CONTROL SYSTEMS

Unit 1 : Modelling of systems :

INTRODUCTION:

When a number of elements & components are connected in a sequence to perform a specific task, the group thus formed is called a "system".

In a system when the output quantity is controlled by varying the input quantity, then the system is called a "control system".

The output quantity is called controlled variable or response and the input quantity is called command signal or excitation.

Types of control systems :

Control systems are broadly classified into 2 categories

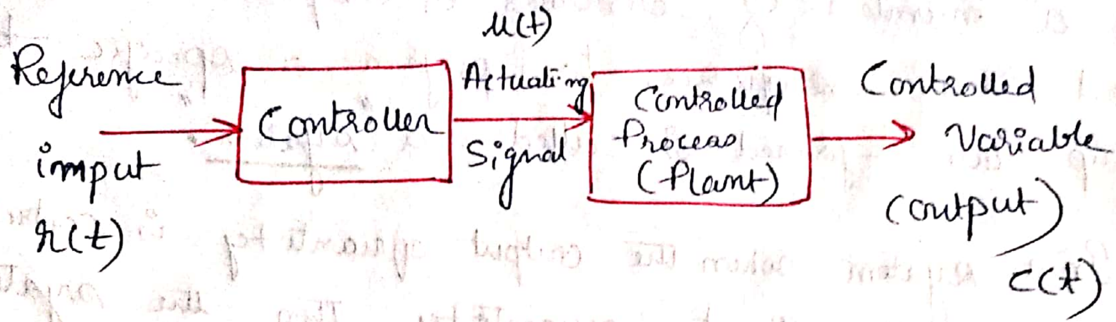
① Open loop system

② closed loop system

① Open loop control systems :

Those systems in which output has no effect on the control action i.e on the input are called "open loop control systems". There fore in open loop control system, the output is neither measured nor feed back for the comparision with the input. Open loop control systems do not have feed back syste

General block diagram of an open-loop control system:



* Reference input r(t) is applied to the controller which generates the actuating signal μ(t) required to control the process which is to be controlled. Process will give the desired result & output c(t).

Advantages: (July 2009, Jan 06)

1) Open loop systems are simple in construction.
2) They do not require many system components and hence system complexicity is less.
3) Maintaince is less owing to less no. of components.
4) Stability of the system is good.
5) Economical in the cost point of view.

Disadvantages:

1) Accuracy of the system is poor.

Scanned by CamScanner

2) Systems are most likly to give error prone results.

3) To maintain the quality and accuracy recaliberation of controller is necessary.

4) Systems cannot sense environmental changes.

5) When high degree of accuracy is required, open loop systems can not be used.

<u>Ex:</u>

1) A good example for an open loop system is an "<u>electric ~~[illegible]~~ switch</u>" controlled room CFL Lamp. Any change in external light (i.e SUN's light) does not have any effect on the CFL Lamp. Lamp here (switch) is controlled by means of a controller over.

2) Second example is "<u>Washing Machine</u>". Here soaking, washing and rinsing in the washer operate on time basis. The machine does not measure the output signal (i.e cleanliness of the clothes.

(Jan 2005, 06, 08, 09, 07, 11)

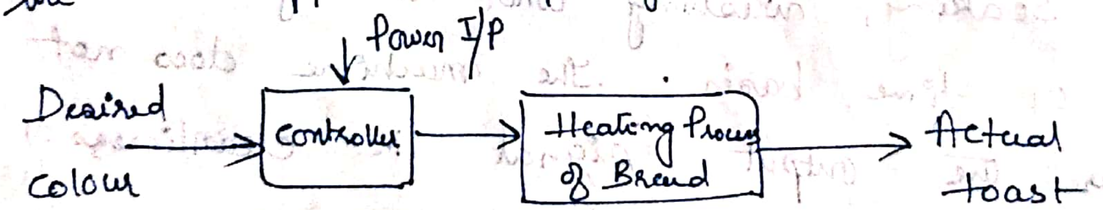
<u>Real time Applications of an Open loop systems</u>

1) <u>Sprinkler used to water a Lawn area</u>:

Here the system is adjusted to water a given area, is done by opening a water valve and Observing the resulting pattern. When we are happy with the results, we keep the valve position to the set point and no further valve adjustment is done.

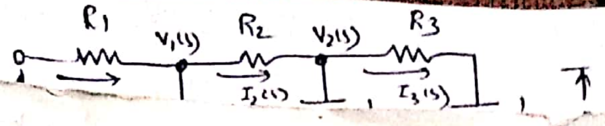
2) Automatic Toaster system :-

Let us assume that we want to toast a bread to a desired colour (usually light brown). The desired colour depends on the time set for toasting. Here the desired colour of timer knob represents the input, quantity and degree of colour of the toasted bread is the o/p quantity. If the colour of the toasted bread is not satisfactory due to some reason, then this condition can not be reverted back automatically, as the heat applied is predefined & cannot be changed.

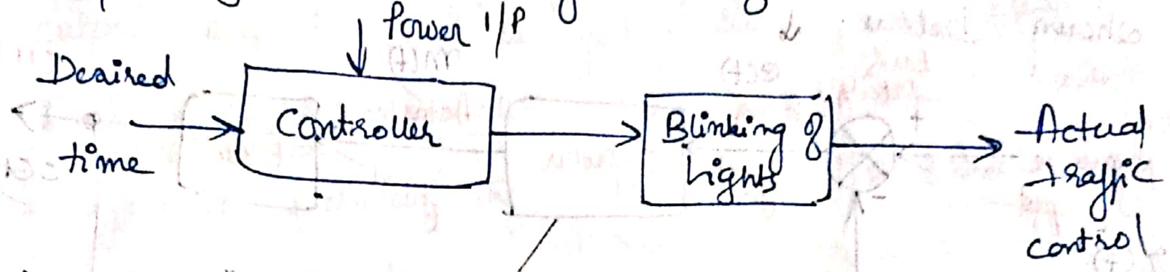


3) Traffic light Controller :

A traffic light control system used on roads is time dependent. The traffic on the road, becomes either dynamic or static, depending on the duration and sequence of lamp glow. At any point



traffic control system can not make out, whether there is traffic or not. That means to say, it may will follow a definite sequence of red light even though the traffic is zero.



4) D.C. shunt motor : (armature control)

For a given field current, a voltage is applied to the armature to produce the desired value of motor speed. If the motor speed is changed, due to change in mechanical load on the shaft, there is no way in the open loop system to change the value of armature voltage to maintain the desired speed.

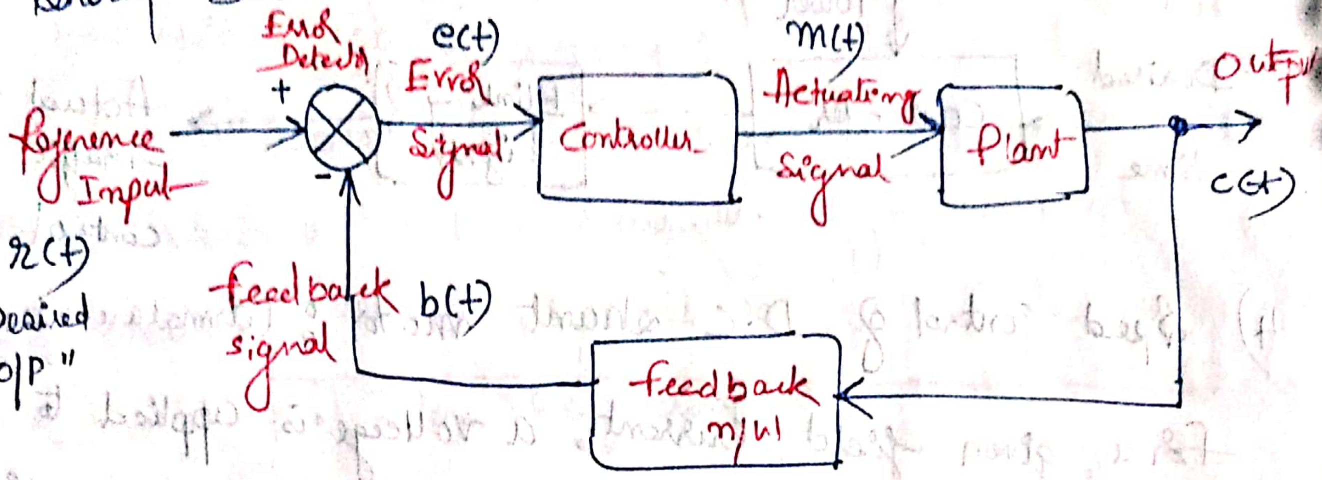
other examples are,
Room heater, fan regulator, automatic coffee server, electric lift, Diming of electric light etc.

closed loop system :

A system in which the controlling action of input is somehow dependent on the output & changes in output is called closed loop system.

In order to have this dependency b/w input and output, a feedback mechanism should be incorporated in the system.

The block diagram of a closed loop system is shown below.



In practice the terms, 'Closed loop control' and 'feedback control' are used interchangeably. In a closed loop control system, the error signal which is the difference between the input signal and the feedback signal is fed to the controller so as to reduce the error and bring the output of the system to a desired value.

"A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system."

The reference input corresponds to desired output. The feedback network will convert the output signal to a signal of the same type, as that of the reference signal. The feedback signal is proportional to the output signal and is fed to the error detector. Error detector generates the error signal which the difference between the Reference i/p signal and the feedback signal.

The controller modifies and amplifies the error signal to produce better control action. The output of the controller is called as the actuating signal which performs corrective actions on the plant so as to correct the output.

Here $r(t)$ = Reference i/p $\qquad e(t)$ = Error signal

$c(t)$ = Controlled output $\qquad m(t)$ = Actuating signal

$b(t)$ = feedback signal.

Advantages: Jan 2005, 06, 08, 09, 2011

1) Accuracy of such system is always high because controller modifies and manipulates the actuating signal such that the error in the system is negligibly small.

2) Such system senses environmental as well as internal disturbances and accordingly modifies error.

3) In such system, there is reduced effect of nonlinearities and distortions.

4) Bandwidth of such system is very high.

5) Automatic control is possible.

Disadvantages:

1) Such systems are complicated and time consuming from design point of view.

2) Costlier due to the design complexity.

3) feedback reduces the overall gain of the system.

4) The feedback ~~m/s~~ in a closed-loop system may lead to oscillations & thus the system may give oscillatory response.

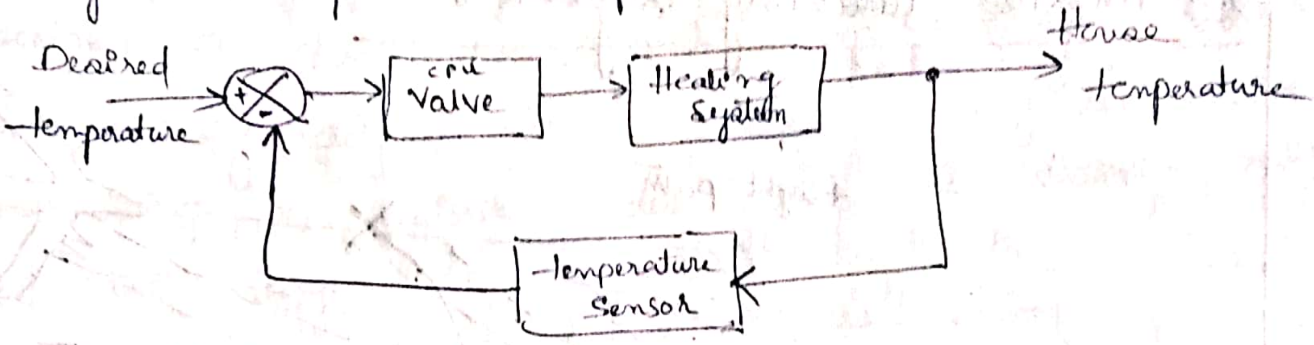
5) They consume more power.

6) They are highly unstable. Hence precautions has to be taken to maintain the stability.

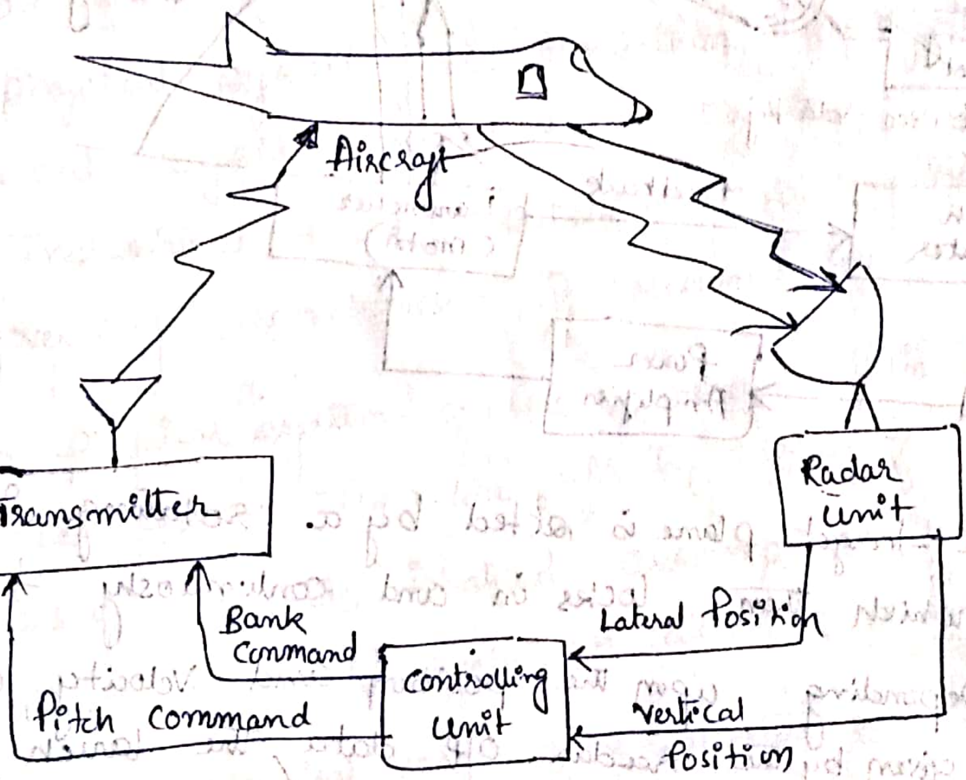
Real time Applications of closed loop system:

1) Home heating system: The Room heating system is operated by a valve. The actual temperature is sensed by a thermal sensor &

compared with the desired temperature. The difference b/w the two, actuates the valve mechanism to change the temperature as per the requirement.



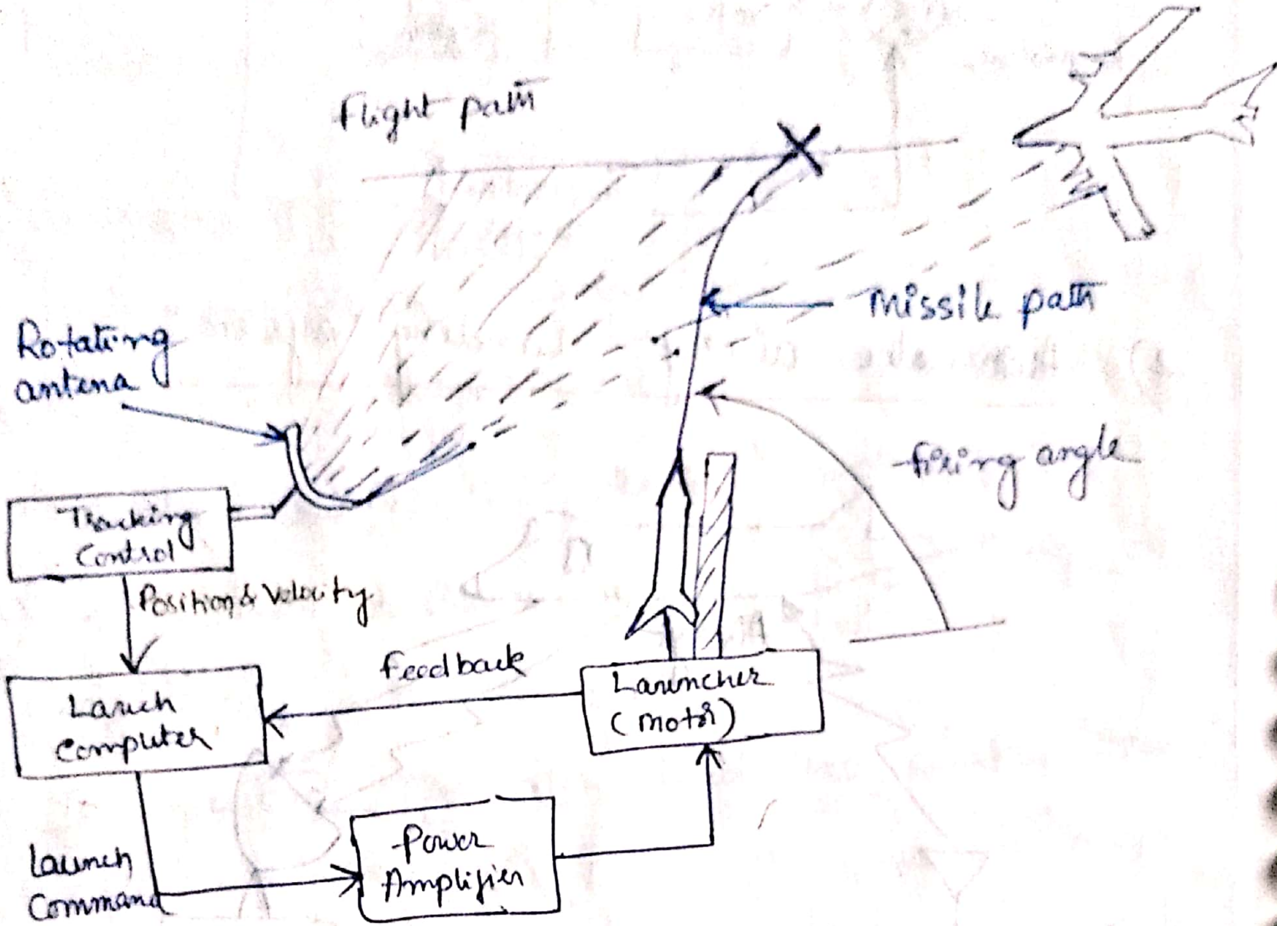
2) Automatic aircraft Landing system :



The system consists of 3 basic parts. The aircraft, the radar unit & the controlling unit.

The radar unit measures the approximate vertical & lateral positions of the aircraft, which are then transmitted to the controlling unit. From these measurements, the controlling unit calculates approximate pitch & bank commands. These commands are then transmitted to the aircraft autopilots which in-turn causes the aircraft to respond.

3) Missile lanching system :



Here the target plane is alted by a rotating radar antena, which then locks in and continuosly tracks the target. Depending upon the position and Velocity of the plane as given by the radar o/p data, the lanch computer calculates the firing angle interms of a launch command signal, which then amplified through Power amplifier. The amplified signal then drives the launcher (D serve motor). The lancher's angular position is fedback to the lanch computer & the missile is triggered as soon as the error b/w the launch command signal & the missile firing angle becomes zero. After being fired, the missile enters the radar beam; which is tracking the target. The control system contained within the missile now receives a guidance signal from the beam, which automatically adjusts the control of the missile such that the missile rides along the beam, finally hits the target.

* Modeling of systems :

Introduction :

A physical system is a collection of physical objects connected together to serve an objective.

ex: Industrial Plant, satellite orbiting the earth, governing mechanism of steam turbine. etc.

No physical system can be represented in its full (intricacies) complicated form. Therefore, for the purpose of analysis and synthesis of systems, idealized assumptions are made. An idealized physical system is called a physical model.

Once the physical model of the system is obtained, then we go for the mathematical model. which is the mathematical representation of the physical model, expressed interms of appropriate physical laws.

ex for mathematical models are;

() Electrical mathematical model :

An electric network may be modelled as a,

* set of nodal equations using KCL.

OR

* set of mesh equation's using KVL.

OR

* State Variable model ; with a help of state variable Vector matrix differential equation.

Once the mathematical model of a system is available, Then with the help of mathematical tools like, the Fourier transform or Laplace transform, analysis & synthesis of the system is done.

Summery:

Mathematical model: A set of mathematical equations, describing the dynamic characteristics of a system.

Purpose of mathematical Model:

To analyze the complicated system and to have a better visualization of the system, a mathematical model is constructed.

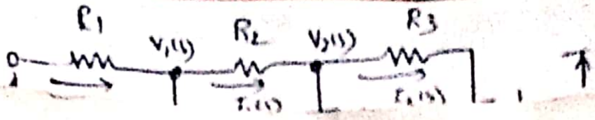
Types of systems:

① Mechanical system.

② Electrical System.

① Mechanical systems:

Mechanical systems and devices can be modelled by means of three ideal translatory and 3 ideal rotory elements. They ~~are~~ are,



Rotary elements	Translatory elements
Inertia (J)	Mass (M)
Damper (f & B)	Damper (f & B)
Spring (K)	Spring (K)

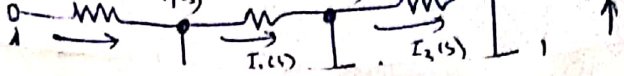
Motion in mechanical systems can be of different types they could be,

1. Rotation motion 
2. Translatory motion 
3. Combination of both

....

" According to newton's law of motion, Sum of forces applied on a system or rigid body must be equal to sum of forces consumed to produce displacement, velocity and acceleration in various elements of the system."

..... Contd....



Variables of Mechanical Elements :-

* Mass / Inertia and the two kinds of springs are the energy storage elements, where in energy can be stored and retrieved without loss. Hence they are called conservative elements.

Energy stored in these elements is expressed as;

Mass : $E = \frac{1}{2} m v^2$ = Kinetic Energy (J); motional energy.

Inertia : $E = \frac{1}{2} J \dot{\omega}^2$ = Kinetic Energy (J); motional energy.

Spring (translatory) : $E = \frac{1}{2} K x^2$ = Potential Energy (J); Deformation energy.

Spring (Torsional) : $E = \frac{1}{2} K \theta^2$ = Potential Energy (J)

; Deformation Energy.

Damper is a dissipative element and power it consumes (lost in the form of heat) is given as

$$\boxed{\begin{aligned} P &= B v^2 \,(W) \rightarrow \text{translational} \\ P &= B \omega^2 \,(W) \rightarrow \text{torsional} \end{aligned}}$$

→ Translational Systems

(1) Mass Element : (M) :

* It stores the kinetic energy of the translational motion
* Mass has no power to store potential energy

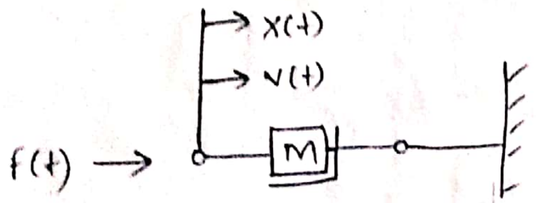


Fig 2.3.1

$$F(t) = M \frac{d\,v(t)}{dt} = \text{Force Velocity}$$

$$f(t) = M \frac{d^2 x(t)}{dt} = M \cdot$$

In the S Domain (Laplace transformation of time Domain functions)

$$\boxed{\begin{aligned} F(s) &= M \cdot s\, V(s) \rightarrow \\ F(s) &= M \cdot s^2 X(s) \rightarrow \end{aligned}}$$

(2) The Spring Element (K) : —

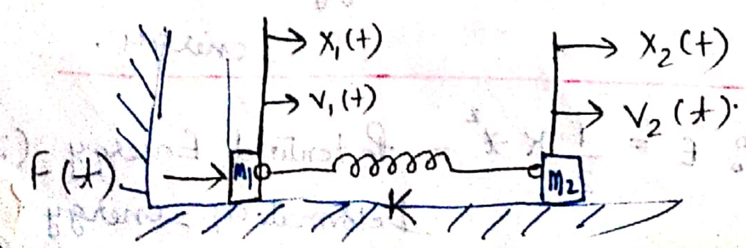
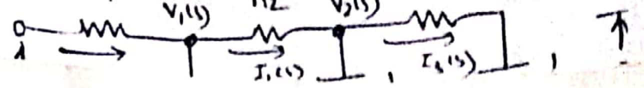


Fig 2.3.2 (a)



Here "K" is the spring constant.

Here force required to cause the displacement in the spring is propotional to the displacement.

In fig 2.3.2 (a), spring is connected between the two moving elements having masses M_1 & M_2.

When a force is applied to mass M_1, mass M_1 will get displaced by $x_1(t)$; but mass M_2 will also get displaced by $X_2(t)$ as spring of constant K will store some potential energy; this is the cause for the change in displacement.

Let's draw the free body diagram of fig 2.3.2(a);

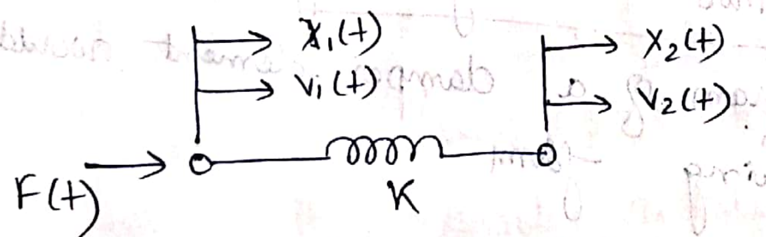


fig 2.3.2 (b)

$$\boxed{F = K \int_{-\infty}^{t} (V_1 - V_2)\, dt}$$

force ⇓ velocity

In fig 2.3.2 (b), the net displacement in the spring is $X_1(t) - X_2(t)$.

time Domain.

Then F(t) of simply or

$$\boxed{F = K\,(X_1(t) - X_2(t))}$$

$$\boxed{F = K\,[X_1(s) - X_2(s)]}$$

S domain

$\boxed{F = K\,[x(t)]}$ if one end is fixed w.r.t. other

(3) The Damper Element (B):

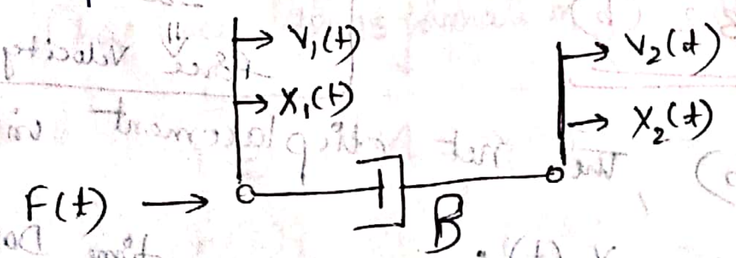
Whenever there is a motion, there exists a friction. friction may be between moving element and fixed support or between two moving surfaces. It can be divided into 3 types,

① Viscous friction ② Static friction ③ Coulomb friction.

Viscous friction is more dominant compared to other 2 & hence they are neglected.

* Viscous friction is assumed to be linear with frictional constant B.
* This has linear relationship with the relative velocity between two moving surfaces.

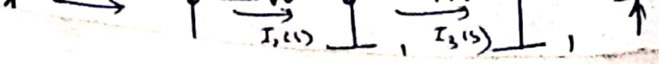
* The freebody diagram of a damper element would take up the following form



Then $F = B(v_1(t) - v_2(t))$

$$\boxed{F = B\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right)}$$ *

If the one end of the damper is fixed w.r.t. the other then F is given as

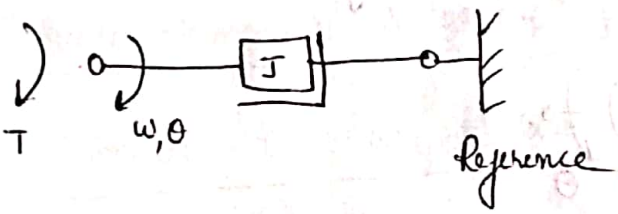


$$F = B \frac{d\,x(t)}{dt} \rightarrow \text{time Domain}$$

$$F(s) = BS\,X(s) \rightarrow \text{'s' domain}$$

Rotational elements

(4) The inertia Element (J)



$M = \frac{W}{g} = \frac{kg}{m/s}$

$= \frac{kg}{\frac{m}{s}}$ kg/m

$$T = J \frac{d^2\theta}{dt^2} \quad \&$$

$$T(s) = J s^2 \cancel{A}\, \theta(s)$$

Also

$$T = J \cdot \frac{d\omega}{dt}$$

$$T(s) = J\, s\, \omega(s)$$

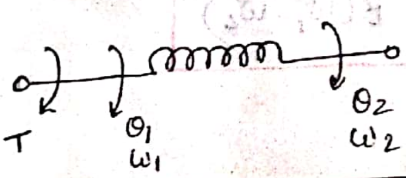
where $\frac{d^2\theta}{dt^2}$ = angular acceleration rad/sec^2

$\omega = \frac{d\theta}{dt}$ = angular Velocity rad/sec

J = Moment of Inertia kg-m^2/rad

θ = angular displacement rad

(5) (a) Spring Element (K): (Spring with both ends free)



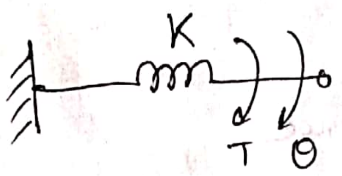
⇒ Torque - Angular Velocity

$$\boxed{T = K\int(\omega_1 - \omega_2)\,dt}$$

⇒ Torque - Angular Displacement

$$\boxed{T = K(\theta_1 - \theta_2)}$$ —— *

*(b) Spring with one end only free (K) :



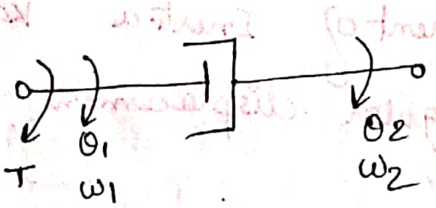
⇒ Torque - Angular Velocity

$$\boxed{T = K\int \omega\,dt}$$

⇒ Torque - Angular Displacement

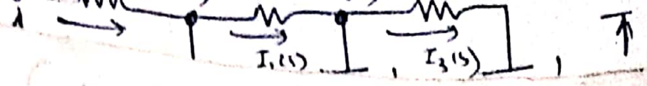
$$\boxed{T = K.\theta}$$ —— *

6) (a) Damper element with both ends Free (B) :-



Torque - Angular Velocity

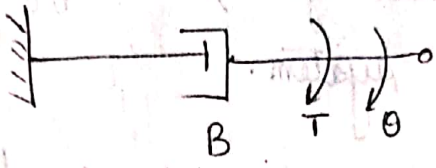
$$\boxed{T = B(\omega_1 - \omega_2)}$$



Torque - angular displacement

$$\boxed{T = B\left[\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right]}$$ → *

*(b) Damper element with one end fixed (B) :



Torque - angular Velocity : $T = B\omega$

Torque - angular displacement : $T = B\frac{d\theta}{dt}$

Table : (1)

translatical.

Sl. No.	Translational Motion	Rotational Motion	Electrical System.
1.	Mass (M)	Inertia (J)	Inductance (L)
2.	friction (B)	friction (B)	Resistance (R)
3.	Spring (K)	Spring (K)	$1/C$
4.	force (F)	Torque (T)	Voltage (V)
5.	Displacement (x)	Angular Displacement (θ)	charge (q)
6.	Velocity $v = \frac{dx}{dt}$	Angular Velocity $\omega = \frac{d\theta}{dt}$	$i = dq/dt$
7.	Acceleration $\frac{d^2x}{dt^2}$	Angular Acceleration $\frac{d^2\theta}{dt^2}$	—

Force - Voltage Analogy

Equivalent mechanical system (Node Basis)

while drawing analogous networks, it is always better to draw the equivalent mechanical system from the given mechanical system.

Steps:

1) Due to the applied force, identify the displacements in the mech. system.

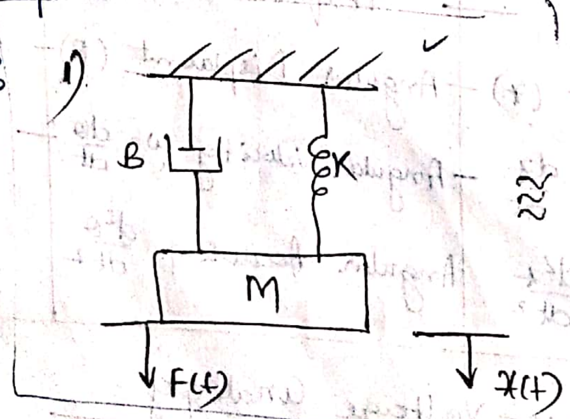
3) Identify the elements which are under the influence of diff. displacements.

2) Represent the each displacement by a seperate node, using nodal analysis.

4) Show all the elements in parallel under the respective nodes which are under the influence of respective displacments.

5) Elements causing the same change in displacemt, will get connected in parallel in between the respective nodes.

ex: 1)



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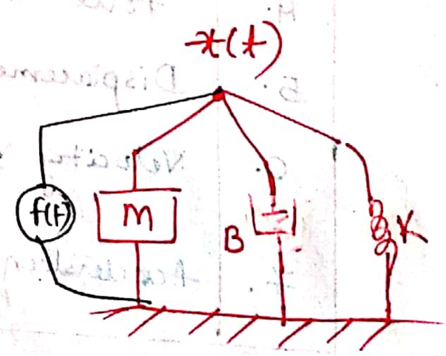
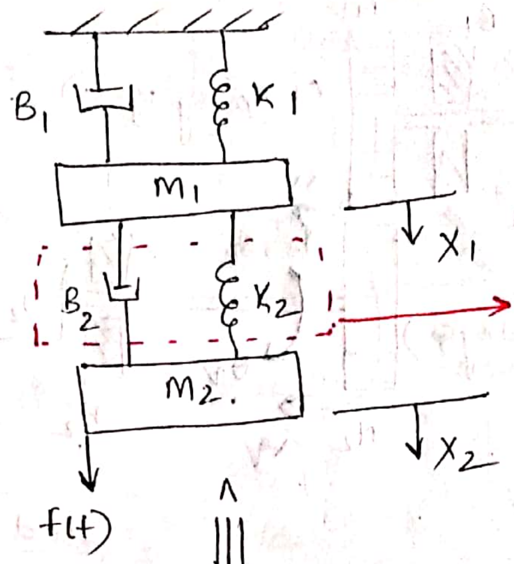
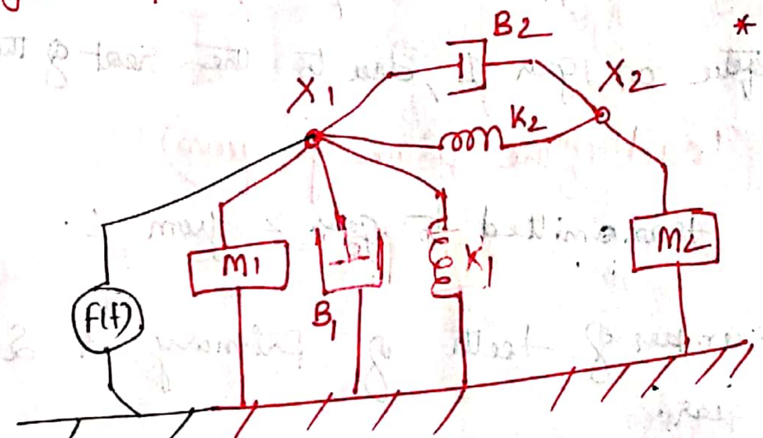


Here all the elements are under the same displacement ex: 2



Here B_2 & K_2 causes displacements x_1 & x_2 & hence to be connected in parallel b/w x_1 & x_2 node :- Step 5

Analogous n/w



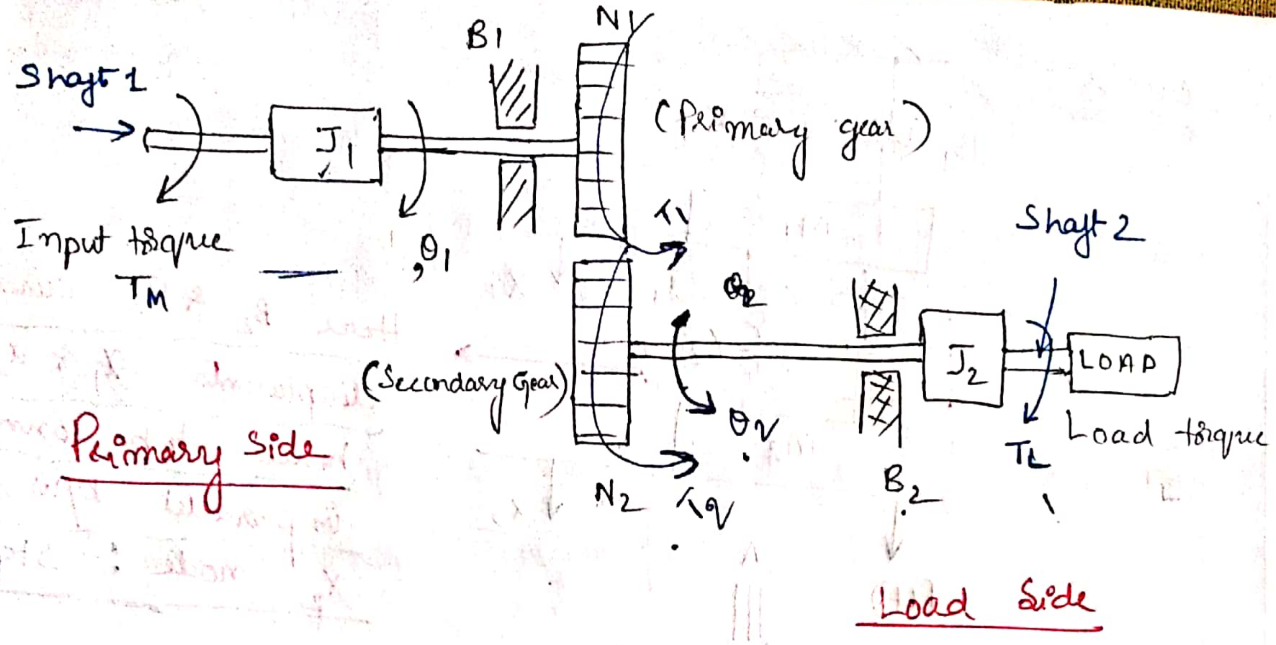
* No mass can be b/w the 2 nodes. Bcoz, mass can't store pot energy & therefore no change in force.

Gear Trains :-

A gear ~~tra~~ train is a mechanical matching device, that transmits energy from one part of the system to another part with maximum power transfer. Transformers are electrical analogous to gears.

For ideal gear trains, friction and inertia is neglected. But in practice, friction & inertia of the gear assembly is accounted.

fig below shows the schematic diagram of 2 gear trains.



Let 'T_m' be the Applied torque of motor torque.

T_1 = Load torque on gear 1, due to the rest of the gear train. (dead load of the primary gear)

T_2 = torque transmitted to gear 2 from 1.

N_1 & N_2 = Number of teeth of primary & Secondary gear.

θ_1, θ_2 = Angular displacements of gear 1 & gear 2.

J_1, J_2 = Inertia of gears 1 & 2.

B_1, B_2 = Friction Co-efficients of gear 1 & 2.

Torque equation of Primary side :

$$T_m = J_1 \frac{d^2\theta_1(t)}{dt^2} + B \frac{d\theta_1}{dt} + T_1 \quad \text{--- (1)}$$

or $T_m = J\ddot{\theta}_1 + B_1\dot{\theta}_1 + T_1$

Torque equation of Secondary side :

$I_1(s) \perp$, $I_3(s) \perp$, ↑

$$T_2 = J_2\frac{d^2\theta_2(t)}{dt^2} + B\frac{d\theta_2(t)}{dt} + T_L \quad — (2)$$

or $$T_2 = J_2\ddot{\theta}_2 + B\dot{\theta}_2 + T_L$$

Let r_1 be the radius of gear 1, &

r_2 " " " " " 2

* It's a known fact that, distance travelled along the surface of each gear same,

we can write $\theta_1 r_1 = \theta_2 r_2$

or $$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} \quad — (3)$$

In a gear system

* $N_1 \propto r_1$

& $N_2 \propto r_2$ $\Rightarrow$ $$\frac{N_1}{N_2} = \frac{r_1}{r_2} \quad — (4)$$

or $\cancel{\frac{N_2}{N_1}} = \cancel{\frac{r_2}{r_1}}$

* Also for an ideal gear system,

work done by gear 1 = work done by gear 2

i.e $T_1\theta_1 = T_2\theta_2$

∵ $P \propto T\omega$

$P \propto T\frac{d\theta}{dt} \propto T\frac{\theta}{t}$

$P \times t = T\theta$

$\boxed{\text{workdone} = T\theta}$

or $$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} \quad — (5).$$

Comparing eq (5), (4) & (3) we can write.

$$\boxed{\frac{T_1}{T_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}} \quad — (6)$$

From eq (6) we can write,

$$T_2 = \frac{N_2}{N_1} \times T_1 \quad \text{--- (7)}$$

substitute eq (2) in eq (7)

$$\frac{N_2}{N_1} \times T_1 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\left[\because \text{taking } \frac{d^2\theta_2(t)}{dt^2} \text{ as } \frac{d^2\theta_2}{dt^2}\right]$$

$$\text{or } T_1 = \left(\frac{N_1}{N_2}\right) J_2 \cdot \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \cdot \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L \quad \text{--- (8)}$$

sub eq (8) in eq (1)

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \cdot \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L \quad \text{--- (9)}$$

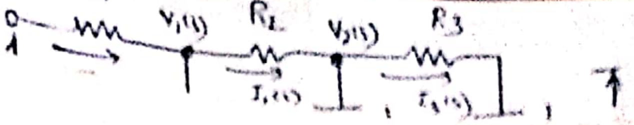
$\theta_2 = \frac{N_1}{N_2} \theta_1$

$\dot{\theta}_2 = \frac{N_1}{N_2} \times \dot{\theta}_1$

$\ddot{\theta}_2 = \frac{N_1}{N_2} \times \ddot{\theta}_1$

substituting $\theta_2 = \frac{N_1}{N_2} \theta_1$ (from eq (6)) in eq (9)

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \left(\frac{N_1}{N_2}\right) \cdot \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \cdot \left(\frac{N_1}{N_2}\right) \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L.$$



$\therefore$

$$T_m = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right]\frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right]\frac{d\theta_1}{dt}$$

$$+ \frac{N_1}{N_2} \cdot T_L \quad \text{—— (10)}$$

Let $J_{1e} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$ = Equivalent inertia referred to primary side.

$B_{1e} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$ = Equivalent friction referred to primary side.

$$\boxed{\therefore \; T_m = J_{1e}\frac{d^2\theta_1}{dt^2} + B_{1e}\frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L}$$

Similarly the equation can be written referred to load side, where applied torque gets transferred to load as $\frac{N_2}{N_1} \cdot T_m$

$$\left(\frac{N_2}{N_1}\right) T_m = J_{2e}\frac{d^2\theta_2}{dt^2} + B_{2e}\frac{d\theta_2}{dt} + T_L$$

where $J_{2e} = J_2 + \left(\frac{N_2}{N_1}\right)^2 J_1$ and

$B_{2e} = B_2 + \left(\frac{N_2}{N_1}\right)^2 B_1$

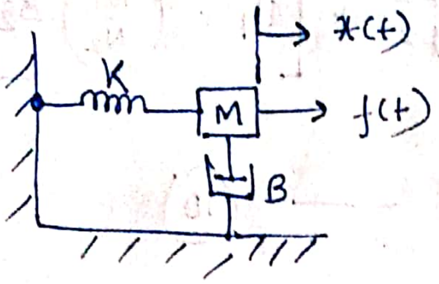
Note: For gear system: $\Rightarrow$ $\left.\begin{array}{l} r_1 \propto N_1 \\ r_2 \propto N_2 \end{array}\right] - (1)$ $\Rightarrow \frac{N_1}{N_2} = \frac{r_1}{r_2}$

$\Rightarrow$ $\left.\begin{array}{l} s_1 = s_2 \\ r_1\theta_1 = r_2\theta_2 \end{array}\right] - (2)$ $\rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$

$\Rightarrow$ Work done 1 = Work done 2
$T_1\theta_1 = T_2\theta_2$ $\Rightarrow$ $\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1}$

* Writing differential equantions:

1



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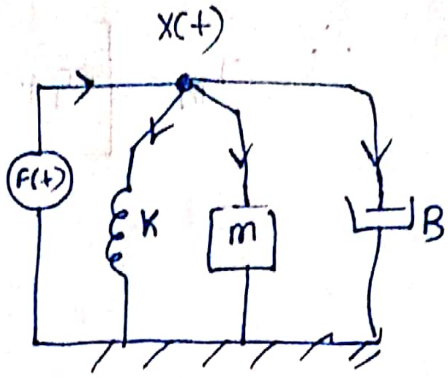


Equilibrium equation will be, in accordance with the Newton's Law of motion. Applied force will cause displacement $x(t)$ in spring, acceleration to mass M against frictional force having constant B.

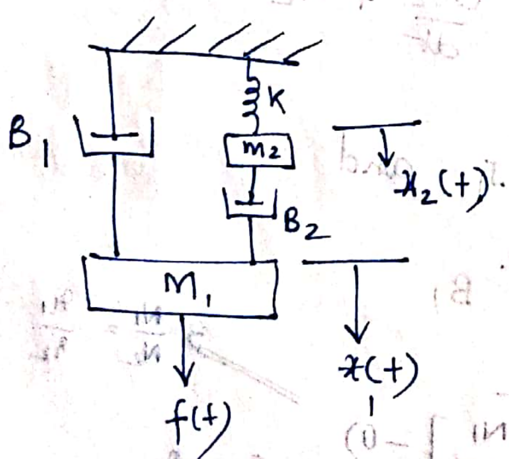
$$\therefore \quad F(t) = \underbrace{Ma}_{\text{acceleration}} + \underbrace{BV}_{\text{Velocity}} + \underbrace{K\,x(t)}_{\text{Displacement}}$$

$$\therefore \quad F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx}{dt} + K\,x(t)$$

taking Laplace transform, $F(s) = Ms^2 X(s) + Bs\,X(s) + K\,X(s)$

i.e $\boxed{F(s) = (Ms^2 + Bs + K)\,X(s)}$

2



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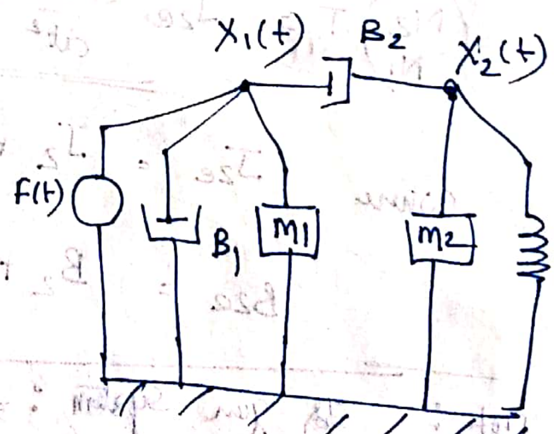


at node 1 i.e $X_1(t)$;

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + B_2 s\,(X_1(s) - X_2(s)) \quad \text{--- (1)}$$

at node ② i.e at $x_2(t)$

$$0 = M_2 s^2 X_2(s) + K X_2(s) + B_2 s(X_2(s) - X_1(s))$$

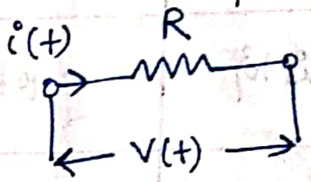
Electrical systems :

The behaviour of electrical systems is given by Ohm's law. Dominant elements of an electrical system are:

(i) Resistor (ii) Inductor (iii) Capacitor.

(i) Resistor :

Resistor is an energy dissipative element. For a resistor as shown in fig below, the relationship b/w current and voltage is given by,

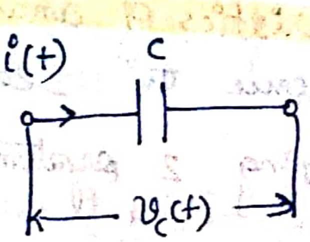


$$v(t) = R\, i(t)$$

(ii) Capacitor :

A capacitor is an electrical element which stores energy in its electrostatic field. If $v_c(t)$ is the voltage across the capacitor & $i(t)$ is the current through the capacitor, then relation b/w $v_c(t)$ & $i(t)$ is given by,

$$v_c(t) = \frac{1}{C}\int i(t)\,dt \quad (Q = CV)$$



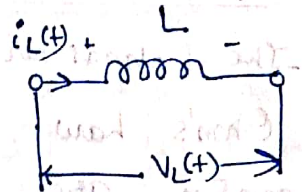
Diff both sides w.r.t. (t)

$$i(t) = C\frac{dv_c(t)}{dt}$$

(iii) Inductor :

An inductor stores energy in a magnetic field. The relationship between $v_L(t)$ and $i_L(t)$ is given by,

$$i_L(t) = \frac{1}{L}\int v_L(t)\, dt$$



or $$v_L(t) = L \cdot \frac{d i_L(t)}{dt}$$

Mathematical models of electrical elements :

Component	Voltage - Current Relation	Current - Voltage relation	Voltage - Charge Relation
Resistor	$v = iR$	$i = v/R$	$v = R\, dq/dt$
Inductor	$v = L \frac{di}{dt}$	$i = \frac{1}{L}\int v\, dt$	$v = L \frac{d^2 q}{dt}$
Capacitor	$v = 1/C \int i\, dt$	$i = C \frac{dv}{dt}$	$v = q/C$

Analogous Systems :

Systems which have identical mathematical models are known as analogous systems. The dynamic characteristics of analogous systems are identical. Hence the equations that represents the corresponding 2 systems are said to be analogous.

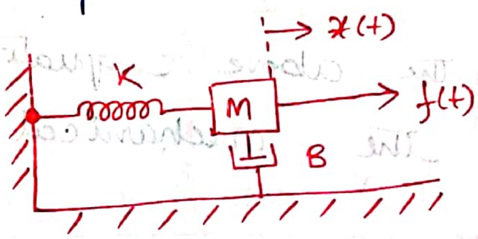
1) Electrical analogous of mechanical translational

Systems :

#1) Force - Voltage analogy :

If the dynamic characteristics of an electrical system is identical to that of a mechanical system, then the electrical system is said to be analogous to the mechanical system.

⇒ Consider a ~~sys~~ simple mechanical system as shown below;



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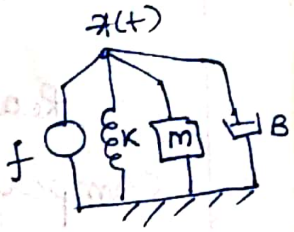


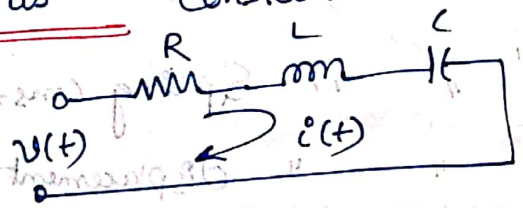
Converting the above mech. model into its equivalent mech. system

$$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

taking Laplace transform

$$\boxed{F(s) = Ms^2 X(s) + BsX(s) + KX(s)} \quad -(1)$$

⇒ Let us consider the analogous electrical network.



Applying K.V.L to the RLC ckt

we have $v(t) = R\,i(t) + L\cdot\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad -(2)$

Laking Laplace transform,

$$V(s) = R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s) \quad -(3)$$

we have $i(t) = \frac{dq}{dt}$

Taking Laplace transformation,

$I(S) = S\,Q(S)$ & $Q(S) = \frac{I(S)}{S}$ — (4)

sub eq (4) in eq (3)

$V(S) = R \cdot S\,Q(S) + LS\,S\,Q(S) + \frac{1}{C}\,Q(S)$

Rearranging the above equation, so as to compare with the mechanical system equation (1).

$$\boxed{V(S) = L\,S^2 Q(S) + R\,S\,Q(S) + \frac{1}{C}\,Q(S)} \quad \text{— (5)}$$

by comparing eq (1) & eq (5) we can list down the analogus parameters.

1) Inductance 'L' is analogous to Mass 'M'

2) Resistance 'R' is " " Friction 'B'

3) Reciprocal of capacitor '$1/C$' " " Spring constant 'K'

4) charge 'q' " " displacement 'x'

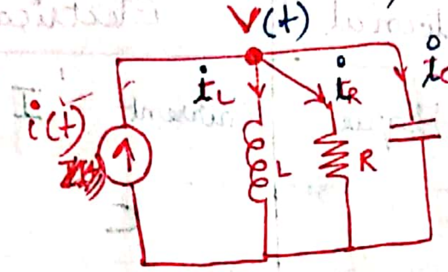
Refer table (1) for complete details...

x x

2) Force Current Analogy (Node Analysis) :-

→ Nodal Analysis circuit is employed here

In this method, Current is treated as analogous quantity to force in the mechanical system, so force equation is replaced by a current equation.
Also series RLC circuit gets reduced to a parallel RLC circuit with a current source connected a/c these 3 elements, as shown below.



Apply KCL at node V,

~~$i(t) = i_L + i_R + i_C$~~ $\quad i(t) = i_L + i_R + i_C$ — (1)

$$\therefore\ i(t) = \frac{1}{L}\int v(t)\,dt + \frac{v(t)}{R} + C\frac{dv(t)}{dt} \quad \text{— (2)}$$

taking Laplace transform of above equation;

$$I(S) = \frac{1}{LS}V(S) + \frac{V(S)}{R} + CS\,V(S) \quad \text{— (3)}$$

Also we have, $V(t) = \frac{d\phi}{dt}$ where ϕ = flux

& $V(S) = S\phi(S)$ & $\phi(S) = \frac{V(S)}{S}$ — (4)

sub eq (4) in eq (3) we have

$$I(S) = \frac{1}{L}\phi(S) + \frac{1}{R}S\phi(S) + CS\,S\phi(S)$$

&
$$\boxed{I(S) = CS^2\phi(S) + \left(\frac{1}{R}\right)S\phi(S) + \left(\frac{1}{L}\right)\phi(S)} \quad \text{— (5)}$$

Comparing equations of F(s) & I(s) we get,

1) Capacitor 'C' is analogous to Mass M

2) Reciprocal of resistance $1/R$ is analogous to friction B

3) Reciprocal of Inductance $1/L$ is " " Spring Constant (K)

Translational	Rotational	Force-Current Electrical	Force-Voltage Electrical
'F' force	'T' torque	Current 'I'	V
M	J	C	L
B	B	$1/R$	R
K	K	$1/L$	$1/C$
X displacement	θ	ϕ	q
$\dot{x}$ = Velocity = $\frac{dx}{dt}$	$\dot{\theta} = \frac{d\theta}{dt} = \omega$	Voltage $e = \frac{d\phi}{dt}$	$i = \frac{dq}{dt}$

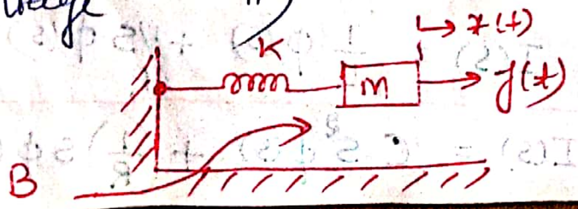
Table 2: Tabular form of Force-Voltage (F-V) and Force-Current (F-I) analogy

Example ① :-

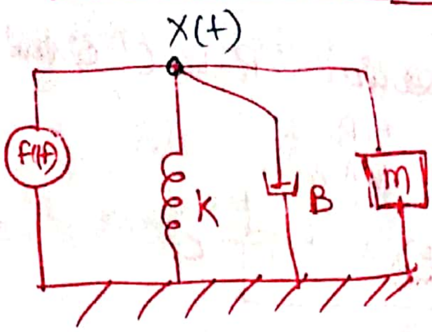
For the physical system shown below, draw its equivalent system and write equilibrium eq'ns. Draw its analogous circuits (Electrical) based on

i) Force - voltage

ii) Force - Current method.



(a) Step 1 : Draw the equivalent Mechanical system



Step 2 : write down the equilibrium force equation

$$F(t) = M \frac{d^2 x(t)}{dt^2} + B \frac{d x(t)}{dt} + K x(t) \quad \text{--- (1)}$$

taking the Laplace transform of eq (1); we get

$$\boxed{F(S) = M S^2 X(S) + BS\, X(S) + K\, X(S) \quad \text{--- (2)}}$$

Step 3 : (i) f-V method : use analogous terms from table (2)

$f \rightarrow V$ $M \rightarrow L$, $B \rightarrow R$, $K \rightarrow 1/C$, $x \rightarrow q$,

$\frac{dx}{dt} \rightarrow \frac{dq}{dt} \rightarrow i$, $x \rightarrow \int i\, dt$, $\frac{d^2 x}{dt^2} = \frac{di}{dt}$

⇒ Here all quantities are expressed in terms of i

eq (1) gets reduced to,

$$\boxed{V(t) = L \frac{di}{dt} + Ri + \frac{1}{C} \int i\, dt \quad \text{--- (3)}}$$

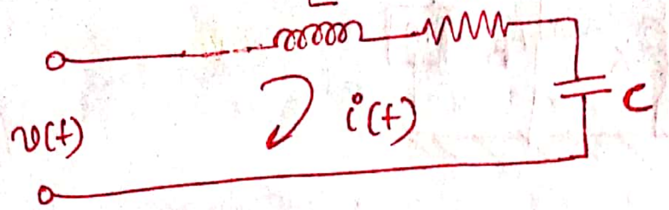
&

$$\boxed{V(S) = LS\, I(S) + R\, I(S) + \frac{1}{CS} I(S) \quad \text{--- (4)}}$$

Equation (3) and eq (4) represents the

Series R-L-C circuit;

Step 4 : Construct the series R-L-C circuit.



(b) Force - Current method : (F-I)

① use table 2, to list out analogous quantities,

Repeat (Step 3) ; but for F-I analogy:

$M \rightarrow C$, $B \rightarrow 1/R$, $K \rightarrow 1/L$, $x \rightarrow \phi$

$\frac{dx}{dt} \Rightarrow d\phi/dt \rightarrow v(t)$, $\quad F \rightarrow I$

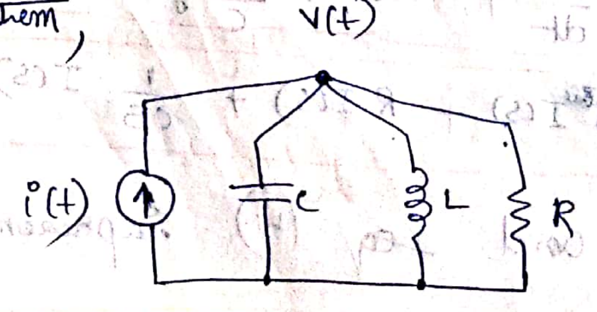
$x \rightarrow \int v(t)\, dt$, $\frac{d^2x}{dt^2} = \frac{dv(t)}{dt}$

• Here all quantities are expressed in terms of V.

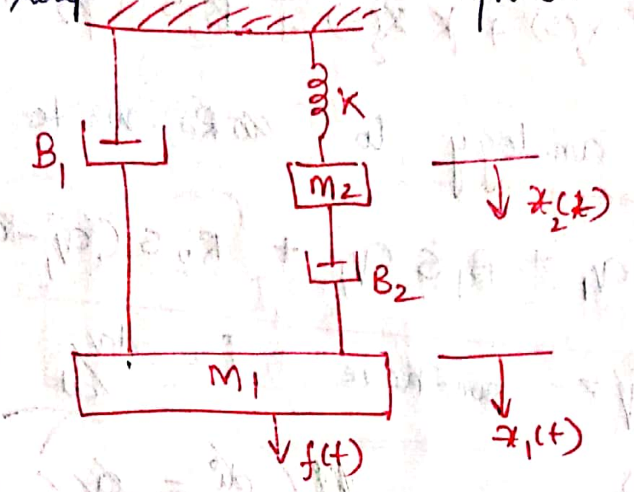
$$\boxed{\therefore \; i(t) = C\frac{dv(t)}{dt} + \frac{1}{R}V(t) + \frac{1}{L}\int V(t)\, dt} \quad -(5)$$

$$\boxed{I(s) = C\,sV(s) + \frac{1}{R}V(s) + \frac{1}{sL}V(s)} \quad -(6)$$

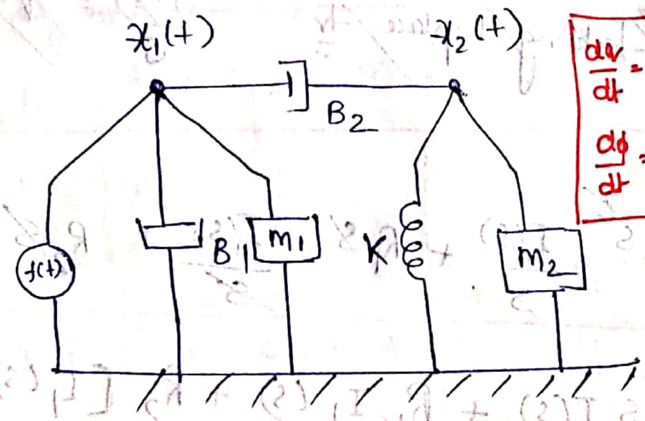
equation (5) and (6) represents a parallel R, L-C circuit with a current source connected across them,



② Draw the equivalent mechanical system of the given system using 1) F-V analogy 2) F-I analogy; also & write the Differential equations and the respective electrical n/w's.



Step 1: Write the equivalent Mech system dig.



$\frac{dv}{dt} = i \rightarrow$ F-V $\rightarrow x \rightarrow q$

$\frac{d\phi}{dt} = v$ F-I $\rightarrow x \rightarrow \phi$

Write down the equations for Force, here since we have 2 nodes & hence we are going to get 2 equations.

at node $x(t)$ we have,

$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{d x_1(t)}{dt} + B_2 \frac{d(x_1(t) - x_2(t))}{dt} \quad -(1)$$

&

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + B_2 s (X_1(s) - X_2(s)) \quad -(2)$$

(3)

Apply Newtons law at node $x_2(t)$

$$0 = m_2 \frac{d^2 x_2(t)}{dt^2} + K x_2(t) + B_2 \frac{d(x_2(t) - x_1(t))}{dt}$$

$$0 = M_2 s^2 X_2(s) + K X_2(s) + B_2 s (X_2(s) - X_1(s)) \quad -(4)$$

(i) use F-V analogy to Re-write eq (2)

$$V(s) = L_1 s^2 q_1 + R_1 s q_1 + R_2 s (q_1 - q_2) \quad -(5)$$

replace q/ we have $i = dq/dt$ (~~Differentiate~~ both sides)

$I(s) = s Q(s)$ $\cancel{\frac{di}{dt} = q}$

$\boxed{Q(s) = \frac{I(s)}{s}}$ ~~taking Laplace tr~~ $\cancel{\frac{I(s)}{s} = q(s)}$

$\therefore$

$$V(s) = L_1 s^{\cancel{2}} \frac{I_1(s)}{\cancel{s}} + R_1 \cancel{s} \frac{I_1(s)}{\cancel{s}} + R_2 \cancel{s} (I_1(s) - I_2(s)) \frac{1}{\cancel{s}}$$

$$\boxed{V(s) = L_1 s I_1(s) + R_1 I_1(s) + R_2 [I_1(s) - I_2(s)]} \quad -(6)$$

(ii) using F-V analogy to Re-write eq (3)

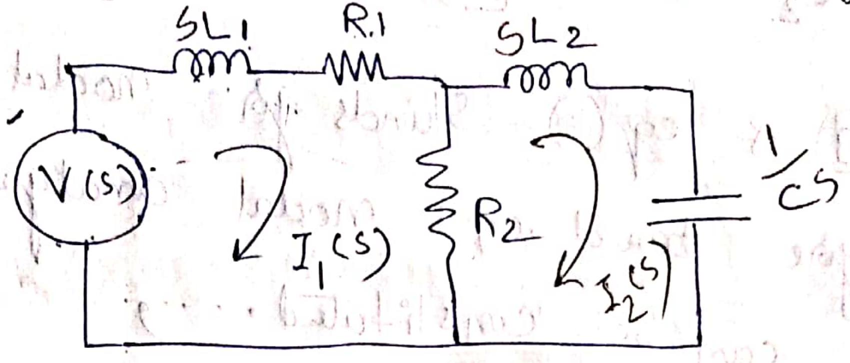
$$0 = L_2 s^2 q_2(s) + \left(\frac{1}{C}\right) q_2 + R_2 s (q_2 - q_1)$$

[replace q with $\frac{di}{dt}$]

$$0 = L_2 s^{\cancel{2}} \frac{I_2(s)}{\cancel{s}} + \left(\frac{1}{C}\right) \frac{I_2(s)}{s} + R_2 \cancel{s} \left(\frac{I_2(s) - I_1(s)}{\cancel{s}}\right)$$

$$\boxed{0 = L_2 s I_2(s) + \left(\frac{1}{Cs}\right) I_2(s) + R_2\left(I_2(s) - I_1(s)\right)}$$

From eq (6) & eq (7) lets form the (7) ckt.



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(ii) Using F I analogy:

using; $F \to I$, $m \to C$, $B \to 1/R$, $K \to 1/L$

$x \to \phi$

Then we have

$$I(s) = C_1 s^2 \underset{\cancel{X_1(s)}}{\phi_1(s)} + \frac{1}{R_1} s \underset{\cancel{X_1(s)}}{\phi_1(s)} + \frac{1}{R_2} s\left(\underset{\cancel{X_1(s)}}{\phi_1(s)} - \underset{\cancel{X_2(s)}}{\phi_2(s)}\right)$$

also
$$0 = C_2 s^2 \underset{\cancel{X_2(s)}}{\phi_2(s)} + \frac{1}{L} \underset{\cancel{X_2(s)}}{\phi_2(s)} + \frac{1}{R_2} s\left(\underset{\cancel{X_2(s)}}{\phi_2(s)} - \underset{\cancel{X_1(s)}}{\phi_1(s)}\right)$$

Replace x with ϕ

$\frac{dx}{dt}$ $\quad \frac{d\phi}{dt} = e \Rightarrow s\phi(s) = V(s)$

$X_2(s) \Rightarrow V_2(s)$

$X_1(s) \Rightarrow V_1(s)$

$X(s) = \phi(s) = \frac{V(s)}{s}$

$$I(s) = C_1 s^{\cancel{2}} \cdot \frac{V_1(s)}{\cancel{s}} + \frac{1}{R_1} \cancel{s} \times \frac{V_1(s)}{\cancel{s}} + \frac{1}{R_2} \times \cancel{s} \times \frac{1}{\cancel{s}} \left(V_1(s) - V_2(s)\right)$$

$$I(s) = C_1 s V_1(s) + \frac{1}{R_1} V_1(s) + \frac{1}{R_2}\left(V_1(s) - V_2(s)\right) \quad (8)$$

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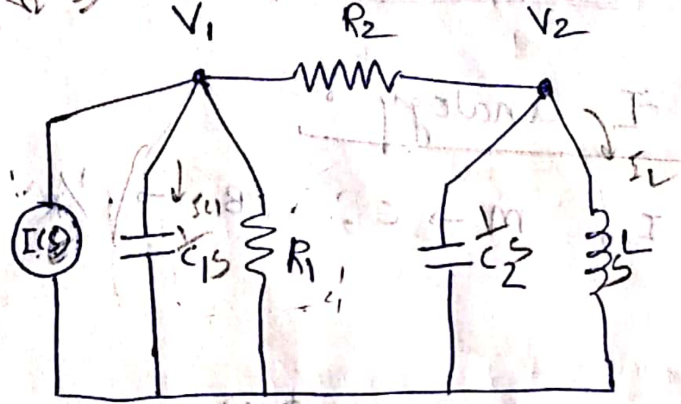
Also,

$$0 = C_2 \cancel{S}^2 \frac{V_2(S)}{\cancel{S}} + \frac{1}{L} \cdot \frac{V_2(S)}{S} + \frac{1}{R_2} \times \cancel{S} \times \frac{1}{\cancel{S}} \left(V_2(S) - V_1(S) \right)$$

$$0 = \overset{I_{c1}}{C_2 S V_2(S)} + \overset{i_L}{\frac{1}{LS} V_2(S)} + \frac{1}{R_2} \left(V_2(S) - V_1(S) \right) \quad \text{—⑨}$$

eq ⑨ & eq ⑧ stands for nodal analysis.
There fore based on nodal analysis a circuit can be constituted....,



no. of node Vtgs are equal to no. of displacements.

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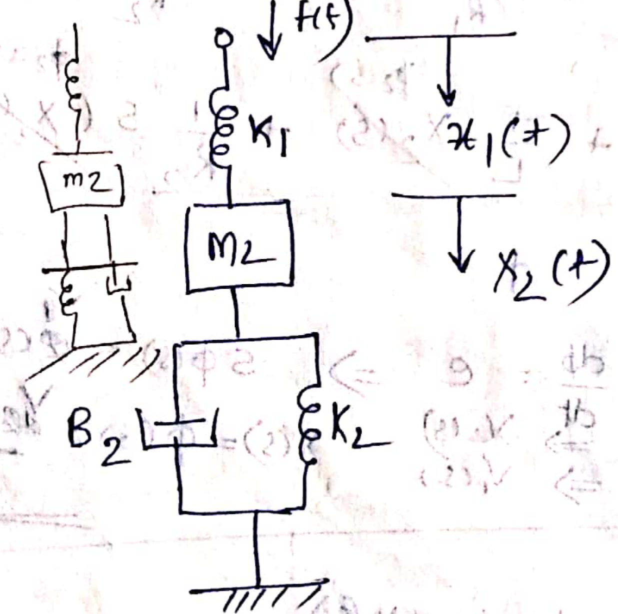
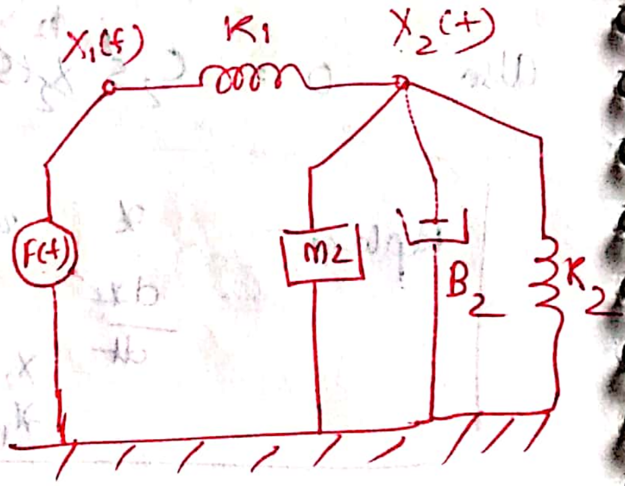
③



given ckt.



equivalent system

Apply Newton's Law at nodes $x_1(t)$ & $x_2(t)$

$$F(t) = K_1(x_1(t) - x_2(t)) \quad — (1) \rightarrow \text{time domain}$$

$$F(s) = K_1(x_1(s) - x_2(s)) \quad — (2) \rightarrow \text{'s' domain}$$

Also,

$$0 = K_1(x_2(t) - x_1(t)) + M_2 \frac{d^2 x_2(t)}{dt^2} + B_2 \cdot \frac{dx_2(t)}{dt} + K_2 x_2(t) \quad (3)$$

$$0 = K_1(X_2(s) - X_1(s)) + M_2 s^2 X_2(s) + B_2 s X_2(s) + K_2 X_2(s) \quad (4)$$

(1) F–V Analogy

In eq (2) & (4) replacing:

$F \rightarrow V$, $B \rightarrow R$, $K \rightarrow \frac{1}{C}$, $M \rightarrow L$

$X \rightarrow q$, also $\frac{dx}{dt} = \frac{dq}{dt} = i$

$\Rightarrow I(s) = s\,Q(s) \Rightarrow Q(s) = \frac{I(s)}{s}$

$$V(s) = \frac{1}{C_1}\left(\frac{I_1(s)}{s} - \frac{I_2(s)}{s}\right)$$

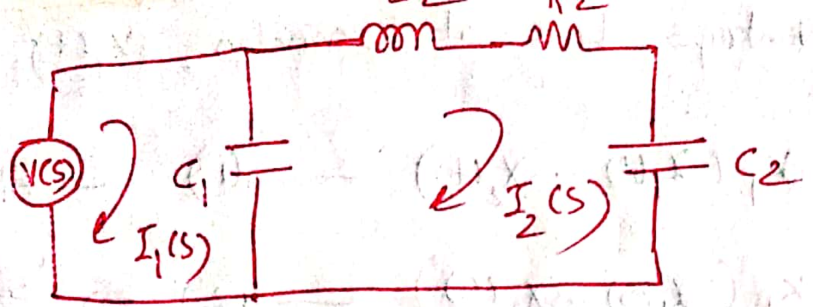
$$\boxed{V(s) = \frac{1}{C_1 s}(I_1(s) - I_2(s))} \quad — (5)$$

Also

$$0 = \frac{1}{C_1 s}(I_2(s) - I_1(s)) + L_2 s^2 \frac{I_2(s)}{s} + R_2 s \frac{I_2(s)}{s} + \frac{1}{C_2}\frac{I_2(s)}{s}$$

$$\boxed{0 = \frac{1}{C_1 s}(I_2(s) - I_1(s)) + L_2 s I_2(s) + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s)} - (6)$$

Eq (6) & (5), represents KVL eq'ns & hence ckt can be formed.



(ii) f–I analogy

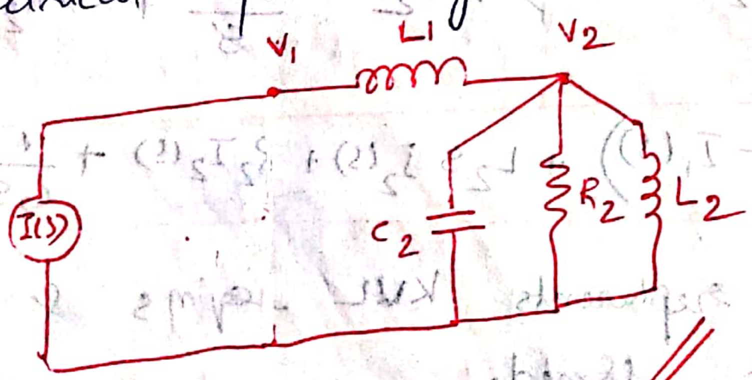
$$\boxed{\begin{array}{l} \text{In eq (2) \& (4) replacing} \\ f \rightarrow I, \quad B \rightarrow \frac{1}{R}, \quad M \rightarrow C, \quad K \rightarrow \frac{1}{L} \\ x \rightarrow \phi \qquad \frac{dx}{dt} = \frac{d\phi}{dt} = V \Rightarrow V(s) = s\phi(s) \\ \qquad \& \ \phi(s) = \frac{V(s)}{s} \end{array}}$$

$$\boxed{I(s) = \frac{1}{L_1 s}\left(V_1(s) - V_2(s)\right) \quad — (7)}$$

$$0 = \frac{1}{L_1 s}\left(V_2(s) - V_1(s)\right) + C_2 s^2 \frac{(V_2(s))}{\cancel{s}} + \frac{1}{R_2}\cancel{s} \cdot \frac{V_2(s)}{\cancel{s}} + \frac{1}{L_2}\frac{V_2(s)}{s}$$

$$\boxed{0 = \frac{1}{L_1 s}\left(V_2(s) - V_1(s)\right) + C_2 s \, V_2(s) + \frac{1}{R_2} \cdot V_2(s) + \frac{V_2(s)}{L_2 s} \quad — (8)}$$

From eq (7) & (8) we can construct the electrical n/w using nodal analysis.



Transfer function :-

The transfer function of a linear time-invarient system is defined as the ratio of the Laplace transform of the output variable to the laplace transform of the input variable, under the assumption that all initial conditions are zero.

Let us ~~define~~ consider a system in its simplest form, as shown below,

$$r(t) \longrightarrow \boxed{\text{System}} \longrightarrow c(t) \qquad\qquad R(S) \longrightarrow \boxed{\text{System}} \longrightarrow C(S)$$

time domain $\qquad\qquad$'s' domain

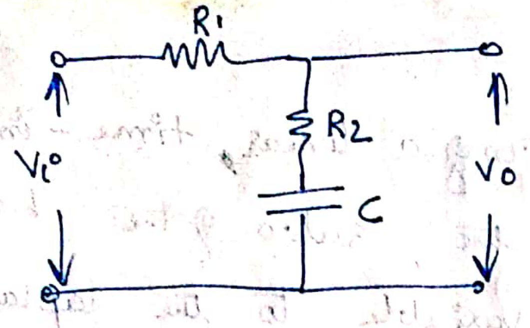
Where $c(t)$ & $C(S)$ is the output variable and $r(t)$ & $R(S)$ is the input variable.

$C(S)$ is the Laplace transform of $c(t)$ &
$R(S)$ is the " " " $r(t)$

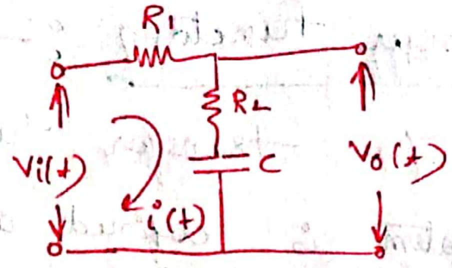
Then the Laplace transfer function $T(S)$ is defined as

$$\boxed{T(S) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(S)}{R(S)}}$$

Ex:(1) Determine the transfer function of $\frac{V_o(S)}{V_i(S)}$ of the ckt shown below,



(a) Given circuit



(b) KVL circuit

Apply KVL to the circuit (b)

$$V_i(t) = R_1\, i(t) + R_2\, i(t) + \frac{1}{C}\int i(t)$$

taking Laplace transform

$$V_i(S) = R_1\, I(S) + R_2\, I(S) + \frac{1}{CS} I(S)$$

$$I(S) = \frac{V_i(S)}{R_1 + R_2 + \frac{1}{CS}} = \frac{V_i(S)}{\frac{sC(R_1+R_2)+1}{CS}}$$

$$I(S) = \frac{sC\,V_i(S)}{sC(R_1+R_2)+1} \quad \text{——— (1)}$$

Then output Voltage of the n/w is,

$$v_o(t) = R_2\, i(t) + \frac{1}{C}\int i(t)$$

$$V_o(S) = R_2\, I(S) + \frac{1}{sC} I(S) \quad \text{——— (2)}$$

$$V_o(S) = I(S)\left[R_2 + \frac{1}{sC}\right] = I(S)\left[\frac{R_2 sC + 1}{sC}\right]$$

~~Subs~~ Sub eq (1) in eq (2) we have,

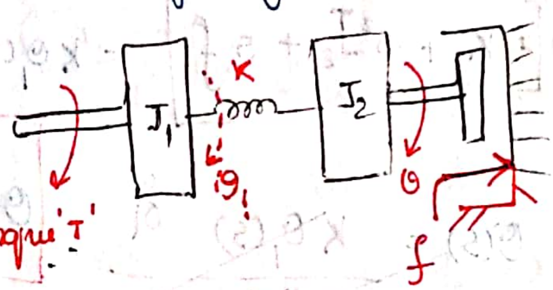
$$V_o(S) = \frac{\cancel{sC}\,V_i(S)}{(sC(R_1+R_2)+1)} \times \left[\frac{R_2 sC + 1}{\cancel{sC}}\right]$$

$$V_0(S) = V_i(S) \cdot \frac{(R_2 SC+1)}{SC(R_1+R_2)+1}$$

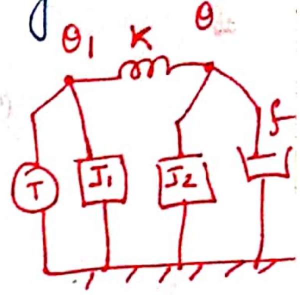
$$\boxed{\frac{V_0(S)}{V_i(S)} = T(S) = \frac{SCR_2+1}{SC(R_1+R_2)+1}}$$

July 2011 (4 marks) $\frac{\theta(s)}{T(s)}$

(2) Obtain the Transfer function of the following system.



$\Leftrightarrow$ Equivalent n/w



Due to the torque T, J_1 is under displacement say $\underline{\theta_1}$. Due to Spring K, J_2 is under the displacement "$\underline{\theta}$". J_2 & θ and are under the influence of displacement θ.

"B" → Nagrath & M. Gopal book

⇒ "f" stands for friction

at θ_1

$$T = J_1 \frac{d\theta_1^2}{dt^2} + K(\theta_1 - \theta) \quad - (1)$$

$$0 = J_2 \frac{d\theta^2}{dt^2} + f\frac{d\theta}{dt} + K(\theta - \theta_1) \quad - (2)$$

taking Laplace transform of eq (1) & (2)

$$T(S) = J_1 S^2 \theta_1(S) + K(\theta_1(S) - \theta(S)) \quad - (3)$$

$$0 = K(\theta(s) - \theta_1(s)) + J_2 s^2 \theta(s) + f s \theta(s) \quad -(4)$$

But $\quad T.F = \dfrac{\mathcal{L}(o/p)}{\mathcal{L}(i/p)} = \dfrac{\theta(s)}{T(s)}$

Re-arranging eq (4)

$$0 = K\theta(s) - K\theta_1(s) + J_2 s^2 \theta(s) + f s \theta(s)$$

$$0 = \theta(s)\left[K + s^2 J_2 + s f\right] - K\theta_1(s)$$

$$\therefore \cancel{T.F =} \quad \cancel{\theta(s) = \frac{K\theta_1(s)}{(K + s^2 J_2 + s f)}} \quad \& \quad \boxed{\theta_1(s) = \frac{\theta(s)\left[K + s^2 J_2 + s f\right]}{K}} \quad (5)$$

$$\therefore \cancel{T.F = \frac{\theta(s)}{T(s)} = \frac{K\theta_1(s)}{(K + s^2 J_2 + s f)}}$$

Sub eq (5) in eq (3) we have

$$T(s) = \frac{(J_1 s^2 + K)\,\theta(s)\,(K + s^2 J_2 + s f)}{K} - K\,\theta(s)$$

$$T(s) = \frac{\theta(s)(J_1 s^2 + K)(J_2 s^2 + f s + K) - K^2 \theta(s)}{K}$$

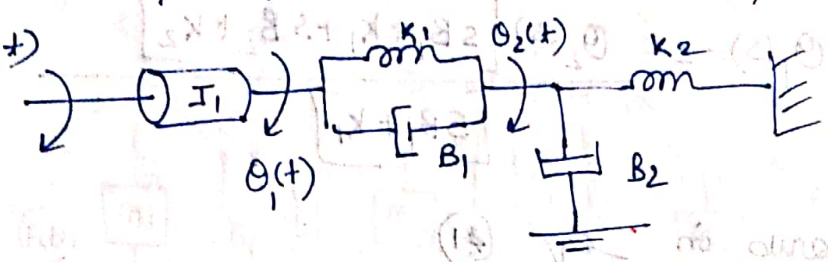
$$T(s) = \frac{\theta(s)\left[\{(J_1 s^2 + K)(J_2 s^2 + f s + K)\} - K^2\right]}{K}$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{(J_2 s^2 + fs + K)(J_1 s^2 + K) - K^2}$$

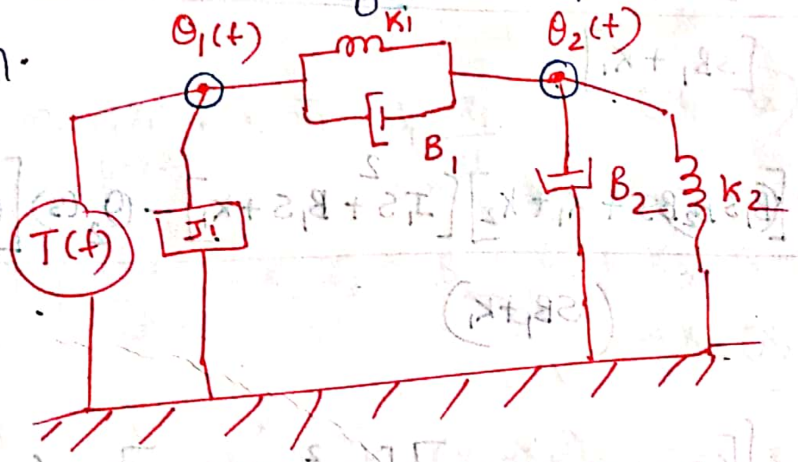
(3) For the system shown below in fig. below find the T. F. $G(s) = \dfrac{\theta_2(s)}{T(s)}$. Consider $B_1 = 1$ Nm/rad/sec, $B_2 = 1$ Nm/rad/sec.

"Dec-2011"

"6 marks"



As the other end of K_2 is fixed & hence no rotation.



$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2) + B_1\left[\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right]$$

$$T(s) = J_1 s^2 \theta_1(s) + K_1\theta_1(s) - K_1\theta_2(s) + sB_1\theta_1(s) - sB_1\theta_2(s)$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + K_1 + sB_1\right] - \theta_2(s)\left[K_1 + sB_1\right] \quad \text{--- (1)}$$

Also $$0 = B_1\left(\frac{d\theta_2}{dt} - \frac{d\theta_1}{dt}\right) + K_1(\theta_2 - \theta_1) + B_2\frac{d\theta_2}{dt} + K\theta_2$$

$$0 = B_1 s\,\theta_2(s) - B_1 s\,\theta_1(s) + K_1\theta_2(s) - K_1\theta_1(s) + B_2 s\theta_2(s) + K\theta_2(s)$$

$$0 = \theta_2(s)\left[sB_1 + K_1 + sB_2 + K_2\right] - \theta_1(s)\left[sB_1 + K_1\right]$$

or $$\theta_1(s) = \frac{\theta_2(s)\left[sB_1 + K_1 + sB_2 + K_2\right]}{\left[sB_1 + K_1\right]} \quad \text{--- (2)}$$

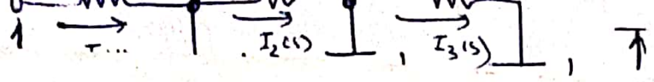
subs in eq (1)

$$T(s) = \frac{\theta_2(s)\left[sB_1 + K_1 + K_2 + sB_2\right]\left[J_1 s^2 + K_1 + sB_1\right]}{\left[sB_1 + K_1\right]} - \theta_2(s)\left[K_1 + sB_1\right]$$

$$T(s) = \frac{\theta_2(s)\left[(B_1 + B_2)s + K_1 + K_2\right]\left[J_1 s^2 + B_1 s + K_1\right] - \theta_2(s)\left[(K_1 + sB_1)^2\right]}{(sB_1 + K_1)}$$

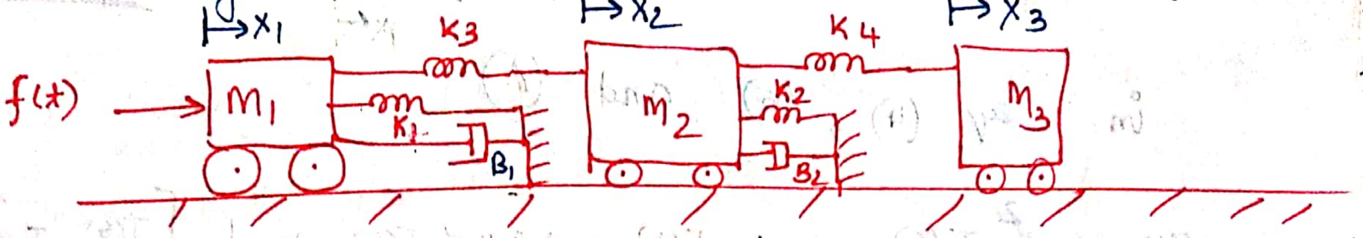
$$T(s) = \frac{\theta_2(s)\left[\left[(B_1 + B_2)s + K_1 + K_2\right]\left[J_1 s^2 + B_1 s + K_1\right] - (K_1 + sB_1)^2\right]}{(sB_1 + K_1)}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{(sB_1 + K_1)}{(J_1 s^2 + B_1 s + K)\left[s(B_1 + B_2) + K_1 + K_2\right] - (sB_1 + K_1)^2}}$$

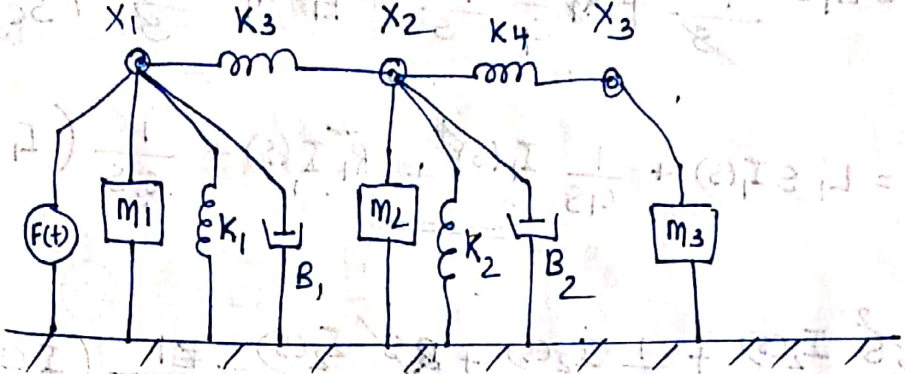


4) Draw the Mechanical n/w for the system shown below. Dec - May 4 10 Marks

- Write the equations of performance & draw its analogous circuit based on f-V analogy



Solution



- Equations

(a) In time domain

at x_1,

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_3(x_1 - x_2) \quad \text{— (1)}$$

at x_2

$$0 = M_2 \frac{d^2x_2}{dt^2} + K_2 x_2 + B_2 \frac{dx_2}{dt} + K_3(x_2 - x_1) + K_4(x_2 - x_3) \quad \text{— (2)}$$

at x_3

$$0 = M_3 \frac{d^2 x_3}{dt^2} + K_4(x_3 - x_2) \quad \text{— (3)}$$

(b) In Laplace form

$$f(s) = M_1 s^2 X_1(s) + K_1 X_1(s) + B_1 s X_1(s) + K_3 X_1(s) - K_3 X_2(s) \quad \text{— (4)}$$

$$0 = M_2 s^2 X_2(s) + K_2 X_2(s) + B_2 s X_2(s) + K_3(X_2(s) - X_1(s)) + K_4(X_2(s) - X_3(s)) \quad \text{— (5)}$$

$$0 = M_3 s^2 X_3(s) + K_4(X_3(s) - X_2(s)) \quad \text{— (6)}$$

- Electrical Analogous network – F-V analogy.

Replacing $M \to L$, $B \to R$, $K \to \frac{1}{C}$

$F \to V$, $x \to q$ $\Rightarrow$ $\frac{dx}{dt} = \frac{dq}{dt} = i$ &

$I(s) = s\,Q(s) \Rightarrow Q(s) = \frac{I(s)}{s}$

in eq (4), (5) and (6)

$$V(s) = L_1 s^{\cancel{2}} \frac{I_1(s)}{\cancel{s}} + \frac{1}{C_1}\frac{I_1(s)}{s} + R_1 \cancel{s} \frac{I(s)}{\cancel{s}} + \frac{1}{sC_3}\left[\frac{I(s)}{\cancel{s}} - I_2(s)\right]$$

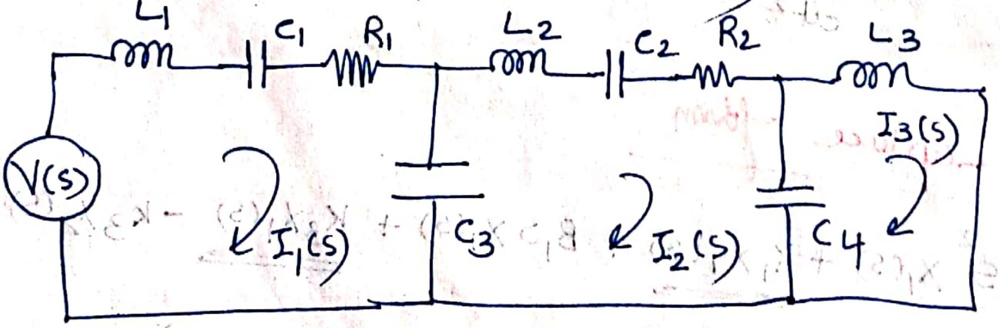
$$V(s) = L_1 s I_1(s) + \frac{1}{C_1 s} I_1(s) + R_1 I_1(s) + \frac{1}{C_3 s}\left(I_1(s) - I_2(s)\right) \quad \text{— (7)}$$

$$0 = L_2 s^{\cancel{2}} \frac{I_2(s)}{\cancel{s}} + \frac{1}{C_2}\frac{I_2(s)}{s} + R_2 \cancel{s} \frac{I_2(s)}{\cancel{s}} + \frac{1}{C_3 s}\left(I_2(s) - I_1(s)\right)$$

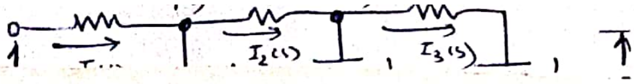
$$+ \frac{1}{sC_4}\left(X_2(s) - X_3(s)\right)$$

$$0 = L_2 s I_2(s) + \frac{1}{C_2 s} I_2(s) + R_2 I_2(s) + \frac{1}{C_3 s}\left(I_2(s) - I_1(s)\right) + \frac{1}{sC_4}\left(X_2(s) - X_3(s)\right) \quad \text{— (8)}$$

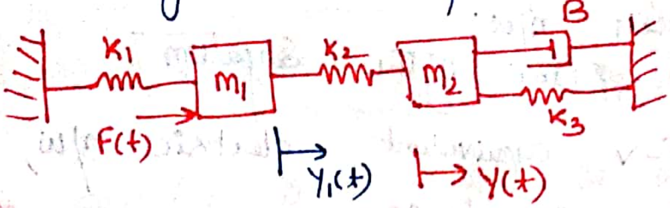
$$0 = L_3 s I_3(s) + \frac{1}{sC_4}\left(I_3(s) - I_2(s)\right) \quad \text{— (9)}$$



Electrical Network

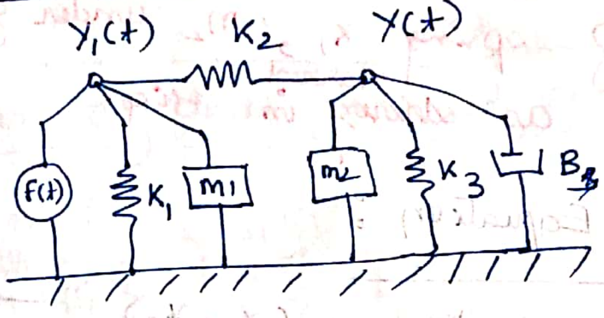


5) For the mech system shown below, write the D.E. "Dec-2010"



Solution :- There is a translation motion i.e displacement at mass m_1 i.e $y_1(t)$.

write the equivalent mech n/w :-



$$F(t) = m_1 \frac{d^2y_1}{dt} + K_1 y_1 + K_2(y_1 - y) \quad \text{--- (1)}$$

$$0 = M_2 \frac{d^2y}{dt^2} + K_3 y + B\frac{dy}{dt} + K_2(y - y_1) \quad \text{--- (2)}$$

(1) and (2): time Domain

In 's' domain

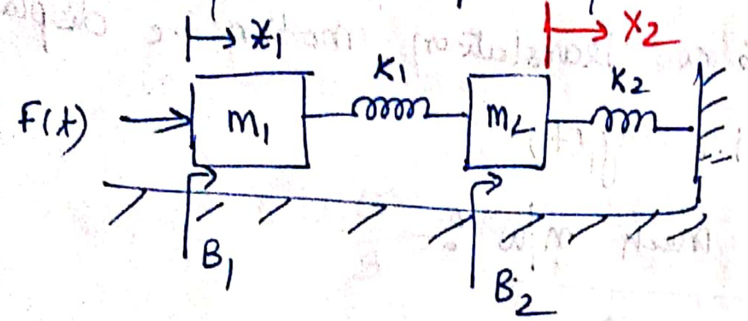
$$F(s) = m_1 s^2 Y_1(s) + K_1 Y_1(s) + K_2(Y_1(s) - Y(s))$$

$$0 = M_2 s^2 Y(s) + K_3 Y(s) + BsY(s) + K_2(Y(s) - Y_1(s))$$

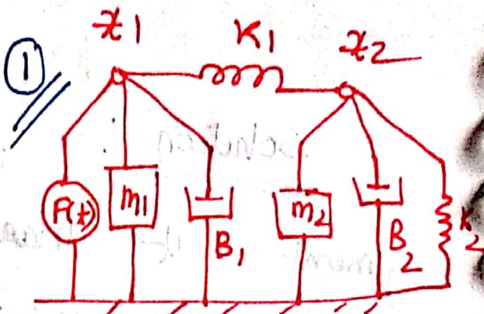
---x---x---

6) For the ckt shown below;

1) Draw Mech n/w.
2) Write the D.O.E of the system.
3) Draw f-V equivalent electric n/w, after writing down the equations.



1) ⟹



Ans: Because of spring K_1, m_2 undergoes a displacement of $x_2(t)$ as marked in fig.

2) **Differential Equation:**

time Domain
$$F(t) = m_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1(x_1 - x_2) \quad \text{— (1)}$$
$$0 = m_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + K_1(x_2 - x_1) \quad \text{— (2)}$$

In laplace form

$$F(s) = m_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 (X_1(s) - X_2(s)) \quad \text{— (3)}$$
$$0 = m_2 s^2 X_2(s) + B_2 s\, X_2(s) + K_2 X_2(s) + K_1 (X_2(s) - X_1(s)) \quad \text{(4)}$$

3) Using F-V analogy, replacing

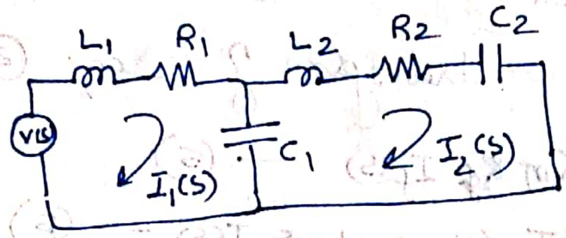
$F \to V$, $m \to L$, $B \to R$, $K \to \frac{1}{C}$

$x \to q$, $Q(s) = \frac{I(s)}{s}$ in eq (3) & (4)

$$V(s) = L_1 s\, I_1(s) + R_1 I_1(s) + \frac{1}{C_1 s}\left(I_1(s) - I_2(s)\right) \quad \text{—} \; (5)$$

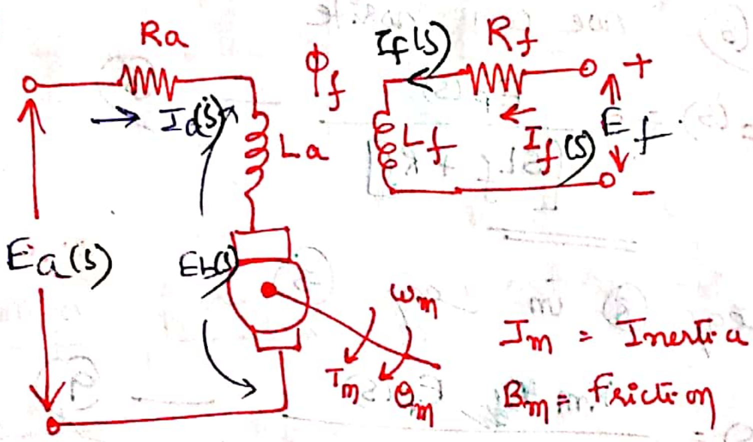
$$0 = L_2 s\, I_2(s) + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) + \frac{1}{C_1 s}\left(I_2(s) - I_1(s)\right) \quad \text{—} \; (6)$$

eq (5) & eq (6) are K.V.L equation, using them lets construct the electrical n/w.



* Transfer function of field controlled D.C. Motor

Ckt :



$$\boxed{T = 0.159\, \phi\, I_a \frac{PZ}{A}}$$

Assumptions made :

- Armature Current I_a remains constant
- Flux produced is propotional to field current $\phi_f \propto I_f$

$$\therefore \quad \phi_f = K_f \cdot I_f \quad \text{—} \; (1)$$

- Torque is propotional to product of flux and armature current.

$$T_m \propto \phi_f I_a$$

$$T_m = K' \phi_f I_a = K' K_f I_f I_a$$

$$T_m = K_m I_f \quad \text{—} \; (2)$$

From eq (1)

$K' I_a = K_m$
$=$ Constant

Apply KVL to field circuit:

$$E_f(t) = R_f I_f + L_f \frac{dI_f}{dt} \quad — (3)$$

torque equation of the motor going to be

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \quad — (4)$$

taking Laplace transform of eq (2), (3) & (4)

$$T_m(s) = K_m \cancel{K_f} I_f(s) \quad — (5)$$

$$E_f(s) = R_f I_f(s) + L_f s I_f(s) \quad — (6)$$

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s \theta_m(s) \quad — (7)$$

From eq (6) we can write

$$I_f(s) = \frac{E_f(s)}{[sL_f + R_f]} \quad — (8)$$

sub eq (8) in eq (5)

$$T_m(s) = \frac{K_m \cancel{K_f} E_f(s)}{(sL_f + R_f)} \quad — (9)$$

~~sub eq~~ comparing eq (9) & eq (7)

$$J_m s^2 \theta_m(s) + B_m s \theta_m(s) = \frac{K_m \cancel{K_f} E_f(s)}{(sL_f + R_f)} \quad — (10)$$

$$\boxed{T.F = \frac{L(O/P)}{L(I/P)} = \frac{\theta_m(s)}{E_f(s)}}$$

$$\theta_m(s)(J_m s^2 + B_m s)(sL_f + R_f) = K_m \cancel{K_f} E_f(s)$$

$$\therefore \quad T.F. = \frac{\Theta_m(s)}{E_f(s)} = \frac{K_m}{R_f\left[1+s\cdot\left(L_f/R_f\right)\right]\cdot s\left(1+s\frac{J_m}{B_m}\right)B_m}$$

$$\boxed{\frac{\Theta_m(s)}{E_f(s)} = \frac{1}{s}\cdot\frac{K_m}{B_m(1+s\tau_m)}\cdot\frac{1}{R_f(1+s\tau_f)}}$$

where $\tau_m = \frac{J_m}{B_m}$ = Motor (Mech.) time constant.

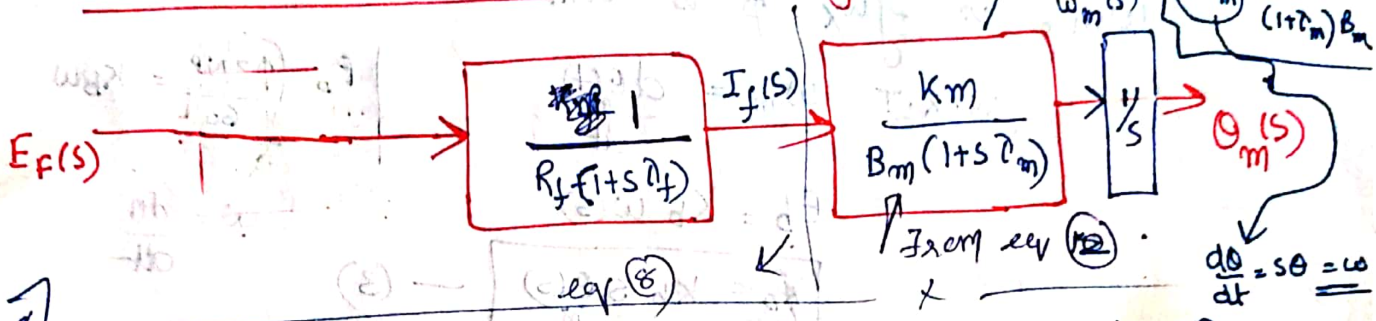
$\tau_f = \frac{L_f}{R_f}$ = field time constant.

$T_m = K_m I_F$

$s(1+\tau_m)B_m\Theta_m = K_m I_F$

$s\Theta_m = \frac{K_m I_F}{(1+\tau_m)B_m}$

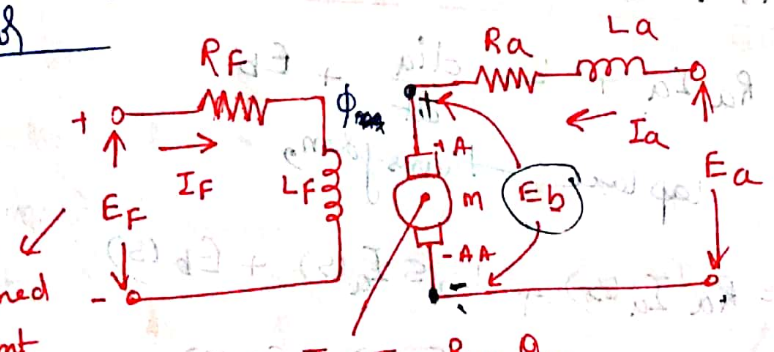
Let's construct the block diagram :-



$\frac{d\theta}{dt} = s\theta = \omega$

* Transfer function of armature controlled D.C. motor

Ckt :



Assumptions :-

1) flux is proportional to the field current $\boxed{\phi \propto I_F = K_f I_F}$ and is constant, (1)

as E_F is held constant.

ii) Torque is propotional to "product of flux and armature current":

$$T \propto \phi I_a = K_m^{'} \phi I_a$$

$$T = K_m^{'} K_f I_f I_a$$

taking $K_m^{'} K_f I_f = K_m$

$$\therefore \boxed{T = K_m I_a} \quad \text{—— (2)}$$

iii) Back emf is directly propotional to shaft velocity ω_m, as flux ϕ is constant.

W.K.T $\omega_m = \dfrac{d\theta_m(t)}{dt}$ $\quad\Big|\ E_b = \left(\dfrac{\phi Z N P}{60 A}\right) = K_b \omega$

$$E_b = K_b \omega(s)$$

$\omega = \dfrac{d\theta}{dt}$

$$\boxed{E_b = K_b \, s \, \theta_m(s)} \quad \text{—— (3)}$$

Apply KVL to armature circuit

$$\Rightarrow \quad E_a = R_a I_a + L_a \frac{di_a}{dt} + E_b$$

taking laplace transform,

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s)$$

$$I_a(s) = \frac{E_a(s) - E_b(s)}{(R_a + s L_a)} \quad \text{—— (4)}$$

sub eq (3) in eq (5)

Also $T_m = K_m I_a$

$$T_m = K_m \cdot \left[\frac{E_a(s) - E_b(s)}{(R_a + s L_a)}\right] \quad \text{—— (5)}$$

Also we have the torque ~~in~~ equation as

$$T_m = J_m \frac{d\theta_m^2}{dt^2} + B_m \frac{d\theta_m}{dt}$$

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s)$$

$$T_m(s) = \left(J_m s^2 + B_m s\right)\theta_m(s) \quad \text{—— (6)}$$

equating eq (6) and eq (5)

for E_b substitute eq (3)

$$\frac{K_m E_a(s)}{(R_a + sL_a)} = \frac{K_m K_b s\, \theta_m(s)}{(R_a + sL_a)} + \left(J_m s^2 + B_m s\right)\theta_m(s)$$

$$\frac{K_m E_a(s)}{(R_a + sL_a)} = \theta_m(s)\left[\left[\frac{K_m \cdot s K_b}{(R_a + sL_a)}\right] + \left(J_m s^2 + B_m s\right)\right]$$

$$T.F = \frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m \cancel{E_a(s)}}{\cancel{R_a(1+s\tau_a)}}}{sB_m(1+s\tau_m) + \dfrac{K_m}{\cancel{R_a(1+s\tau_a)}} \cdot sK_b}$$

$s^2+2s+s+1$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m}{\underbrace{sB_mR_a(1+s\tau_m)(1+s\tau_a)}_{B} + K_m \cdot sK_b}$$

$+\dfrac{s+1}{s^2+3s+1}$

take (B) common

$$= \frac{K_m}{\left[sB_mR_a(1+s\tau_m)(1+s\tau_a)\right]\left[1 + \dfrac{K_m \cdot (sK_b)}{sB_mR_a(1+s\tau_m)(1+s\tau_a)}\right]}$$

take it to Nr

$$T.F = \frac{\Theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{S R_a B_m (1+S\tau_m)(1+S\tau_a)}}{1 + \dfrac{K_m}{S R_a B_m (1+S\tau_m)(1+S\tau_a)} \cdot (S K_b)} \quad \text{—} (7)$$

Equation (7) of the form

$$T.F = \frac{G(S)}{1+G(S)\cdot H(S)} \quad \text{—} (8)$$

where $G(S) \rightarrow$ open loop Transfer function (T.F.)

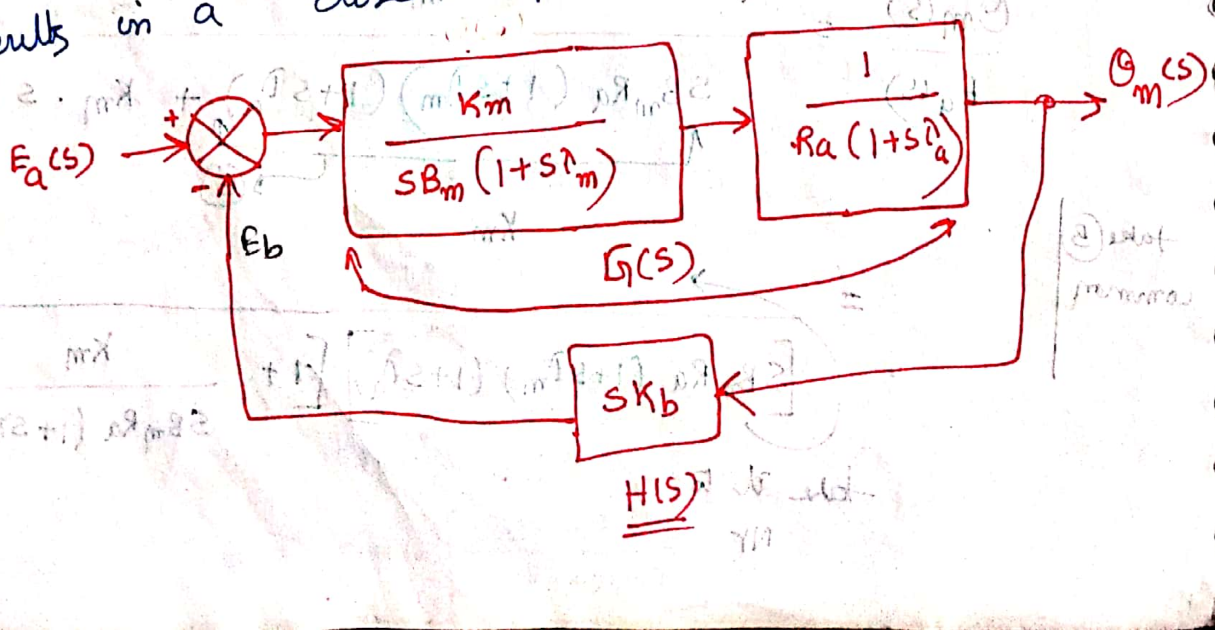
$H(S) \rightarrow$ feed back factor.

Comparing eq (7) & (8) we get

$$G(S) = \frac{K_m}{S R_a B_m (1+S\tau_m)(1+S\tau_a)}$$

$$H(S) = S K_b$$

∴ A block diag can be constructed; which results in a closed loop Block diagram,



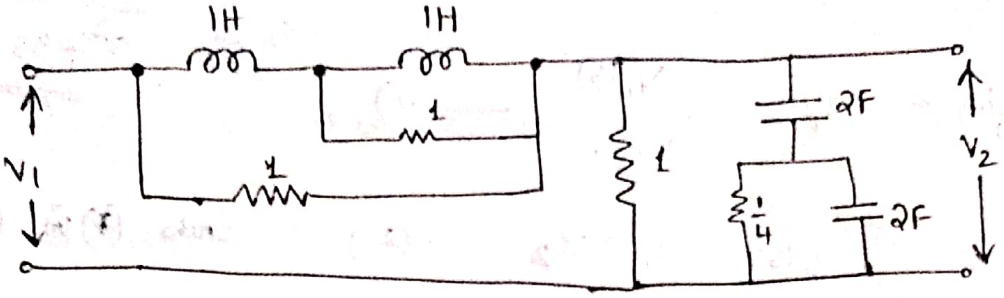
"6 hrs"

③ :- Related to unit 1

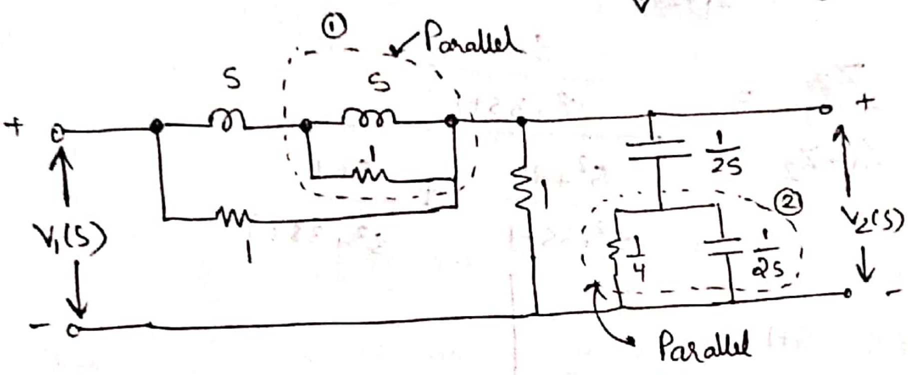
Module - 1

For the 2 port n/w shown below find (i) $\frac{V_2(S)}{V_1(S)}$

(ii) $\frac{V_1(S)}{I_1(S)}$

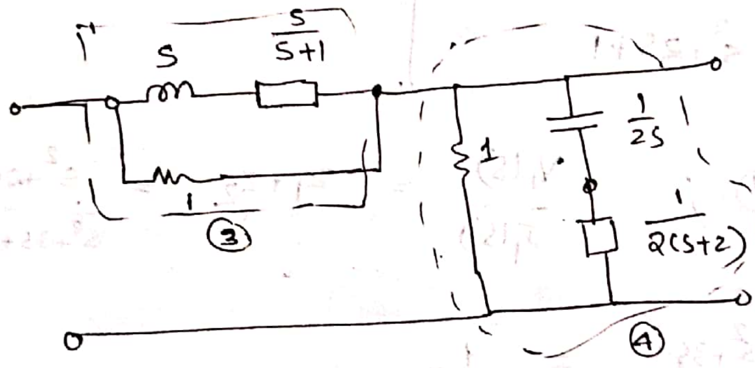


$\Downarrow$ converting it to Laplace Domain



① $\frac{S\times 1}{S+1} = \frac{S}{S+1}$

② $\frac{\frac{1}{4}\times\frac{1}{2S}}{\frac{1}{4}+\frac{1}{2S}} = \frac{\frac{1}{8S}}{\frac{2S+4}{8S}} = \frac{1}{2(S+2)}$

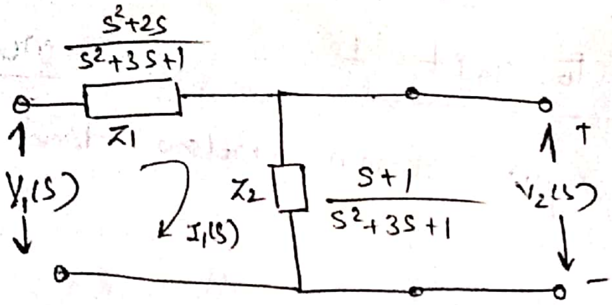


③ $\Rightarrow S + \frac{S}{S+1} = \frac{S^2+S+S}{S+1} = \frac{S^2+2S}{S+1}$

$$\frac{\frac{S^2+2S}{S+1}\times 1}{1+\frac{S^2+2S}{S+1}} = \frac{S^2+2S}{S+1+S^2+2S} = \frac{S^2+2S}{S^2+3S+1}$$

④ $\Rightarrow \frac{1}{2S}+\frac{1}{2(S+2)} = \frac{2(S+2)+2S}{2S\times 2(S+2)} = \frac{2S+4+2S}{4S^2+8S} = \frac{4S+4}{4S^2+8S}$

$= \frac{S+1}{S^2+2S}$



$$\frac{\frac{s+1}{s(s+2)}}{1+\frac{s+1}{s(s+2)}} \quad s^2+2s+s+$$

$$= \frac{s+1}{s^2+3s+1}$$

$$I_1(s) = \frac{V_1(s)}{Z_1+Z_2} \quad \text{--- (1)}$$

$$V_2(s) = I_1(s)\, Z_2 \quad \text{--- (2)}$$

Sub (1) in (2)

$$V_2(s) = \frac{V_1(s)}{Z_1+Z_2} \times Z_2$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{s+1}{s^2+3s+1}}{\frac{s^2+2s}{s^2+3s+1}+\frac{s+1}{s^2+3s+1}}$$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{s+1}{s^2+3s+1}}$$

From eq (1);

$$\frac{V_1(s)}{I_1(s)} = Z_1+Z_2 = \frac{s^2+2s}{s^2+3s+1}+\frac{s+1}{s^2+3s+1}$$

$$\frac{V_1(s)}{I_1(s)} = \frac{\cancel{s^2+3s+1}}{\cancel{s^2+3s+1}} = 1$$

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