HW 1

Q1:

Let P denotes "passing the class"; Q denotes "answering question quickly"

By the assumption,

$$Pr(P) = 0.9 \; ; \; Pr(Q|P) = 0.6 \; ; \; Pr(Q|P^c) = 0.3.$$

goal: $Pr(P|Q)$.

$$Pr(P|Q) = \frac{Pr(P \cap Q)}{Pr(Q)} = \frac{Pr(P) \cdot Pr(Q|P) \quad ①}{Pr(Q \cap P) + Pr(Q \cap P^c) \quad ②}$$

① $= Pr(P) \cdot Pr(Q|P) = 0.9 \cdot 0.6 = 0.54$

② $= Pr(Q \cap P) + Pr(Q \cap P^c) =$ ① $+ Pr(Q|P^c) \cdot (1 - Pr(P)) = 0.54 + 0.03 = 0.57$.

$$Pr(P|Q) = \frac{0.54}{0.57} \approx 0.947 = 94.7\%$$

Q2:

The posterior distribution, which is $P(\theta|x, n, a)$, follows a Dirichlet distribution.

Since $p(x|\theta, n) =$ Multinomial, then $P(x|\theta, n) = \frac{n!}{x_1! x_2! \dots x_k!} \prod_{i=1}^{k} \theta_i^{x_i}$

Since $p(\theta|a) =$ Dirichlet, then $P(\theta|a) = \frac{1}{B(a)} \prod_{i=1}^{k} \theta_i^{a_i - 1}$

$$P(\theta|x, n, a) = \frac{P(x|\theta, n) P(\theta|a)}{C} = \frac{\prod_{i=1}^{k} \theta_i^{x_i} \prod_{i=1}^{k} \theta_i^{a_i - 1}}{C}$$

Since $\frac{1}{B(a)}$ and $\frac{n!}{x_1! x_2! \cdots x_k!}$, don't depend on θ, I removed them.

$$= \frac{\prod_{i=1}^{k} \theta^{(x_i + a_i) - 1}}{C} \Rightarrow P(\theta|x, n, a) \propto \prod_{i=1}^{k} \theta_i^{(x_i + a_i) - 1}$$

Thus we conclude $P(\theta|x, n, a)$ follows a dirichlet distribution with parameters $a = (a_1 + x_1, a_2 + x_2, \dots, a_k + x_k)$