Mathematical analysis. Lesson 10. Homework

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1

$\int(2x^2 - 2x - 1 + \sin x - \cos x + \ln x + e^x)dx =$
$= 2\int x^2 dx - 2\int xdx - \int dx + \int \sin xdx - \int \cos xdx + \int \ln xdx + \int e^x dx =$
$= 2\frac{x^3}{3} - 2\frac{x^2}{2} - x - \cos x - \sin x + x\ln x - x + e^x + C$

2

$\int(2x + 6xz^2 - 5x^2y - 3\ln z)dx =$
$= 2\int xdx + 6z^2\int xdx - 5y\int x^2dx - 3\ln z\int dx =$
$= 2\frac{x^2}{2} + 6z^2\frac{x^2}{2} - 5y\frac{x^3}{3} - 3x\ln z + C$

3

$\int_0^\pi 3x^2\sin(2x)dx$
$3\int x^2\sin(2x)dx = 3\int udv = *$
$u = x^2$, $du = 2xdx$
$dv = \sin(2x)dx$, $v = \int\sin(2x)dx = \frac{1}{2}\int\sin(2x)d(2x) = -\frac{1}{2}\cos(2x)$
$* = 3(uv - \int vdu) = 3(x^2(-\frac{1}{2}\cos(2x) - \int(-\frac{1}{2}\cos(2x)2xdx))) =$
$= 3(-\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)dx) = **$
$\int x\cos(2x)dx = \int udv = *$
$u = x$, $du = dx$
$dv = \cos(2x)dx$, $v = \int\cos(2x)dx = \frac{1}{2}\int\cos(2x)d(2x) = \frac{1}{2}\sin(2x)$
$* = uv - \int vdu = \frac{1}{2}x\sin(2x) - \frac{1}{2}\int\sin(2x)dx =$
$= \frac{1}{2}x\sin(2x) - \frac{1}{2}(-\frac{1}{2}\cos(2x)) = \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)$
$** = 3(-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)) + C =$
$= -\frac{3}{2}x^2\cos(2x) + \frac{3}{2}x\sin(2x) + \frac{3}{4}\cos(2x) + C$
$\int_0^\pi 3x^2\sin(2x)dx =$
$= (-\frac{3}{2}x^2\cos(2x) + \frac{3}{2}x\sin(2x) + \frac{3}{4}\cos(2x))|_0^\pi = F(\pi) - F(0) =$
$= -\frac{3}{2}\pi^2\cos(2\pi) + \frac{3}{2}\pi\sin(2\pi) + \frac{3}{4}\cos(2\pi) - (\frac{3}{4}\cos 0) =$
$-\frac{3}{2}\pi^2\cdot 1 + \frac{3}{2}\pi\cdot 0 + \frac{3}{4}\cdot 1 - \frac{3}{4}\cdot 1 = -\frac{3}{2}\pi^2$

4

$\int \frac{dx}{\sqrt{x+1}} = *$
$t = x + 1\ ,\ dt = dx\ ,\ x = t - 1$
$* = \int \frac{dt}{\sqrt{t-1+1}} = \int t^{-\frac{1}{2}} dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{t} + C = 2\sqrt{x+1} + C$