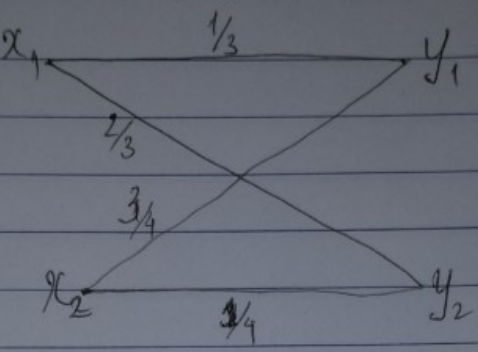
Thứ ngày

Bài 1:



$p(x_1) = 0,25$

$\Rightarrow p(x_2) = 0,75$

$\Rightarrow p(y_1) = p(x_1, y_1) + p(x_2, y_1)$

$= p(x_1) \cdot p(y_1|x_1) + p(x_2) \cdot p(y_1|x_2)$

$= 0,25 \cdot \frac{1}{3} + 0,75 \cdot \frac{3}{4} = \frac{31}{48}$

$\Rightarrow p(y_2) = p(x_1, y_2) + p(x_2, y_2) = \frac{17}{48}$

$\Rightarrow p(x_1, y_1) = p(x_1) \cdot p(y_1|x_1) = 0,25 \cdot \frac{1}{3} = \frac{1}{12}$

$p(x_2, y_1) = p(x_2) \cdot p(y_1|x_2) = 0,75 \cdot \frac{3}{4} = \frac{9}{16}$

$p(x_1, y_2) = p(x_1) \cdot p(y_2|x_1) = 0,25 \cdot \frac{2}{3} = \frac{1}{6}$

$p(x_2, y_2) = p(x_2) \cdot p(y_2|x_2) = 0,75 \cdot \frac{1}{4} = \frac{3}{16}$

$\Rightarrow p(x_1|y_1) = \frac{p(x_1, y_1)}{p(y_1)} = \frac{\frac{1}{12}}{\frac{31}{48}} = \frac{4}{31}$

$p(x_2|y_1) = \frac{p(x_2, y_1)}{p(y_1)} = \frac{\frac{9}{16}}{\frac{31}{48}} = \frac{27}{31}$

$p(x_1|y_2) = \frac{p(x_1, y_2)}{p(y_2)} = \frac{8}{17}$

$p(x_2|y_2) = \frac{p(x_2, y_2)}{p(y_2)} = \frac{9}{17}$

Thứ ngày

- $H(X|Y) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l) \cdot \log\left(p(x_k|y_l)\right)$

$= 0,711576\,3823 \approx 0,71157$ (✓)

- $p(y_1) = \frac{31}{48} \approx 0,6458333\ldots \approx 0,69583$ (✓)
- $H(X,Y) = I(X,Y)$

$= -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l) \cdot \log\left(p(x_k, y_l)\right)$

$= 1,679\,310\,676 \approx 1,69931$ (✓)

- $p(x_2) = 0,75000$
- $I(X;Y) = \sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l) \cdot \log\left(\frac{p(x_k, y_l)}{p(x_k) \cdot p(y_l)}\right)$

$= 0,099\,701\,742\,13 \approx 0,09970$ (✓)

- $H(Y|X) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l) \cdot \log\left(p(y_l|x_k)\right)$

$= 0,838\,032\,5519 \approx 0,83803$ (✓)

- $H(Y) = -\sum_{l=1}^{2} p(y_l) \cdot \log(p(y_l)) = 0,937\,734\,294 \approx 0,93773$ (✓)

- $H(X) = -\sum_{k=1}^{2} p(x_k) \cdot \log(p(x_k)) = 0,811\,278\,1245$

$\approx 0,81127$ (✓)