2(b) Signal flow graph [SFG]

The graphical representation of the variables of a set of linear algebraic equations representing the system is called Signal flow graph representation.

This method is developed "S.J. Mason".

* It should be noted that, SFG approach and block diagram approach, yield the same information.

* But in SFG, by using Mason's gain formula, the overall gain of the system can be computed very easily. This method is very simple compared to tedious block diagram reduction technique.

* The signal flow graph depicts the flow of signals from one to another in a given system. It also gives the relationships among the signals.

* Properties of Signal flow graph :-

Jun - 2007

1) It is applicable to linear time invariant systems.

2) The signal in the system flows along the branches, only in the marked direction of the arrow heads.

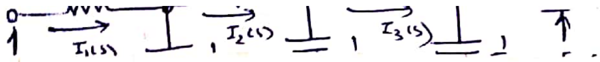
Fig 1

ex: $x_1 \circ \!\longrightarrow\! \circ\, x_2$

$\longleftarrow$ Direction of signal flow

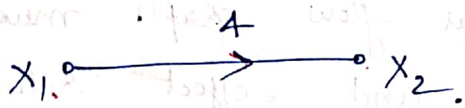
where x_1 and x_2 are nodes

and $x_1 x_2$ is a branch.



3) When the signal travels along the direction of the arrow head, the signal gets multiplied by the branch gain or branch transmittance.

ex:



When the signal travels along the branch $x_1 x_2$, it gets multiplied by 4. Hence value at x_2 is equal to 4 times the value of x_1.

Fig 2

i.e. $x_2 = 4x_1$

It shows the dependence of x_2 on x_1.

4) A node in the signal flow graph represents the variable or signal.

5) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.

ex:

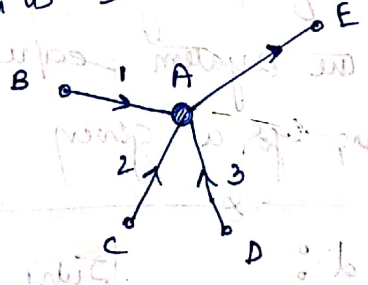


Fig 3

Consider the variable A (i.e. Node A); at A, 3 signals are entering from nodes B, C, D along the branches BA, CA and DA respectively.

∴ Value of A is = algebraic sum of all entering signals

$= B + 2C + 3D$

Finally value of A is transmitted along the branch AE as shown in the fig.

6) The algebraic equations which are used to construct signal flow graph must be in the form of cause and effect relationship.

for example in the form of KVL ;

i.e $\Sigma E = \Sigma IR$

cause effect

is a node

7) "A mixed node" which has both incoming and outgoing branches.

8) A branch indicates functional dependence of one signal on the other.

ex: fig ② where $X_2 = 4x_1$

9) Signal flow graph of a system is not unique. By rearranging the system equations, diff types of SFG can be drawn for a given system

$E = IR$

or $I = E/R$

* Terminology used : [July - 2009, 8 marks]

Node : It represents a system variable or signal. which is equal to the sum of all incoming signals at that node.

"I/P Node"

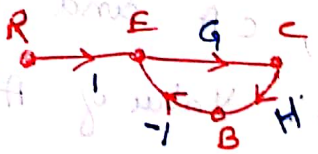
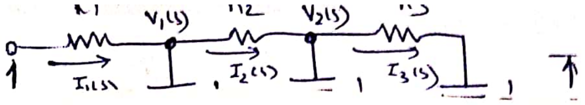


Fig : 4



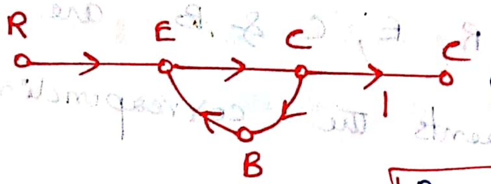
In fig (H); R, E, C & B are nodes. They also represents the corresponding node variables & signals.

Branch: A branch is a directed line segment joining 2 nodes. The arrow on the branch indicates the direction of signal flow. The gain of a branch is called as the transmittance.

Transmittance: The gain aquired by the signal when it travels from one node to another is called as a transmittance. The transmittance can be real or complex.

Input Node (source): It is a node, which has only outgoing branches. For example 'R' is an input Node in fig 4.

Output Node (Sink): It is a node, which has only incoming branches. However this condition is not always met. For ex: in fig 4; Node 'C' can not be an output node, as it has one outgoing branch. But node C can be made as an output Node by introducing an additional brach with "unit" transmittance as shown in fig 5.



output Node

fig 5

Mixed Node :

It is a Node that has both incoming and outgoing branches. For ex in fig 4 E, C & B Nodes are "mixed Nodes".

Path : It is the traversal of connected branches in the direction of the branch arrows such that, no node is traversed more than once.

Forward Path : It is a path from an I/P node to an output node; that does not cross any node more than once.

ex: In fig (5) Path R-E-C is the Fwd path.

Loop : It is a path, which originates and terminates at the same node.

ex: In fig $\boxed{5}$, loop is E-C-B-E

Non touching loops : Loops are said to be non-touching if they do not possesses any common node.

Forward path gain : It is the product of the branch transmittances (gains) of a forward path. In fig 4

Forward path gain = G.

$= \frac{1}{s^2+2s}$

1 $I_1(s)$ ⊥ , $I_2(s)$ ⊥ , $I_3(s)$ ⊥ , ↑

Loop gain: It is the product of the branch transmittances (gains) of a loop.

In fig 4: loop gain is of E - C - B - E is; "$-GH$".

Mason's Gain formula:

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system.

Let $R(s)$ = Input to the system.
$C(s)$ = Output of the system.

Now transfer function of the system, $T(s) = \frac{C(s)}{R(s)}$ —(i)

⇒ Mason's gain formula states that, the overall gain of the system [transfer function] as follows, is, eq

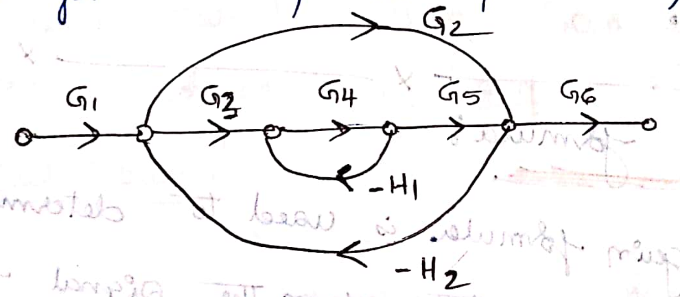
$$\boxed{\text{Overall gain, } T = \frac{1}{\Delta} \sum_K T_K \Delta_K}$$

where $T = T(s)$ = Transfer function of the system.
T_K = Forward path gain of K^{th} forward path.
K = No. of forward paths in the signal flow graph.

Δ = 1 - (Sum of Individual loop gains)
+ (Sum of gain products of all possible combinations of 2 non-touching loops)
- (Sum of gain products of all possible combinations of 3 non-touching loops) + ...

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path. (List out the loops which is not touching the Fwd path under consideration)

ex: 1) Find the overall T.F. by using Mason's gain formula for the SFG shown below;



Solution:

1) Identify the forward paths:

K = No. of forward paths = 2

say $T_1 = G_1 G_3 G_4 G_5 G_6$

& $T_2 = G_1 G_2 G_6$

2) List out the Individual loops gains

1) $L_1 = -G_4 H_1$

2) $L_2 = -G_3 G_4 G_5 H_2$

3) $L_3 = -G_2 H_2$

3) Note down the Non-touching loops

(a) 2 Non touching loops: $L_1 = -G_4 H_1$ & $L_3 = -G_2 H_2$

$\therefore \Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$

Δ_K

as $K=2$, ∴ we have Δ_1 and Δ_2.

$\Delta_1 = 1$; as L_1, L_2, L_3 are all touching the forward path T_1, ∴ eliminate L_1, L_2 & L_3.

$\Delta_2 = 1 - [L_1]$; as loops L_2 & L_3 are touching the forward path T_2, & hence gets eliminated.

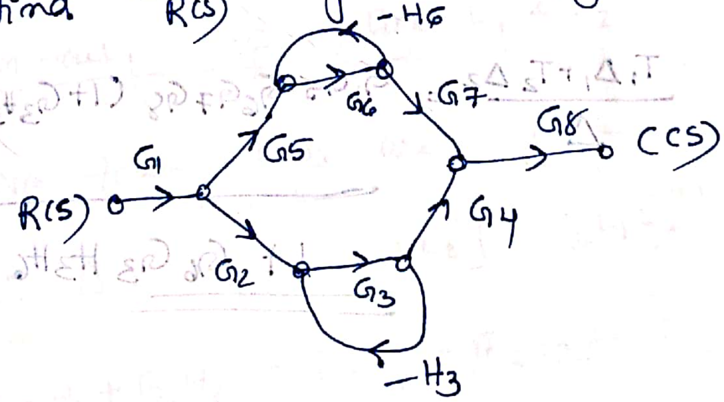
substitute all the values in Mason's gain formula;

$$T.F = \frac{1}{\Delta}\sum_K T_K \Delta_K$$ where $K=2$

$$\therefore T.F = \frac{1}{\Delta}\left[T_1\Delta_1 + T_2\Delta_2\right]$$

$$T.F = \frac{G_1G_3G_5G_4G_6[1] + G_1G_2G_6(1+G_4H_1)}{1+G_4H_1+G_3G_4G_5H_2+G_2H_2+G_4H_1G_2H_2}$$

2) Find $\frac{C(s)}{R(s)}$ by mason's gain formula; June-July 09



1) No. of forward paths $K=2$

$T_1 = G_1G_5G_6G_7G_8$

$T_2 = G_1G_2G_3G_4G_8$

2) **Loops** (Individual loop gains)

$L_1 = -G_6H_6$

$L_2 = -G_3H_3$

3) No of 2 non-touching loop $= 2$

i.e L_1 & L_2 ; they do not touch each other.

$$\therefore \Delta = 1 - [L_1 + L_2] + [L_1 L_2]$$

$$\Delta = 1 + [G_6G_3H_3H_6] + G_6H_6 + G_3H_3$$

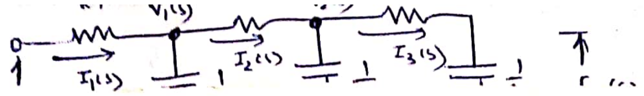
4) $\Delta_1 = 1 - [L_2]$ for T_1, L_2 is not touching

$\Delta_2 = 1 - [L_1]$ for T_2, L_1 is not touching.

$$\therefore \Delta_1 = 1 + G_3H_3$$

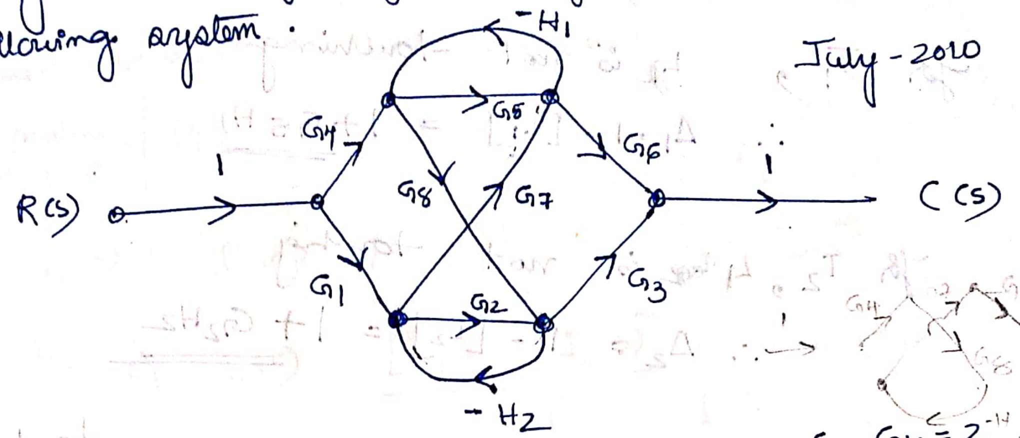
$$\Delta_2 = 1 + G_6H_6.$$

$$\therefore TF = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{G_1G_5G_6G_7G_8(1+G_3H_3) + G_1G_2G_3G_4G_8(1+G_6H_6)}{1 + G_6G_3H_3H_6 + G_6H_6 + G_3H_3}$$



3) Using Mason's gain formula, find the gain of the following system.

July - 2010



Find the gain if $G_1 = 3$, $G_2 = 20$, $G_3 = 6$, $G_4 = 2$, $H_1 = 2$, $H_2 = 1$, $G_5 = 10$, $G_6 = 4$, $G_7 = 2$, $G_8 = 2$

1) Forward paths:

$T_1 = G_4 G_5 G_6 = 80$ $T_2 = G_1 G_2 G_3$

$T_3 = G_4 G_8 G_3$ $T_4 = G_1 G_7 G_6$

$T_6 = G_1 G_7 (-H_1) G_8 G_3$

$T_5 = G_4 G_8 (-H_2) G_7 G_6$

2) Individual loop gains:

(a) $L_1 = -G_5 H_1$ $L_2 = G_2(-H_2)$ $L_3 = G_7 H_1 G_8 H_2$

3) 2 Non touching loops are L_1 & L_2

3 Non touching loops are nil

$\therefore \Delta = 1 - [L_1 + L_2] + [L_1 L_2]$

$\Delta = 1 + G_5 H_1 + G_2 H_2 + G_5 H_1 + G_2 H_2$

4) as $K = 6$, $\Delta_K = 6$

for T_1, L_2 is not touching

$\therefore \Delta_1 = 1 - [L_1] = 1 + G_5 H_1$

for T_2, L_1 loop is not touching

$\therefore \Delta_2 = 1 - [L_2] = 1 + G_2 H_2$

for T_3, T_4 T_5 T_6 all loops are touching

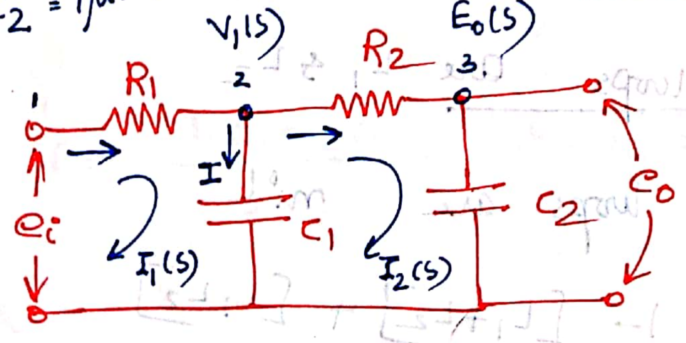
$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$\therefore$ Gain $= \dfrac{\Sigma T_K \Delta_K}{\Delta} = \dfrac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4 + T_5\Delta_5 + T_6\Delta_6}{\Delta}$

Gain $= 21.0438$

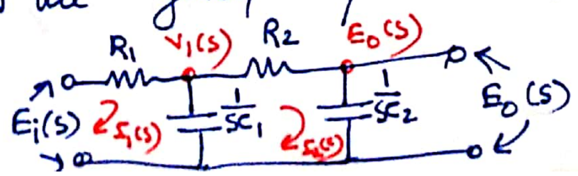
5) For the network shown below, construct the SFG and obtain the T.F, if $R_1 = 100k\Omega$, $R_2 = 1M\Omega$, $C_1 = 10\mu F$ and $C_2 = 1\mu F$.

V.T.U : Aug 95, Mar-2001, Jan 2006



Solution: Assume loop currents as shown in the above ckt.

Step 1: Re-draw the given n/w in Laplace domain



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Step 2 : Write down the equations for the different branch currents & node voltages.

Nodal analysis b/w 1–2

$$\boxed{I_1(s) = \frac{E_i(s) - V_1(s)}{R_1}} \quad \text{— (1)}$$

$V_1(s)$ = Drop a/c $\frac{1}{sC_1}$ = $I \times \frac{1}{sC_1}$

i.e

$$\boxed{V_1(s) = \frac{[I_1(s) - I_2(s)]}{sC_1}} \quad \text{— (2)}$$

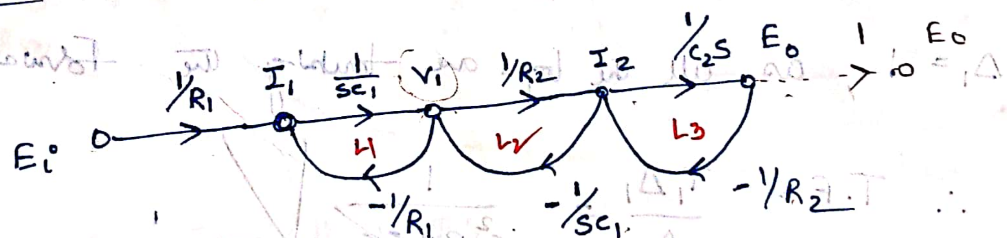
Nodal Anay b/w 2–3

$$\boxed{I_2(s) = \frac{V_1(s) - E_0(s)}{R_2}} \quad \text{— (3)}$$

$E_0(s)$ = Drop a/c C_2

$$\boxed{E_0(s) = I_2(s) \times \frac{1}{C_2 s}} \quad \text{— (4)}$$

Step 3 : Construct SFG



Step 5 : SFG steps

1) Forward paths ; $T_1 = \frac{1}{R_1} \cdot \frac{1}{sC_1} \cdot \frac{1}{R_2} \cdot \frac{1}{sC_2}$

$$T_1 = \frac{1}{s^2 R_1 R_2 C_1 C_2}$$

∴ Node Value = Value of the node obtained after adding the signals algebraically, ~~leaving~~ meeting at the node.

of node under consideration

2) Individual Loops

$L_1 = -\frac{1}{SR_1C_1}$; $L_2 = -\frac{1}{SR_2C_1}$; $L_3 = -\frac{1}{SR_2C_2}$

3) Δ Calculation of $\Delta = 1 - [\Sigma$ indi loop gains$] + [$Product of 2 Non-touching loops$]$ (Summation)

2 Non touching loops :-

1) Loop $L_1 L_3$ are non tchg.

2) $\therefore \Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$

$$\Delta = 1 + \left[\frac{1}{SR_1C_1} + \frac{1}{SR_2C_1} + \frac{1}{SR_2C_2}\right] + \left[\frac{1}{S^2R_1C_1R_2C_2}\right]$$

4) Calculation of Δ_K :

as $K = 1$ $\quad \Delta_K = \Delta_1$

$\Delta_1 = 1$, as all the loops are touching the Forward Path.

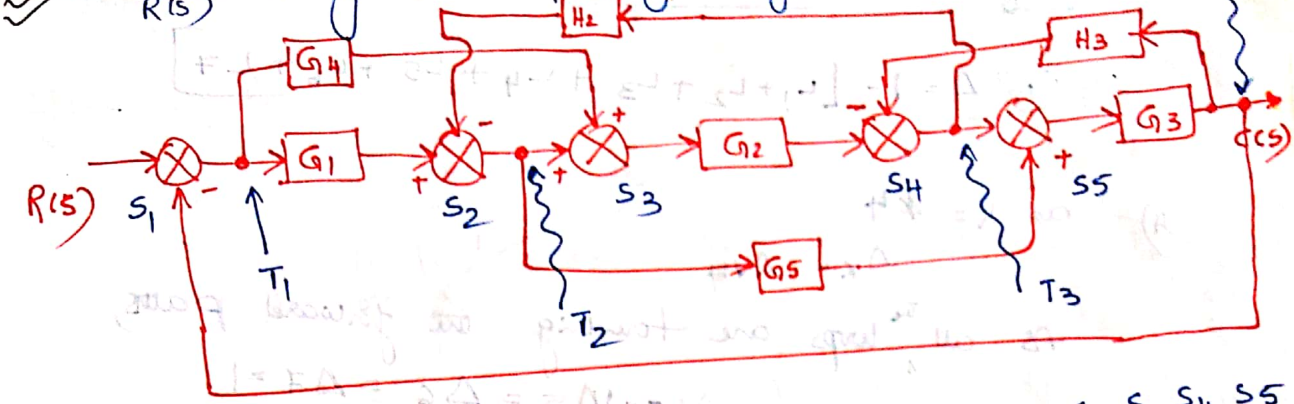
$$\therefore T.F = \frac{T_1\Delta_1}{\Delta} = \frac{\frac{1}{S^2R_1R_2C_1C_2}}{1 + \frac{1}{SR_1C_1} + \frac{1}{SR_2C_1} + \frac{1}{SR_2C_2} + \frac{1}{S^2R_1R_2C_1C_2}}$$

$$= \frac{1}{SR_2C_2 + SR_1C_2 + SR_1C_1 + S^2R_1R_2C_1C_2 + 1}$$

$$T.F = \frac{1}{S^2R_1R_2C_1C_2 + S(R_2C_2 + R_1C_1 + R_1C_2) + 1}$$

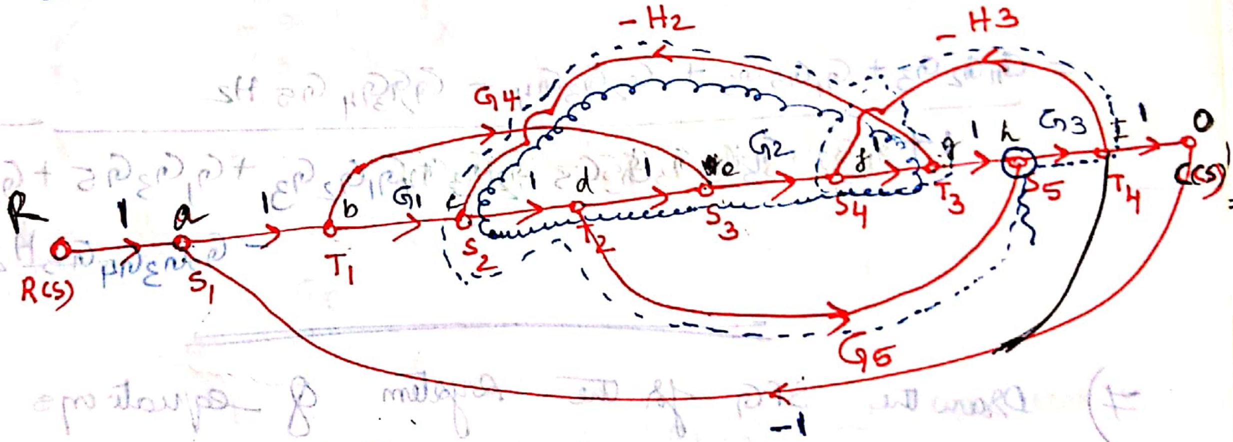
$= \frac{}{S^2 + 2\zeta\omega_n}$

6) Draw the SFG from the BD shown below, also find $\frac{C(s)}{R(s)}$ using Mason's gain formula.



Solution: Name the summing points as $S_1\ S_2\ S_3\ S_4\ S_5$ and take off points as T_1, T_2, T_3 & T_4.

⇒ Now construct the SFG :-

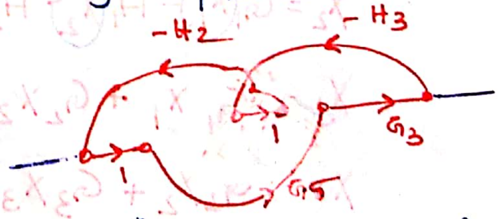


Loops

Step 1: Forward paths

$T_1 = G_1 G_2 G_3$, $T_2 = G_1 G_5 G_3$, $T_3 = G_4 G_2 G_3$

$T_4 = G_4 G_2 (-H_2) G_5 G_3$



Step 2:

Individual loops

$L_1 = -G_2 H_2$ $L_2 = -G_3 H_3$

$L_3 = (-H_2) G_5 G_3 (-H_3)$

& $L_3 = G_3 G_5 H_3 H_2$

$L_4 = -G_1 G_2 G_3$ $L_5 = -G_1 G_5 G_3$

3) Calculation of Δ

$L_6 = -G_4 G_2 G_3$ $L_7 = +G_4 G_2 H_2 G_5 G_3$

No of 2 non-touching loops :→ zero

$$\therefore \Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7]$$

4) as $K = 4$

$\Delta_K = \Delta_4$

As all the loops are touching the forward paths,

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

Then using Mason's gain formula;

$$T.F = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3 + T_4 \Delta_4}{\Delta}$$

$$= \frac{G_1 G_2 G_3 + G_1 G_5 G_3 + G_2 G_3 G_4 - G_2 G_3 G_4 G_5 H_2}{1 + G_2 H_2 + G_3 H_3 + G_3 G_5 H_2 H_3 + G_1 G_2 G_3 + G_1 G_3 G_5 + G_2 G_3 G_4 - G_2 G_3 G_4 G_5 H_2}$$

7) Draw the SFG for the system of equations given below & obtain the overall T.F. $\frac{X_6}{X_1}$ using MGF. VTU 2003, 2007

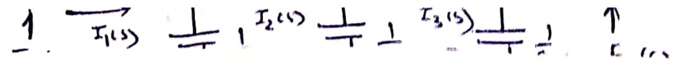
$X_2 = G_1 X_1 - H_1 X_2 + H_2 X_3 - H_6 X_6$ — ①

$X_3 = G_1 X_1 + G_2 X_2 - H_3 X_3$ — ②

$X_4 = G_2 X_2 + G_3 X_3 - H_4 X_5$ — ③

$X_5 = G_4 X_4 - H_5 X_6$ — ④

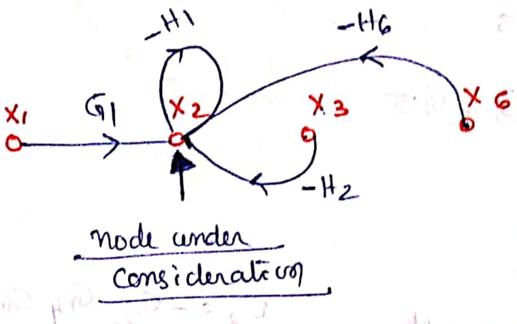
$X_6 = G_5 X_5$ — ⑤



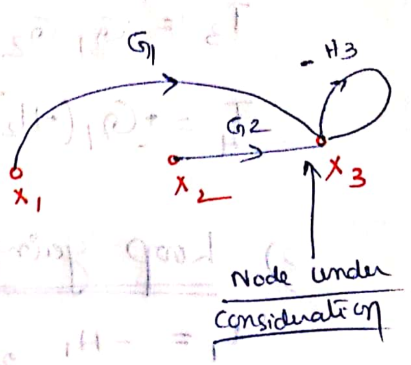
Solution :-

From the (5) equations, let us construct the SFG:

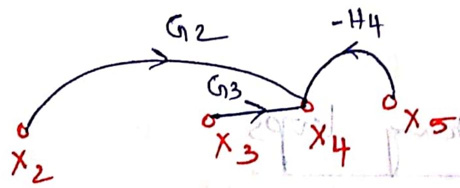
From eq (1)



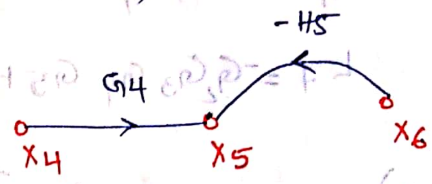
From eq (2)



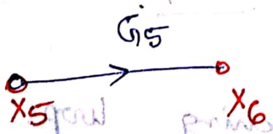
From eq (3)



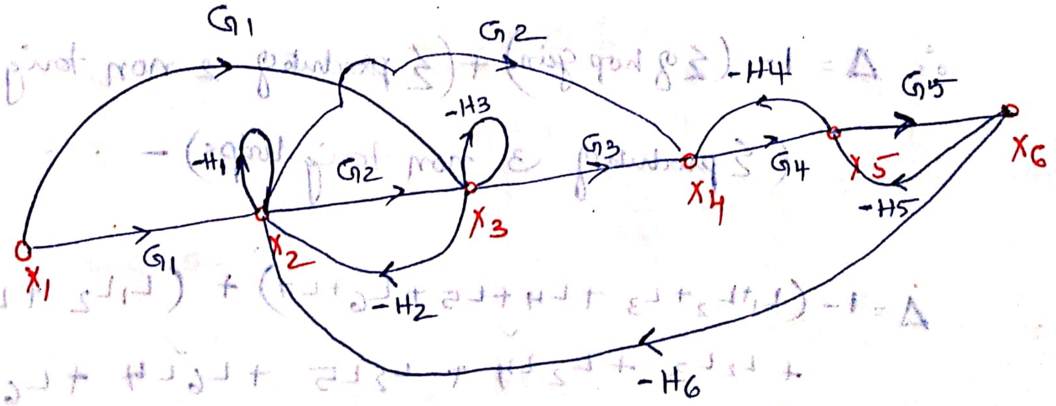
From (4)



From (5)



From the (5) individual SFG, lets construct 1 SFG.



1) forward path gains :

$T_1 = G_1 G_2 G_3 G_4 G_5$

$T_2 = G_1 G_3 G_4 G_5$

$T_3 = G_1 G_2 G_4 G_5$

$T_4 = +G_1(-H_2) G_2 G_4 G_5$

2) Loop gains

$L_1 = -H_1$, $L_2 = -H_3$, $L_3 = -G_2 G_4 G_5 H_6$

$L_4 = -G_4 H_4$ $L_5 = -G_5 H_5$ $L_6 = -G_2 H_2$

$L_7 = -G_2 G_3 G_4 G_5 H_6$

3) ⓐ combination of 2 non-touching loops

$L_1 L_2$ ~~$L_1 L_3$~~ $L_1 L_4$ $L_1 L_5$ ~~$L_1 L_6$~~ ~~$L_1 L_7$~~

$L_2 L_3$ $L_2 L_4$ $L_2 L_5$ $L_6 L_4$ $L_6 L_5$

ⓑ combination of 3 non-touching loops

$L_1 L_2 L_4$ $L_1 L_2 L_5$

$\therefore \Delta = 1 - (\Sigma$ of loop gain$) + (\Sigma$ products of 2 non touching loops$) - (\Sigma$ products of 3 non touching loops$) - \ldots$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7) + (L_1 L_2 + L_1 L_4 + L_1 L_5 + L_2 L_3 + L_2 L_4 + L_2 L_5 + L_6 L_4 + L_6 L_5) - (L_1 L_2 L_4 + L_1 L_2 L_5)$

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4) as No. of fward paths are 4,

$\Delta_k = \Delta_4$ $(\because k=4)$

all loops touching T_1 $\therefore \Delta_1 = 1$

for T_2; loop L_1 is non touig $\therefore \Delta_2 = 1 - \boxed{L_1}$

for T_3; loop L_2 is non touching $\therefore \Delta_3 = 1 - \boxed{L_2}$

for T_4; all loops are touching $\therefore \Delta_4 = 1$

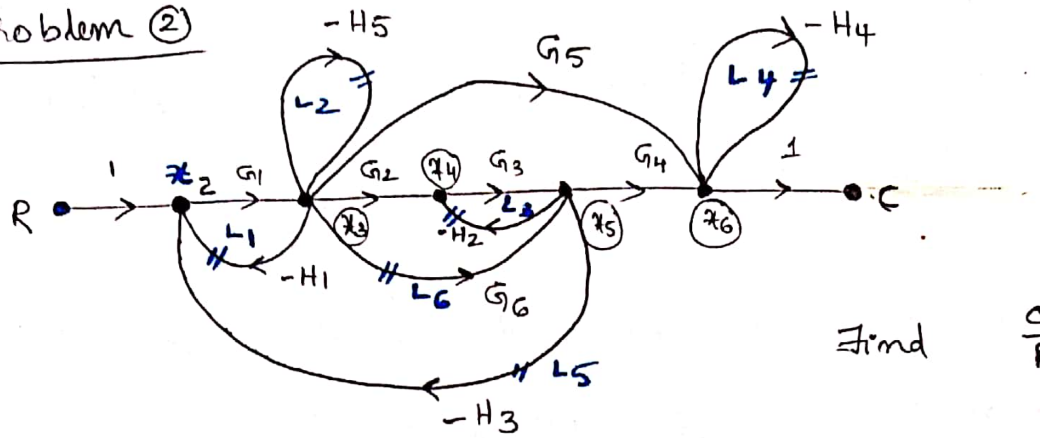
$\therefore$ using mason's gain formula;

$$\frac{X_6}{X_1} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_3G_4G_5(1+H_1) + G_1G_2G_4G_5(1+H_3) + G_1G_2G_4G_5(-H_2)}{1-(-H_1-H_3-G_2G_4G_5H_6-G_4H_4-G_5H_5-G_2H_2-G_2G_3G_4G_5G_6) + (H_1H_3 + H_1G_2H_4 + G_5H_1H_5 + G_2G_4G_5H_6H_3 + G_4H_3H_4 \quad G_5H_3H_5 + G_2G_4H_2H_4 + G_2G_5H_2H_5) + G_4H_1H_3H_4 + G_5H_1H_3H_5}$$

—— END ——

Problem ②



Find $\frac{C}{R}$

Solution :

1) Forward paths

$T_1 = 1 \times G_1 \times G_2 \times G_3 \times G_4 \times 1 = G_1 G_2 G_3 G_4$

$T_2 = 1 \times G_1 \times G_5 \times 1 = G_1 G_5$

$T_3 = 1 \times G_1 \times G_6 \times G_4 \times 1 = G_1 G_6 G_4$

2) Loops :

a) Individual Loops :

$L_1 = -G_1 H_1$

$L_2 = -H_5$

$L_3 = -G_3 H_2$

$L_4 = -H_4$

$L_5 = -G_1 G_2 G_3 H_3$

$L_6 = -G_1 G_6 H_3$

6 Loops

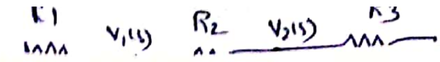
b) Two non touching loops :

$L_1 L_3$ $L_1 L_4$ $L_2 L_3$ $L_2 L_4$ $L_3 L_4$

$L_4 L_5$ $L_4 L_6$ 7 sets of 2 non touching loops.

c) 3 non touching loops :

$L_1 L_3 L_4$ $L_2 L_3 L_4$ ⇒ 2 3 non touching loops



3)

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4 + L_3L_4 + L_4L_5 + L_4L_6) - (L_1L_3L_4 + L_2L_3L_4)$$

$$\Delta = 1 + G_1H_1 + H_5 + G_3H_2 + H_4 + G_1G_2G_3H_3 + G_1G_6H_3 + G_1G_3H_1H_2 + G_1H_1H_4 + G_3H_2H_5 + H_4H_5 + G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_6H_3H_4 - (-G_1H_1G_3H_2H_4 - H_5G_3H_2H_4)$$

4) $\Delta_k = ?$ $K = 3$

Δ_1 for $T_1 = 1 - (0) = \underline{\underline{1}}$

Δ_2 for $T_2 = 1 - (L_3) = \underline{\underline{1 + G_3H_2}}$

Δ_3 for $T_3 = 1 - (0) = \underline{\underline{1}}$

$$\therefore \quad T(s) = \frac{C}{R} = \frac{\sum_{K=3} T_K \Delta_K}{\Delta} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$= \frac{G_1G_2G_3G_4 + G_1G_5 + G_1G_3G_5H_2 + G_1G_6G_4}{\Delta}$$ ← Plz. Substitute.

Say Suppose if you have been asked to find $\frac{x_2}{R} = T(s)$

???

Solution: Between R & x_2; we have only 1 forward path.

$\underline{\underline{T_1 = 1}}$ $\quad \therefore \quad T(s) = \frac{T_1\Delta_1}{\Delta}$

Δ will be the same only.

with T_1 forward path:-
The following loops are non touching

<u>individual loops</u> :→ L_2, L_3, L_4

<u>2 non-touching loops</u> :→ $L_2 L_3$ $L_2 L_4$, $L_3 L_4$

3 non-touching loops :→ $L_2 L_3 L_4$

$$\therefore \cancel{T(s)} = \quad \Delta_1 = 1 - (L_2 + L_3 + L_4) + (L_2L_3 + L_2L_4 + L_3L_4) - (L_2L_3L_4)$$

$$\therefore \Delta_1 = 1 + H_5 + G_3H_2 + H_4 + G_3H_2H_5 + H_4H_5 + G_3H_2H_4 + G_3H_2H_4H_5$$

$$\therefore \quad T(s) = \frac{x_2}{R} = \frac{1 \times \Delta_1}{\Delta}$$

← Substitute Δ_1

← Substitute Δ.

---x ---x ---x ---

④ Draw the SFG for the system of equations given below & obtain overall T.F. $\frac{x_6}{x_1}$ using MGF.

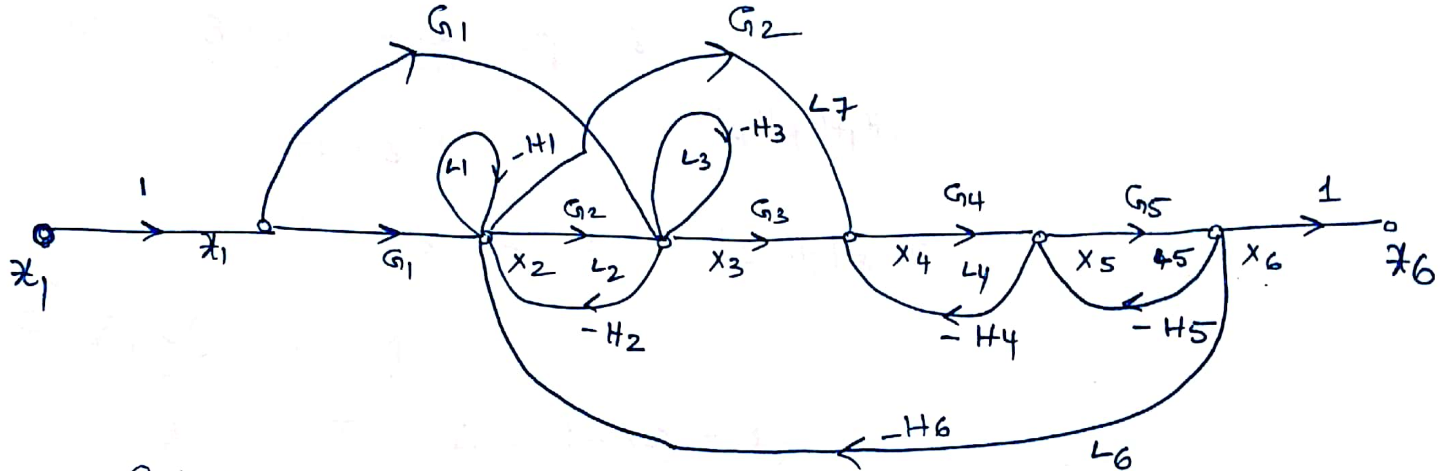
$$X_2 = G_1X_1 - X_2H_1 - H_2X_3 - X_6H_6$$

$$X_3 = G_1X_1 + G_2X_2 - H_3X_3$$

$$X_4 = G_2X_2 + G_3X_3 - H_4X_5$$

$$X_5 = G_4X_4 - H_5X_6$$

$$X_6 = G_5X_5$$



Solution :

① Forward paths:

$$T_1 = G_1G_2G_3G_4G_5$$

$$T_2 = G_1G_3G_4G_5$$

$$T_3 = G_1G_2G_4G_5$$

$$T_4 = G_1(-H_2)G_2G_4G_5$$

② Loops: (a) Individual loops

$L_1 = -H_1$

$L_2 = -G_2H_2$

$L_3 = -H_3$

$L_4 = -G_4H_4$

$L_5 = -G_5H_5$

$L_6 = -G_2G_3G_4G_5H_6$

$L_7 = -G_2G_4G_5H_6$

(b) 2 non touching loops

$L_1 L_3 = H_1 H_3$

$L_3 L_4 = + G_4 H_4 H_3$

$L_1 L_4 = + H_1 H_4 G_4$

$L_3 L_5 = + G_5 H_5 H_3$

$L_3 L_7 = + G_2 G_4 G_5 H_6 H_3$

$L_1 L_5 = + G_5 H_5 H_1$

$L_2 L_4 = + G_2 H_2 G_4 H_4$

$L_2 L_5 = + G_2 H_2 G_5 H_5$

(c) 3 non tong loops :

$L_1 L_3 L_4 = - G_4 H_4 H_1 H_3$

$L_1 L_3 L_5 = - G_5 H_1 H_3 H_5$

(3) $\Delta_K = ?$ $K = 4$

Δ_1 for T_1 $\Delta_1 = 1 - (0) = 1$

Δ_2 for T_2 $\Delta_2 = 1 - (-H_1) = 1 + H_1$

Δ_3 for T_3 $\Delta_3 = 1 - (-H_3) = 1 + H_3$

Δ_4 for T_4 $\Delta_4 = 1 - (0) = 0$.

(4) $\Delta = 1 + H_1 + G_2 H_2 + H_3 + G_4 H_4 + G_5 H_5 + G_2 G_3 G_4 G_5 H_6$
$+ G_2 G_4 G_5 H_6 + H_1 H_3 + H_1 H_4 G_4 + G_5 H_5 H_1 + G_2 H_2 G_4 H_4$
$+ G_4 H_4 H_3 + G_5 H_5 H_3 + G_2 G_4 G_5 H_6 H_3 + G_2 H_2 G_5 H_5$
$+ G_4 H_4 H_1 H_3 + G_5 H_1 H_3 H_5$ —— (i)

$$\therefore \quad \frac{X_6}{X_1} = \frac{G_1G_2G_3G_4G_5 + G_1G_3G_4G_5(1+H_1) + G_1G_2G_4G_5(1+H_3) - G_1G_2G_4G_5H_2}{\Delta}$$

Where Δ is given by equation (1)

———— x ———— x ————

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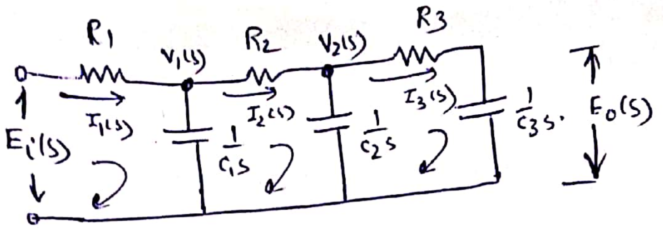


$I_3(s)$

$V_1(s)$ $\quad$ $I_2(s)$ $\quad$ $V_2(s)$

~~$V_1(s)$~~   Signals : $E_i(s)$ $\quad$ $I_1(s)$ $\qquad$ $E_0(s)$

Variables //

node Value

$$\Rightarrow I_1(s) = \frac{E_i(s) - V_1(s)}{R_1} = \left(\frac{1}{R_1}\right) E_i(s) - \left(\frac{1}{R_1}\right) V_1(s) \quad -(1)$$

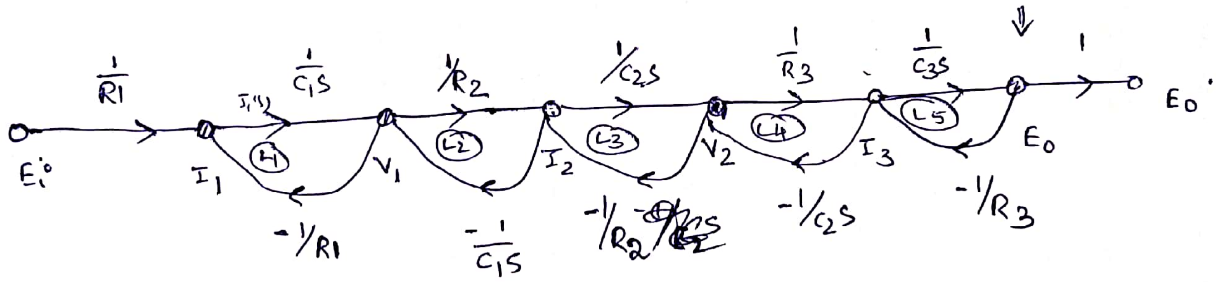
$$V_1(s) = \left(I_1(s) - I_2(s)\right)\frac{1}{C_1 s} = \left[\frac{1}{C_1 s}\right] I_1(s) - \left[\frac{1}{C_1 s}\right] I_2(s) \quad -(2)$$

$$I_2(s) = \frac{V_1(s) - V_2(s)}{R_2} = \left[\frac{1}{R_2}\right] V_1(s) - \left[\frac{1}{R_2}\right] V_2(s) \quad -(3)$$

$$V_2(s) = \left[\frac{1}{C_2 s}\right] I_2(s) - \left[\frac{1}{C_2 s}\right] I_3(s) \quad -(4)$$

$$I_3(s) = V_2(s)\left(\frac{1}{R_3}\right) - \left(\frac{1}{R_3}\right) E_0(s) \quad -(5)$$

$$E_0(s) = \frac{1}{C_3 s} \times I_3(s)$$



Forward path : $T_1 = \frac{1}{(R_1 R_2 R_3 C_1 C_2 C_3) s^3}$

Loops : L_1, L_2, L_3, L_4, L_5

2 non-tay loops $\quad L_1 L_3 \quad L_1 L_4 \quad L_1 L_5$

$L_2 L_4 \quad L_2 L_5$

$L_3 L_5$

3 non-tay loops : $L_1 \times L_3 L_5$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1L_2 + L_1L_4 + L_1L_5 + L_2L_4 + L_2L_5 + L_3L_5) - (L_1L_3L_5)$$

$$\Delta = (1 + \frac{1}{R_1C_1s} + \frac{1}{R_2C_1s} + \frac{1}{R_2C_2s} + \frac{1}{R_3C_2s} + \frac{1}{R_3C_3s}$$

$$+ \frac{1}{s^2 R_1R_2C_1C_2} + \frac{1}{s^2 R_1R_3C_1C_2} + \frac{1}{s^2 C_1C_3R_1R_3} + \frac{1}{s^2 R_2R_3C_1C_2} + \frac{1}{s^2 R_2R_3C_1C_3}$$

$$+ \frac{1}{s^2 C_2C_3 R_2R_3} + \frac{1}{s^3 C_1C_2C_3R_1R_2R_3}$$

<u>Calculation of Δ_K:</u>

$K = 1 \quad \therefore \quad \Delta_1 = 1 - (0) = \underline{\underline{1}}.$

$$\therefore \quad T(S) = \frac{\dfrac{1}{s^3 (R_1R_2R_3C_1C_2C_3)}}{1 + \dfrac{1}{R_1}}$$

$=$