Mathematical analysis. Lesson 2. Homework

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1 Introduction to Mathematical analysis

1.1 Execrise 1

Elements of sequence are elements of some set. Sequence is function of natural numbers.

1.2 Execrise 2

1. $\forall y \in [0;1] : sgn(y) = 1$

   For every y, if y in [0;1], then $sgn(y) = 1$.

   This statement is false, because if $y = 0$, then $sgn(y) = 0$.

   Negative: $\exists y \in [0;1] : sgn(y) \neq 1$

2. $\forall n \in \mathbb{N} > 2 : \exists x, y, z \in \mathbb{N} : x^n = y^n + z^n$

   For every natural number n, there exist natural numbers x,y,z such that $x^n = y^n + z^n$.

   This statement is false. It's Fermat's Last Theorem.

   Negative: $\forall n \in \mathbb{N} > 2 : \nexists x, y, z \in \mathbb{N} : x^n = y^n + z^n$

3. $\forall x \in \mathbb{R}\ \exists X \in \mathbb{R} : X > x$

   For every real number x, there exists real number X such that $X > x$.

   This statement is true. An X may always be equals to $x + 1$.

   Negative: $\forall x \in \mathbb{R}\ \ \nexists X \in \mathbb{R} : X > x$

4. $\forall x \in \mathbb{C}\ \ \nexists y \in \mathbb{C} : x > y || x < y$

   For every complex number x, there does not exist complex number y such that $x > y || x < y$.

   This statement is true. We can't compare two complex numbers.

   Negative: $\forall x \in \mathbb{C}\ \exists y \in \mathbb{C} : x > y || x < y$

5. $\forall y \in [0; \frac{\pi}{2}]\ \exists \varepsilon > 0 : \sin y < \sin(y + \varepsilon)$

   For every y in $[0; \frac{\pi}{2}]$, there exist $\varepsilon > 0$ such that $\sin y < \sin(y + \varepsilon)$.

   This statement is false, because maximum value of sine is when $y = \frac{\pi}{2}$.

   Negative: $\exists y \in [0; \frac{\pi}{2}]\ \ \nexists \varepsilon > 0 : \sin y < \sin(y + \varepsilon)$

6. $\forall y \in [0; \pi)\ \exists \varepsilon > 0 : \cos y > \cos(y + \varepsilon)$

   For every y in $[0; \pi)$, there exist $\varepsilon > 0$ such that $\cos y > \cos(y + \varepsilon)$.

   This statement is true. If y in $[0; pi)$, then $\cos y$ is greater than -1. If $\varepsilon = \pi - y$, then $\cos(y + \varepsilon) = -1$.

   Negative: $\forall y \in [0; \pi)\ \ \nexists \varepsilon > 0 : \cos y > \cos(y + \varepsilon)$

7. $\exists x : x \notin \{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}\}$

   There exist x that does not belong to any set of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$.

   This statement may be true, if we consider sets, that contains not numbers.

   Negative: $\forall x : x \in \{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}\}$