

Mathematical analysis. Lesson 8. Homework

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1

$$z = \sqrt{1 - x^3} + \ln(y^2 - 1)$$
$$\begin{cases} 1 - x^3 \geq 0 \\ y^2 - 1 > 0 \end{cases} \quad \begin{cases} 1 \geq x^3 \\ y^2 > 1 \end{cases} \quad \begin{cases} x \leq 1 \\ y > 1 \end{cases}$$

2

$$z = \left(1 + \frac{\ln x}{\ln y}\right)^3$$
$$z'_x = \left(\left(1 + \frac{\ln x}{\ln y}\right)^3\right)'_x = 3 \left(1 + \frac{\ln x}{\ln y}\right)^2 \left(1 + \frac{\ln x}{\ln y}\right)'_x =$$
$$= 3 \left(1 + \frac{\ln x}{\ln y}\right)^2 \frac{1}{\ln y} \frac{1}{x} = \frac{3}{x \ln y} \left(1 + \frac{\ln x}{\ln y}\right)^2$$
$$z'_y = \left(\left(1 + \frac{\ln x}{\ln y}\right)^3\right)'_y = 3 \left(1 + \frac{\ln x}{\ln y}\right)^2 \left(1 + \frac{\ln x}{\ln y}\right)'_y =$$
$$= 3 \left(1 + \frac{\ln x}{\ln y}\right)^2 \ln x \cdot ((\ln y)^{-1})'_y =$$
$$= 3 \left(1 + \frac{\ln x}{\ln y}\right)^2 \ln x \cdot (-1)(\ln y)^{-2} \cdot \frac{1}{y} =$$
$$= -\frac{3 \ln x}{y (\ln y)^2} \left(1 + \frac{\ln x}{\ln y}\right)^2$$

3

$$z = \sqrt{2xy + \cos \frac{x}{y}}$$
$$z'_x = \left((2xy + \cos \frac{x}{y})^{\frac{1}{2}}\right)'_x = \frac{1}{2}(2xy + \cos \frac{x}{y})^{-\frac{1}{2}} (2xy + \cos \frac{x}{y})'_x =$$
$$= \frac{1}{2}(2xy + \cos \frac{x}{y})^{-\frac{1}{2}} (2y - \sin \frac{x}{y} \cdot \frac{1}{y})$$
$$z'_y = \left((2xy + \cos \frac{x}{y})^{\frac{1}{2}}\right)'_y = \frac{1}{2}(2xy + \cos \frac{x}{y})^{-\frac{1}{2}} (2xy + \cos \frac{x}{y})'_y =$$
$$= \frac{1}{2}(2xy + \cos \frac{x}{y})^{-\frac{1}{2}} (2x - \sin \frac{x}{y} \cdot x(-1)y^{-2})$$
$$dz = z'_x dx + z'_y dy =$$
$$= \frac{2y - \frac{1}{y} \sin \frac{x}{y}}{2\sqrt{2xy + \cos \frac{x}{y}}} dx + \frac{2x + \frac{x}{y^2} \sin \frac{x}{y}}{2\sqrt{2xy + \cos \frac{x}{y}}} dy$$
$$dz(1; 1) = \frac{2 - \sin 1}{2\sqrt{2 + \cos 1}} dx + \frac{2 + \sin 1}{2\sqrt{2 + \cos 1}} dy$$

4

$$z = x^2 + xy + y^2 - 6x - 9y$$

$$z'_x = 2x + y - 6$$

$$z'_y = x + 2y - 9$$

$$z''_{xx} = 2$$

$$z''_{yy} = 2$$

$$z''_{xy} = 1$$

$$z''_{yx} = 1$$

$$\begin{cases} 2x + y - 6 = 0 \\ x + 2y - 9 = 0 \end{cases} \quad \begin{cases} y = 6 - 2x \\ x + 2(6 - 2x) - 9 = 0 \end{cases}$$

$$\begin{cases} y = 6 - 2x \\ -3x + 3 = 0 \end{cases} \quad \begin{cases} y = 6 - 2 = 4 \\ x = 1 \end{cases}$$

M(1;4)

$$\Delta = \begin{vmatrix} z''_{xx} & z''_{xy} \\ z''_{xy} & z''_{yy} \end{vmatrix} = z''_{xx}z''_{yy} - (z''_{xy})^2 = 2 \cdot 2 - 1 = 3 > 0$$

$$z''_{xx} > 0$$

M(1;4) is local minimum