EE538 Neural Networks

Homework 2

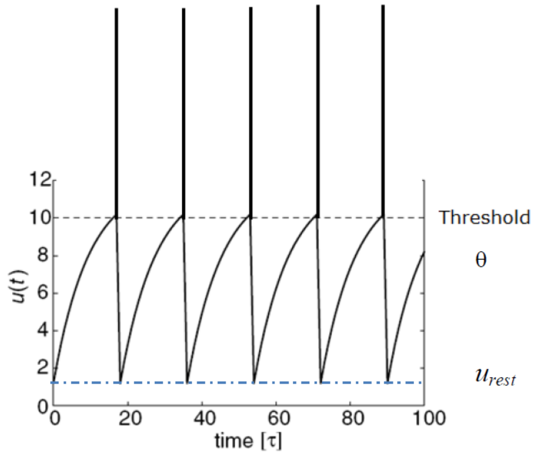
Due: 01:00 pm, March 31st, 2021

Please submit a pdf file for the answers and program codes in a zip file.

1. Let's assume *200* samples are randomly distributed with a probability density $p_1(x1, x2, x3) = 1/\pi$ for $x1^2 + x2^2 < 1$ and $0 < x3 < 1$, *200* samples with $p_2(x1, x2, x3) = 1/\pi$ for $(x1-2)^2 + x2^2 < 1$ and $0 < x3 < 1$, and *200* samples with $p_3(x1, x2, x3) = 1/\pi$ for $x1^2 + (x2-2)^2 < 1$ and $0 < x3 < 1$ in a ($x1$,$x2$,$x3$) 3-dimensional space.

   (a) Calculate three eigenvectors using any library or open source. (10 points)
   (b) Derive the following learning rule to obtain the first principal component. (10 points)
   $$w_i(n+1) \approx w_i(n) + \eta y(n) x'_i(n) \quad \text{with} \quad x'_i(n) \equiv x_i(n) - y(n) w_i(n)$$
   (c) Develop a program for (b) and calculate the first principal component using the program. It is required to use fundamental operations only without any neural networks/machine learning library. (10 points)
   (d) Derive a learning rule to obtain the second principal components. (10 points)
   (e) Develop a program for (d) and calculate the second principal component using the learning rule. It is required to use fundamental operations only without any neural networks/machine learning library. (10 points)

2. The sigmoid and rectified linear unit (ReLU) nonlinear functions for deep learning neuron models may be related to the integrate-and-fire (IF) model, which is the simple approximation of the Hodgkin-Huxley model of biological neurons. The Integrate-and-Fire neuron model is defined as follows:



- Unless a spike is generated, the membrane potential $u(t)$, i.e., the difference between internal potential from the external potential of a neuron, is modeled as a leaky integrator with an injection current $I(t)$. Here, τ_m and R are time constant and resistance parameters, respectively.

$$\tau_m \frac{du(t)}{dt} = -u(t) + RI(t)$$

with $u(0)$=0. Here, the second term in the right side denotes the generated voltage by external currents from other neurons, and the first term denotes the membrane potential will go to zero without external currents. For simplicity, let's assume $RI(t)$ is a constant V_i.

- When the membrane potential $u(t)$ reaches a threshold value θ, it becomes very high, i.e., generating a spike, and quickly goes down to u_{rest}. For simplicity, let's assume that u_{rest} =0.

(a) Before the membrane potential reaches the threshold, derive an equation for the membrane potential $u(t)$ as a function of time t. (10 points) [Hint: You may find an analytic solution of the differential equation.]

(b) Develop a computer program to calculate the membrane potential as a function of time, and plot the membrane potential versus time for several different values of V_i , θ, and τ_m. (10 points)

(c) Derive an equation for the number of spikes per unit time in terms of V_i , θ, and τ_m. (10 points) [Hint: Calculate the time interval between two spikes first.]

(d) Develop a computer program to obtain the number of spikes per unit time and plot as a function of V_i for several different values of θ and τ_m. (10 points)

(e) Let's assume 200 neurons receive the same resistance R and injection currents $I(t)$, i.e. the same V_i, but the threshold level θ and time constant τ_m have random disturbance with Gaussian distribution (standard deviation is 1/10 of the mean value). Repeat the above and plot the total number of spikes per unit time versus V_i . (10 points)