Mathematical analysis. Lesson 9. Homework

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1

$U = 3 - 8x + 6y$, for $x^2 + y^2 = 36$
$L = f(x,y) + \lambda\varphi(x,y) = 3 - 8x + 6y + \lambda(x^2 + y^2 = 36)$
$L'_x = -8 + \lambda 2x$
$L'_y = 6 + \lambda 2y$
$\begin{cases} L'_x = 0 \\ L'_y = 0 \\ \varphi(x,y) = 0 \end{cases} \quad \begin{cases} -8 + \lambda 2x = 0 \\ 6 + \lambda 2y = 0 \\ x^2 + y^2 = 36 \end{cases} \quad \begin{cases} x = \frac{8}{2\lambda} \\ y = -\frac{6}{2\lambda} \\ (\frac{8}{2\lambda})^2 + (-\frac{6}{2\lambda})^2 = 36 \end{cases}$
$64 + 36 = 36 \cdot 4\lambda^2$
$\lambda^2 = \frac{100}{144}$
$\lambda_1 = \frac{10}{12}$; $x = \frac{24}{5}$; $y = -\frac{18}{5}$
$\lambda_2 = -\frac{10}{12}$; $x = -\frac{24}{5}$; $y = \frac{18}{5}$
$M_1(\frac{24}{5}; -\frac{18}{5})$; $M_2(-\frac{24}{5}; \frac{18}{5})$
$U(M_1) = 3 - \frac{8 \cdot 24}{5} - \frac{6 \cdot 18}{5} = \frac{15-192-108}{5} = -\frac{285}{5} = -57$ - is conditional minimum
$U(M_2) = 3 + \frac{8 \cdot 24}{5} + \frac{6 \cdot 18}{5} = \frac{15+192+108}{5} = \frac{315}{5} = 63$ - is conditional maximum

2

$U = 2x^2 + 12xy + 32y^2 + 15$, for $x^2 + 16y^2 = 64$
$U(-4\sqrt{2}; -\sqrt{2}) = U(4\sqrt{2}; \sqrt{2}) = 239$ - is conditional maximum
$U(-4\sqrt{2}; \sqrt{2}) = U(4\sqrt{2}; -\sqrt{2}) = 47$ - is conditional minimum

3

$U = x^2 + y^2 + z^2$, $\vec{c}(-9; 8; -12)$, $M(8; -12; 9)$
$U'_x = (x^2 + y^2 + z^2)'_x = 2x$
$U'_y = 2y$
$U'_z = 2z$
$U'_x(M) = 2 \cdot 8 = 16$
$U'_y(M) = -24$
$U'_z(M) = -18$
$\vec{c}_0 = \frac{\vec{c}}{|\vec{c}|} = \frac{-9i+8j-12k}{\sqrt{(-9)^2+8^2+(-12)^2}} = \frac{-9i+8j-12k}{\sqrt{81+64+144}} = -\frac{9}{17}i + \frac{8}{17}j - \frac{12}{17}k$

$\frac{\partial U}{\partial l_c} = U'_x(M_0)\cos\alpha + U'_y(M_0)\cos\beta + U'_z(M_0)\cos\gamma =$
$= 16(-\frac{9}{17}) - 24(\frac{8}{17}) + 9(-\frac{12}{17}) =$
$\frac{-144-192-108}{17} = -\frac{144}{17}$

4

$U = e^{x^2+y^2+z^2},\ \vec{d}(4;-13;-16),\ L(-16;4;-13)$
$U'_x = 2xe^{x^2+y^2+z^2}$
$U'_y = 2ye^{x^2+y^2+z^2}$
$U'_z = 2ze^{x^2+y^2+z^2}$
$U'_x(L) = 2(-16)e^{x^2+y^2+z^2} = -32e^{441}$
$U'_y(L) = 8e^{441}$
$U'_z(L) = -13e^{441}$
$\vec{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{4i-13j-16k}{\sqrt{16+169+256}} = \frac{4}{21}i - \frac{13}{21}j - \frac{16}{21}k$
$\frac{\partial U}{\partial l_d} = -32e^{441}\cdot\frac{4}{21} - 8e^{441}\cdot\frac{13}{21} + 13e^{441}\cdot\frac{16}{21} =$
$= \frac{e^{441}}{21}(-31\cdot 4 - 8\cdot 13 + 13\cdot 16) =$
$= \frac{e^{441}}{21}(-128-104+208) = -\frac{24}{21}e^{441}$