. 8 show the comparison of the overall cost and CPU time for these four models. As the length of time period decreases, the optimal objective value of the problem also decreases gradually. This in turn gives sufficient evidence of the result of Theorem 2 that the optimal objective value of the original model is always an upper bound for the reformulated model obtained by shortening the length of the time period. Notice that

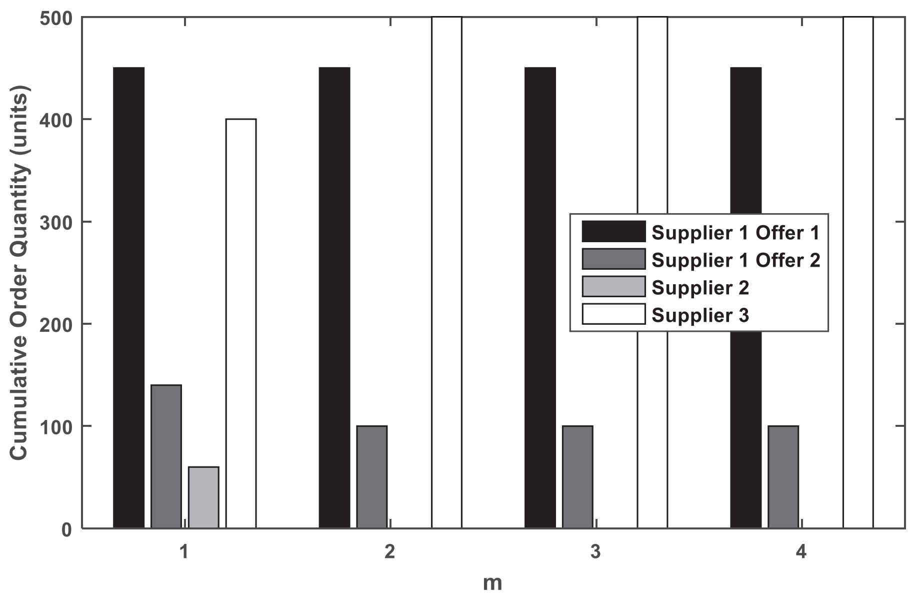

Fig. 10. Comparison of purchasing strategies when shortening the length of time period.

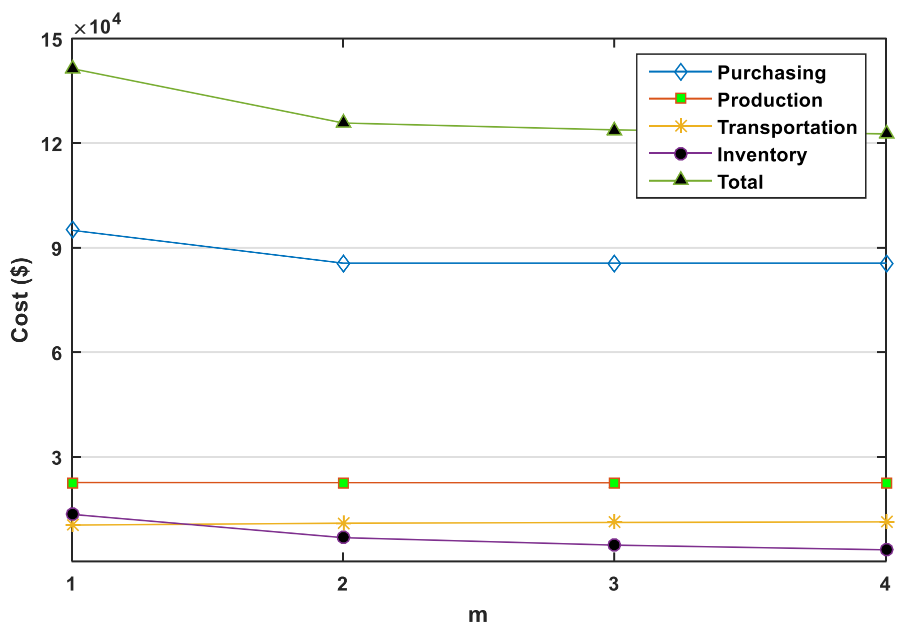

Fig. 11. Analysis of the incurred costs when splitting the time period and the demand.

there are slight cost savings when the length of the time period is shortened from 6 days to 4 days. Whereas, when this parameter is set to 6 days instead of 12 days, the overall cost across entire supply chain goes down abruptly by 7.3%. This indicates the selection and determination of appreciate length of the time period plays an important role on the efficiency of the operational planning of purchasing, manufacturing, inventory, and distributing, which consequently influences the overall incurred cost across the entire supply chain. When it comes to the CPU time, as the number of time periods increases, the CPU time also increases gradually. Note that the CPU time goes up sharply when the length of the time period goes down to 3 days, while at the same time, the model size of P_4 is getting much larger with 731 decision variables and 918 constraints. Thus there is a trade-off between cost savings and CPU processing time as well as the problem formulation, which is deemed to be a critical issue in the process of operations planning in a supply chain design.

Furthermore, now let us consider that the demand in each period in the original model is evenly distributed within m periods as one period is split into m sub-periods, three additional MILP formulations are developed. The reformulated models with evenly divided demand when splitting the length of time period are denoted as P_{2d}, P_{3d}, and P_{4d}. The results of these four models when splitting both the length of time period and the demands are also shown in Table 8 and Fig. 8. It can be seen that the overall incurred cost poses a decreasing trend as parameter m grows larger, which has been discussed previously. Similarly, the largest cost drop takes place when the length of the time period is shortened by half. Besides, for each given length of time period, the total cost to the model with divided demand is less than that of the model with aggregate demand, which has been shown in Theorem 3. Moreover, it is irrational to shorten the length of time period infinitely because the problem becomes more and more complicated as the length of time period gets smaller. Particularly in this example, the

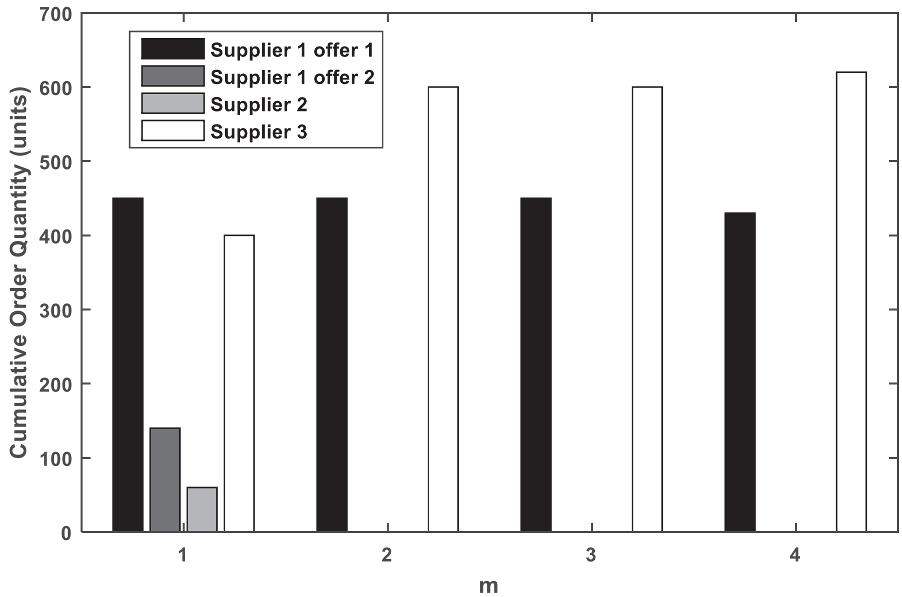

Fig. 12. Comparison of purchasing strategies when splitting the time period and the demand.

Table 4
Supplier characteristics.

Supplier	Minimal quantity for first order (units)	Minimal quantity for orders afterwards (units)	Capacity of each order (units)	Ordering Cost	
				Primary ($)	Secondary ($)
1	50	20	500	550	1000
2	50	20	500	500	1000
3	50	20	500	600	1050

Table 5
Summary of parameters for the illustrative example.

Period	Demand (units)	Production Cost		Holding cost ($/unit/period)	Capacity (units/period)		
		Fixed ($)	Variable ($/unit)		Inventory	Production	Transportation
1	100	2500	10	5	200	270	300
2	200	2500	10	5	200	270	300
3	250	3000	12	5	200	270	300
4	300	3000	12	6	200	270	300
5	200	3500	13	6	200	270	300

number of decision variables increases by a factor of 4 when the length of time period is shortened from 12 days to 3 days. Hence it is impossible to set the length of time period as short as possible given the complexity of the reformulated MILP. Furthermore, it may be hard for the manufacturer to control the production efficiently or for the distribution center to keep running in a smooth manner given frequent order arrivals or departures of the items. Thus, an optimal length of the time period can be concluded for the purpose of both demand forecasting and operational decisions making.

Table 9 and Fig. 9 show the influence of shortening the length of time period on incurred costs and purchasing strategies. It can be seen that the total number of orders is slightly increased as the number of time periods increases by 5. This indicates the trade-off between variable ordering cost and fixed order cost. Considering the availability of each supplier, it is obvious that supplier 1 remains full capacity when m increases. But when m increases, more units of raw materials are allocated to supplier 3 while supplier 2 is ruled out from purchasing strategies (See Fig. 10). The possible cause is less price advantage of supplier 2 when the length of time period is shortened from 12 days to 6 days. Moreover, no-

Table 6
Nominal and actual freight rates for the illustrative example (Ventura et al., 2013).

Nominal freight rate			Actual freight rate		
Number of units	Freight rate		Number of units	Freight rate	
1–31	$	519	1–31	$	519
32–62	$/unit	16.2	32–48	$/unit	16.2
63–124	$/unit	12.5	49–62	$	789
125–312	$/unit	11.3	63–112	$/unit	12.5
			113–124	$	1411
			125–254	$/unit	11.3
			255–312	$	2876

Table 7
Purchasing strategies.

Period	Supplier 1 (units)		Supplier 2 (units)	Supplier 3 (units)
	1st offer	2nd offer		
1	270			
2	180		60	
3				400
4		140		

Table 8
Comparison of the overall cost and processing CPU time.

m	Length per period (days)	Number of periods	P_m		P_{md}	
			Cost ($)	CPU time (seconds)	Cost ($)	CPU time (seconds)
1	12	5	141,404	0.23	141,404	0.23
2	6	10	131,092	0.32	125,785	0.45
3	4	15	129,847	0.86	123,813	3.83
4	3	20	127,897	2.86	122,654	11.52

Table 9
Comparison of the optimal strategies and costs when splitting time period.

m	Purchasing cost ($)	(a)	Supplier 1				Supplier 2		Supplier 3		Prod. cost ($)	Transp. cost ($)	Invent. cost ($)
			Offer 1		Offer 2								
			(b)	Units	(b)	Units	(b)	Units	(b)	Units			
1	95,000	4	2	450	1	140	1	60	1	400	22,580	10,374	13,450
2	90,600	6	2	450	1	100	0	0	3	500	22,540	10,852	7100
3	91,650	6	2	450	1	100	0	0	4	500	22,540	10,852	4805
4	90,600	6	2	450	1	100	0	0	3	500	22,540	10,732	4025

Column (a) indicates Number of periods, where raw material orders were placed, while (b) represents Number of periods ordered from the specific supplier.

Table 10
Comparison of the optimal strategies and costs when splitting time period and demand.

m	Purchasing cost ($)	(a)	Supplier 1				Supplier 2		Supplier 3		Prod. Cost ($)	Transp. Cost ($)	Invent. Cost ($)
			Offer 1		Offer 2								
			(b)	Units	(b)	Units	(b)	Units	(b)	Units			
1	95,000	4	2	450	1	140	1	60	1	400	22,580	10,374	13,450
2	85,550	5	2	450	0	0	0	0	3	600	22,566	10,909	6760
3	85,550	5	2	450	0	0	0	0	3	600	22,520	11,125	4617
4	85,550	5	2	430	0	0	0	0	3	620	22,540	11,294	3270

Column (a) indicates Number of periods, where raw material orders were placed, while (b) represents Number of periods ordered from the specific supplier.

Table 11
Comparison of overall cost in problems with initial condition and without initial condition.

m	Length per period (days)	Number of periods	P_m cost ($)	P_{md}			
				With initial condition		Without initial condition	
				Cost ($)	Deviation to P_m ($)	Cost ($)	Deviation to P_m ($)
1	12	5	141,404	141,404	0	141,404	0
2	6	10	131,092	125,785	−5307	127,033	−4059
3	4	15	129,847	123,813	−6034	125,673	−4174
4	3	20	127,897	122,654	−5243	124,393	−3504

tice that the purchasing strategies for P_2 and P_4 are almost the same except different delivery date in terms of the first order from supplier 3. The first order from supplier 3 arrives on day 7 for P_2 while on day 10 for P_4. Despite the same purchasing cost, the difference in delivery date result to different transportation cost and inventory cost. From column (2) and (12)–(14) in Table 9, it can be easily concluded that purchasing cost and inventory cost are two major factors that vary significantly as the length of time period gets shorter. This directly demonstrates the necessity of considering supplier selection, lot sizing allocation, and inventory planning together as an integrate model, which has been discussed a lot in previous research. In addition, the fact that purchasing cost and inventory cost vary as one period is split into several periods indicates the importance of appropriate setting of the length of time period for the purpose of decreasing the expenditure in procurement and inventory. Unlike inventory cost that gradually goes down when shortening the length of time period, the purchasing cost decreases when one time period is split into two sub-periods while further split doesn't contribute to obvious pattern.

Notice that the production costs remain unchanged while the production schedule is adjusted for different length of time period. This situation attributes to the stable production capacity constraints over time in this numerical example. For similar reason, the transportation cost also appears slight difference as the length of the time period varies.

The incurred costs and purchasing strategies with divided demand are shown in Table 10 and Fig. 11. Compared to Table 9, the purchasing cost demonstrates a noticeable decrease when the length of time period shortens from 12 days to 6 days. Similar to the scenario of aggregate demand, supplier 1 is still the most preferred raw materials provider. However, when the length of time period is shortened, offer 2 of supplier 1 is no longer selected and then more units are acquired from supplier 3 (See Fig. 12.). This is due to the less price advantage with respect to a newly started offer under the scenario of divided demand. Besides, unlike the almost stable transportation cost in Table 9, the overall transportation cost gradually goes up as the parameter m grows in Table 10, which is the result of the more frequent demand requirement at the last stage. To conclude, the length of time period plays an essential rule on the operational planning across the entire supply. Slight change with respect to the length of time period will lead

Table 12
Manufacturing capacity and cost parameters.

Period	Fixed cost ($)		Variable cost ($/unit)		Capacity (units/period)	
	Production site 1	Production site 2	Production site 1	Production site 2	Production site 1	Production site 2
1	2500	3000	10	15	270	300
2	2500	3000	10	15	270	300
3	3000	3000	12	15	270	300
4	3000	3000	12	16	270	300
5	3500	3200	13	16	270	300

Table 13
Comparison of optimal solutions with various setting and original demand.

Number of stages	m	P_m		P_{md}	
		Cost ($)	CPU time (seconds)	Cost ($)	CPU time (seconds)
3	1	130,329	0.21	130,329	0.21
	2	122,791	0.24	117,430	0.30
4	1	141,404	0.23	141,404	0.23
	2	131,092	0.32	125,785	0.45
5	1	169,922	0.25	169,922	0.25
	2	159,317	0.51	154,285	0.70

to significant decrease in the overall cost, particularly in procurement cost and inventory cost.

Without the initial condition of $h_{n_K}^{(t-1)m+n} = 0,\ n = 1, 2, \ldots, m-1$ in Theorem 3, problems P_{md} are recalculated and compared to all previous problems. Table 11 shows a comparison of the overall cost with regard to P_m and P_{md}. It can be seen that the cost of P_{md} is still less than that of P_m. In addition, the total cost of P_{md} goes down gradually as the length of each time period is shortened even without the initial condition. Furthermore, our model can also be adapted to numerous setting, such as having additional manufacturing stages, and adding or removing regional warehouses or distribution centers, considering a different number of planning periods, and so on. In reality, it is also possible to prepare some scenarios where costs of P_{md} without the initial condition in column (7) are higher than the corresponding costs of P_m in column (4) considering less intermediate stages and higher inventory cost.

Now, in order to explore the application of the proposed models and methodologies, we consider and compare two design variations of the original example in Section 5. The first design variation is a five-stage serial supply chain with two capacitated production stages. Multi-stage manufacturing has been widely used in the pharmaceutical industry due to its expensive building cost of manufacturing facilities and the short profitable life span of pharmaceuticals (Kaminksy & Simchi-levi, 2003). In our five-stage serial supply chain, raw materials from selected suppliers are first processed at manufacturing site 1 (stage 1) and the semi-processed products are held at the local warehouse (stage 2). Additional manufacturing is completed at manufacturing site 2 (stage 3). The finished products are stored at the corresponding local warehouse (stage 4) and then transported to a regional warehouse (stage 5), where the final demand from customers is satisfied. We assume the same demand, set of suppliers, inventory cost, and inventory capacity as shown in Tables 4 and 5. In addition, we are using the same third-party carrier for transporting the products from stage 2 to 3 and from stage 4 to 5. In doing so, we consider the transportation capacity and cost detailed in Tables 5 and 6, respectively. Similarly, lead time between stages 2 and 3 is 1. Otherwise, Table 12 shows the parameters at the two manufacturing sites.

In the second design variation, we investigate the case of removing one regional warehouse from the serial supply chain analyzed in Section 5. Besides, we analyze the situation where one period is split into two sub-periods in each design alternative. Each scenario has been formulated as an MILP and the resulting optimal solutions are illustrated in Table 13. Furthermore, the models are implemented in a new demand setting, in which the demand of periods 3 and 4 is adjusted to 130 units and 240 units, respectively. The computational results are shown in Table 14. As it was expected, the overall costs across the serial supply chain decrease when shortening the length of a time period. Meanwhile, when dividing the demand in one period into 2 sub-periods, we always observe an obvious decrease in cost. All of these results provide numerical validations of Theorems 2 and 3. Notice that, the CPU time goes up with both the increasing number of stages and the increasing number of periods. Further observation also indicates the significant impact of the length of the time period on the operational decisions, especially on the procurement inventory policies under different system structures. In sum, the proposed models can be applied to various systems in the real world to analyze the appropriate length of time period and provide an applicable operational plan for decision makers.

7. Conclusion

In this paper, an integrated MILP model dealing with the process of purchasing, production, inventory management, and transportation in a serial supply chain has been developed. The model aims to minimize the overall incurred cost across the entire supply chain, combining the decisions of supplier selection, lot-sizing, inventory control, and transportation. A new supplier price break scheme has been introduced and incorporated in the model, taking into account order frequency and lead time constraints. Under this scheme, each supplier claims his own supply schedule with several price break points. Every price break point is paired with the cumulative amount of available quantity as well as the earliest delivery time. Unlike previous research, where the cost of purchasing accounts for the quantity of each individual order, in our price break scheme, the cost of the raw materials to the buyer is related to the total quantity procured from each supplier. This scheme, on the one hand, offers sufficient flexibility for the buyer to decide the order quantities, order frequencies and delivery times from suppliers, on the other hand, ensures that the buyer achieves minimal purchasing cost by taking advantage of economies of scale. Incorporating this new scheme into the considered serial supply chain model, the optimal solution of this integrated model provides a procurement plan showing order quantities and corresponding time periods for the orders placed to

Table 14
Comparison of optimal solutions with various setting and adjusted demand.

Number of stages	m	P_m		P_{md}	
		Cost ($)	CPU time (seconds)	Cost ($)	CPU time (seconds)
3	1	107,525	0.22	107,525	0.22
	2	102,671	0.24	101,381	0.32
4	1	116,885	0.28	116,885	0.28
	2	109,795	0.56	108,420	0.53
5	1	142,559	0.31	142,559	0.31
	2	135,497	0.85	134,050	0.70

each supplier, as well as optimal decisions conforming to production, inventory and transportation planning to meet the predetermined time-varying demand at the beginning of each time period.

Considering the length of a time period that is normally treated as a constant, this paper for the first time proposes the idea of splitting one time period into multiple sub-periods and then investigates the corresponding influence on the operational planning decisions of the entire supply chain. A new MILP formulation has been developed for the derived problem by shortening the length of the time period. A useful theorem shows that, under certain conditions, the optimal solution to the original model can be transferred to a feasible solution to the derived model and, thus, the optimal objective value of the original model is an upper bound on the objective value of the derived model. The numerical example demonstrates the new supplier price break scheme and the proposed MILP model, providing operational decisions on procurement, manufacturing, inventory, and transportation. Meanwhile, the length of time period is considered as a variable and decreased gradually, three more MILP formulation are derived and implemented. A comparison of the optimal solution and objective values shows the fact that the overall cost decreases gradually as the length of time periods is shortened. Among these, purchasing cost varies on a large scale as the length of the time period is shortened. Thus, it is noticeable that the length of the time period has a significant impact on the supplier selection and lot sizing decisions. In addition, shortening of the length of time period has a great contribution to inventory cost saving. This demonstrates the importance of the selection of the optimal length of time period when designing a supply chain system. The trade-off between model complexity and cost efficiency may be conducted utilizing the methodology in multi-criteria optimization.

In this paper, the demand is assumed to be deterministic. Hence a direct extension of this research is to include uncertain, stochastic demand into the model. On the other hand, it may be a meaningful direction of research to consider shortages and service level when formulating the problem, exploring the influence of shortages on operational planning decisions. Considering multiple products is also a possible future research direction. In addition, multiple manufacturing facilities, multiple local warehouses, and multiple regional warehouses or distribution centers at the corresponding stages can be included in the serial supply chain.

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