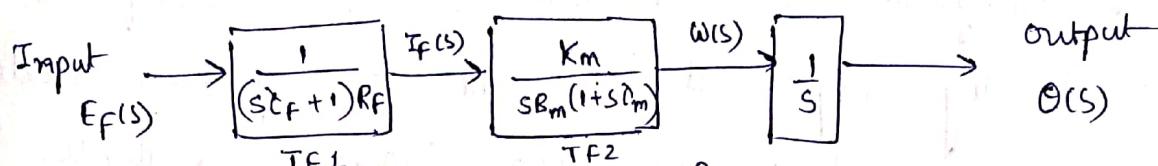


## 2A - Block Diagrams

①

A block diagram of a system is a pictorial representation of an actual system.

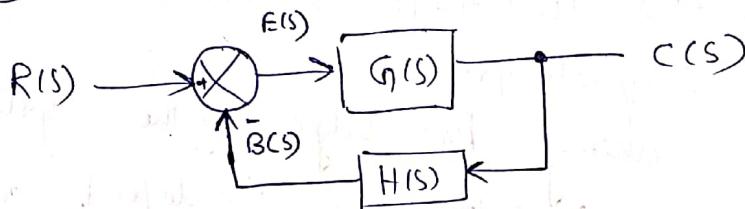
Ans: DC motor Block Diagram (field control)



What does it consists of?

- Open loop gains of various systems (Ans: Field System, mechanical system etc.)
- Signal flow (Ans:  $I_F(s)$ ,  $\theta(s)$  etc.)

⇒ Block diagram of a closed loop system



$$E(s) = R(s) - B(s) \quad \text{--- (1)}$$

$$G(s) = \frac{C(s)}{E(s)} ; \quad E(s) = \frac{C(s)}{G(s)} \quad \text{--- (2)}$$

$$H(s) = \frac{B(s)}{C(s)} ; \quad B(s) = C(s) \cdot H(s) \quad \text{--- (3)}$$

Sub (3), (2) in eq (1)

$$\therefore \frac{G(s)}{C(s)} = R(s) - C(s)H(s)$$

$$\frac{C(s)}{G(s)} + C(s)H(s) = R(s)$$

$$C(s) \left[ \frac{1}{G(s)} + H(s) \right] = R(s)$$

$$C(s) \left[ \frac{1 + G(s)H(s)}{G(s)} \right] = R(s) \Rightarrow$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

A block diagram contains information regarding dynamic behaviour, but it does not include any information on the physical construction of the system.

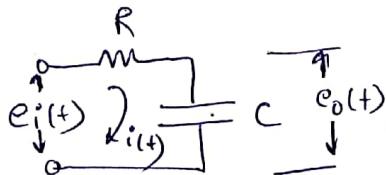
~~A~~ Block diagram of a system is not unique, it is totally dependent on the assumptions made. Hence a physical system can have multiple block diagrams based on user perspective.

### Procedure for Drawing Block diagram :

1. Write the DE equations that describe the dynamic behaviour of system.
2. Take the Laplace transforms, assuming zero initial conditions to obtain algebraic equations.
3. From the working knowledge of the system, input & output variables are identified and the block diagram for each equation can be drawn.

Example :

Consider an RC circuit, as shown below.



(a) Given circuit

Step 1 : DE  $i(t) = \frac{e_i(t) - e_o(t)}{R}$  — (1)

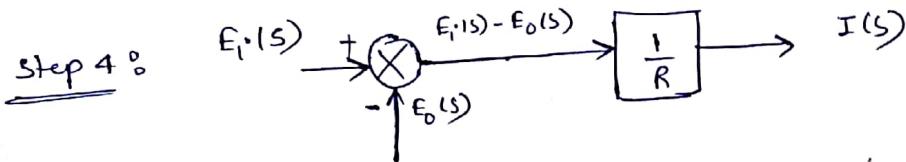
$$e_o(t) = \frac{1}{C} \int i(t) dt - (2)$$

Step 2 : Laplace transform

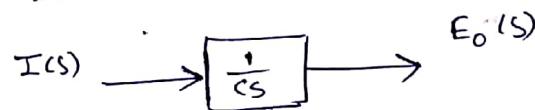
$$I(s) = \frac{E_i(s) - E_o(s)}{R} \quad \text{--- (3)}$$

$$E_o(s) = \frac{1}{Cs} I(s) \quad \text{--- (4)}$$

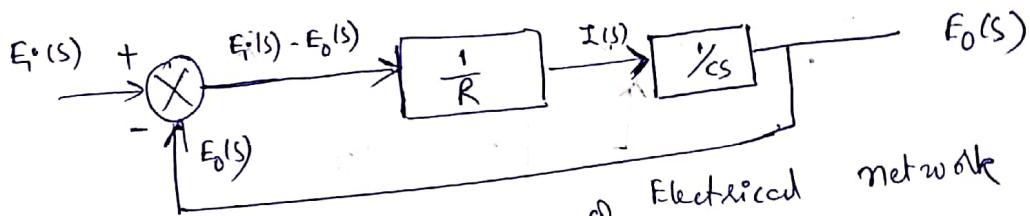
Step 3 : Input Variable is  $E_i(s)$   
Output Variable is  $E_o(s)$



(b) Block diagram for  $I(s)$



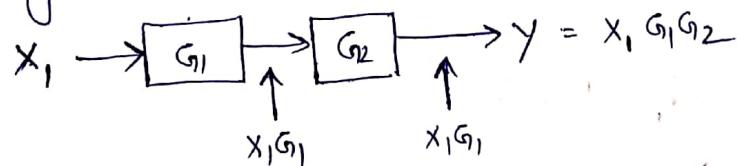
(b) Block diagram for  $E_o(s)$



(c) Block diagram of Electrical network

Block diagram Reduction :-

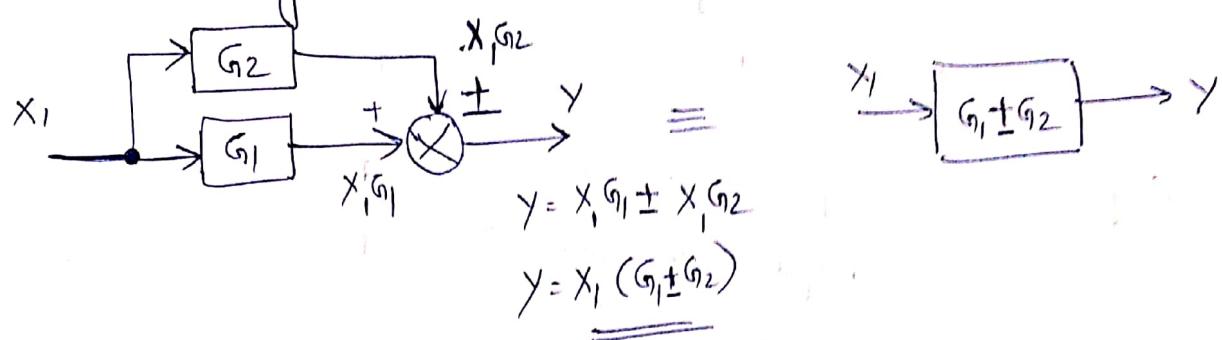
1. Combining blocks connected in cascade.



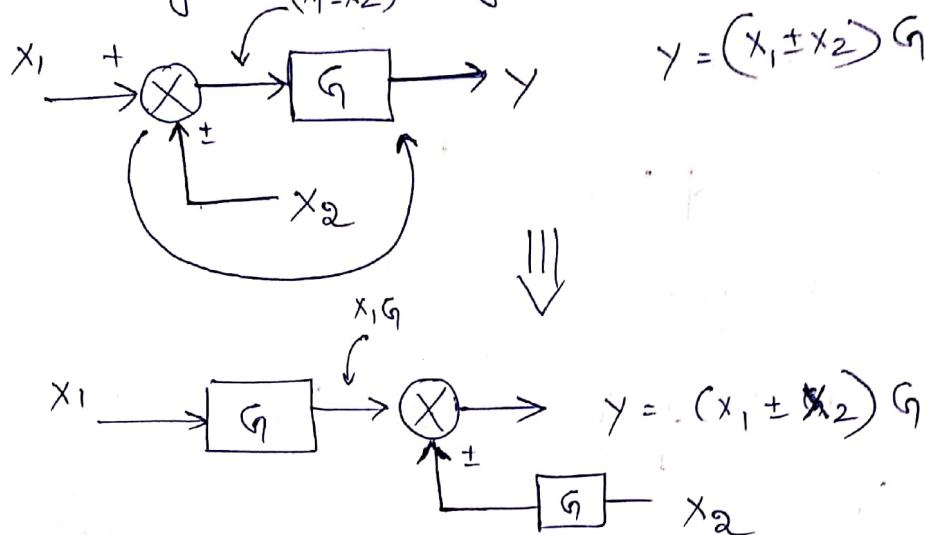
||

$$x_1 \rightarrow [G_1 G_2] \rightarrow y = x_1 G_1 G_2$$

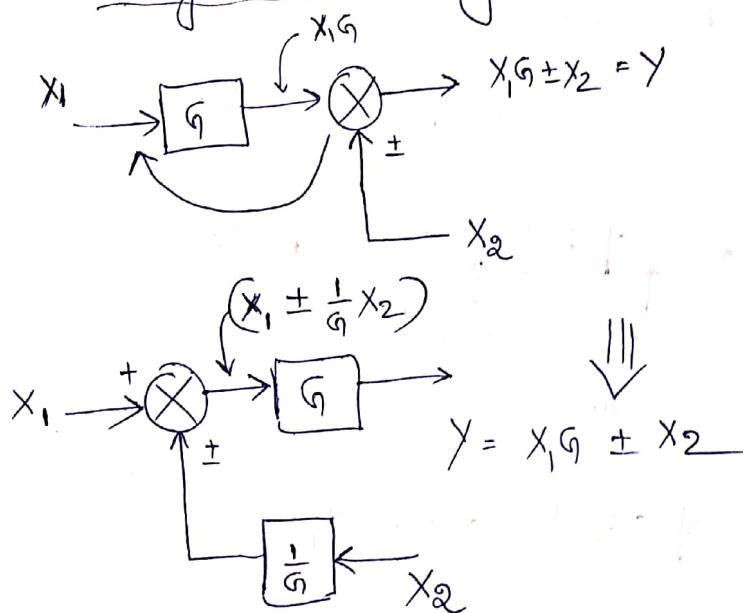
2. Combining blocks connected in parallel -



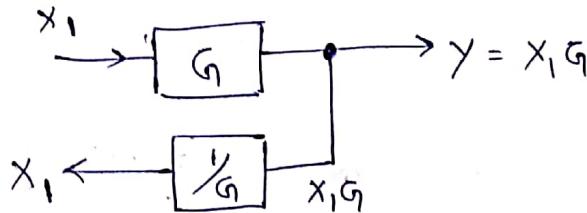
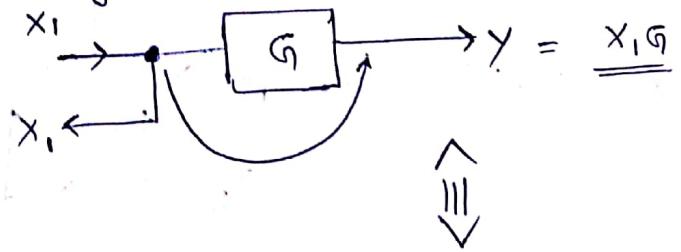
3. Moving a summing point after a block



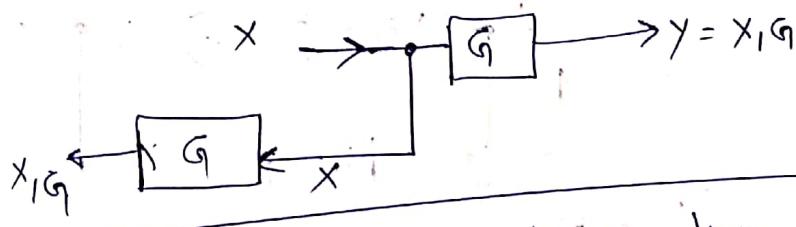
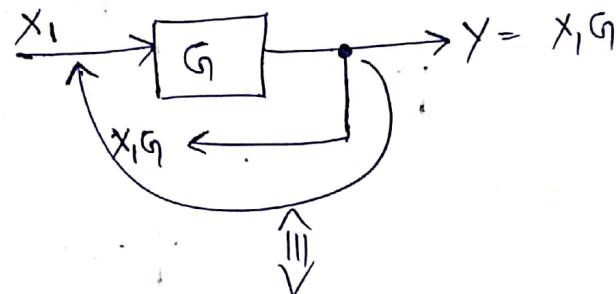
4. Moving a summing point ahead of a block



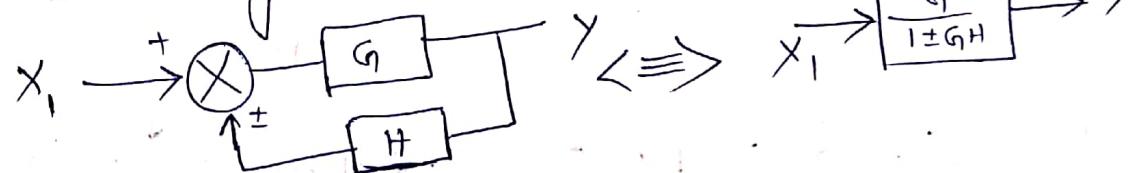
5. Moving a take off point after a block



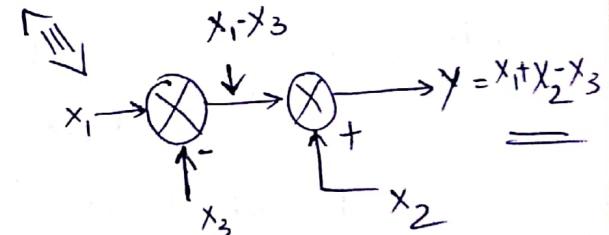
6. Moving a take off point ahead of a block



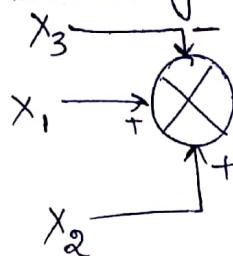
7. Eliminating feedback loop



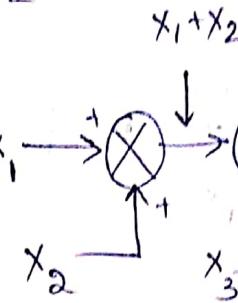
8. Interchanging summing points:



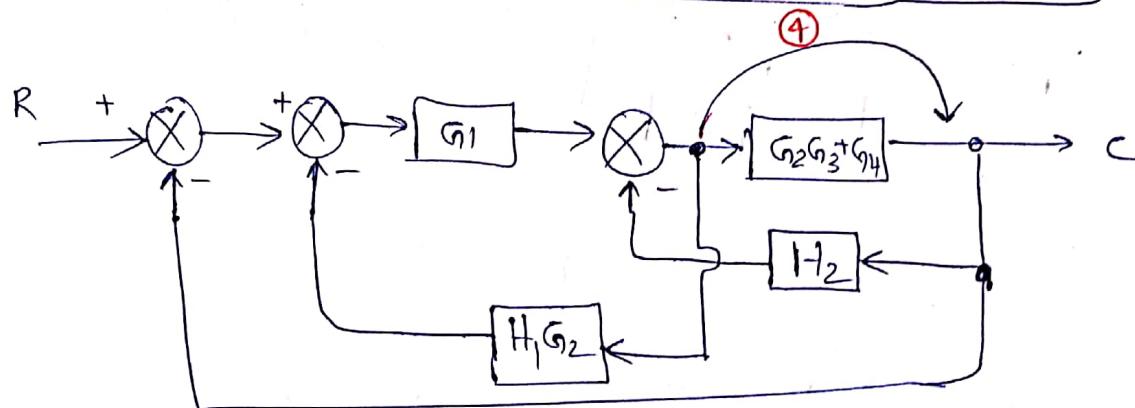
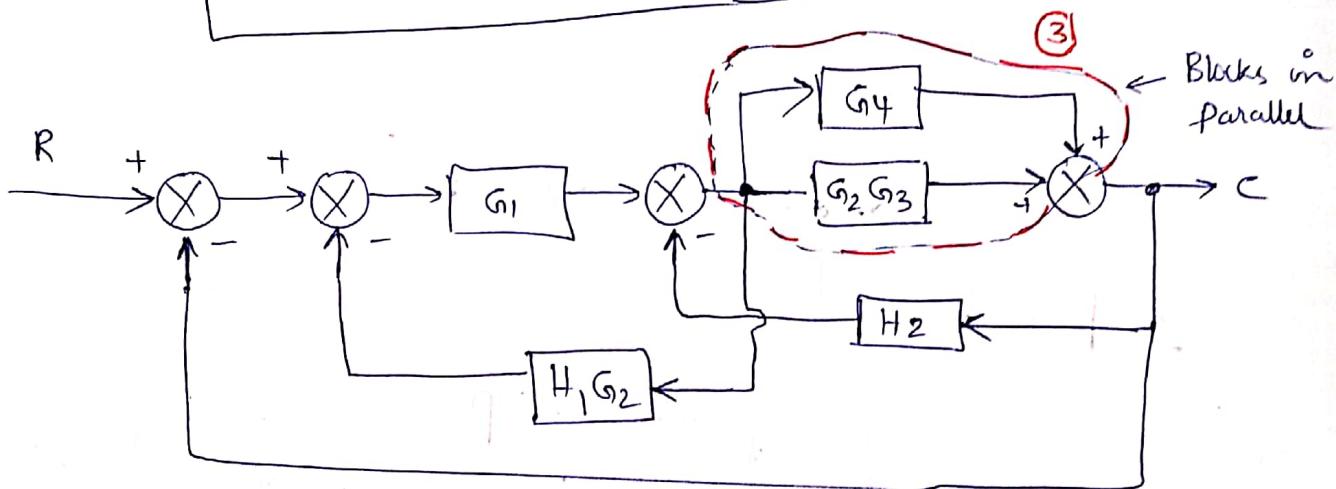
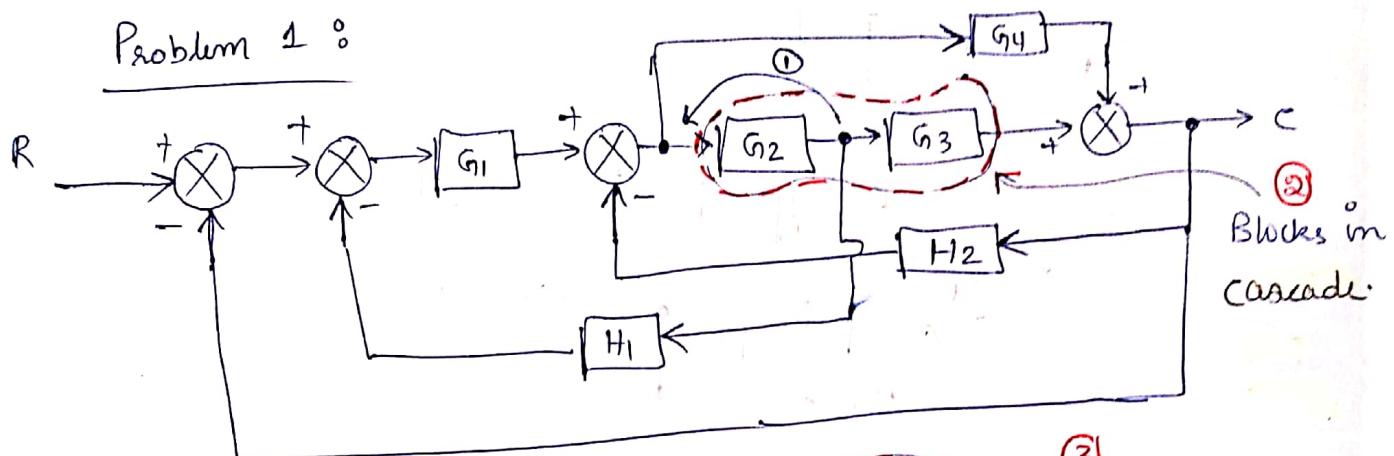
### 9. Splitting a Summing point :-



$$y = x_1 + x_2 - x_3 \Leftrightarrow$$



### Problem 1 :-

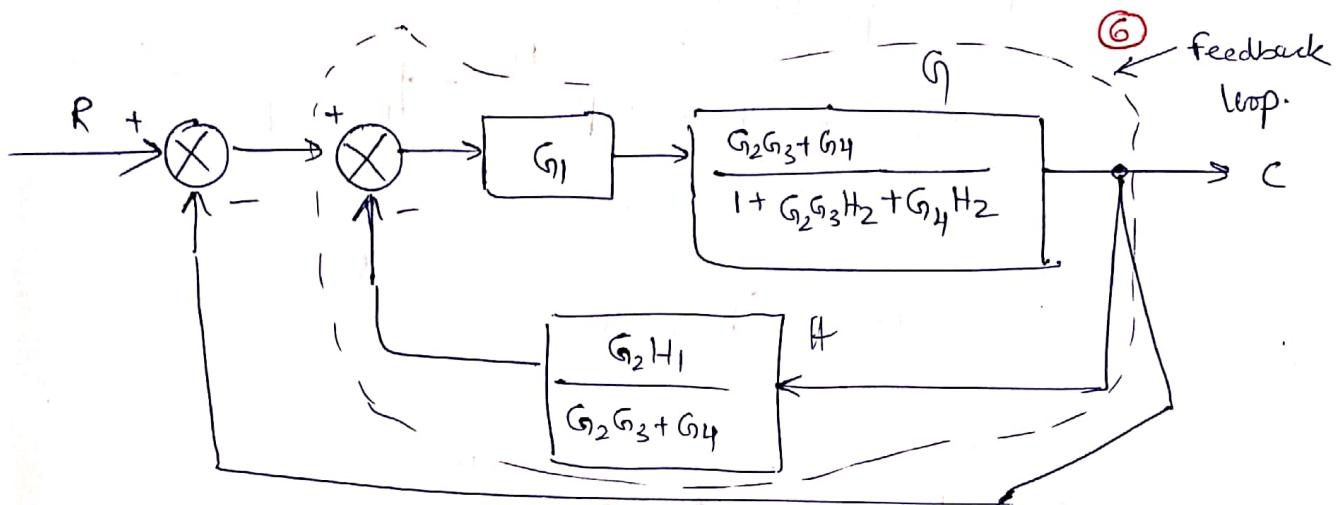
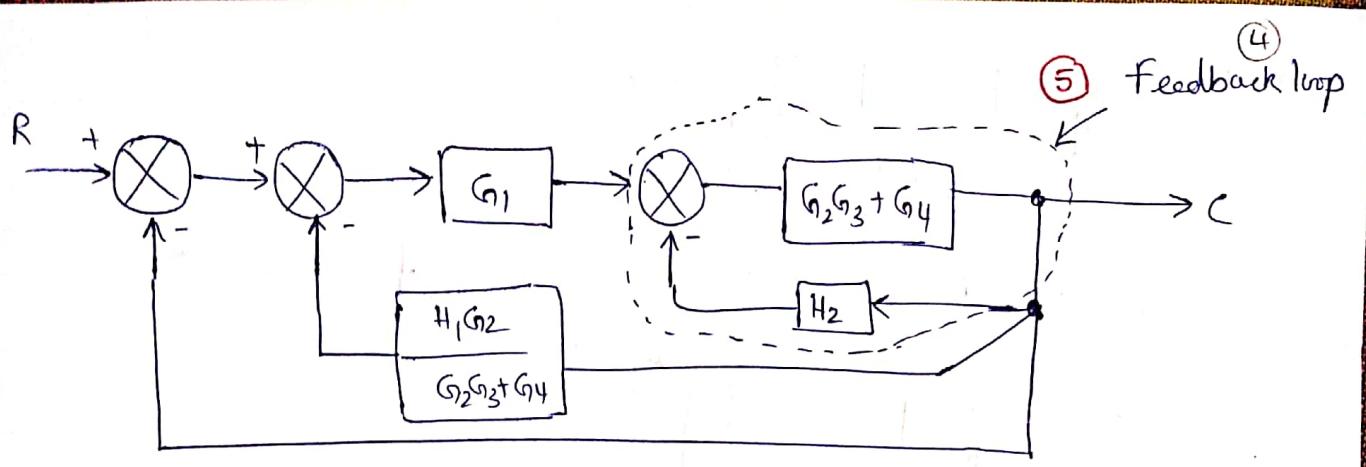


→ ① Shifting "take-off" point before  $G_2$

→ ② cascaded blocks  $\rightarrow [G_2 G_3] \rightarrow$

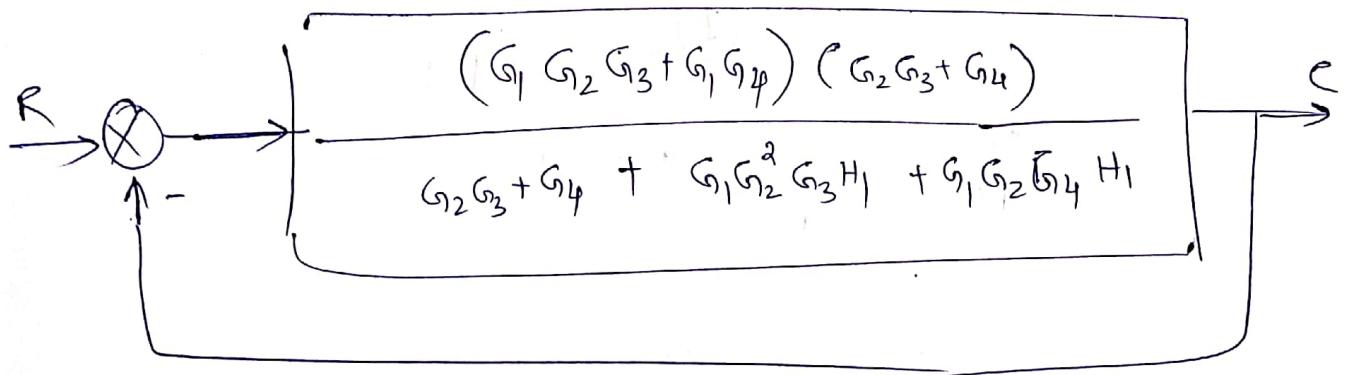
→ ③ Parallel Blocks  $\rightarrow [G_4 + G_2 G_3] \rightarrow$

→ ④ Shifting "take-off" point after a block.

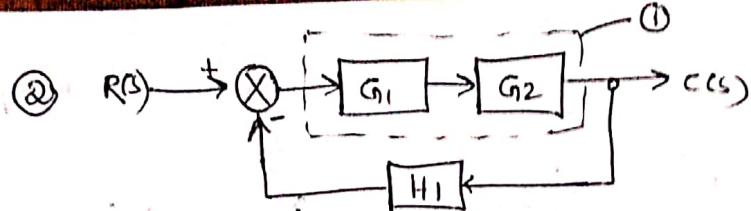


$$\frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_2 G_3 H_2 + G_4 H_2}$$

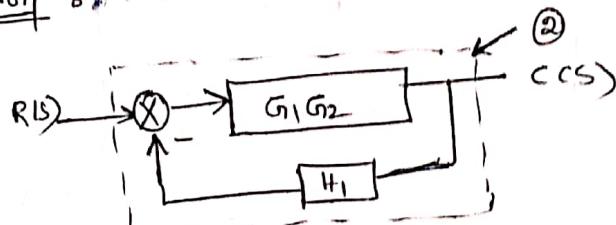
$$1 + \left( \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_2 G_3 H_2 + G_4 H_2} \right) \left( \frac{G_2 H_1}{G_2 G_3 + G_4} \right)$$



$$\frac{G_1 G_2^2 G_3 + G_1 G_2 G_3 G_4 + G_1 G_2 G_3 G_4 + G_1 G_4^2}{G_2 G_3 + G_4 + G_1 G_2^2 G_3 H_1 + G_1 G_2 G_4 H_1 + (G_1 G_2 G_3 + G_1 G_4)(G_2 G_3 + G_4)}$$



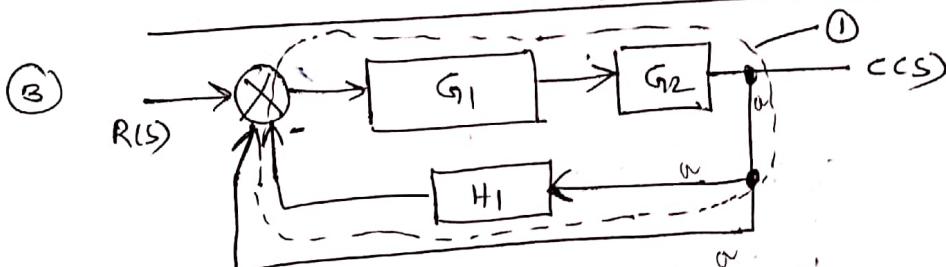
Solution :- ① Connected in cascade  $\rightarrow$   $G_1 G_2$   $\rightarrow$



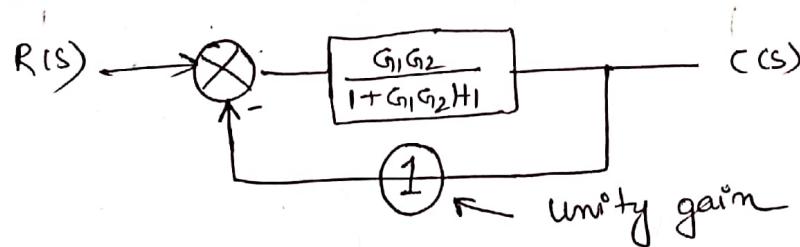
$$R(s) \rightarrow \boxed{\frac{G_1 G_2}{1 + G_1 G_2 H_1}} \rightarrow c(s)$$

② negative Feedback loop

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_1} \quad \longrightarrow \quad (3)$$

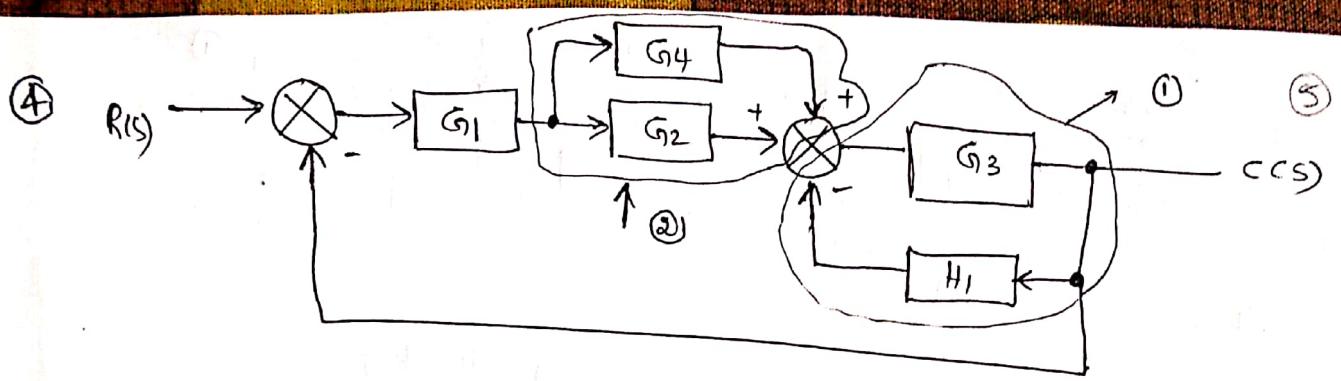


Solution: (1)  $\rightarrow$  loop transfer is given by eq (3)



$$\frac{C_{CS}}{R(S)} = \frac{G_1 G_2 / (1 + G_1 G_2 H_1)}{1 + \frac{G_1 G_2}{1 + G_1 G_2 H_1} \cdot 1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_1 + G_1 G_2}$$



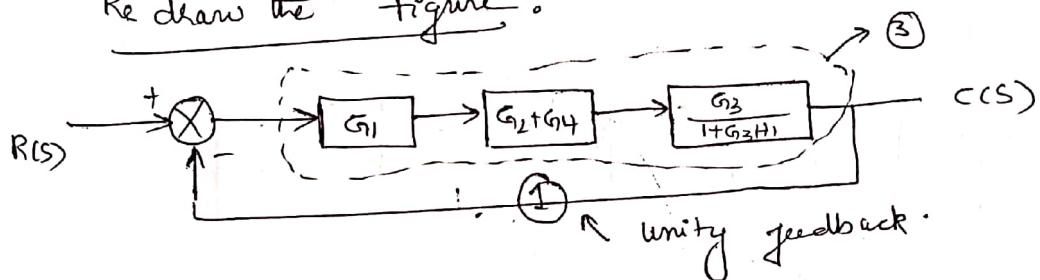
Solution : ① As much as possible, try to identify minor feedback loops first.

$$\rightarrow \frac{G_3}{1+G_3H_1} \rightarrow$$

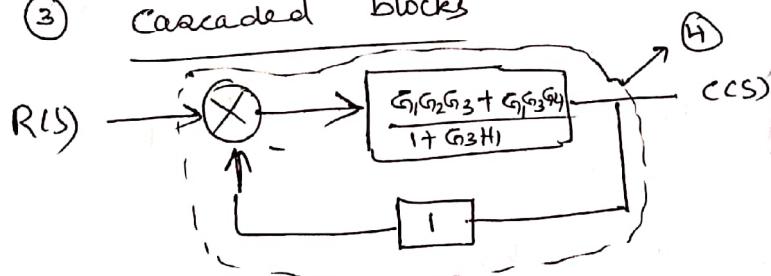
② Blocks connected parallel

$$\rightarrow G_2 + G_{14} \rightarrow$$

Redraw the figure :



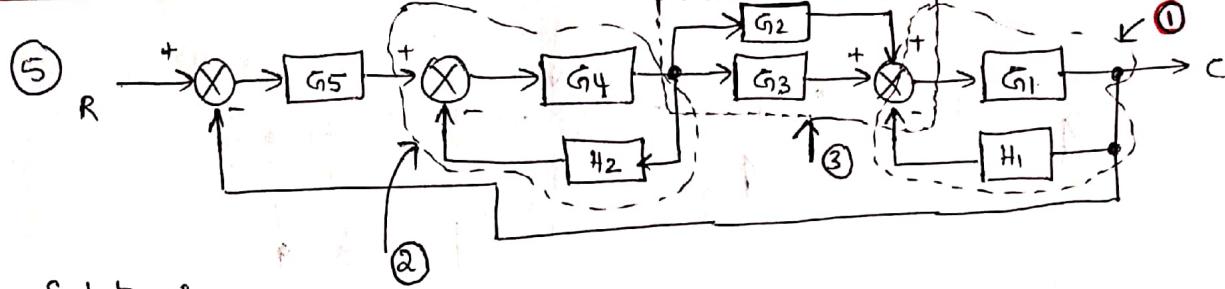
③ Cascaded blocks



④ Minor feedback loop (-ve f/B)

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3 + G_1G_3G_4}{1+G_3H_1}}{1 + \frac{G_1G_2G_3 + G_1G_3G_4}{1+G_3H_1}}$$

$$\frac{C(s)}{R(s)} = \frac{\underline{\underline{G_1G_2G_3 + G_1G_3G_4}}}{\underline{\underline{1+G_3H_1 + G_1G_2G_3 + G_1G_3G_4}}}$$



Solution:

① Minor feedback loop (-negative feedback)

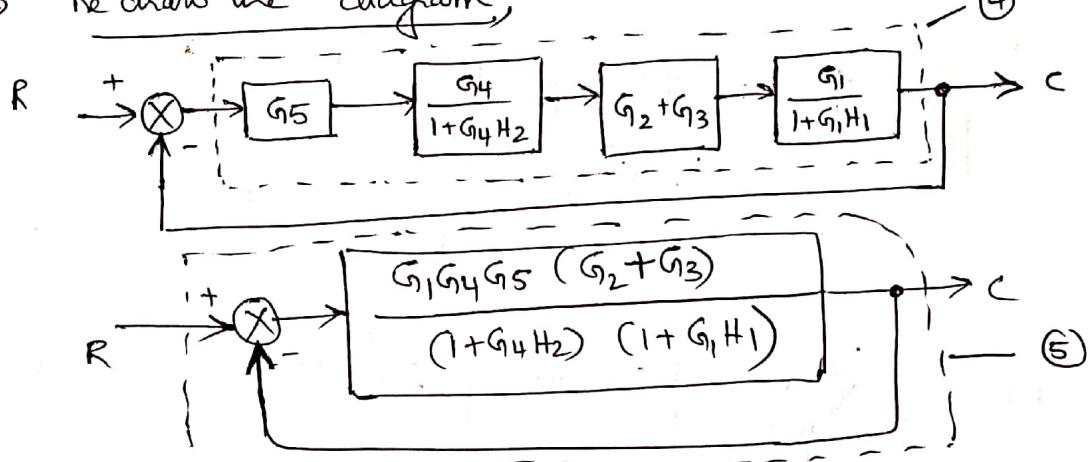
$$\rightarrow \boxed{\frac{G_1}{1+G_1H_1}} \rightarrow$$

② Minor feedback loop (-ve feedback)

$$\rightarrow \boxed{\frac{G_{14}}{1+G_{14}H_2}} \rightarrow$$

③ Parallel blocks  $\rightarrow \boxed{G_2+G_3} \rightarrow$

⇒ Re draw the diagram;

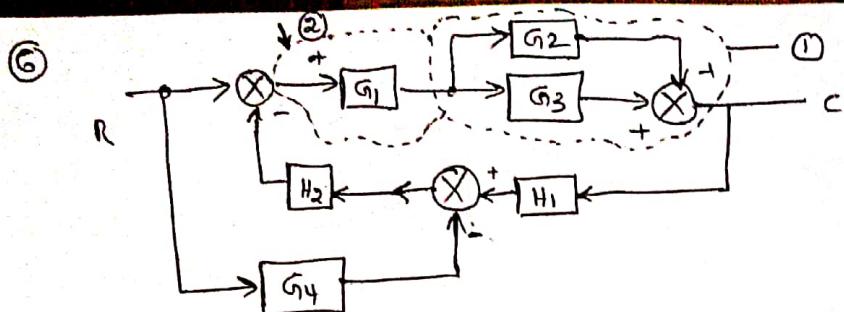


④ Cascaded blocks

⑤ Minor negative feedback

$$\frac{C}{R} = \frac{(G_1G_4G_5)(G_2+G_3)}{G_1G_4G_5(G_2+G_3) + (1+G_4H_2)(1+G_1H_1)}$$

$$T.O.F = \frac{C}{R} = \frac{G_1G_2G_4G_5 + G_1G_3G_4G_5}{1 + G_1G_2G_4G_5 + G_1G_3G_4G_5 + G_1H_1 + G_4H_2 + G_1G_4H_1H_2}$$



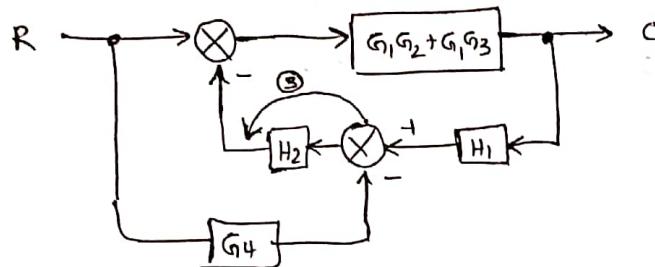
Solution:

① Parallel cascaded blocks

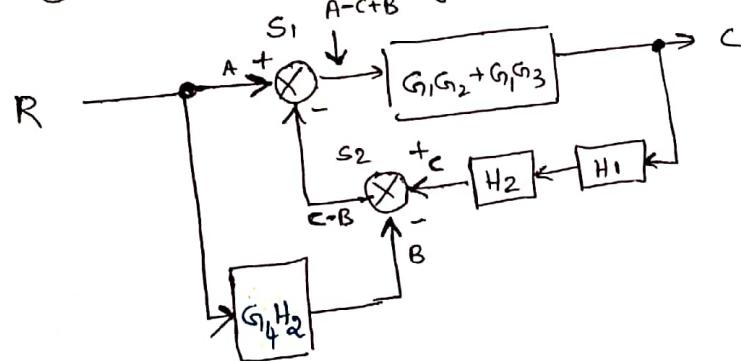
$$\rightarrow G_2 + G_3 \rightarrow$$

②  $G_1$  &  $(G_2 + G_3)$  Blocks are connected in cascade

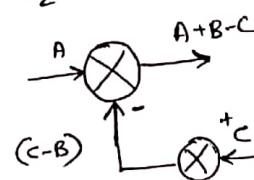
$$\rightarrow G_1 G_2 + G_1 G_3 \rightarrow$$



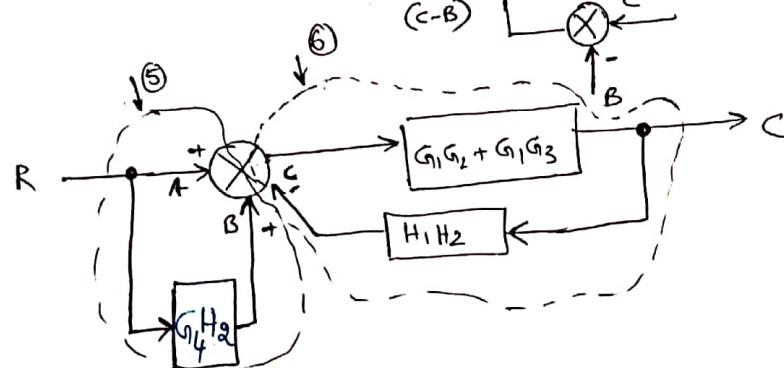
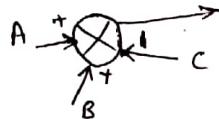
③ move summing point ahead of  $H_2$



④ combining  $S_1$  &  $S_2$  into a single summer



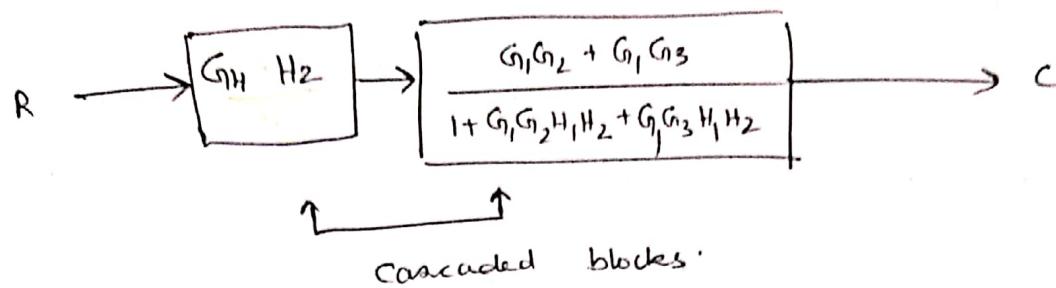
$\Rightarrow$



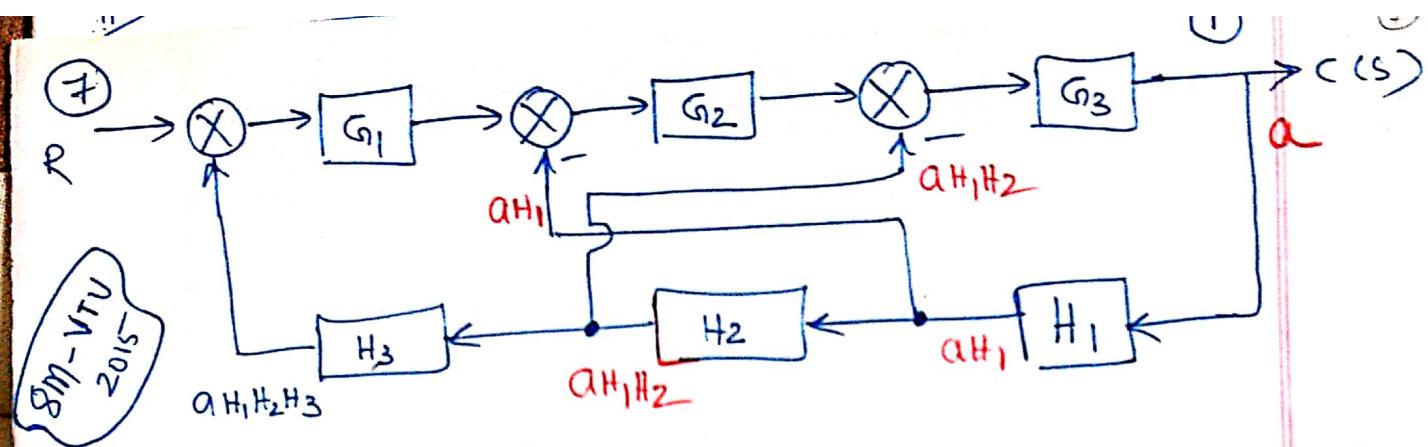
⑤ Parallel connected blocks

$$\rightarrow 1 + G_4 H_2 \rightarrow = \boxed{1 + G_4 H_2}$$

$$G_1 G_2 + G_1 G_3 \\ \hline 1 + G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2$$

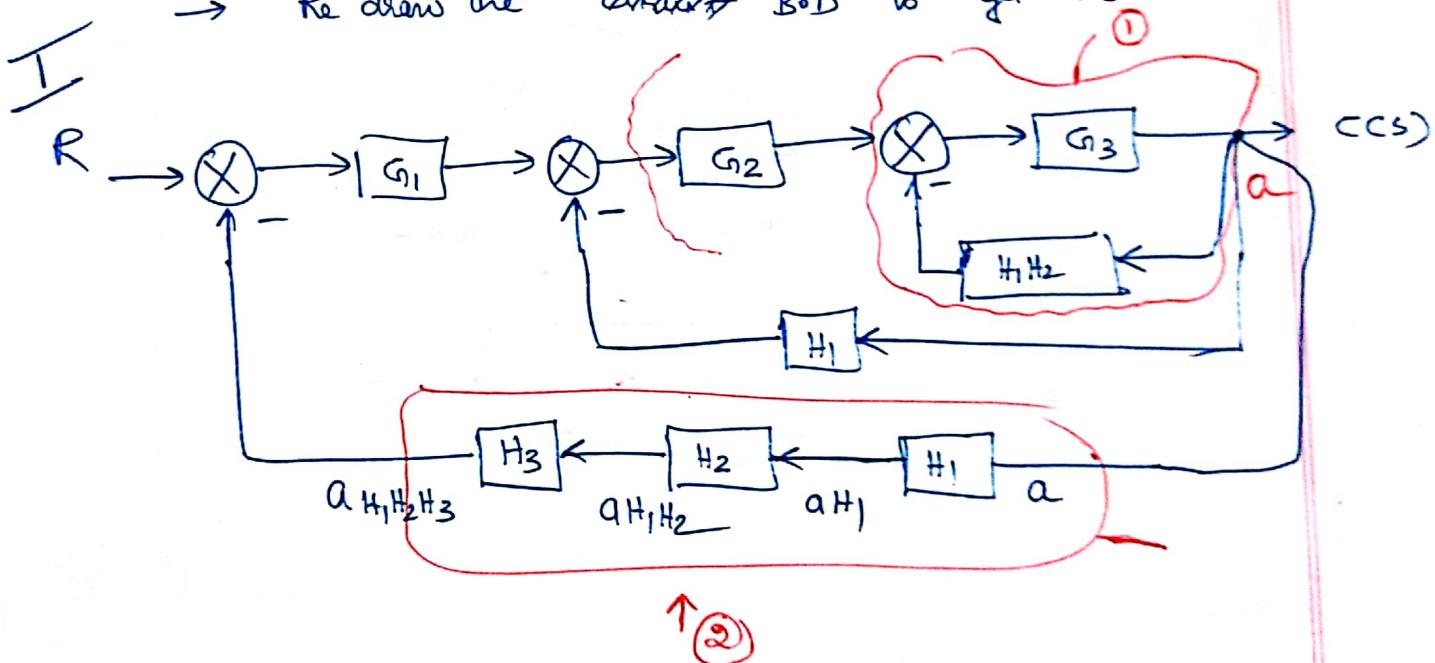


$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_4 + G_1 G_3 G_4 + G_1 G_2 H_2 + G_1 G_3 H_2}{(G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2 + 1) H_2}$$



Solution: Let us think of separating out the feedback paths.

- Say output signal  $C(s) = \text{some } 'a'$
- write down various signal values @ summing points.
- Re-draw the circuit B.O.D to get the same output.



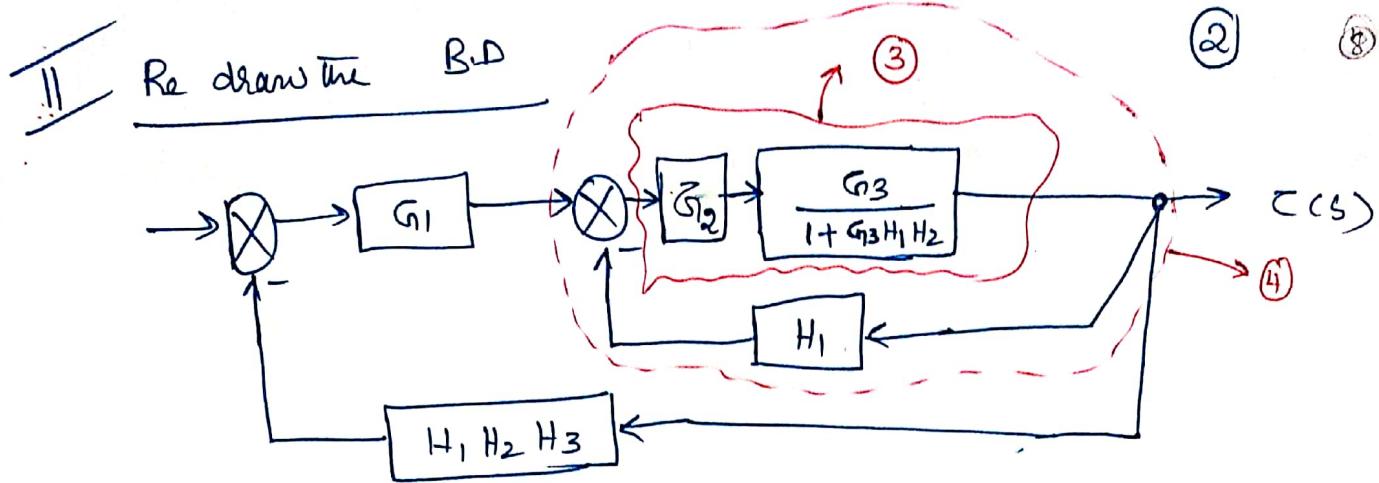
⇒ ① Blocks connected in -ve feedback manner

$$\frac{G_3}{1 + H_1 H_2 G_3}$$

ZD

⇒ ② Blocks connected in cascaded manner

$$H_1 H_2 H_3$$



⇒ ③ cascaded blocks

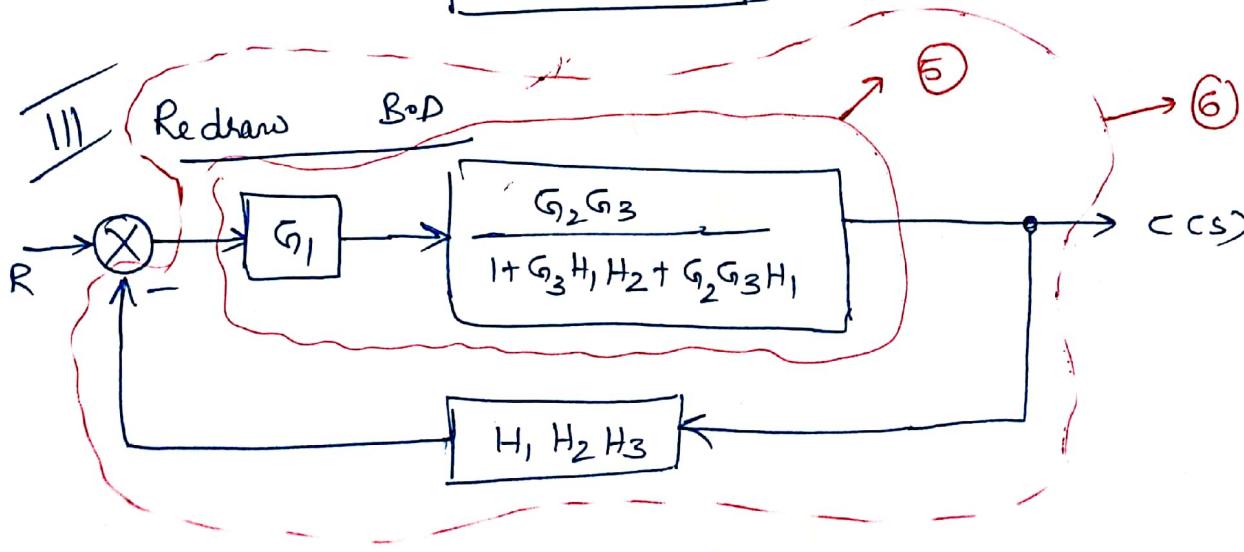
$$\frac{G_2 G_3}{1 + G_3 H_1 H_2}$$

⇒ ④ Negative feedback loop.

$$\frac{\frac{G_2 G_3}{1 + G_3 H_1 H_2}}{1 + \frac{G_2 G_3}{1 + G_3 H_1 H_2} \times H_1}$$

taking L.C.M

$$\frac{G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1}$$



⇒ ⑤ Blocks connected in cascade

(3) (3)

$$\frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_1 G_3 H_1}$$

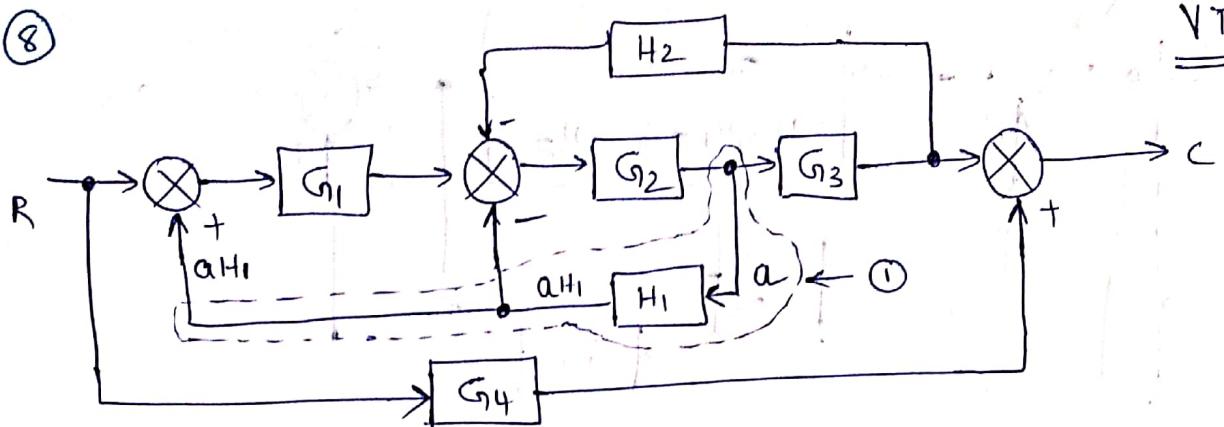
43

⇒ ⑥ Negative feedback loop.

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_1 G_3 H_1}}{1 + \frac{G_1 G_2 G_3 H_1 H_2 H_3}{1 + G_3 H_1 H_2 + G_1 G_3 H_1}}$$

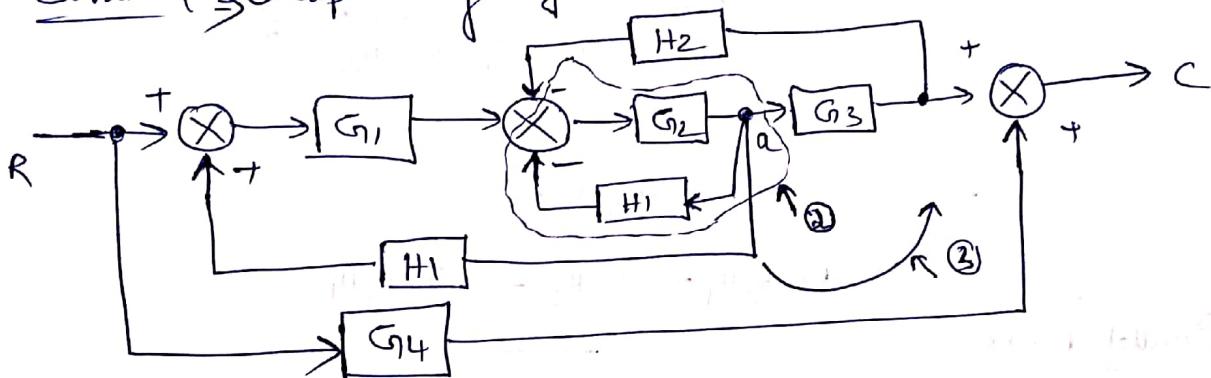
$$\frac{C(s)}{R(s)} = \frac{\cancel{G_1 G_2 G_3}}{1 + \cancel{G_3 H_1 H_2} + \cancel{G_1 G_3 H_1} + \cancel{G_1 G_2 G_3 H_1 H_2 H_3}}$$

$$\frac{\cancel{G_1 G_2 G_3}}{\cancel{1 + G_3 H_1 H_2} + \cancel{G_1 G_3 H_1} + \cancel{G_1 G_2 G_3 H_1 H_2 H_3}} \times \frac{\cancel{1 + G_3 H_1 H_2 + G_1 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}}{\cancel{1 + G_3 H_1 H_2 + G_1 G_3 H_1}}$$

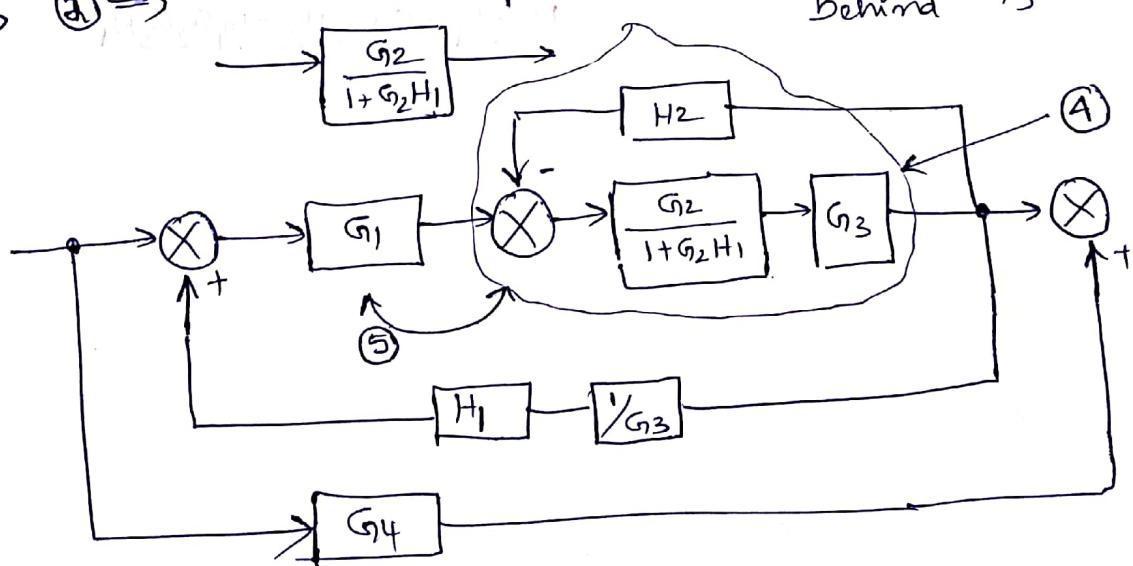


Find  $\frac{C}{R}$  using B.D.R technique.

Solution  $\Rightarrow$  ① Separating feedback paths.

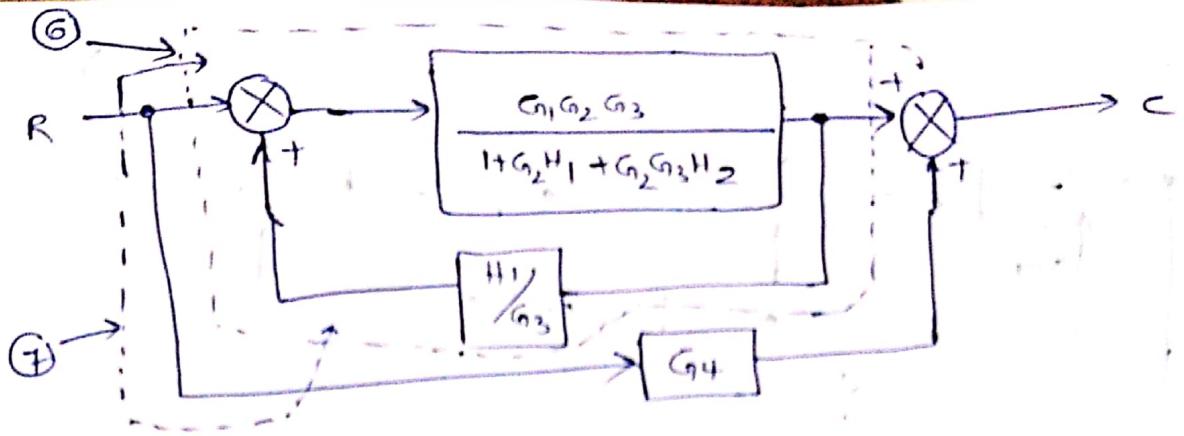


$\Rightarrow$  ②  $\Rightarrow$  feedback loop      ③ (take off) shift it behind  $G_3$



$$\Rightarrow \text{④ } \begin{matrix} \text{"-ve"} \\ \text{f/B loop} \end{matrix} \rightarrow \frac{\frac{G_2 G_3}{1 + G_2 H_1}}{1 + \frac{G_2 G_3 H_2}{1 + G_2 H_1}} = \frac{G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2}$$

$$\Rightarrow \textcircled{5} \quad \text{cascaded blocks} \quad \Rightarrow \quad \left\{ \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2} \right\}$$



⑥  $\rightarrow$  positive F/B

$$\frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2}$$

$$1 - \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2} \times \frac{H_1}{G_3}$$

$$= \frac{(G_1 G_2 G_3) G_3}{1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 G_3 H_1}$$

Parallel Blocks

⑦

$$\frac{C(s)}{R(s)} = G_4 + \frac{(G_1 G_2 G_3) G_3}{1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 G_3 H_1}$$

(11)

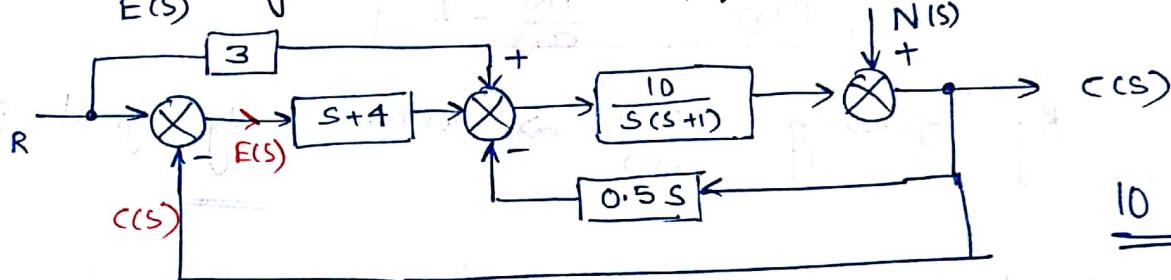
(9)

The system B·D is given below. Find

$$(i) \frac{C(s)}{E(s)} \text{ if } N(s) = 0$$

$$(ii) \frac{C(s)}{R(s)} \text{ if } N(s) = 0$$

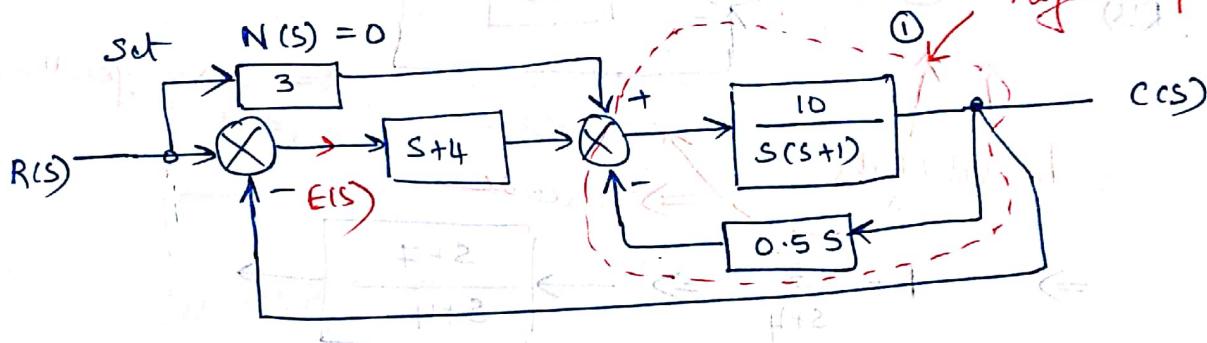
$$(iii) \frac{C(s)}{N(s)} \text{ if } R(s) = 0$$



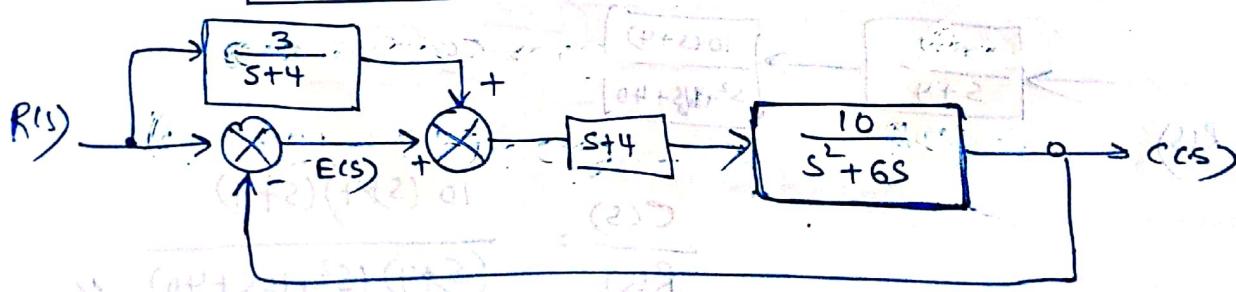
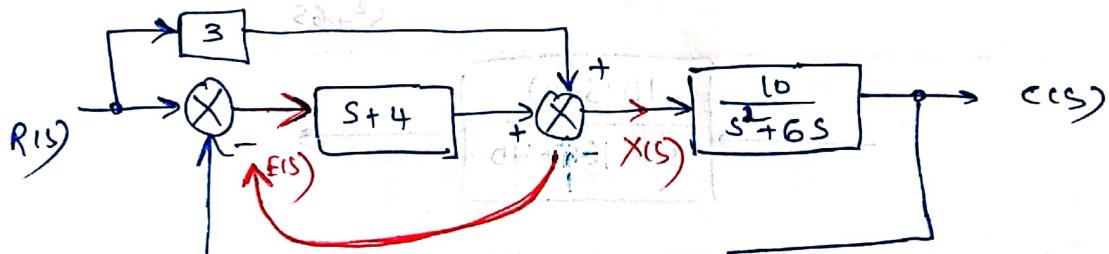
10 Marks

Solution:

$$(i) \text{ if } N(s) = 0 \text{ find } \frac{C(s)}{E(s)}$$



$$\textcircled{1} \Rightarrow \frac{\frac{10}{s^2+s}}{1 + \frac{10}{s^2+s} \times 0.5s} = \frac{10}{s^2+6s}$$

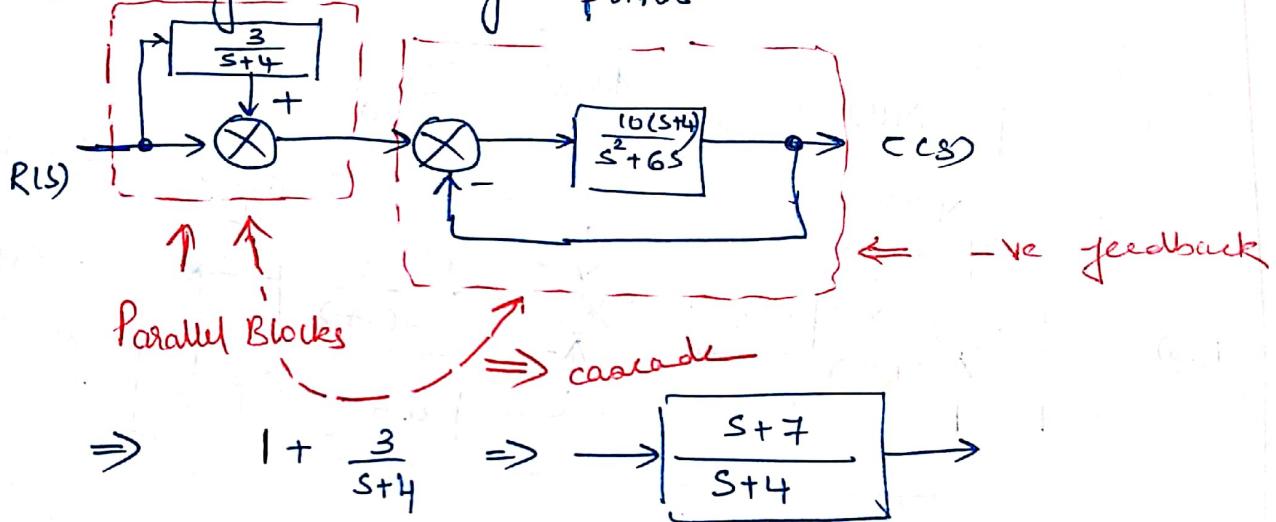


$$\text{Also } \frac{C(s)}{E(s)} = \frac{C(s)}{R(s) - C(s)} = \frac{1}{\frac{R(s)}{C(s)} - 1}$$

$$\boxed{\frac{C(s)}{E(s)} = \frac{1}{\frac{R(s)}{C(s)} - 1}}$$

By finding  $\frac{R(s)}{C(s)}$   $\Rightarrow$  we can easily find  $\underline{\frac{C(s)}{E(s)}}$

$\Rightarrow$  enacting summing points.



$\Rightarrow$  negative f/B

$$\frac{\frac{10(s+4)}{s^2+6s}}{1 + \frac{10(s+4)}{s^2+6s}} = \frac{10(s+4)}{s^2+6s+10s+40}$$

$$\frac{10(s+4)}{s^2+16s+40}$$

$$R(s) \rightarrow \frac{s+7}{s+4} \rightarrow \frac{10(s+4)}{s^2+16s+40} \rightarrow C(s)$$

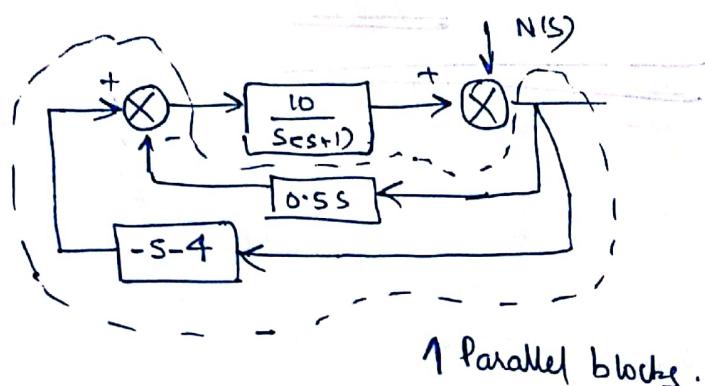
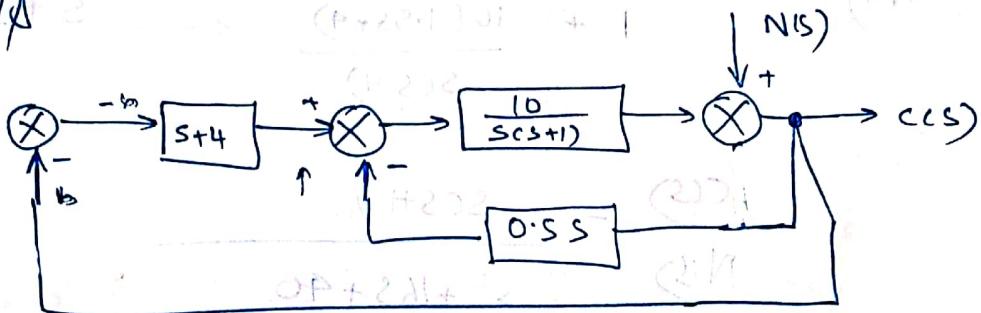
$$\frac{C(s)}{R(s)} = \frac{10(s+4)(s+7)}{(s+4)(s^2+16s+40)} //$$

$$\frac{C(s)}{E(s)} = \frac{\frac{1}{s+4}}{\frac{10(s+7)}{s^2+16s+40} - 1} = \frac{\frac{1}{s+4}}{\frac{10(s+7) - s^2 - 16s - 40}{s^2+16s+40}} = \frac{s^2+16s+40}{10s+70 - s^2 - 16s - 40}$$

$$\frac{C(s)}{E(s)} = \frac{1}{\frac{s^2+16s+40}{10(s+7)}} = \frac{10(s+7)}{s^2+16s+40 - 10s - 70}$$

$\frac{C(s)}{E(s)} = \frac{10(s+7)}{s^2+6s-30}$

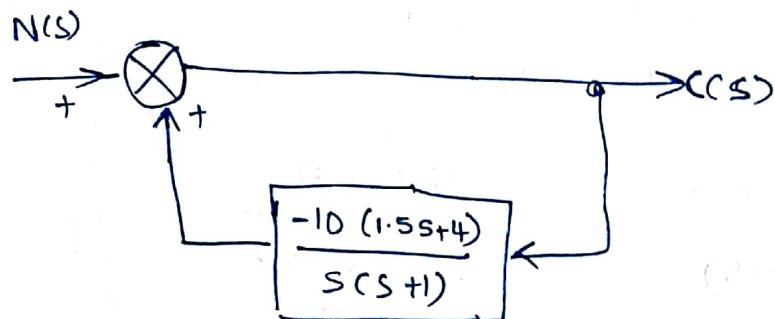
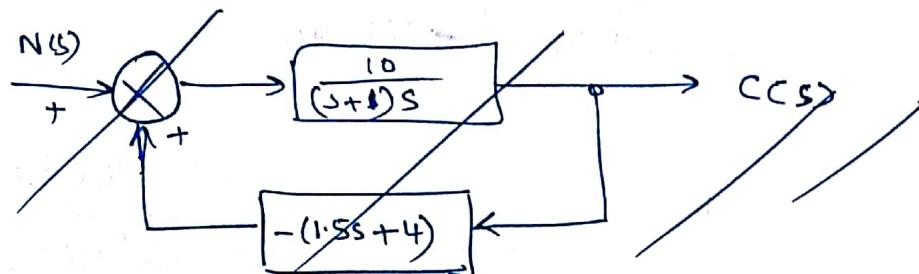
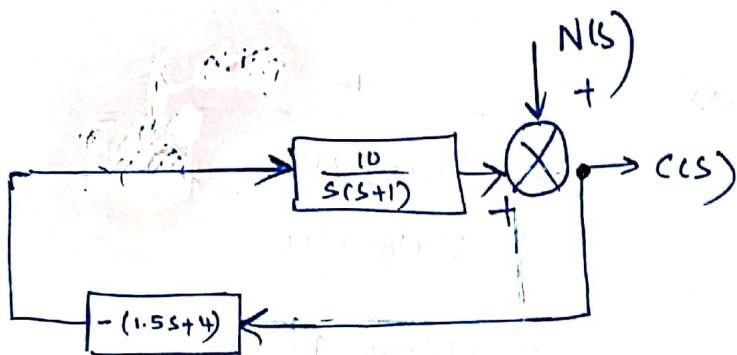
(iii) if  $s = 0$  with  $R(s) = 0$



$$-s - 4 - 0.5s$$

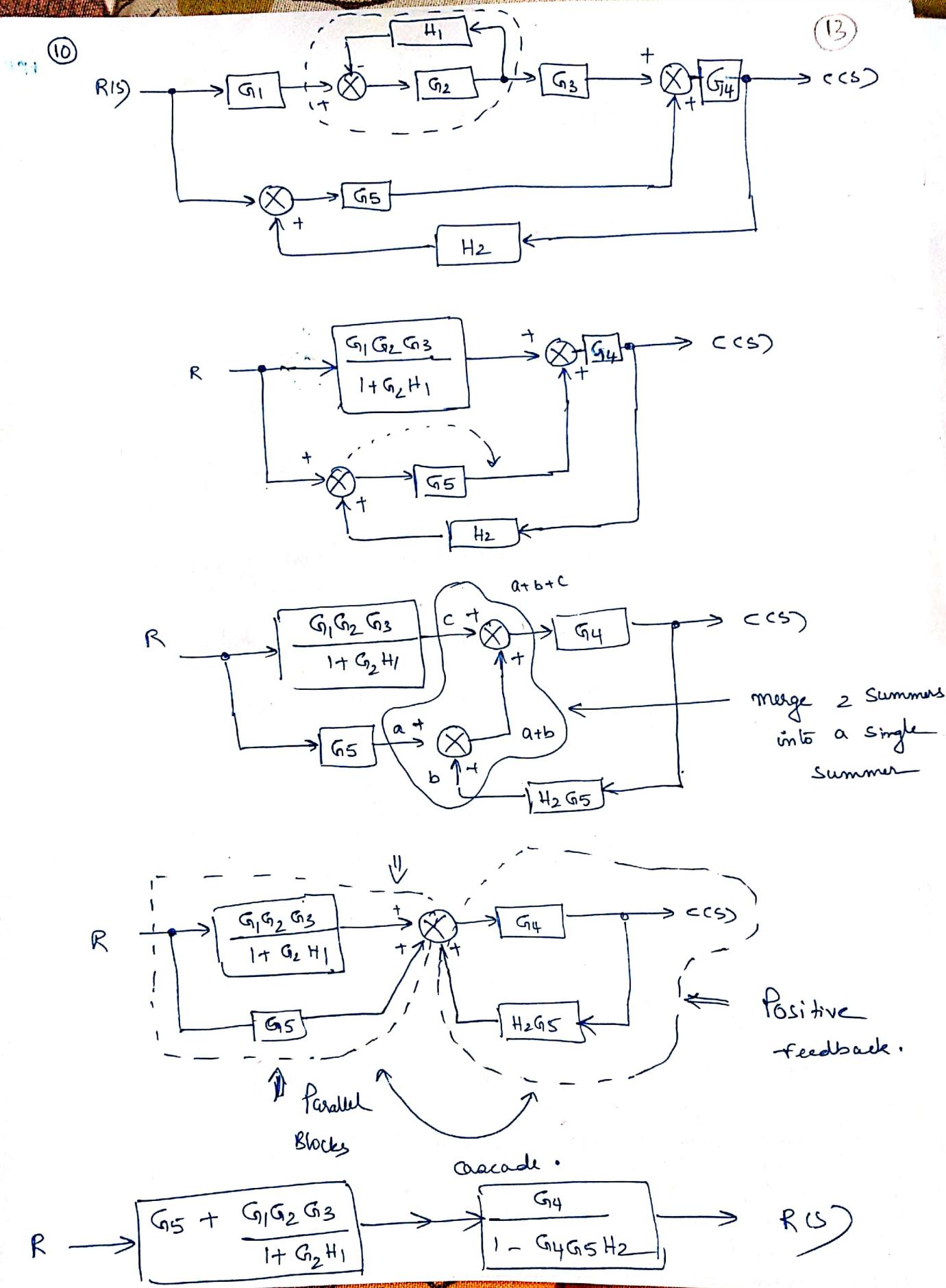
$$= -1.5s - 4$$

$$= -\underline{(1.5s + 4)}$$

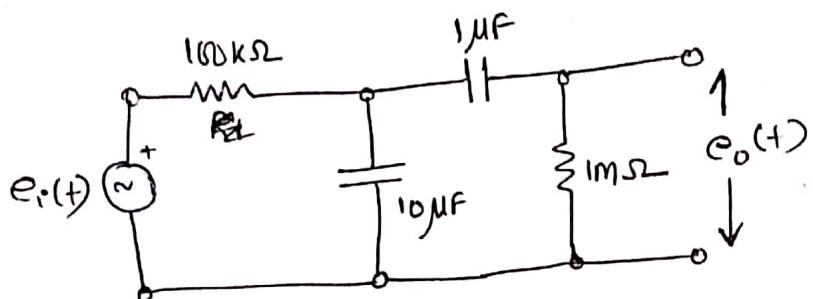


$$\frac{C(s)}{N(s)} = \frac{1}{1 + \frac{10(1.5s+4)}{s(s+1)}} = \frac{s(s+1)}{s^2 + 15s + 40}$$

$$\frac{C(s)}{N(s)} = \frac{s(s+1)}{s^2 + 16s + 40}$$

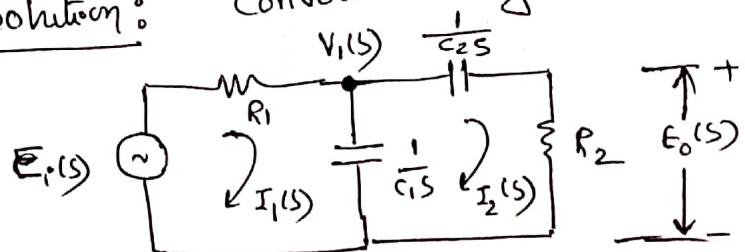


(14)



- Draw block diagram & evaluate  $\frac{E_o(s)}{E_i(s)}$

Solution: Convert the given circuit into Laplace domain



$$\begin{aligned} C_1 &= 10\mu F \\ C_2 &= 1\mu F \\ R_1 &\approx 100k\Omega \\ R_2 &= 1m\Omega \end{aligned}$$

Apply Nodal analysis

Variables are  $E_i(s)$ ,  $I_1(s)$ ,  $V_1(s)$ ,  $I_2(s)$  &  $E_o(s)$

$$I_1(s) = \frac{E_i(s) - V_1(s)}{R_1} = E_i(s) - V_1(s) \left[ \frac{1}{R_1} \right] \quad (1)$$

$$V_1(s) = [I_1(s) - I_2(s)] \times \frac{1}{C_1 s} \quad (2)$$

$$I_2(s) = \frac{V_1(s) - E_o(s)}{\left( \frac{1}{C_2 s} \right)} = C_2 s (V_1(s) - E_o(s)) \quad (3)$$

$$E_o(s) = I_2(s) R_2 \quad (4)$$

$\Rightarrow$  Constructing Block diagram from 4 algebraic Equations.

