

Mathematical analysis. Lesson 3. Homework

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1 Limit of a function

1.1

$$f(x) = \sin \frac{1}{x} + \sin x$$

1.2

$$f(x) = \begin{cases} \sin \frac{1}{x}, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \end{cases}$$

$$\mathbf{1.3} \quad f(x) = x^3 - x^2$$

1.3.a

$$\text{domain} = \text{codomain} = (-\infty; +\infty)$$

1.3.b

$$\begin{aligned} f'(x) &= 3x^2 - 2x \\ f''(x) &= 6x - 2 \end{aligned}$$

1. Root $x = 1$ $f(1) = 0$

$$f'(1) \neq 0$$

The multiplicity of a root = 1.

2. Root $x = 0$

$$f(0) = f'(0) = 0$$

$$f''(0) \neq 0$$

The multiplicity of a root = 2.

1.3.c

$$f(x) > 0, \text{ for } x \in (1; +\infty)$$

$$f(x) < 0, \text{ for } x \in (-\infty; 0) \cup (0; 1)$$

1.3.d

$$f'(x) = 3x^2 - 2x = 0$$

$$D = b^2 - 4ac = 4$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{2+2}{6} = \frac{4}{6}$$

$$x_2 = \frac{2-2}{6} = 0$$

Function is increasing on $(-\infty; 0) \cup (\frac{4}{6}; +\infty)$.

Function is decreasing on $(0; \frac{4}{6})$.

1.3.e

Function is neither even nor odd.

1.3.f

Function isn't limited.

1.3.g

Function isn't periodic.

1.4**1.4.a**

$$\lim_{x \rightarrow 0} \frac{3x^3 - 2x^2}{4x^2} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{x^2(3x - 2)}{4x^2} = \lim_{x \rightarrow 0} \frac{3x - 2}{4} = 0$$

1.4.b

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{2}} - 1}{(1+x)^{\frac{1}{3}} - 1} = *$$

$$1+x = t^6, \quad \lim_{x \rightarrow 0} t = \lim_{x \rightarrow 0} \sqrt[6]{1+x} = 1$$

$$* = \lim_{t \rightarrow 1} \frac{t^{\frac{6}{2}} - 1}{t^{\frac{6}{3}} - 1} = \lim_{t \rightarrow 1} \frac{t^3 - 1}{t^2 - 1} = \lim_{t \rightarrow 1} \frac{(t-1)(t^2+t+1)}{(t-1)(t+1)} =$$

$$\lim_{t \rightarrow 1} \frac{t^2+t+1}{t+1} = \frac{3}{2}$$

2 Theorems of limits

2.a

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{4x} = \lim_{x \rightarrow 0} \frac{1}{2} \cdot \frac{\sin 2x}{2x} = \frac{1}{2}$$

2.b

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

2.c

$$\lim_{x \rightarrow 0} \frac{x}{\arcsin x} = \{\arcsin x \equiv x, \text{ for } x \rightarrow 0\} = 1$$