EE538 Neural Networks

Homework 2

Due: 12:59 pm, April 7th, 2021

1. Let's assume the data in Region A (for Class A) are randomly distributed with a probability $p(x,y) \propto \exp[-((x-1)^2 + (y-2)^2)/2\sigma_1^2]$, and the data in Region B (for Class B) with a probability $p(x,y) \propto \exp[-(x^2+y^2)/2\sigma_2^2]$. Also, assume the numbers of data in Class A and B are big number N_1 and N_2, respectively. Unless specified otherwise, use $N_1 = N_2 = 1000$.

(a) With the class labels A and B, estimate analytically the decision boundary based on Bayes classifier, and sketch the decision boundary in (x,y) 2D space. Assume $\sigma_1 = \sigma_2 = \sigma$ =1. (20 points)
(b) For (a), develop and run a computer program to learn a Single-layer Perceptron and plot the decision boundary. (20 Points)
(c) Repeat (a) for data with different distribution, i.e., $\sigma_1 = 1, \sigma_2 = \sqrt{2}$. (10 points)
(d) For the data in (c), run a computer program to learn a Single-layer Perceptron and plot the decision boundary. (10 Points)
(e) Repeat (c) for N_1 =700 and N_2 =1400. (10 points)
(f) For the data in (e), run a computer program to learn a Single-layer Perceptron and plot the decision boundary. (10 Points)
(g) Repeat from (a) to (f) for $p(x,y)$ is a constant for $(x-1)^2 + (y-2)^2 < 2\sigma_1^2$, and the data in Region B (for Class B) with a probability $p(x,y)$ is another constant for $(x^2+y^2) < 2\sigma_2^2$. No data exists in other positions. Please discuss the differences from the above. (20 points)

2. For a 3-layers (2 hidden layers) Perceptron with N input neurons, K and L hidden neurons, and M output neurons, derive the backpropagation learning algorithm based on the following 2 approaches of gradient calculation.

(a) Chain rule (10 points)
(b) Sensitivity analysis, i.e., calculating changes of the loss function from a small change of a synaptic weight W_{ml}, W_{lk} and W_{kn}.(10 points)