Mathematical analysis. Lesson 6. Homework

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1

1.1

$y' = (\sin x \cdot \cos x)' = (\sin x)' \cdot \cos x + \sin x(\cos x)' = \cos^2 x - \sin^2 x$

1.2

$y' = \left(\ln (2x+1)^3\right)' = \frac{\left((2x+1)^3\right)'}{(2x+1)^3} = \frac{3(2x+1)^2 \cdot 2}{(2x+1)^3} = \frac{6}{2x+1}$

1.3

$y' = \left(\sqrt{\sin^2 (\ln x^3)}\right)' = \left(\left|\sin (\ln x^3)\right|\right)' = \frac{\sin (\ln x^3)}{|\sin (\ln x^3)|} \left(\sin (\ln x^3)\right)' = *$
$\left(\sin (\ln x^3)\right)' = \left(\cos (\ln x^3)\right) \cdot (\ln x^3)' = \left(\cos (\ln x^3)\right) \cdot \frac{1}{x^3} \cdot (x^3)' =$
$\cos (\ln x^3) \cdot \frac{1}{x^3} \cdot 3x^2 = \cos (\ln x^3) \cdot \frac{3}{x}$
$* = \frac{3 \cdot \sin (\ln x^3) \cdot \cos (\ln x^3)}{x \cdot |\sin (\ln x^3)|}$

1.4

$y' = \left(\frac{x^4}{\ln x}\right)' = \left(x^4 \cdot (\ln x)^{-1}\right)' = \frac{(x^4)'}{\ln x} + x^4 \left((\ln x)^{-1}\right)' =$
$\frac{4x^3}{\ln x} + x^4(-1)(\ln x)^{-2}(\ln x)' = \frac{4x^3}{\ln x} + \frac{x^4}{\ln^2 x} \cdot \frac{1}{x} = \frac{4x^3}{\ln x} + \frac{x^4}{x \ln^2 x}$

2

$f'(x) = \left(\cos (x^2+3x)\right)' = -\sin (x^2+3x) \cdot (x^2+3x)' =$
$= -\sin (x^2+3x)(2x+3)$
$f'(\sqrt{\pi}) = -(2\sqrt{\pi}+3) \sin (\pi + 3\sqrt{\pi})$

3

$f'(x) = \left((x^3 - x^2 - x - 1)(1 + 2x + 3x^2 - 4x^3)^{-1}\right)' =$
$\frac{(x^3-x^2-x-1)'}{1+2x+3x^2-4x^3} + (x^3-x^2-x-1)(-1)(1+2x+3x^2-4x^3)^{-2}(1+2x+3x^2-4x^3)' =$
$\frac{3x^2-2x-1}{1+2x+3x^2-4x^3} - \frac{x^3-x^2-x-1}{(1+2x+3x^2-4x^3)^2}(2+6x-12x^2)$

$f'(0) = \frac{-1}{1} - \frac{-1}{1}(2) = -1 + 2 = 1$

4

$f'(x) = \left((3x)^{\frac{1}{2}} \ln x\right)' = \left((3x)^{\frac{1}{2}}\right)' \ln x + \sqrt{3x}(\ln x)' =$
$= \frac{1}{2}(3x)^{-\frac{1}{2}}(3x)' \ln x + \frac{\sqrt{3x}}{x} = \frac{3 \ln x}{2\sqrt{3x}} + \frac{\sqrt{3x}}{x} = \frac{3x \ln x + 6x}{2x\sqrt{3x}} = \frac{3 \ln x + 6}{2\sqrt{3x}}$
$f'(1) = \frac{3 \ln 1 + 6}{2\sqrt{3}}$