# Mathematical analysis. Lesson 11. Homework

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## 1

$$\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$$

$\lim\limits_{n\to\infty} \frac{(n+1)^{n+1}}{((n+1)!)^2} : \frac{n^n}{(n!)^2} = \lim\limits_{n\to\infty} \frac{(n+1)^{n+1}}{((n+1)!)^2} \cdot \frac{(n!)^2}{n^n} = \lim\limits_{n\to\infty} \frac{(n+1)^n(n+1)(n!)^2}{(n!(n+1))^2 n^n} =$
$\lim\limits_{n\to\infty} \frac{(n+1)^n}{(n+1)n^n} = \lim\limits_{n\to\infty} \frac{1}{n+1} \cdot (\frac{n+1}{n})^n = \lim\limits_{n\to\infty} \frac{1}{n+1} \cdot (1+\frac{1}{n})^n = 0 < 1$
This series is convergent

## 2

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

$\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1$
This series is convergent

## 3

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\ln n}$$

$\lim\limits_{n\to\infty} \frac{(-1)^n}{n+\ln n} = 0$
This series is convergent

## 4

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n}$$

$\lim_{n\to\infty}(n(\frac{3^n}{2^n} : \frac{3^{n+1}}{2^{n+1}} - 1)) = \lim_{n\to\infty}(n(\frac{3^n}{2^n} \cdot \frac{2^{n+1}}{3^{n+1}} - 1)) =$
$\lim_{n\to\infty}(n(\frac{2}{3} - 1)) = \lim_{n\to\infty}(n(-\frac{1}{3})) = -\infty < 1$
This series is divergent

# 5

$f(x) = \ln(16x^2)$
$f'(x) = \frac{1}{16x^2}32x = \frac{2}{x}$
$f''(x) = -\frac{2}{x^2}$
$f'''(x) = \frac{4}{x^3}$
$f^{(4)}(x) = -\frac{12}{x^4}$
$f^{(n)}(x) = (-1)^{n+1}\frac{1}{x^n}2(n-1)!$
$\frac{f^{(n)}(a)}{n!}(x-a)^n = (-1)^{n+1}\frac{1}{a^n}2(n-1)!\frac{1}{n!}(x-a)^n =$
$= (-1)^{n+1}2(n-1)!\frac{1}{n!}(x-1)^n =$
$= (-1)^{n+1}2\frac{1}{n}(x-1)^n =$
$= -1 \cdot ((-1)(x-1))^n \cdot \frac{2}{n} =$
$= -1 \cdot (1-x)^n \cdot \frac{2}{n}$