Filter Design Assignment

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Filter 1

1. Specifications

Filter type: Band pass filter.
Pass band nature: equiripple
Stop band nature: monotonic
Passband tolerance, del1 = 0.1 (in magnitude)
Stopband tolerance, del2 = 0.1 (in magnitude)
Transition band = 1 KHz on either side.
Sampling frequency = 90 KHz
Filter number = 102
Pass band edge 1, $B_L(m)$ = 8.1KHz
Pass band edge 2, $B_H(m)$ = 13.1 KHz

2. Normalized Filter Specifications

This is calculated by multiplying the given specifications by the 2π /sampling frequency .
Normalized pass band edges (w_{Np1} and w_{Np2}) are 0.5655 and 0.9145 radians.
Normalized pass band edges (w_{Ns1} and w_{Ns2}) are 0.4956 and 0.9843 radians.

3. IIR Filter Design

3.1 Analog Filter Specifications:

Here we make use of the transformation function Ω = tan (ω/2) to get the analog filter specifications corresponding to the above digital filter. So after the transformation,
pass band edges are at 0.2905 and 0.4920
stop band edges are at 0.2530 and 0.5362

Because of the requirement for equiripple in pass band and monotonic stop band, we need to use the Chebyschev filter. While designing low pass filter pass band edge is taken as ΩLp = 1 and stop band edge is taken as more stringent of the two obtained from the below formula.

$$\Omega Ls = \text{minimum of } \{ (\Omega_{S1}^2-\Omega_0^2)/B*\Omega_{S1}, (\Omega_{S2}^2-\Omega_0^2)/B*\Omega_{S2}\} = 1.33765$$

where, $\Omega_0^2 = \Omega_{P1} * \Omega_{P2} = 0.142957$,

$B = \Omega_{P2} - \Omega_{P1} = 0.201534$

The Transform used for transformation of analog low pass filter to analog band pass filter is

$$S_L = (S^2+\Omega_0^2)/BS$$

Corresponding Ω transform is

$$\Omega_L = (\Omega^2 - \Omega_0^2)/B\Omega \text{ (by putting } S = j\,\Omega)$$

The values for D1 and D2 are obtained as
D1 = 0.2346
D2 = 99.000

From this, the order is obtained using the formula
N = ceil ((acosh(sqrt(D2/D1)))/(acosh(ΩLs / ΩLp))
The value for N is obtained as, N = 5.

Analog Filter Transfer Functions and Pole-Zero Plots:

The analog low pass filter transfer function is obtained as,

$H_{analog,\ LPF}\ (S_L) =$

$$\frac{0.1290}{s^5 + 0.9667\ s^4 + 1.717\ s^3 + 1.014\ s^2 + 0.5995\ s + 0.129}$$

The pole-zero plot of the equivalent low pass filter is as shown below.

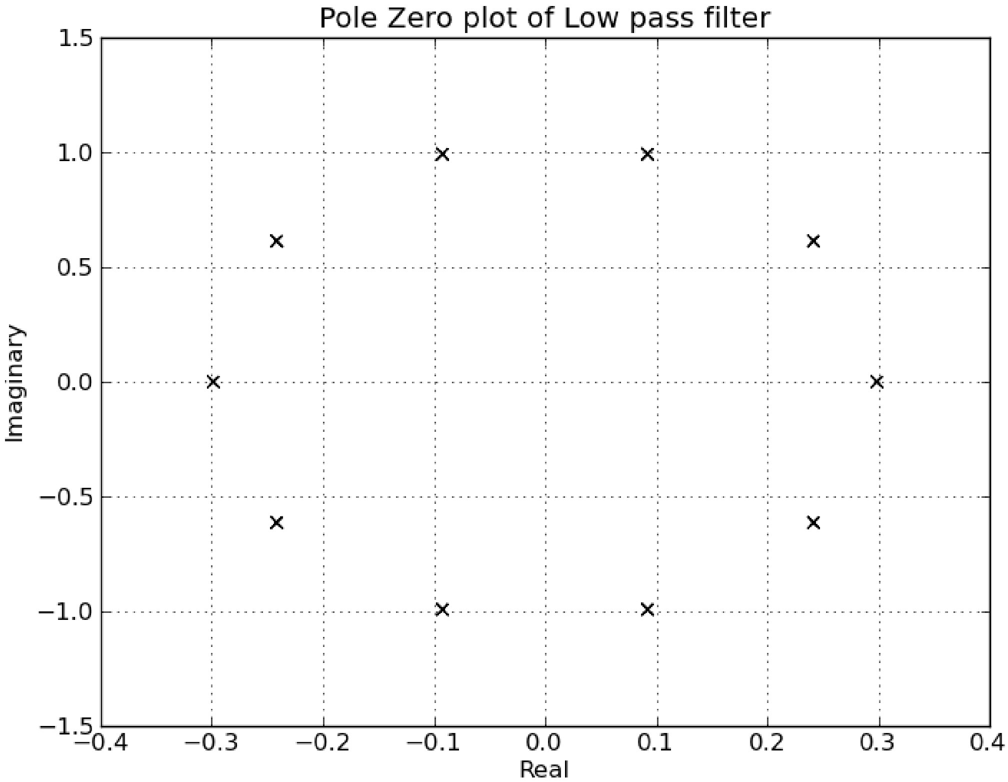


Figure: 1

The analog band pass filter transfer function is obtained as

$H_{analog,\ BPF}\ (S) =$

$$\frac{4.29e{-}05\ s^5}{(s^{10} + 0.1948\ s^9 + 0.7845\ s^8 + 0.1197\ s^7 + 0.2353\ s^6 + 0.0263\ s^5 + 0.03363\ s^4 + 0.002446\ s^3 + 0.002292\ s^2 + 8.137e{-}05\ s + 5.971e{-}05)}$$

The pole-zero plot of the analog band pass filter is as shown below.

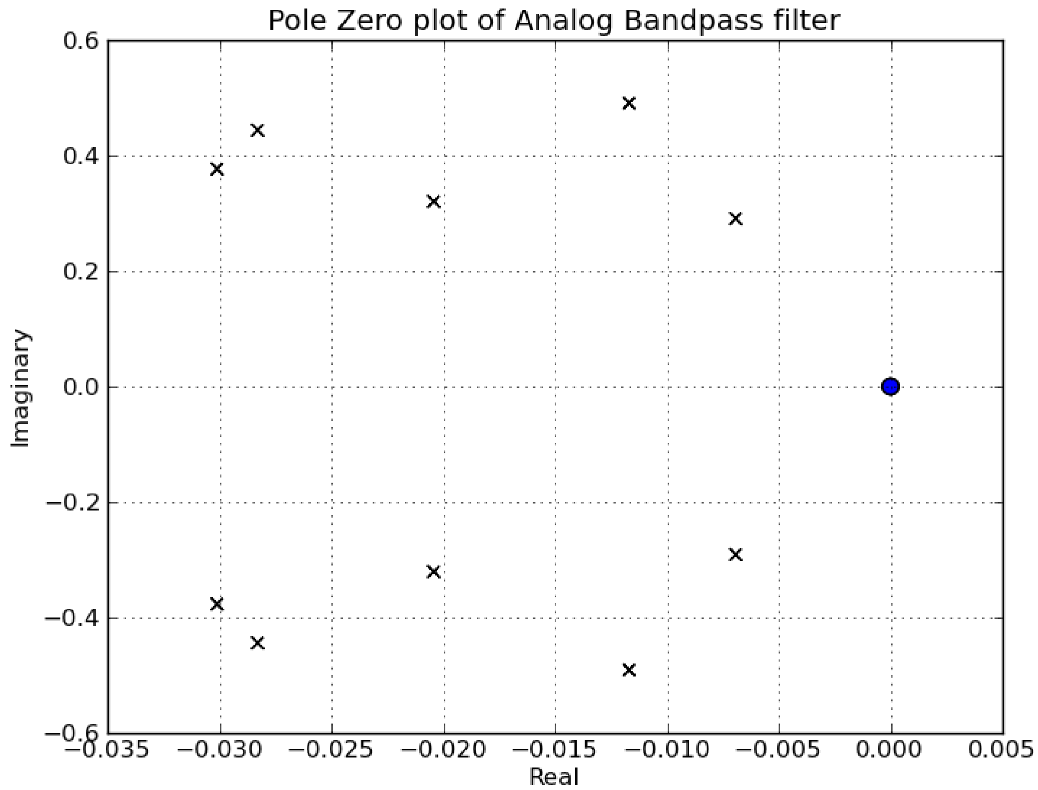


Figure: 2

3.2 Digital Filter:

The transformation used to transform the analog filter to Discrete Band Pass filter is
$S = (1-Z^{-1})/ (1+Z^{-1})$. The Discrete filter transform function is

H (Z) =

1.79231265e-05 – 8.96156323e-05 z^-2 + 1.78814448e-04 z^-4 -1.78814448e-04 z^-6 + 8.96156323e-05 z^-8 -1.79231265e-05z^-10

(1 -7.13716068 z^-1 +24.97644276 z^-2 -55.45620734 z^-3 +85.978283014 z^-4 -96.91070615 z^-5 + 80.36067708 z^-6 -48.44596698 z^-7 + 20.39381777 z^-8
-5.44746669 z^-9 0.71377184z^-10)

The frequency response of the digital filter is as shown in the figure below.

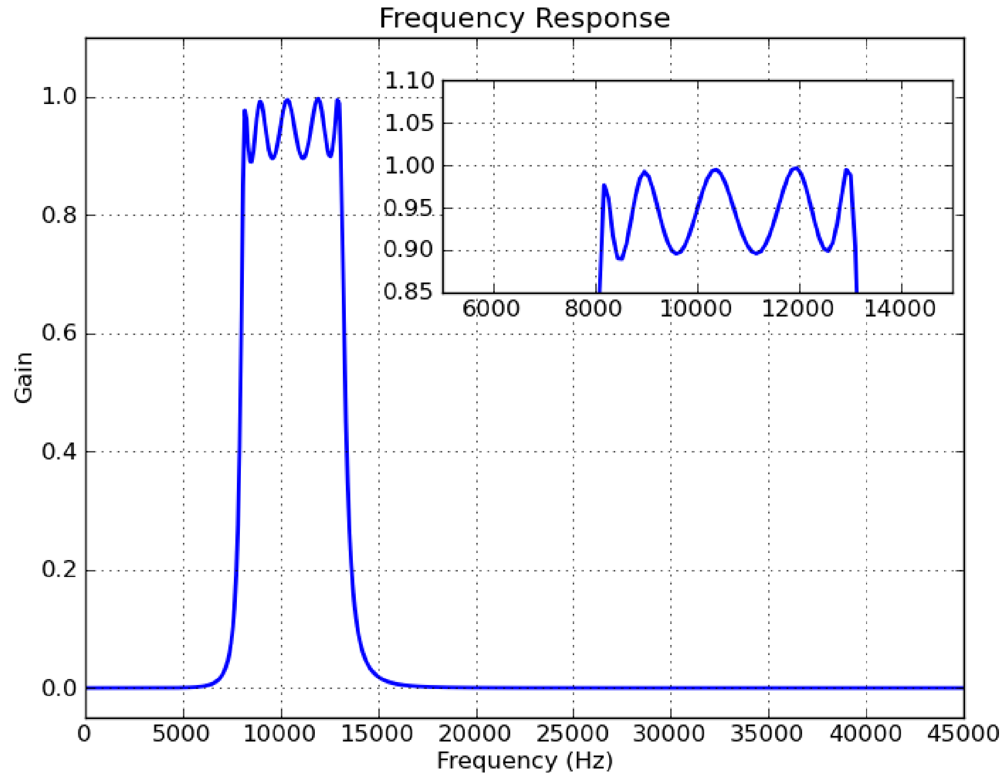


Figure: 3

The pole-zero plot (zoomed) of the filter is shown below.

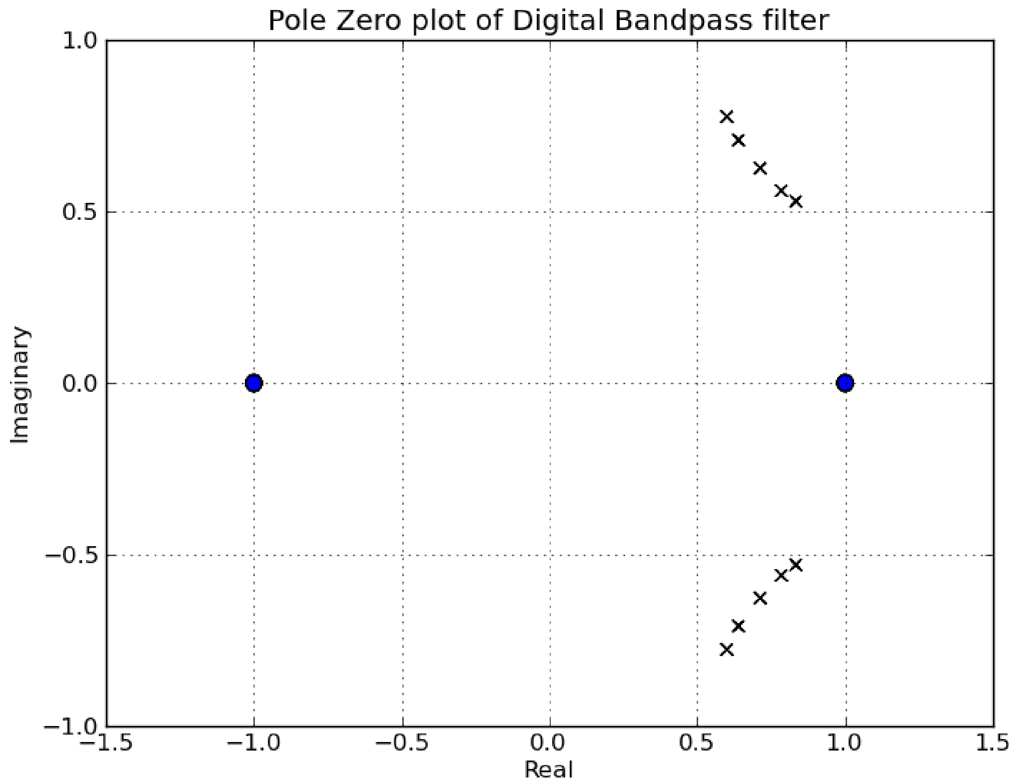


Figure: 4

The lattice parameters for the IIR filter with the specifications given, is mentioned below.
Lattice Coefficients:Kn
[-0.80394235 0.98669483 -0.7320443   0.97957285 -0.73884154 0.97801281
 -0.74639071 0.9618184 -0.71996118 0.71377184]

Lattice Coefficients:Cn
[-9.63179219e-06   7.09115852e-05   1.47157755e-04 -4.05023699e-04
 -4.44746042e-04   5.30712296e-04   6.17162961e-04 -1.15070766e-04
 -3.09979012e-04 -1.27920234e-04 -1.79231265e-05]

4. FIR Filter Design

The order of the filter required to satisfy the specifications mentioned in section 1, is calculated with the help of the equation,

$(2*N + 1) > 1+ ((A-8)/2.285*\Delta w_T)$,

where, $\Delta w_T = w_s - w_p$, $A = -20*\log_{10}(\varsigma)$, ς is the tolerance, = w_s and w_p are the normalized frequencies. Here the more stringent condition is taken with respect to the value of Δw_T .The ideal impulse response of the band [ass filter is generated using the equation

h_fir (n) = (sin(n* w_{c2})-sin(n*w_{c1}))*(1/(n*pi)), $n \neq 0$

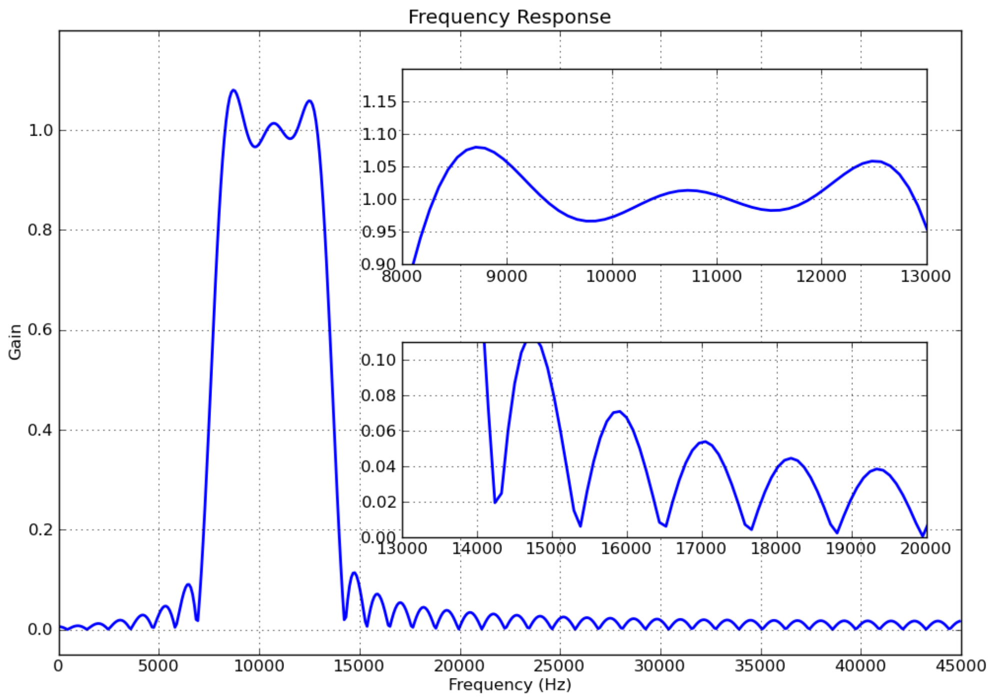
= $(w_{c2}-w_{c1})$/pi, n = 0

n ranges from –N to N, w_{c2} and w_{c1} are the cutoff frequencies calculated from the normalised frequencies as $w_{c1}= 0.5*(w_{Ns1}+w_{Np1})$ and $w_{c2}= 0.5*(w_{Ns2}+w_{Np2})$

This ideal impulse response is then multiplied with the Kaiser window (in this case it turned out to be a rectangular window) to obtain the FIR impulse response. The transfer function of the FIR filter so obtained is as given below.

H(Z) =
[-1.64652462e-02   -1.07222548e-02 z^-1   1.05602433e-03 z^-2   1.13312183e-02 z^-3
 1.39093033e-02 z^-4   8.58377315e-03 z^-5   9.58102165e-04 z^-6   -2.48549064e-03 z^-7
 -3.62925785e-09 z^-8   3.93664996e-03 z^-9   2.73463289e-03 z^-10   -5.90091109e-03 z^-11
 -1.68232011e-02 z^-12   -2.07231892e-02 z^-13   -1.16878358e-02 z^-14   7.03177064e-03 z^-15
 2.41904432e-02 z^-16   2.84277421e-02 z^-17   1.69716064e-02 z^-18   -1.91029894e-03 z^-19
 -1.51542721e-02 z^-20   -1.52300827e-02 z^-21   -6.18619628e-03 z^-22   -3.87836852e-10 z^-23
 -5.61037716e-03 z^-24   -1.95388560e-02 z^-25   -2.66728040e-02 z^-26   -1.21432739e-02 z^-27
 2.41686590e-02 z^-28   6.25493016e-02 z^-29   7.39835450e-02 z^-30   4.07807432e-02 z^-31
 -2.71367517e-02 z^-32   -9.35106807e-02 z^-33   -1.16332089e-01 z^-34   -7.54128058e-02 z^-35
 1.17340416e-02 z^-36   9.77424620e-02 z^-37   1.33333337e-01 z^-38   9.77424620e-02 z^-39
 1.17340416e-02 z^-40   -7.54128058e-02 z^-41   -1.16332089e-01 z^-42   -9.35106807e-02 z^-43
 -2.71367517e-02 z^-44   4.07807432e-02 z^-45   7.39835450e-02 z^-46   6.25493016e-02 z^-47
 2.41686590e-02 z^-48   -1.21432739e-02 z^-49   -2.66728040e-02 z^-50   -1.95388560e-02 z^-51
 -5.61037716e-03 z^-52   -3.87836852e-10 z^-53   -6.18619628e-03 z^-54   -1.52300827e-02 z^-55
 -1.51542721e-02 z^-56   -1.91029894e-03 z^-57   1.69716064e-02 z^-58   2.84277421e-02 z^-59
 2.41904432e-02 z^-60   7.03177064e-03 z^-61   -1.16878358e-02 z^-62   -2.07231892e-02 z^-63
 -1.68232011e-02 z^-64   -5.90091109e-03 z^-65   2.73463289e-03 z^-66   3.93664996e-03 z^-67
 -3.62925785e-09 z^-68   -2.48549064e-03 z^-69   9.58102165e-04 z^-70   8.58377315e-03 z^-71
 1.39093033e-02 z^-72   1.13312183e-02 z^-73   1.05602433e-03 z^-74   -1.07222548e-02 z^-75
 -1.64652462e-02 z^-76]

The magnitude and phase response of the FIR filter is as shown below.



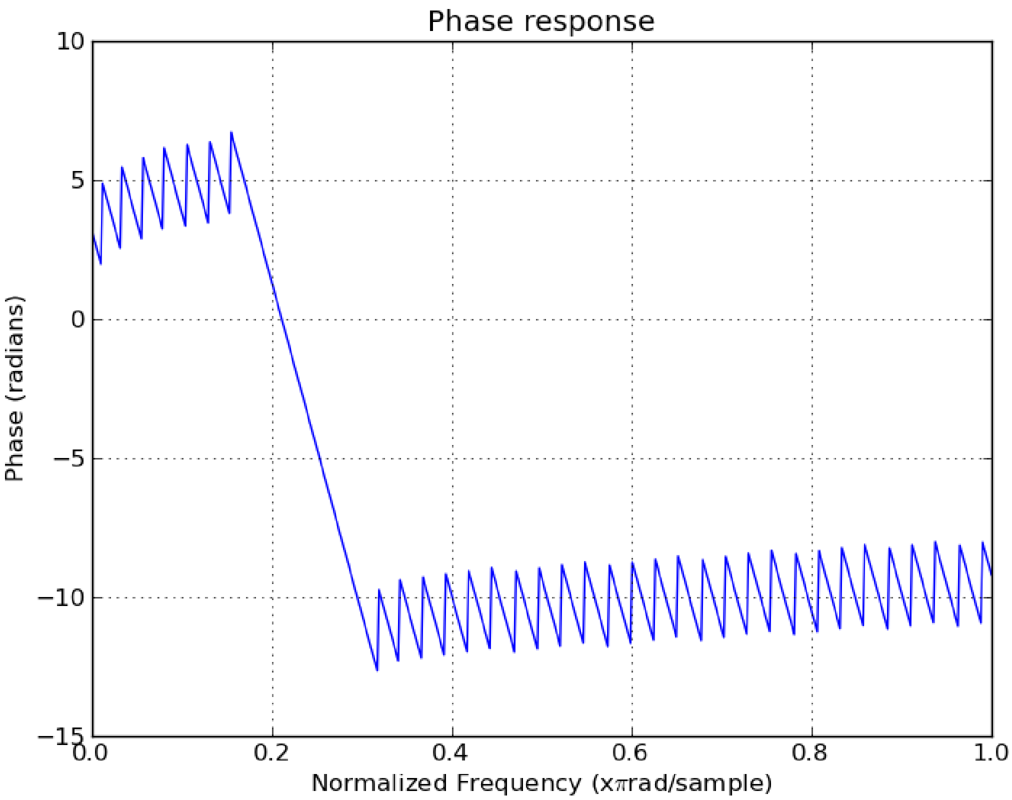


Figure: 6

The impulse response of the filter is also shown below.

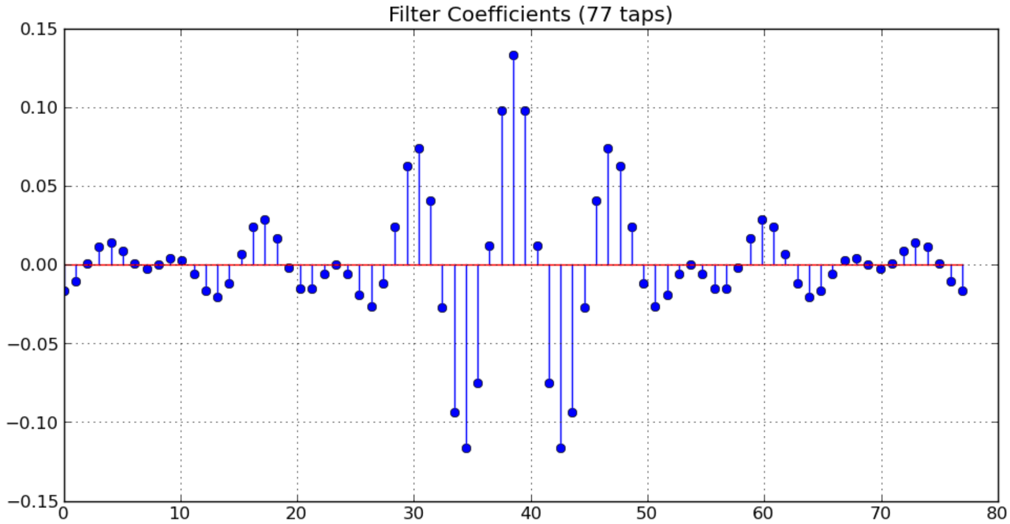


Figure: 7

Filter 2

1. Specifications

Filter type: Band stop filter.
Pass band nature: monotonic
Stop band nature: monotonic
Passband tolerance, del1 = 0.1 (in magnitude)
Stopband tolerance, del2 = 0.1 (in magnitude)
Transition band = 1 KHz on either side.
Sampling frequency = 70 KHz
Filter number = 102
Stop band edge 1, $B_L(m)$ = 6.8 KHz
Stop band edge 2, $B_H(m)$ = 9.8 KHz

2. Normalized Filter Specifications

This is calculated by dividing the given specifications by the sampling frequency and multiplying with 2π.
Normalized pass band edges (w_{Np1} and w_{Np2}) are 0.52060676 and 0.96940577 radians.
Normalized stop band edges (w_{Ns1} and w_{Ns2}) are 0.61036658 and 0.87964594 radians.

3. IIR Filter Design

3.1 Analog Filter Specifications:

Here we make use of the transformation function Ω = tan (ω/2) to get the analog filter specifications corresponding to the above digital filter. So after the transformation,
pass band edges are at 0.26634642 and 0.5266031
stop band edges are at 0.3150247 and 0.47056428

Because of the requirement for monotonic reponse in pass band and stop band, we need to use the Butterworth filter. While designing low pass filter pass band edge is taken as Ω_{Lp} = 1 and stop band edge is taken as the more stringent of the two obtained from the below formula.

Ω_{Ls} = minimum of { B* Ω_{S1} /($-\Omega_{S1}^2+\Omega_0^2$), B* Ω_{S2} /($-\Omega_{S2}^2+\Omega_0^2$)} = 1.6713

where, $\Omega_0^2 = \Omega_{P1} * \Omega_{P2}$ = 0.140259,

B $= \Omega_{P2} - \Omega_{P1}$ = 0.260257

The transform used for transformation of analog low pass filter to analog band pass filter is

$S_L = BS/(S^2+\Omega_0^2)$

Corresponding Ω transform is

$\Omega_L = B\Omega/(\Omega_0^2 - \Omega^2)$ (by putting S = j Ω)

The values for D1 and D2 are obtained as

D1 = 0.2346

D2 = 99.0

From this, the order is obtained using the formula

N = ceil ((log(sqrt(D2/D1)))/(log(Ω_{Ls} / Ω_{Lp}))

The value for N is obtained as, N = 8.

3.2 Analog Filter Transfer Functions and Pole-Zero Plots:

The analog low pass filter transfer function is obtained as,

$$H_{analog,\,LPF}(S_L) = \frac{2.36304069157}{s_L^8+5.708\, s_L^7 + 16.29 s_L^6 + 30.16 s_L^5 + 39.49 s_L^4 + 37.39 s_L^3 + 25.04 s_L^2 + 10.88 s_L + 2.363}$$

The pole-zero plot of the equivalent low pass filter is as shown below.

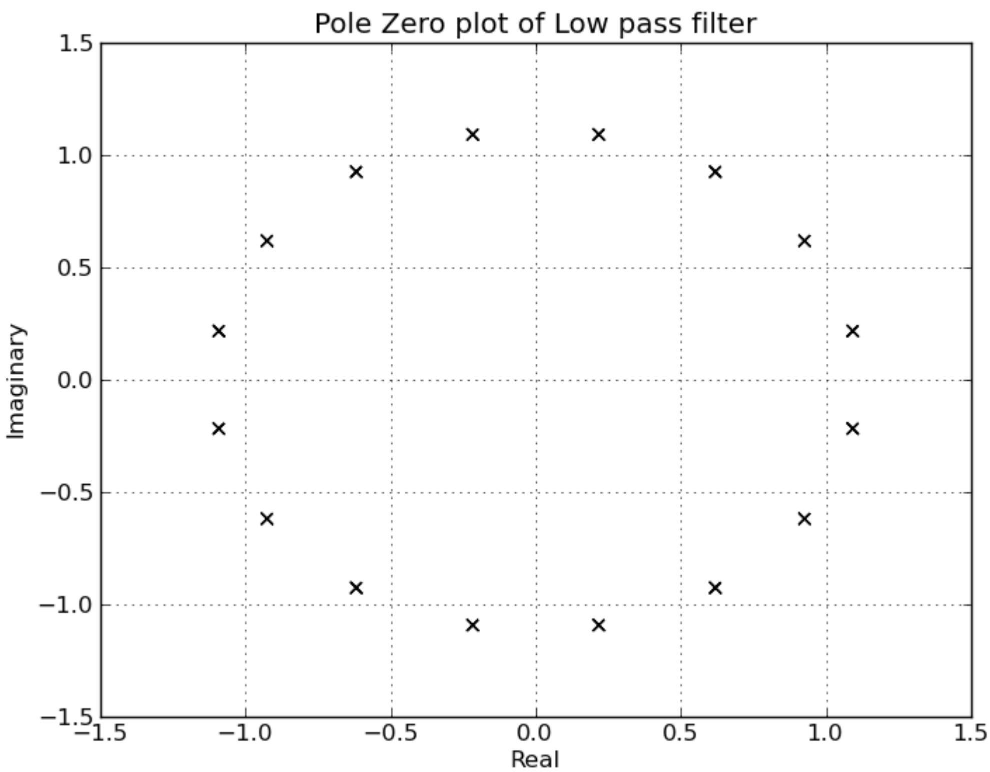


Figure: 8

The analog band stop filter transfer function is obtained as

$H_{analog,\ BSF}(S)$ =

2.363 s^16 +2.651 s^14 +1.302 s^12 +0.3651 s^10 +0.06402 s^8 +0.007183 s^6 +0.0005037 s^4 +2.019e-05 s^2 + 3.539e-07

(2.363 s^16+2.831 s^15+4.347 s^14 + 3.439 s^13 + 2.91 s^12 + 1.668 s^11 + 0.9723 s^10 + 0.4187 s^9 + 0.1804 s^8 + 0.05873 s^7 + 0.01913 s^6 + 0.004602 s^5 + 0.001126 s^4 + 0.0001867 s^3 + 3.31e-05 s^2 + 3.023e-06 s + 3.539e-07)

The pole-zero plot of the analog band stop filter is as shown below.

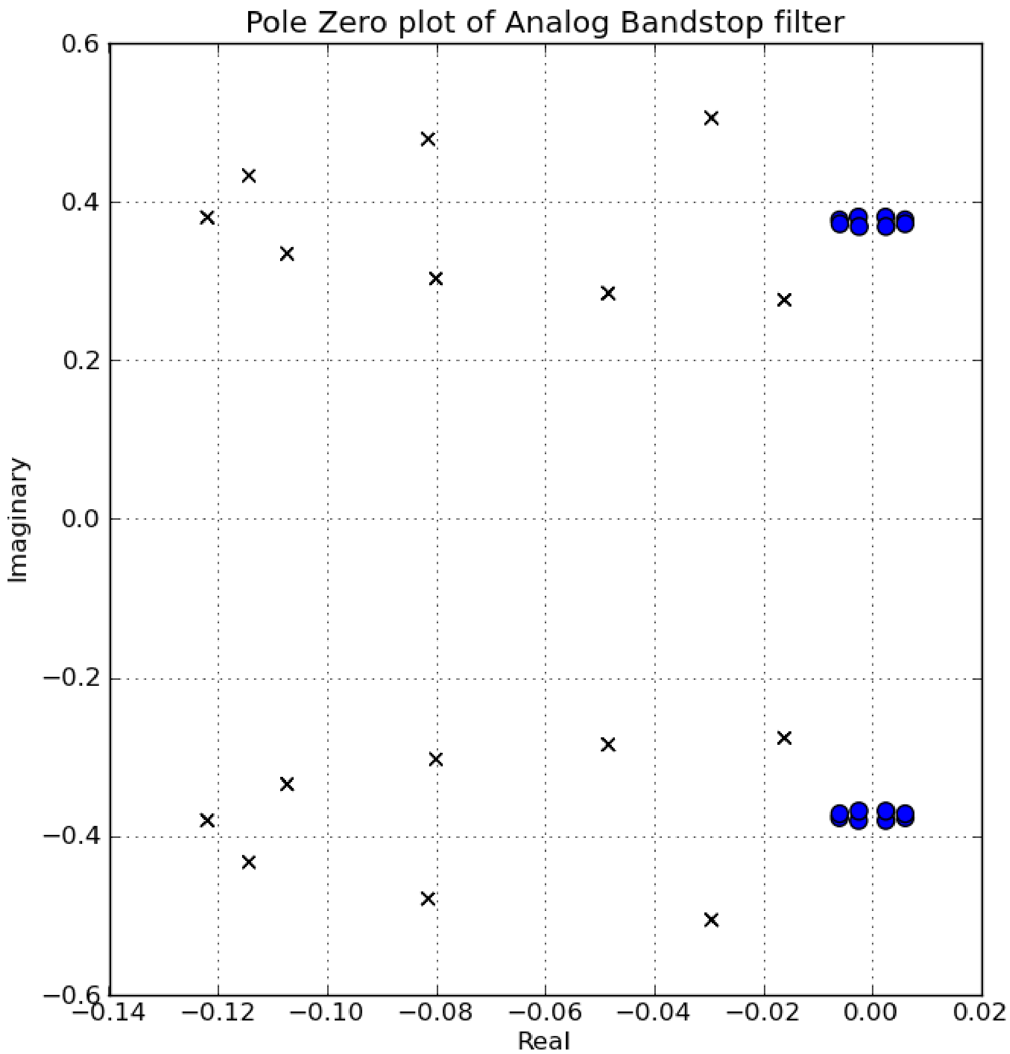


Figure: 9

3.3 Digital Filter:

The transformation used to transform the analog filter to Discrete Band stop filter is $S = (1-Z^{-1})/(1+Z^{-1})$. The Discrete filter transform function is

H (Z) =

[3.51474696e-01 -4.24012161e+00z^-1 + 2.51907944e+01z^-2 -9.71748050e+01z^-3 + 2.71339301e+02 z^-4 - 5.79992887e+02z^-5 + 9.79986107e+02 z^-6 - 1.33364452e+03z^-7

+ 1.47637053e+03 z^-8 - 1.33364452e+03 z^-9 + 9.79986107e+02z^-10 - 5.79992887e+02z^-11
+ 2.7133930e+02z^-12   -9.71748050e+01z^-13 + 2.51907944e+01z^-14 -4.24012161e+00z^-15
+3.51474696e-01z^-16]

[1.00000000e+00 -1.05021168e+01z^-1   +5.43016449e+01z^-2 -1.82415221e+02z^-3
+4.44024096e+02z^-4 -8.28438149e+02 z^-5   +1.22354053e+03 z^-6   -1.45763585e+03 z^-7
+1.41474176e+03 z^-8   -1.12216421e+03 z^-9   +7.25138641e+02 z^-10   -3.77953489e+02 z^-11
+1.55934659e+02 z^-12   -4.93124268e+01 z^-13   +1.13010130e+01 z^-14   -1.68321756e+00 z^-15
+1.23534469e-01 z^-16]

The frequency response of the digital filter is as shown in the figure below.

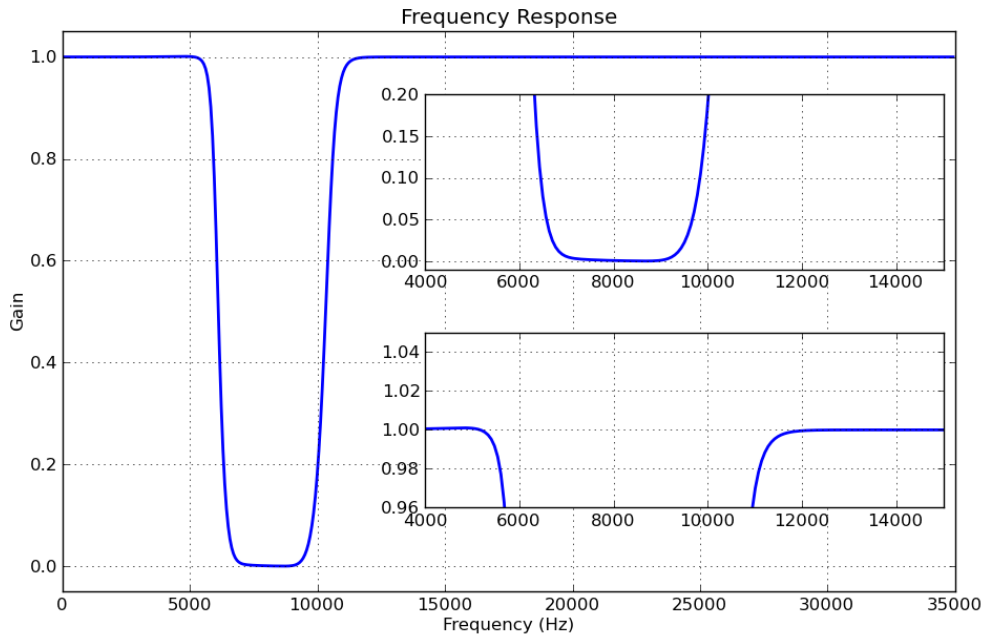


Figure: 10

The pole-zero plot of the digital filter is as shown below.

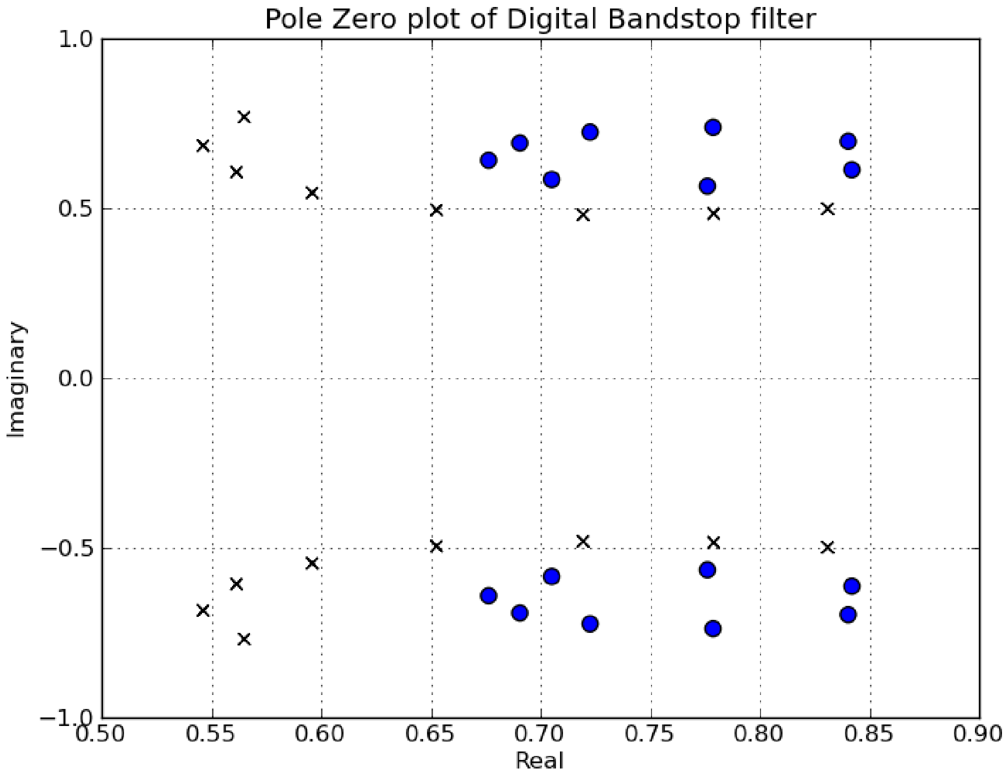


Figure: 11

The lattice coefficients of the above bandstop filter is as given below

Lattice Coefficients:Kn

[-0.8391467 0.99350699 -0.77475231 0.97142235 -0.75398259 0.96393553
-0.77147338 0.94865473 -0.79211542 0.91526198 -0.80625706 0.85446582
-0.75878201 0.67108138 -0.39182368 0.12353447]

Lattice Coefficients:Cn

[-5.43535423e-07 -1.37543584e-06 -1.94805615e-05 -1.07484305e-05
-3.52191890e-05 -3.25312554e-05 -3.30919281e-04 2.14131566e-03
1.40468356e-03 3.20313254e-03 4.47523364e-02 3.89253954e-02
-3.53392528e-02 1.70796460e-01 3.67167141e-01 -5.48893310e-01
3.51474696e-01]

4. FIR Filter Design

The order of the filter required to satisfy the specifications mentioned in section 1, is calculated with the help of the equation,

$(2*N + 1) > 1 + ((A-8)/2.285*\Delta w_T)$,

where, $\Delta w_T = w_s - w_p$, $A = -20*\log_{10}(\xi)$, ξ is the tolerance, w_s and w_p are the normalized frequencies. Here the more stringent condition is taken with respect to the value of Δw_T .The ideal impulse response of the band pass filter is generated using the equation

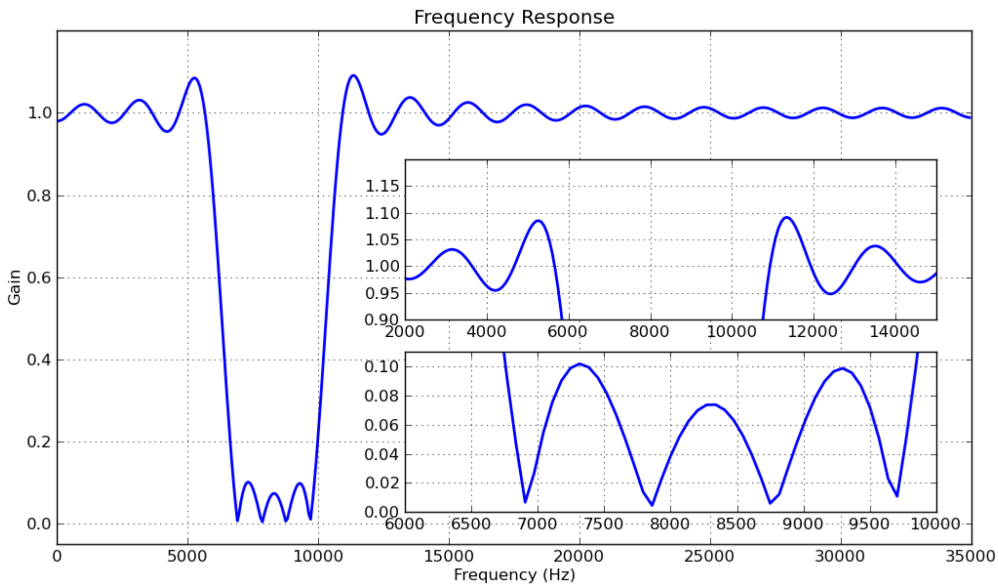
h_fir (n) = (sin(n* w_{c1})-sin(n*w_{c2}))*(1/(n*pi)), n ≠ 0
= (w_{c1}-w_{c2})/pi, n = 0

n ranges from –N to N, w_{c2} and w_{c1} are the cut-off frequencies calculated from the normalised frequencies as $w_{c1}= 0.5*(w_{Ns1}+w_{Np1})$ and $w_{c2}= 0.5*(w_{Ns2}+w_{Np2})$.

This ideal impulse response is then multiplied with the Kaiser window (in this case it turned out to be a rectangular window) to obtain the FIR impulse response. The transfer function of the FIR filter obtained, is

H(Z)=
[-1.55330474e-02 -1.79090850e-02z^-1 -9.20689178e-03 z^-2 7.02046690e-03 z^-3
2.12201024e-02 z^-4 2.42038900e-02 z^-5 1.38003106e-02 z^-6 -3.30616351e-03 z^-7
-1.62348489e-02 z^-8 -1.77836892e-02 z^-9 -9.54423699e-03 z^-10 -1.60014632e-04 z^-11
2.10724204e-03 z^-12 -3.34047500e-03 z^-13 -8.45058850e-03 z^-14 -3.28805349e-03 z^-15
1.43217153e-02 z^-16 3.42033853e-02 z^-17 3.91755423e-02 z^-18 1.78017974e-02 z^-19
-2.43934243e-02 z^-20 -6.44687685e-02 z^-21 -7.47759818e-02 z^-22 -4.16690393e-02 z^-23
2.24228714e-02 z^-24 8.30783287e-02 z^-25 1.03350443e-01 z^-26 6.70944800e-02 z^-27
-9.02552778e-03 z^-28 -8.35590602e-02 z^-29 8.85714277e-01 z^-30 -8.35590602e-02 z^-31
-9.02552778e-03 z^-32 6.70944800e-02 z^-33 1.03350443e-01 z^-34 8.30783287e-02 z^-35
2.24228714e-02 z^-36 -4.16690393e-02 z^-37 -7.47759818e-02 z^-38 -6.44687685e-02 z^-39
-2.43934243e-02 z^-40 1.78017974e-02 z^-41 3.91755423e-02 z^-42 3.42033853e-02 z^-43
1.43217153e-02 z^-44 -3.28805349e-03 z^-45 -8.45058850e-03 z^-46 -3.34047500e-03 z^-47
2.10724204e-03 z^-48 -1.60014632e-04 z^-49 -9.54423699e-03 z^-50 -1.77836892e-02 z^-51
-1.62348489e-02 z^-52 -3.30616351e-03 z^-53 1.38003106e-02 z^-54 2.42038900e-02 z^-55
2.12201024e-02 z^-56 7.02046690e-03 z^-57 -9.20689178e-03 z^-58 -1.79090850e-02 z^-59
-1.55330474e-02 z^-60]

The magnitude and phase response of the FIR filter is as shown below.



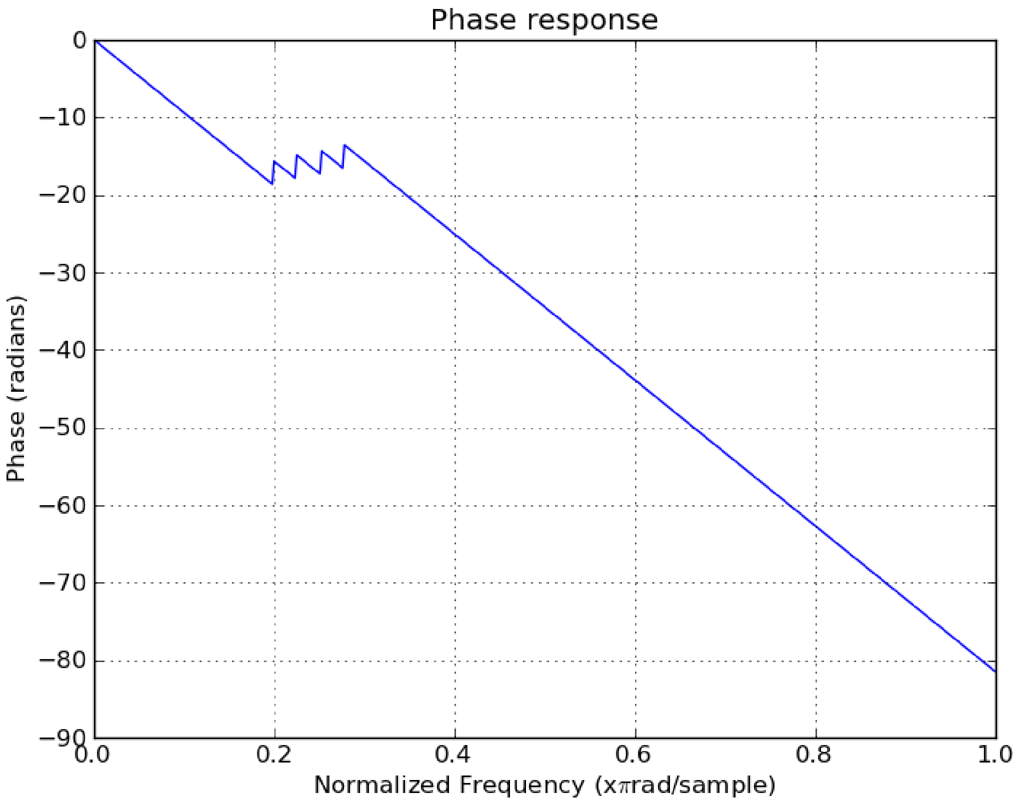


Figure: 12

The impulse response of the filter is also shown below.

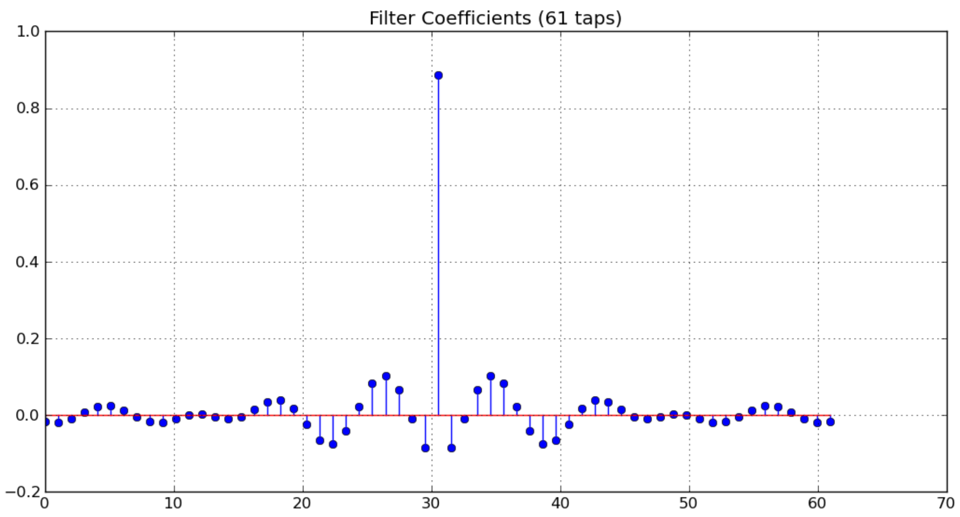


Figure: 13