Assignment 3

Design: A Small Numerical Library

Linhao Chen
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1 Introduction

This program can print the chart(s) for sin, cos, tan, exponent, and all of them by following the command-line option (-s, -c, -t, -e, or -a). The program can only choose one option at one time. The chart will compute the sin, cos, tan by using Padé approximants and use Taylor series approximations to calculate the exponent. What the program also needs to do is printing the value by using Math library in C and compare their differences. Expect the program may use Function, Boolean, Getopt, Switch, Loop, etc.

2 Calculation

sin(x)

By using Taylor series, we can calculate sin(x) by the formula below:

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} + \frac{x^{13}}{6227020800} - \frac{x^{15}}{1307674368000} + O(x^{16})$$

By using Padé Approximant, in the program we can calculate sin(x) by doing this:

Numerator =
x*(133684890792694976170345029120000.0+x*x*(-20494411297105391300731453977600.0+
x*x*(828214724158018950289951814400.0+x*x*(-13571254880131693253451022080.0+
x*x*(104761946712028505958458400.0+x*x*(-385435322571107751366060.0+
x*x*55584614664995265467 9.0))))))

Denominator =
133684890792694976170345029120000.0+x*x*(17864038350104380609927175424 00.0+
x*x*(1190794005396715903586282880 0.0+x*x*(5148308962802722249653120 0.0+
x*x*(1551121263885194981179 20.0+x*x*(315769438704063484140.0+
x*x*348811434161499491.0)))))

sin(x) = Numerator / Denominator

cos(x)

By using Taylor series, we can calculate cos(x) by the formula below:

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} + \frac{x^{12}}{479001600} - \frac{x^{14}}{87178291200} + O(x^{16}).$$

By using Padé Approximant, in the program we can calculate cos(x) by doing this:

```
Numerator =
567804334259460858741817413313536000.0+
x*x*(27669479708166778446757678005120000000.0+
x*x*(2010090281040533208617049911689728000.0+
x*x*(-51115232112849702194564011965120000.0+
x*x*(589521544439298390578817718560000.0+
x*x*(-33893899028536219679025124560000.0+
x*x*(95704952302489373336525592000.0-
10737096533191631696695750.0*x*x))))))

Denominator =
567804334259460858741817413313536000.0 +
x*x*(72073700480626449033319266055680000.0+
x*x*(460739069591187569274035316172800.0+
x*x*(1946779310366782428666671942400.0+
x*x*(5956981238166831424568016000.0+
x*x*(13425143673825939828558240.0+
x*x*(21053833464601381951920.0+
18164636218281257111.0*x*x))))))

cos(x) = Numerator / Denominator
```

tan(x)

As we all know from middle school, tan(x) equals to sin(x) / cos(x). So, we can use same theory in the program.

e^x

By using Maclaurin series, we can calculate e^x by the formula below:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \ \ldots\ + \frac{x^n}{n!}$$

So, we need to consider what is a good n for our program. If the last term is smaller than 0.01, I think it could be good enough. Just think about domain is change from 0 to 10. So, we should use 10 to test. $10^n / (n!) < 0.01$, this n should be 30. So, we can use n = 30 to calculate the value of e^x. For each e^x, we should use for loop to do calculate each term and add them together.

3 Program

Main {
  Read the command-line option
  if more than one options, or the command is unknown: Print Error and program ends
  Switch to each case depend on the command-line option
  In case ‘s’:
    Print the heading information (x, sin, library, difference)
    For (x equals -2π ; after each loop, x adds $\pi/16$ until x bigger than 2 π) {
        Print the value of x;
        Print the value of sin by calling the function (Use parameter x)
        Print the value of sin by using the library
        Print the difference between two values
        Print the new line
    }
    After the loop, program ends

  In case ‘c’:
    Print the heading information (x, cos, library, difference)
    For (x equals -2π ; after each loop, x adds $\pi/16$ until x bigger than 2 π) {
        Print the value of x;
        Print the value of cos by calling the function (Use parameter x)
        Print the value of cos by using the library
        Print the difference between two values
        Print the new line
    }
    After the loop, program ends

  In case ‘t’:
    Print the heading information (x, tan, library, difference)
    For (x equals -($\frac{\pi}{2} - 0.001$) ; after each loop, x adds $\pi/16$ until x bigger than ($\frac{\pi}{2} -$ 0.001)) {
        Print the value of x;
        Print the value of tan by calling the function (Use parameter x)
        Print the value of tan by using the library
        Print the difference between two values
        Print the new line
    }
    After the loop, program ends

  In case ‘e’:
    Print the heading information (x, tan, library, difference)

For (x equals 0; after each loop, x adds 0.1 until x bigger than10) {
Print the value of x;
Print the value of e^x by calling the function (Use parameter x)
Print the value of e^x by using the library
Print the difference between two values
Print the new line
}
After the loop, program ends

In case 'a':
Print the heading information (x, sin, library, difference)
For (x equals -2π ; after each loop, x adds $\pi/16$ until x bigger than 2 π) {
Print the value of x;
Print the value of sin by calling the function (Use parameter x)
Print the value of sin by using the library
Print the difference between two values
Print the new line
}

Print the heading information (x, cos, library, difference)
For (x equals -2π ; after each loop, x adds $\pi/16$ until x bigger than 2 π) {
Print the value of x;
Print the value of cos by calling the function (Use parameter x)
Print the value of cos by using the library
Print the difference between two values
Print the new line
}

Print the heading information (x, tan, library, difference)
For (x equals -($\frac{\pi}{2} - 0.001$) ; after each loop, x adds $\pi/16$ until x bigger than ($\frac{\pi}{2} - 0.001$)) {
Print the value of x;
Print the value of tan by calling the function (Use parameter x)
Print the value of tan by using the library
Print the difference between two values
}

Print the heading information (x, tan, library, difference)
For (x equals 0; after each loop, x adds 0.1 until x bigger than10) {
Print the value of x;
Print the value of e^x by calling the function (Use parameter x)
Print the value of e^x by using the library
Print the difference between two values

```
        Print the new line
    }
    After the loop, program ends
}

Function Sin {
    Receive the parameter x from main
    Use the formula provided in Calculation part to get the value of sin(x)
    Return the value of sin(x) to main
}

Function Cos {
    Receive the parameter x from main
    Use the formula provided in Calculation part to get the value of cos(x)
    Return the value of cos(x) to main
}

Function Tan {
    Receive the parameter x from main
    Call Function Sin(x) and get the value of sin(x)
    Call Function Cos(x) and get the value of cos(x)
    Tan(x) will be equal to sin(x)/cos(x)
    Return the value of tan(x) to main
}

Function Exp {
    Receive the parameter x from main
    Use the formula provided in Calculation part to get the value of e^x
    Return the value of e^x to main
}
```