# SIGNAL PROCESSING

FT

1. CTFS periodic continuous
2. CTFT a " "
3. DTFS Periodic discrete
4. DTFT a " " → O/P continuous
5. DFT / IDFT aperiodic " → op discrete

Laplace Transform

Z - transform

Application →

1. Filter
2. Noise cancellation

Theory Application

. LTI ⇒ assumption
→ Convolution

exponential
→ Fair assumption
because real
world signals behave
this way.
Also, good ~~~~ because
it is an eigen function.

Focus of the course →

→ Filter

→ DFT ⇒ FFT.

several hands on activity

Maybe Noise cancellation → adaptive filter

Main Reference:
Signal Processing - Orfanidis

→ Appreciate convolution better, when seeing real world examples.

→ How did we get frequency domain.

→ $e^{-at} u(t) \xrightarrow{F} \frac{1}{a+jw}$

$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-jwt}\, dt$$

$$= \int_{0}^{\infty} e^{-t(a+jw)}\, dt$$

$$= 0 - \left(\frac{-1}{a+jw}\right) = \frac{1}{a+jw}. \quad //$$

→ Note : If time period increases in the time domain, the frequency coefficients in frequency domain get closer.

→ $C_k = \frac{1}{N_0} \sum_{N_0} x[n] \, e^{-jk\Omega_0 n}$

$C_0 = \frac{1}{4} \sum_{n=0}^{3} x[n] = \frac{1}{4}(0+1+2+3) = \frac{3}{2}$

1. Compute 4pt DFT of $[0\ 1\ 2\ 3]^T$ after expressing in matrix form

$$\cancel{\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}}$$

$\left(e^{-j\frac{2\pi}{N}}\right)$

2. $\begin{bmatrix} j \\ \\ \\ \end{bmatrix}$

64

36

16 + 8 ⇒ 24

1024 x 1024

→ Complexity of IDFT

→ How is Butterfly structure going to be different.

IDFT →

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\left(\frac{2\pi}{N}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N] \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[n] \end{bmatrix}$$

$$\begin{bmatrix} & & \\ & & \\ & \downarrow & \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X \vdots \\ X[N] \end{bmatrix} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N] \end{bmatrix}$$

$$\begin{bmatrix} \cancel{e^{j\frac{2\pi}{N}0}}\ \cancel{e}\ W_N^{0\cdot0} & W_N^{0\cdot1} & \cdots & W_N^{0\cdot N} \\ W_N^{1\cdot0} & & & \\ \vdots & & & \\ W_N^{N\cdot0} & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8 & W_8^2 & W_8^3 & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \end{bmatrix}$$

→ Textbook → Chapter 9.5,6 read, spot what they are trying to do differ in IFFT & justify it.

$$\begin{bmatrix} (1-j)/\sqrt{2} \\ -j \\ -(1+j)/\sqrt{2} \end{bmatrix}$$



→ [5 0 -3 4]

$$\begin{bmatrix} 5 \\ 0 \\ -3 \\ 4 \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ -3 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 8+4j \\ -2 \\ 8-4j \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \end{bmatrix} \begin{bmatrix} 1 \\ -j \end{bmatrix} \begin{bmatrix} 4 \\ 4j \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

$e^{-j2\pi/?}$

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} \begin{bmatrix} 1 \\ -j \end{bmatrix} \begin{bmatrix} 4 \\ 2j \end{bmatrix}$$

→

$$\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ -3 \\ 0 \\ 4 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ 0 \\ -3 \\ 4 \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ -3 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

→ Try out dft, fft in octave.
See the time difference.

→ Derive the 8 point FFT.

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} x[0] \\ \vdots \\ x[8] \end{bmatrix} = \begin{bmatrix} X[0] \\ \vdots \\ X[8] \end{bmatrix}$$

$$\Downarrow$$

$$\begin{array}{c|c}
 & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\
\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} &
\begin{bmatrix}
W_8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & W_8 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\
1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\
1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\
1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{56}
\end{bmatrix}
\end{array}$$

$$\Downarrow$$

$$W_8 = e^{-j\frac{2\pi}{8}} \times 4$$

$$\Rightarrow W_8^2 = e^{-j\frac{2\pi}{4}} = W_4$$

$$W_8^8 = 1$$

$$W_8^4 = W_4^2 = e^{-j\pi} = 0\ 1$$

$$W_8^5$$

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & W_8 & W_4 & W_8^3 & W_4^2 & W_8^5 & W_4^3 & W_8^7 \\
1 & W_4 & W_4^2 & W_4^3 & 1 & W_4 & W_4^2 & 1 \\
1 & W_8^3 & W_4^3 & W_8 & W_4^2 & W_4 & W_4 & W_8^5 \\
1 & W_4^2 & 1 & W_4^2 & 1 & & & \\
4 & & & & & & & \\
1 & & & & & & & \\
j & & & & & & &
\end{bmatrix}$$

| Signal | Fs | Window of analysis | #Sample |
|---|---|---|---|
| Audio signal | 44.kHz | 0.1 s | 4400 |
| Speech | 8 kHz | 0.1 s | 800 |

$$\rightarrow X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}nk}.$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, e^{-j\frac{2\pi}{N}2n.k} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, e^{-j\frac{2\pi}{N}(2n+1)k}.$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, e^{-j\frac{2\pi}{N}2n.k} + e^{-j\frac{2\pi}{N}k} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, e^{-j\frac{2\pi}{N}.k.2n}$$

$$\underset{k=0 \text{ to } N-1}{X[k]} = \sum_{n=0}^{\frac{N}{2}-1} x(2n)\left(W_{\frac{N}{2}}\right)^{n.k} + e^{-j\frac{2\pi}{N}k} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\left(W_{\frac{N}{2}}\right)^{nk}.$$

$$\underset{k=0 \text{ to } \frac{N}{2}-1}{X[k]} = \sum_{n=0}^{\frac{N}{2}-1} x(2n)\left(W_{\frac{N}{2}}\right)^{n.k} + e^{-j\frac{2\pi}{N}k.} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\left(W_{\frac{N}{2}}\right)^{n.k}$$

$$\underset{\frac{N}{2}-1 \text{ to } N-1}{x[k]} = \sum_{n=0}^{\frac{N}{2}-1} x(2n)\left(W_{\frac{N}{2}}\right)^{n.(k+\frac{n}{2})}$$

# Quiz-1 preparation:

## DTFT computation:

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

→ DTFT can be thought of as evaluation of Z-transform of $x(n)$ on unit circle.

DFT ⇒

$$X(\omega_k) = \sum_{n=0}^{L-1} x(n) e^{-j\omega_k n}$$

→ DFT & zero padding.

No change in DFT when zeros are padded at the end.

In the beginning, it will correspond to a delay, ⇒ $X_D(\omega_k) = e^{-j\omega_k P} X(\omega_k)$

→ in DFT, we get the peaks corresponding to only

$$f_k = \frac{k}{N} f_s.$$

So, we must note that if $f_s$ isn't good enough ⇒ aliasing, biasing

if N not good enough ⇒ miss the peaks

CTFS →

~~$X(j\Omega)=$~~ $\underline{C_k} = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\Omega_0 t}\, dt$

$x(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{jk\Omega_0 t}.$

CTFT →

$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt.$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$

DTFS

$C_k = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x(t)\, e^{-jk\Omega_0 n}$

$x(t) = \sum_{k=0}^{N_0-1} C_k\, e^{jk\Omega_0 n}.$

DTFT

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$x[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega.$

$$DFT(x(n)) = \sum_{n=0}^{N_0-1} x(n) W_N^{k.n}$$

$$X(k) = \sum_{n=0}^{N_0-1} W_N^{k(2n)} x(2n) + \sum_{n=0}^{N_0-1} W_N^{k(2n+1)} x(2n+1)$$

change limits

$$= \sum_{n=0}^{\frac{N_0}{2}-1} W_N^{k(2n)} x(2n) + \sum_{n=0}^{\frac{N_0}{2}-1} W_N^{k(2n+1)} x(2n+1)$$

$$g(n) = x(2n)$$

$$h(n) = x(2n+1)$$

$$G(k) = \sum_{n=0}^{\frac{N_0}{2}-1} g(n) \cdot W_{\frac{N}{2}}^{kn}$$

$$H(k) = \sum_{n=0}^{\frac{N_0}{2}-1} W_{N/2}^{kn} h(n)$$

$$W_N^2 \Rightarrow \left(e^{-j\frac{2\pi}{N}}\right)^2 = e^{-j\frac{2\pi}{\frac{N}{2}}} = W_{\frac{N}{2}}$$

$\Rightarrow$

$$G(k) + W_N^k H(k)$$

$$X(k) = G(k) + W_N^k H(k)$$

$$X(k+N/2) = G(k+N/2) - W_N^k H(k+N/2)$$

$$\frac{1}{N} AA^* = I$$

$$\Rightarrow A^{-1} = \frac{1}{N} A^*$$

$$A^* x = (Ax^*)^*$$

$$\Rightarrow IFFT(x) = \frac{1}{N}(FFT(x^*))^*$$

$$\rightarrow \quad X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$H(z) = \frac{5+2z^{-1}}{1-0.8z^{-1}}$$

$$= -2.5 + \frac{7.5}{1-0.8z^{-1}}$$ ~~$-0.8t$~~

~~$-2.5\delta(t) + 7.5u(t)$~~

$$\Rightarrow h(n) = -2.5\delta(n) + 7.5(0.8)^n u(n)$$

~~$y(n) = 7.5x(n) - 2$~~

$$y(n) = -2.5x[n] + 7.5x[n] +$$

$\frac{7.5}{1-0.8z^{-1}}$ ) is represented by that $w[n]$ loop.

$H$

$$= \frac{W(z)}{X(z)}$$

→ Problem: $f_s = 20k$ $f_{pass} = 5kHz$

$f_{stop} = 6kHz$

$A_{pass} = 0.1 dB$, $A_{stop} = 60 dB$

$$\delta_{pass} = \frac{10^{0.1/20}-1}{10^{0.1/20}+1} = 0.0058$$

$$\delta_{stop} = 10^{-60/20} = 10^{-3} = 0.001$$

$$\delta = 0.001. \Rightarrow A = -20\log_{10}\delta$$

FIR filter → stable because it doesn't have poles

$\alpha = 0.1102(60-8.7) = 5.6532$

$D = \frac{60-7.95}{14.36} = 3.62$

$\Delta F = 1kHz$, $N = \frac{3.62 \times 20\cancel{k}}{1\cancel{k}}$ 14

$= 73.4$

$\Rightarrow N = 75$. $M = \frac{1}{2} 74 = 37$

$h(n) = w(n)\, d(n-M)$

$= \frac{I_0\left(7.857\sqrt{n(75-n)}/51\right)}{I_0(7.857)} \cdot \frac{\sin(0.55\pi(n-37))}{\pi(n-37)}$

$\omega_c = \frac{2\pi f_c}{f_s}$ $f_c = 5.5kHz$

$= \cancel{\frac{2\pi \cdot 5.5k}{20k}}$ ~~7.27~~ $\pi$

$\frac{2\pi \times 5.5\cancel{k}}{20\cancel{k}} = 0.55\pi$

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