

# Unsupervised Machine Learning to Unravel Long-range Predictive Structure In Global Seismicity

Ishanu Chattopadhyay

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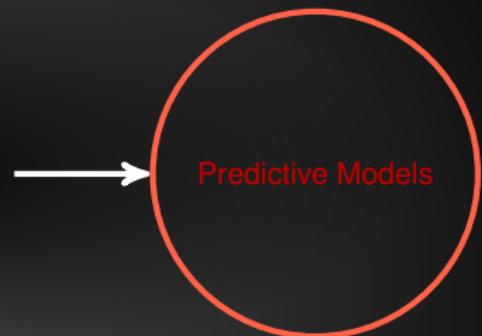
Department of Medicine  
Computation Institute

University of Chicago



# Learning Patterns from Seismic Catalogs

1973-04-30T23:32:36.000Z,51.595,177.786,61.4,8,"Rat Islands, Aleutian Islands, Alaska"  
1973-04-30T22:37:47.900Z,36.227,141.417,51.4,8,"near the east coast of Honshu, Japan"  
1973-04-30T21:00:41.000Z,4.993,-77.96,23.5,"near the west coast of Colombia"  
1973-04-30T16:39:49.800Z,29.93,131.612,62.4,4,"southeast of the Ryukyu Islands, Japan"  
1973-04-30T15:55:21.100Z,-17.85,167.649,22.4,4,"Vanuatu"  
1973-04-30T15:48:24.800Z,43.343,43.766,46.4,2,"Caucasus region, Russia"  
1973-04-30T12:41:15.900Z,-10.867,166.812,33.4,2,"Santa Cruz Islands"  
1973-04-30T11:55:29.200Z,60.951,151.131,33.3,4,"Kenai Peninsula, Alaska"  
1973-04-30T08:39:07.700Z,-17.514,179.623,613.5,4,"Fiji"  
1973-04-30T07:29:48.700Z,50.993,89.734,33.4,6,"southwestern Siberia, Russia"



**Marian Anghel, LANL**

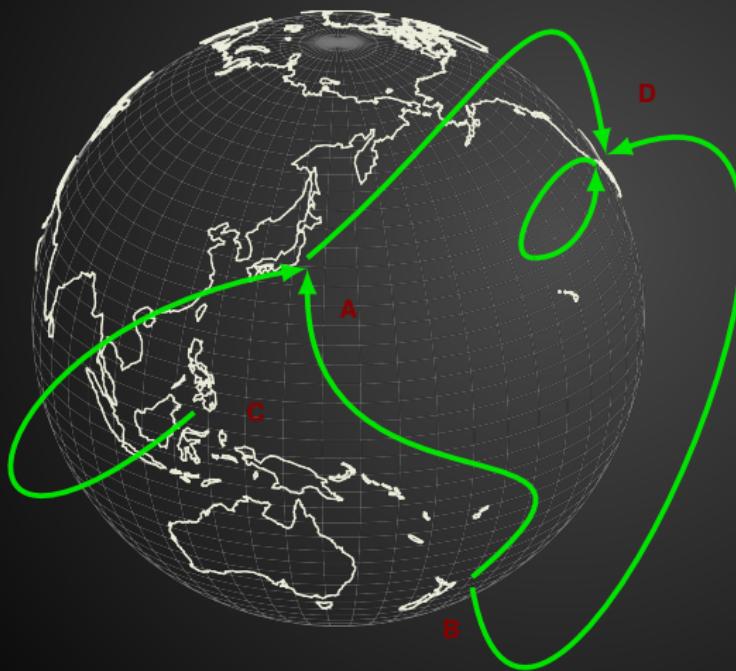
**Yehuda Ben Zion, Geophysics, USC**

**Hod Lipson, Data Science Institute, Columbia University**



# Towards Seismic Prediction

We Need To Model Cross-dependencies

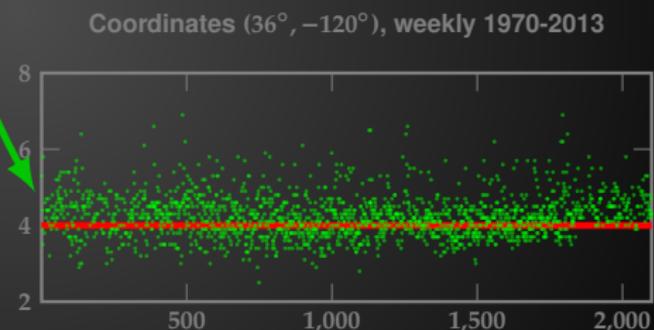


Do Faults Interact  
Over Vast Distances  
& Over Extended Time-periods?



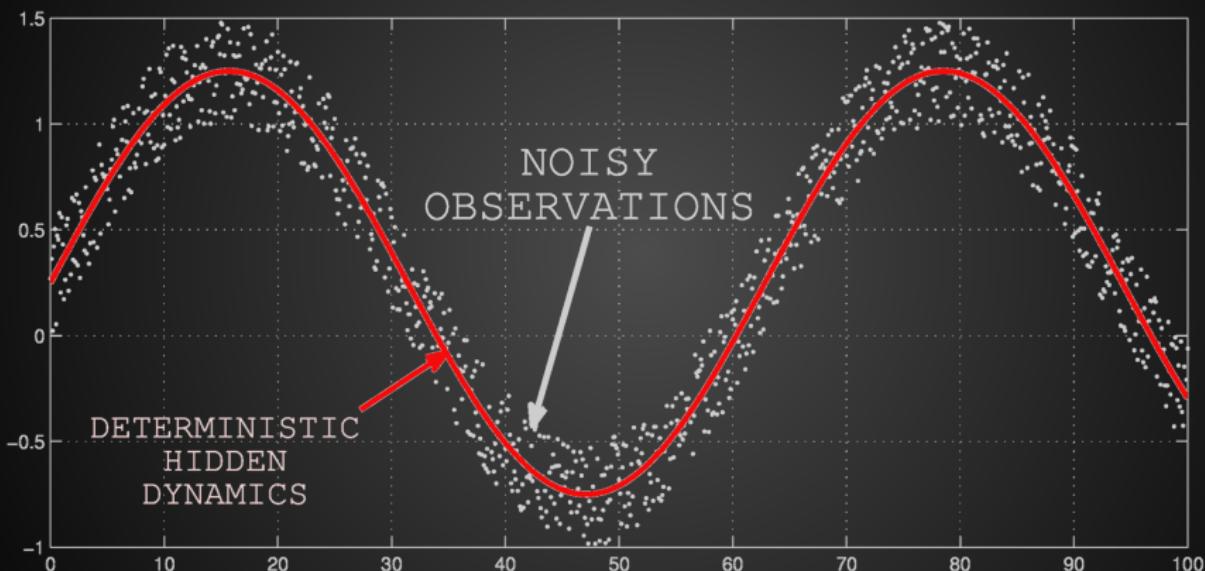
## Predicting Seismic Events

- Complex spatio-temporal stochastic processes drive seismic events
- Difficult to impossible to know parametric values for detailed simulations
- Can we simply mine the catalogs?



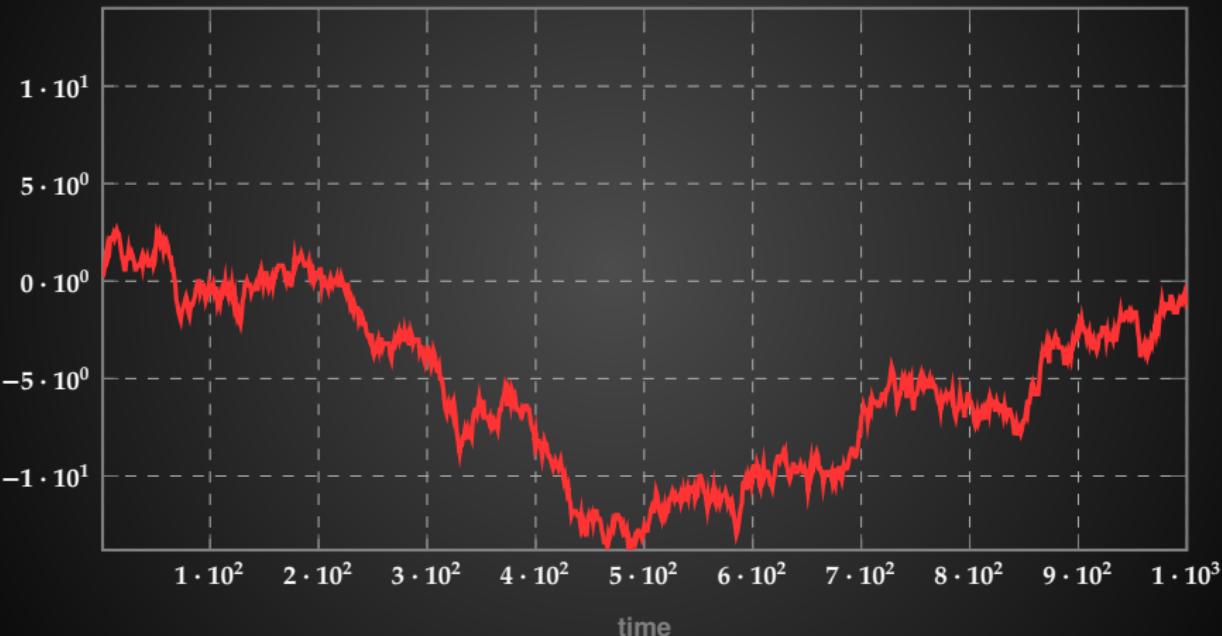


## Stochastic Process $\neq$ Deterministic + Noise



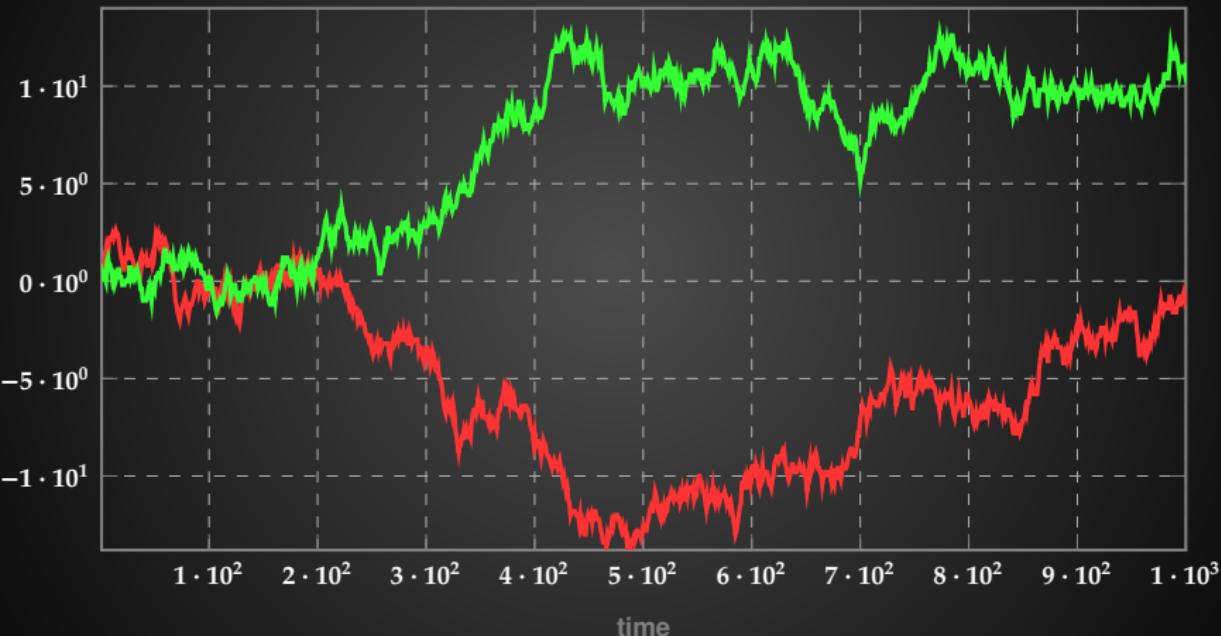


## Stochastic Process $\neq$ Deterministic + Noise





## Stochastic Process $\neq$ Deterministic + Noise





# Causality Not Correlation

Granger's Notion of Causality

“Unlike art, causality is a concept (for) whose definition people know what they do not like, but few know what they do like.”

- C. W. J. Granger

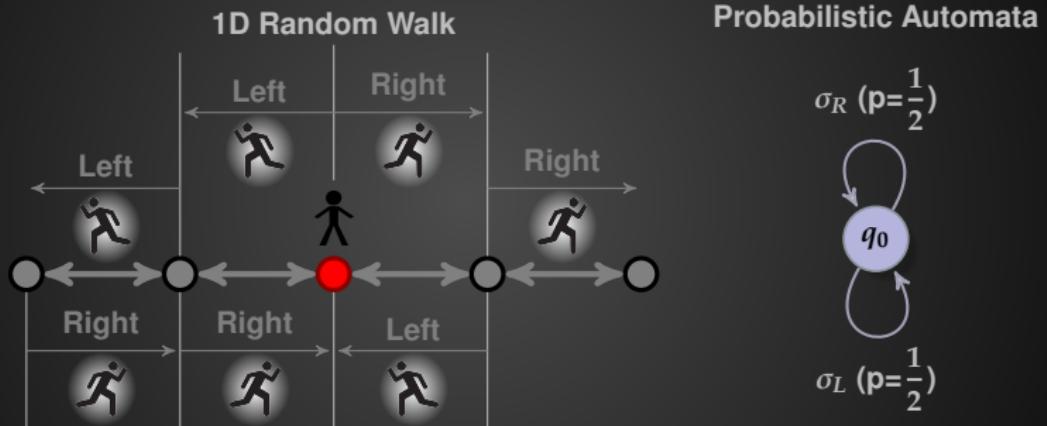
$Y$  is a cause of  $X$ ,  
if  $Y$  has unique information that better predicts future of  $X$ .





# Can We Learn Models “Non-parametrically”

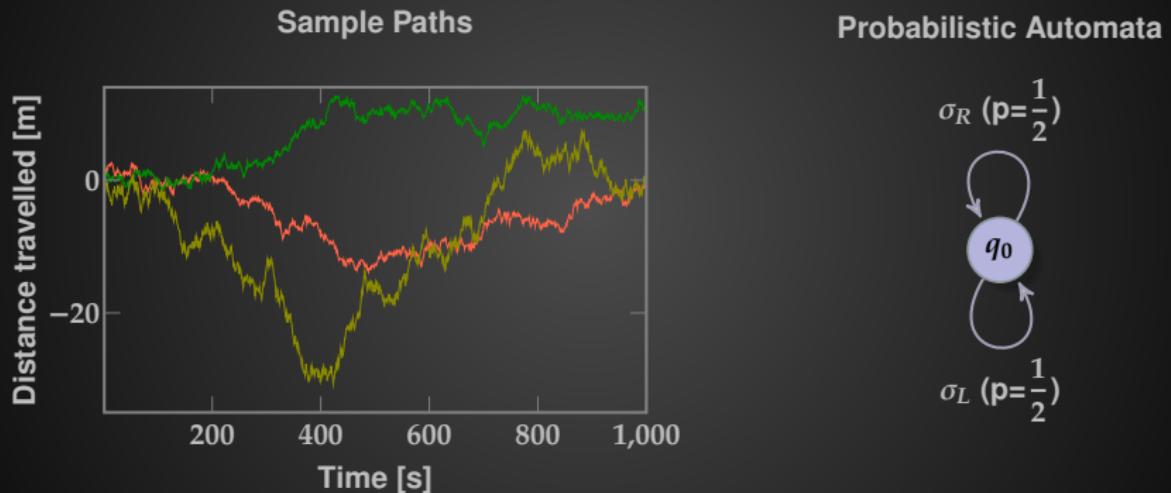
Probabilistic Automata As Stochastic Process Representations





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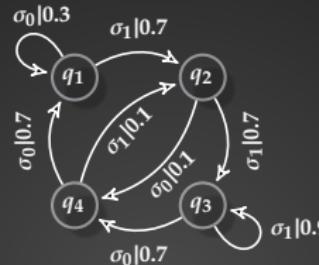
Probabilistic Automata As Stochastic Process Representations





# Probabilistic Finite State Automata

Models For Quantized Stationary Ergodic Stochastic Processes

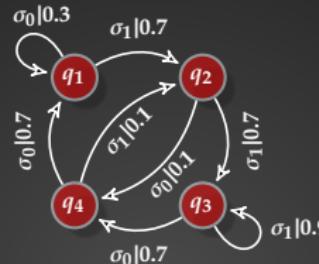


|                         |                                                      |                                                                                  |
|-------------------------|------------------------------------------------------|----------------------------------------------------------------------------------|
| State Set               | $Q$                                                  | $q_1, \dots, q_4$                                                                |
| Alphabet                | $\Sigma$                                             | $\sigma_0, \sigma_1$                                                             |
| Morph probabilities     | $\tilde{\pi} : Q \times \Sigma^* \rightarrow [0, 1]$ | $\begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \\ 0.1 & 0.9 \\ 0.7 & 0.3 \end{pmatrix}$ |
| Stationary distribution | $\varphi^*$                                          | $\varphi^* P = \varphi^*$                                                        |



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Models For Quantized Stationary Ergodic Stochastic Processes

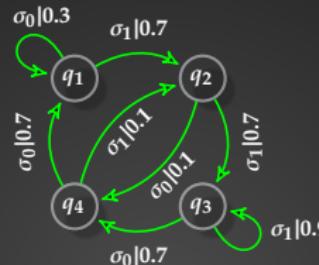


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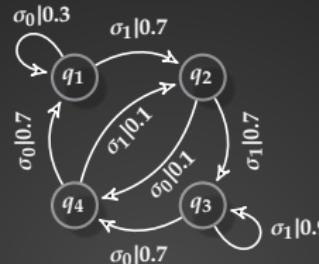


| State Set               | $Q$                                                  | $q_1, \dots, q_4$                                                                |
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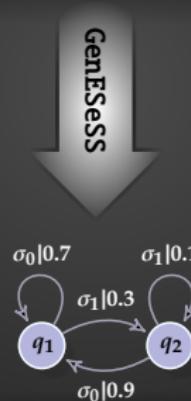
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## Explicit Construction

Algorithm GenESeSS

010010001001000010000010000101000010000101000010100100000000100000100000010010010000000000010  
00101000101000100010001000000010001010110000101000101000110010000001000000010101000000010001  
001010010001100011001010001000000001000100001001000011001000110010000100100000010100000000101  
010001000000000001111000000001000010100010001000000000100000100000101000101010100101001000101  
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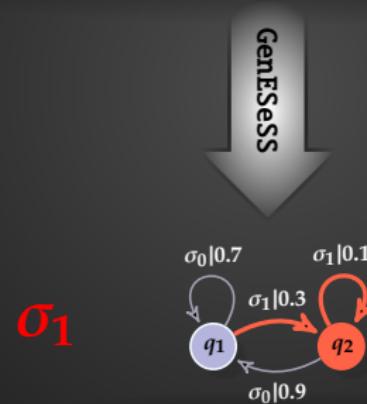
generator Extraction from Self-Similar Semantics



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010010001001000010000001000010000101000010000101000010100100000000100000100000010010010000000000010  
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00101001000110001100101000100000000100010000100100001100100010000110010000100100000010100000000101  
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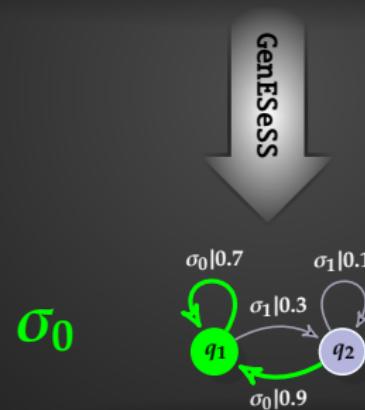


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## Explicit Construction

Algorithm GenESeSS



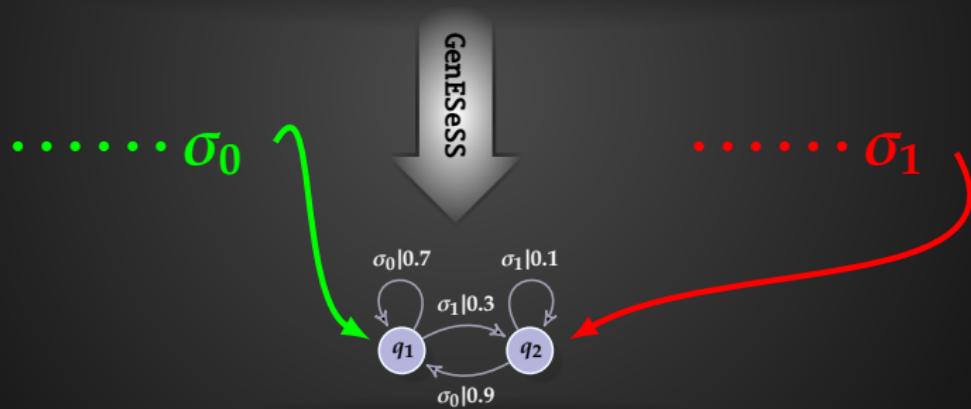
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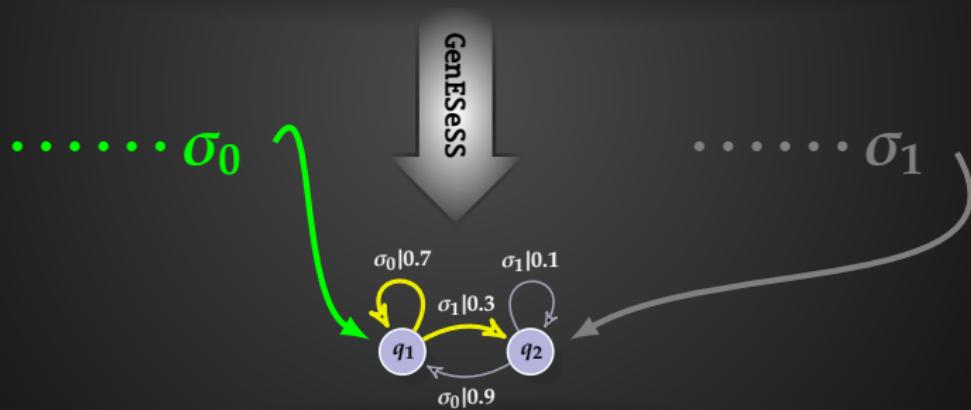


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## Explicit Construction

### Algorithm GenESS



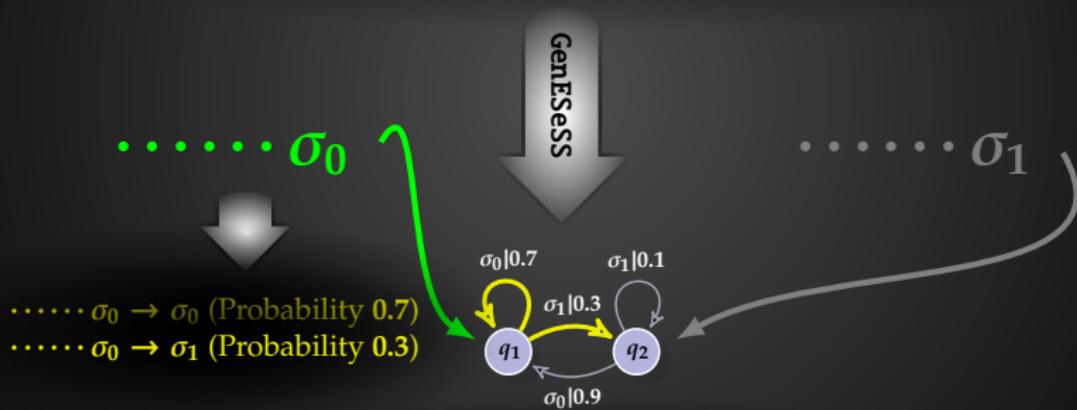
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## Explicit Construction

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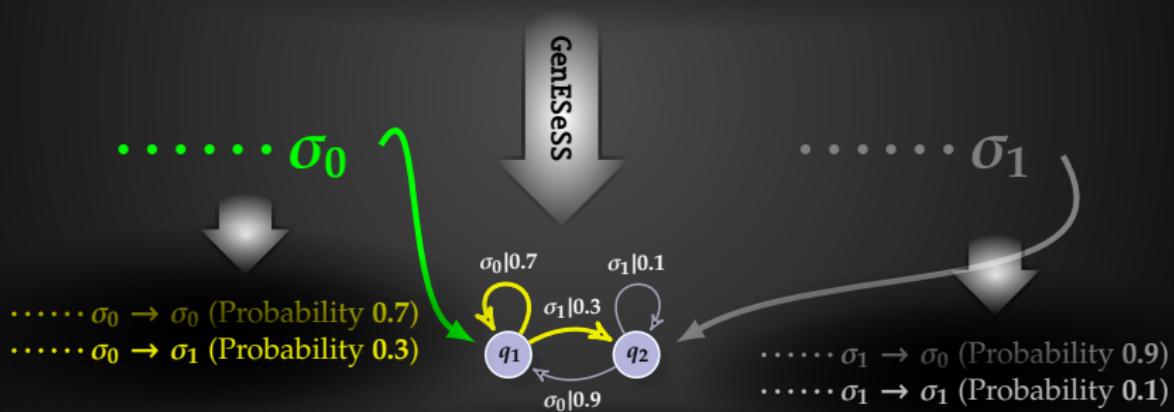
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## Explicit Construction

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010010001001000010000010000101000010000101000010100100000000100000100000010010010000000000010  
00101000101000100010001000000010001010110000101000101000110010000001000000010101000000010001  
001010010001100011001010001000000001000100001001000011001000110010000100100000010100000000101  
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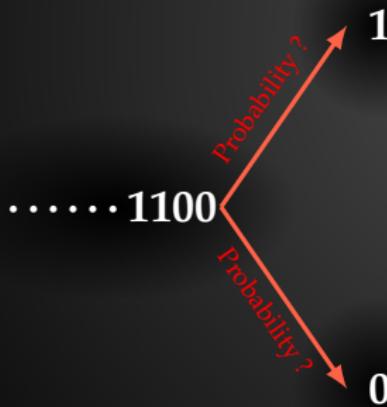
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## Symbolic Derivatives

## Estimating Average Immediate Future

01001000100100001000000100001000010100001000010100001010010000000100000100000010010010000000000010  
0010100010100010001000100000001000101011000010100010100011001000000100000001010100000010001  
00101001000110001100101000100000000100010000100100001100100001100100000101000000101000000000101  
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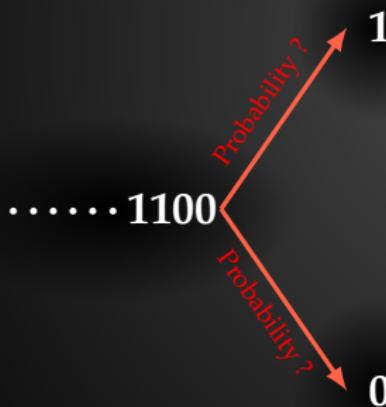
$$\phi_{\omega}^s = \begin{pmatrix} Pr(\sigma_0) \\ Pr(\sigma_1) \end{pmatrix}$$

where  $\omega = \sigma_1\sigma_1\sigma_0\sigma_0$



## Symbolic Derivatives

## Estimating Average Immediate Future



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# Symbolic Derivatives

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Estimating Average Immediate Future

|           |          |           |
|-----------|----------|-----------|
| $\lambda$ | 0.750285 | 0.249715  |
| 0         | 0.700112 | 0.299888  |
| 1         | 0.901009 | 0.0989909 |
| 00        | 0.699844 | 0.300156  |
| 01        | 0.899111 | 0.100889  |
| 10        | 0.700711 | 0.299289  |
| 11        | 0.918285 | 0.0817152 |
| 000       | 0.699004 | 0.300996  |
| 001       | 0.898769 | 0.10123   |
| 010       | 0.701038 | 0.298962  |
| 011       | 0.917181 | 0.0828194 |
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| 101       | 0.899911 | 0.100089  |
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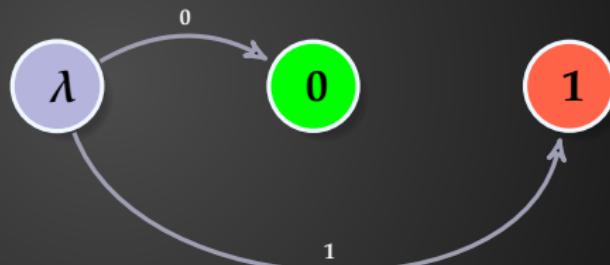




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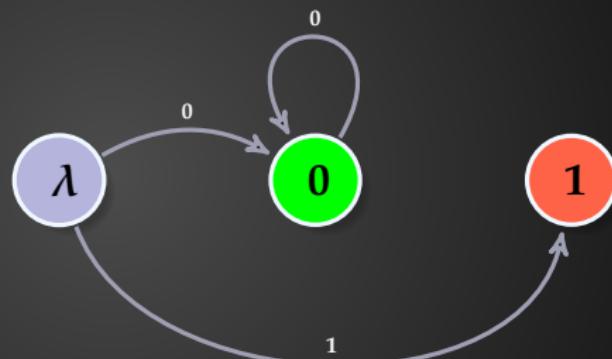




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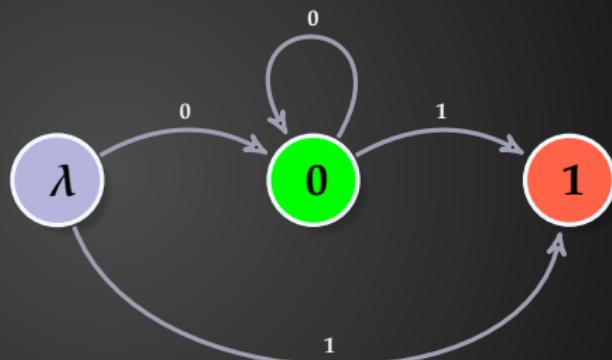




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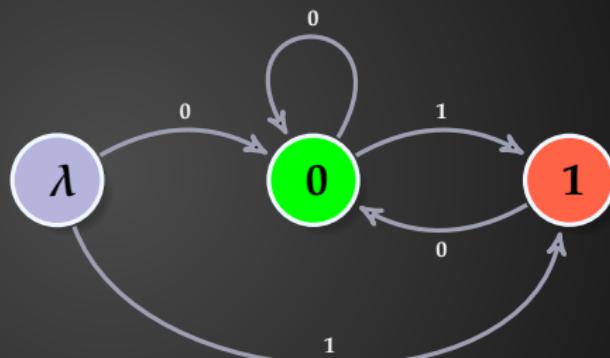




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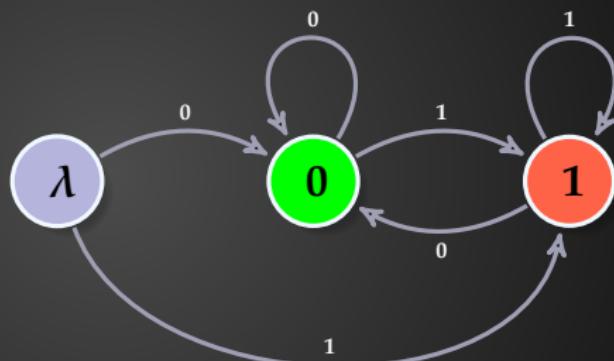




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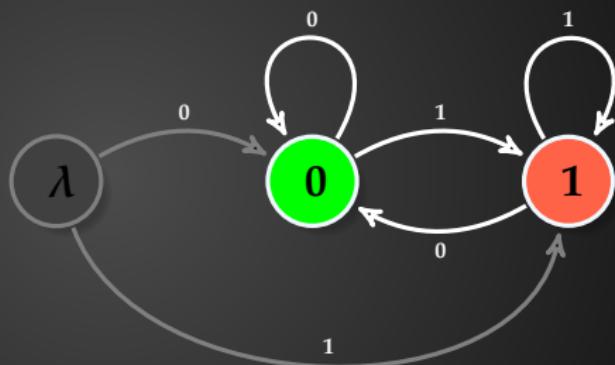




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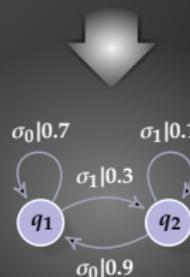
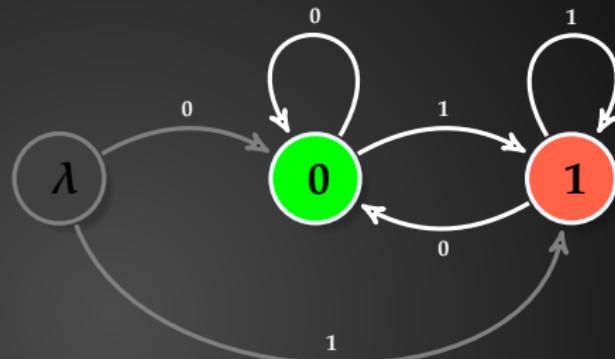




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| 000       | 0.699004 | 0.300996  |
| 001       | 0.898769 | 0.10123   |
| 010       | 0.701038 | 0.298962  |
| 011       | 0.917181 | 0.0828194 |
| 100       | 0.701763 | 0.298237  |
| 101       | 0.899911 | 0.100089  |
| 110       | 0.697797 | 0.302203  |
| 111       | 0.930693 | 0.0693069 |
| 0000      | 0.699284 | 0.300716  |
| 0001      | 0.902025 | 0.0979754 |
| :         | :        | :         |

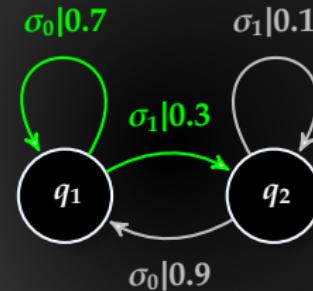




# Symbolic Derivatives

Estimating Average Immediate Future

|           |                 |                 |
|-----------|-----------------|-----------------|
| $\lambda$ | 0.750285        | 0.249715        |
| 0         | <b>0.700112</b> | <b>0.299888</b> |
| 1         | 0.901009        | 0.0989909       |
| 00        | <b>0.699844</b> | <b>0.300156</b> |
| 01        | 0.899111        | 0.100889        |
| 10        | <b>0.700711</b> | <b>0.299289</b> |
| 11        | 0.918285        | 0.0817152       |
| 000       | <b>0.699004</b> | <b>0.300996</b> |
| 001       | 0.898769        | 0.10123         |
| 010       | <b>0.701038</b> | <b>0.298962</b> |
| 011       | 0.917181        | 0.0828194       |
| 100       | <b>0.701763</b> | <b>0.298237</b> |
| 101       | 0.899911        | 0.100089        |
| 110       | <b>0.697797</b> | <b>0.302203</b> |
| 111       | 0.930693        | 0.0693069       |
| 0000      | <b>0.699284</b> | <b>0.300716</b> |
| 0001      | 0.902025        | 0.0979754       |
| :         | :               | :               |
| :         | :               | :               |



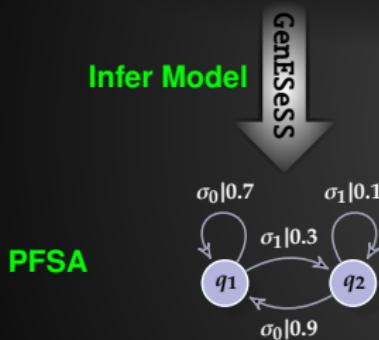
Merging of Symbolic Derivatives  
Under  $\epsilon$  Resolution



# Minimizing Modeling Error

Combining **GenESeSS** with **data smashing**

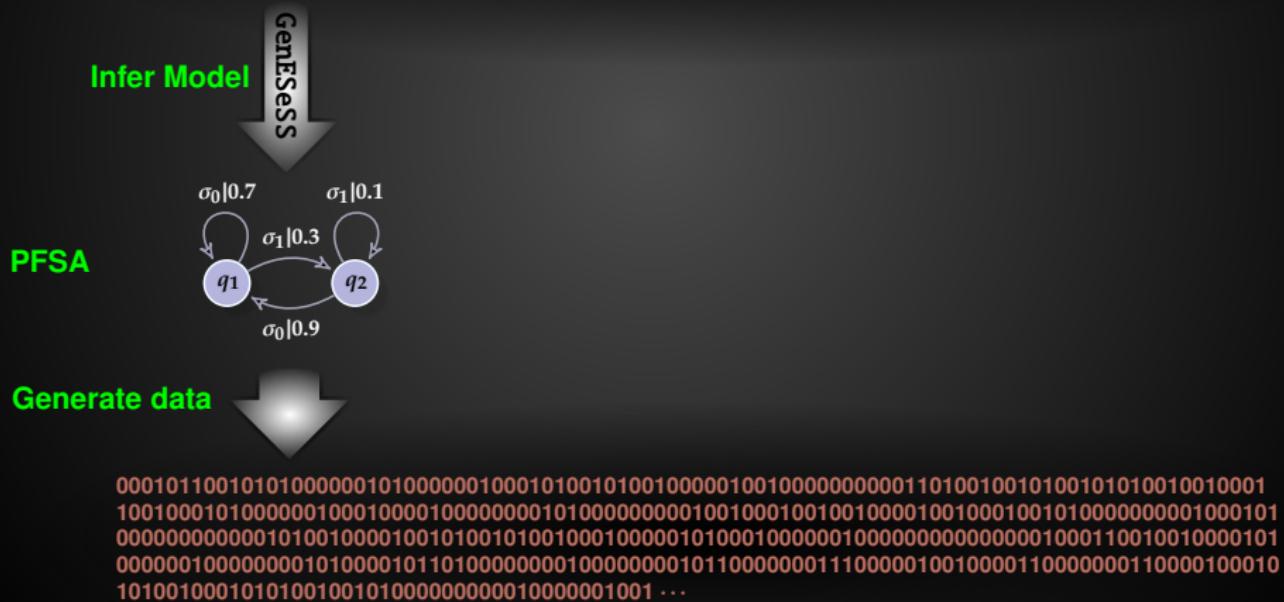
01001000100100001000000100001000010100001000010100001010010000000100000100000010010010000000  
00101000101000100010000001000101011000010100010100011001000000100000001000000010101000000  
0010100100011000110010100010000000010001000010010000110010001000100001100100001001000000101000000  
010001000000000000001111000000001000010100010001000000001000001000001010001010101001001010010  
0010000000100001100010000000010010100000000010000001000000010010100001000010001000 ...





## Minimizing Modeling Error

## Combining GenESeSS with data smashing

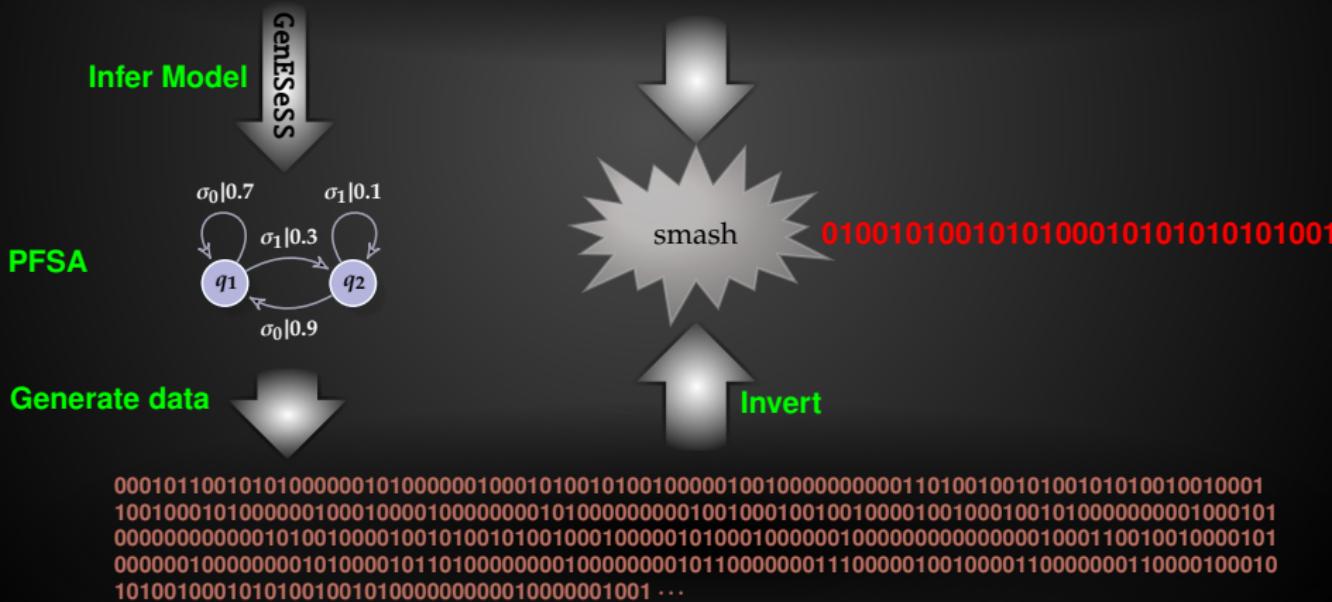




## Minimizing Modeling Error

## Combining GenESeSS with data smashing

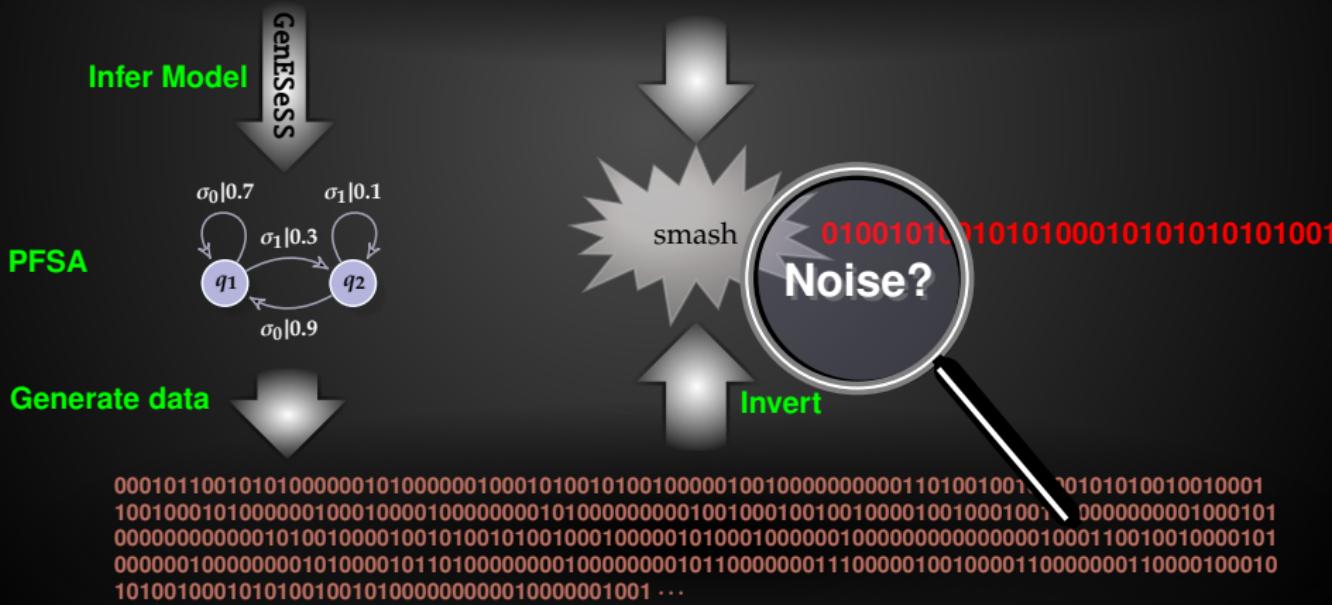
0100100010010000100000010000101000010000101000010100100000000100000100000010010010000000  
00101000101000100010001000000100010101100001010001010001100100000010000000100000010101000000  
001010010001100011001010001000000001000100001001000011001000100001100100001001000000101000000  
010001000000000001111000000001000010100010001000000001000001000001000001010001010101001010010  
00100000000010000110001000000001001010000000001000000100000000100101000001000010001000 ...





## Minimizing Modeling Error

## Combining GenESeSS with data smashing





# Probably Approximately Correct

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## How Hard Is It To Learn PFSAs

### Time Complexity

Assuming  $|s| > |\Sigma|$ , the asymptotic time complexity of GenESeSS is:

$$\mathcal{T} = O\left(\frac{|s||\Sigma|}{\epsilon}\right)$$

### PAC-Learnability

Ergodic, stationary quantized stochastic processes with finite number of causal states has the following property:

For  $\epsilon, \eta > 0$ , and for every sufficiently long sequence  $s$  generated by QSP  $\mathcal{H}$ , GenESeSS computes  $\mathcal{P}'_{\mathcal{H}}$  as an estimate for  $\mathcal{P}_{\mathcal{H}}$  with:

$$Pr(\Theta(\mathcal{P}_{\mathcal{H}}, \mathcal{P}'_{\mathcal{H}}) \leq \epsilon) \geq 1 - \eta$$

Asymptotic runtime is polynomial in  $1/\epsilon, 1/\eta, |s|$ , and sample complexity is:

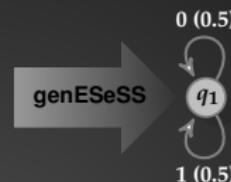
$$|s| = O(C_0^{|Q|} \frac{1}{\epsilon} \log \frac{1}{\eta})$$



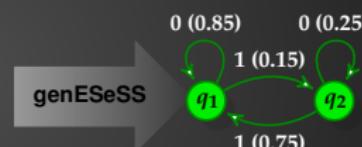
# Learning Quantized Stochastic Processes

### Algorithm GenESeSS

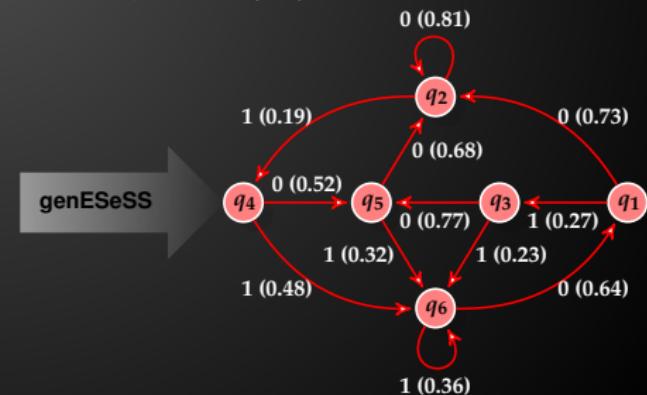
## Flat White Noise (Fair Coin Toss Sequence)



## Stochastic process with 2 causal states



## Stochastic process with 6 causal states

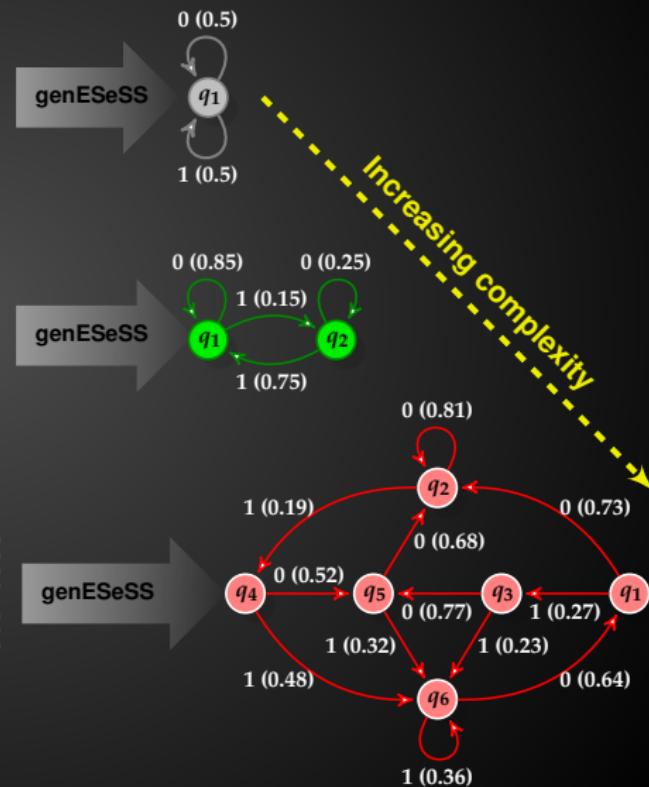




# Learning Quantized Stochastic Processes

### Algorithm GenESeSS

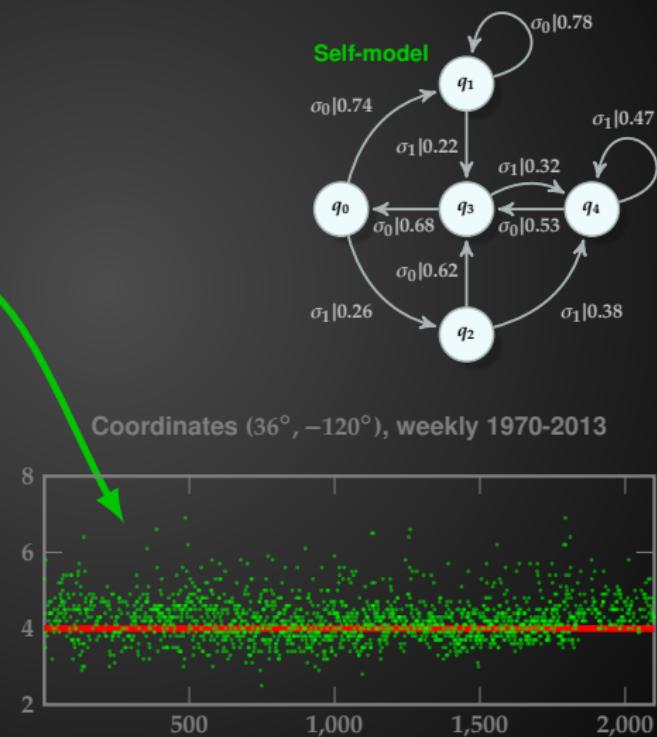
## Flat White Noise (Fair Coin Toss Sequence)





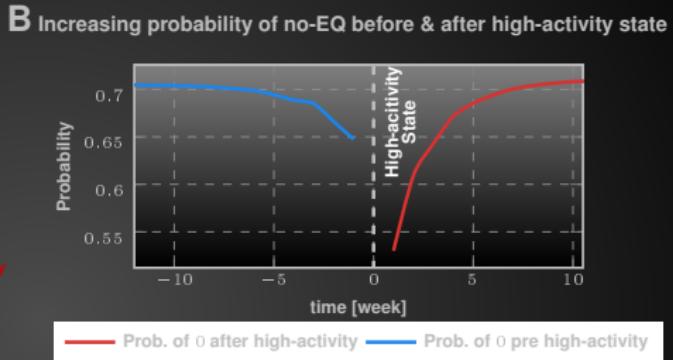
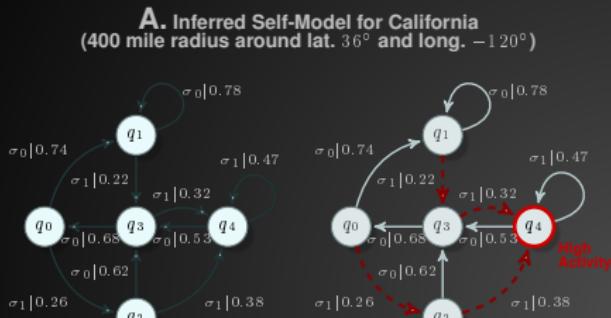
# Predicting Seismic Events

With Both Space & Time Quantization

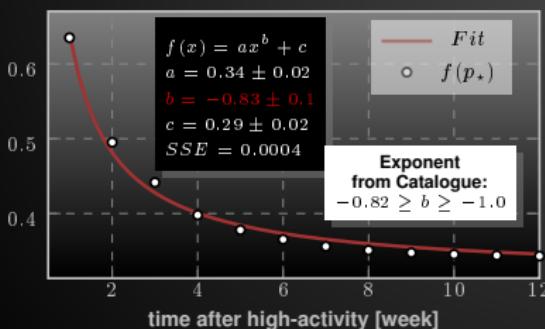




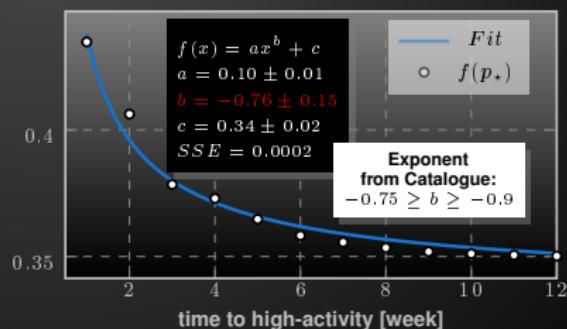
# Validating The Omori-Utsu Law



**D. Indirect Validation of Omori-Utsu Law (aftershocks)**

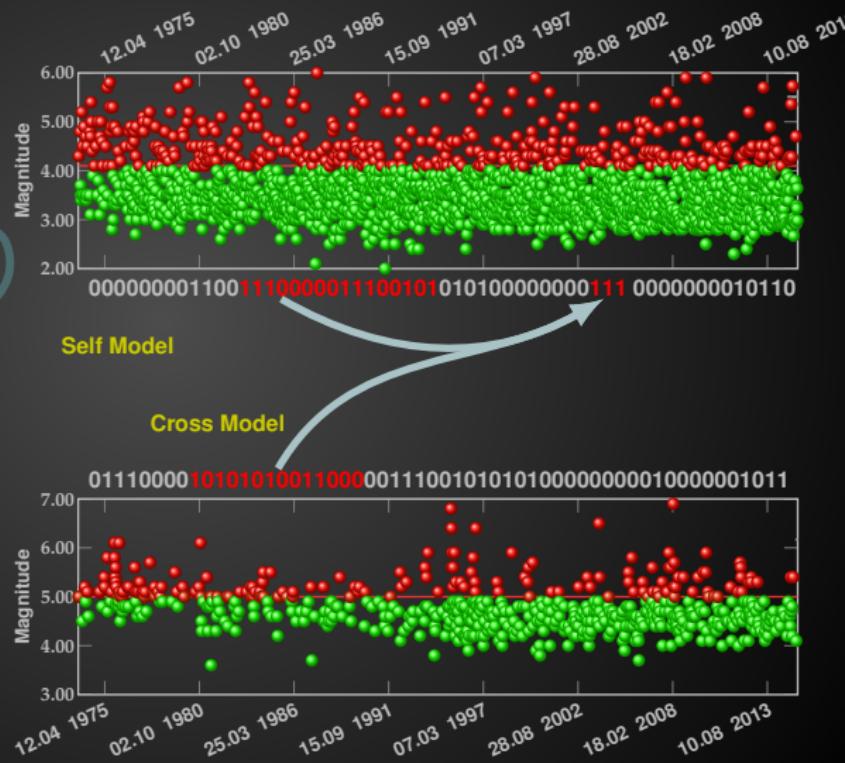
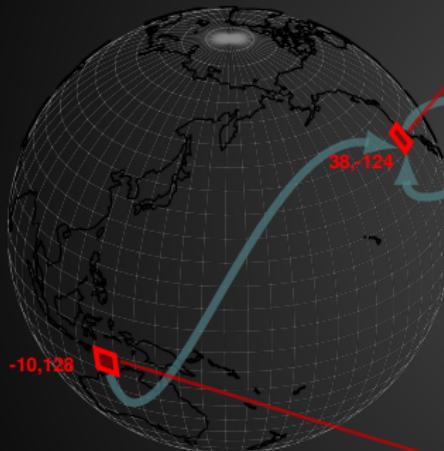


**D. Indirect Validation of Omori-Utsu Law (foreshocks)**





# Seismic Causality Network

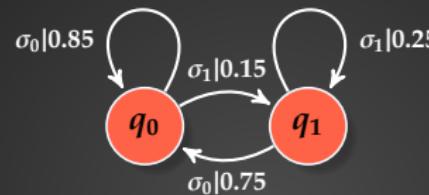




# Inferring Cross-dependence

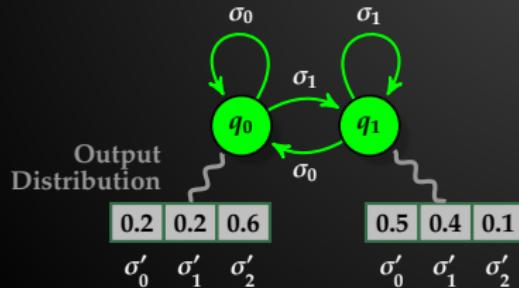
Probabilistic Transducers

## A. Probabilistic Finite State Automata

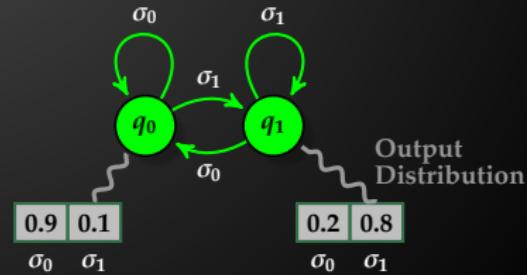


## Crossed Probabilistic Finite State Automata (XPFSA)

### B. (3-letter output alphabet)



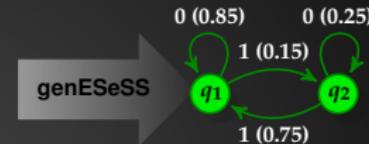
### C. (2-letter output alphabet)





## Inferring Causality

### Algorithm XgenESeSS



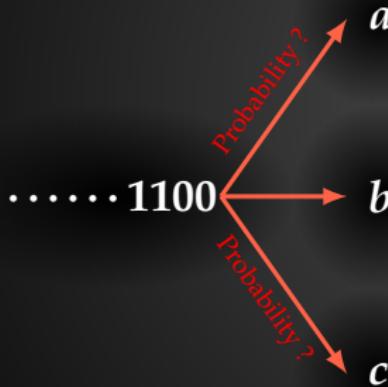
## A. Stochastic process on two letter alphabet

aaabbabbacababababcacacacccaaaabbbbaccacabbbbabbbbaccacaccccabbacc  
cabbbaccccabbacacbaccacabababccacababcacaccababacaccabbbbacab  
abcdbcbbcaccacabcbcbabababababacbabacacccccaaaabbbbaccacabbb  
ababbbcacacccabbbaccacccabbbaccacbabacacccabbaaca  
caabbacaccabbbacacabcbcbcbccaccacabcbababacacacccccaaaabbbbaccac  
cababbcacccaabbababacaccabbbacacbcbbcbccaccacabcbcbabababbacabab

## B. Stochastic process on 3 letter alphabet



## Symbolic Cross Derivatives



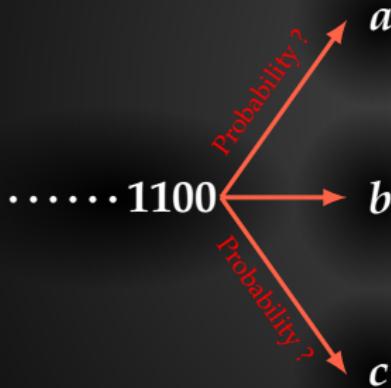
$$\phi_{\omega}^{s_1 \rightarrow s_2} = \begin{pmatrix} Pr(a) \\ Pr(b) \\ Pr(c) \end{pmatrix}$$

where  $\omega = 1100$

ababbaabbabbbcbcbccaccacccabacaccacacabcbcbccaccacccacaccabbbcbcbccacacacacaaaabbccbbcbccbaaa  
caccaccaabbcbbcbccaccacccacaabbaaccacacacacaaabbccbbcbccaccaccaacbcbbcbccbababbababacbbccbccaccac  
babbbabbccbbcbccaccacccacabbbcbcbcccccacccaaaabbbabbbabcbcbccbbcbccacaccacccaaaaabbabbccbbcbccbb  
bcbbcbcbccbbcbccbbcbccbbccaccacaccacccaaaabbbabbbabbaaacccaccaababbbcbcccccacabbbabbabacc  
babbbabbccbbccaccacccababbaaccacccaaaabbccbbcbccaccaccaabbaabbababacbbccbbcbcccccacacaca...



## Symbolic Cross Derivatives



$$\phi_{\omega}^{s_1 \rightarrow s_2} = \begin{pmatrix} Pr(a) \\ Pr(b) \\ Pr(c) \end{pmatrix}$$

where  $\omega = 1100$

ababbaabbababbcbcbccaccacccabbacaccacacabcbcbccaccaccccccacaccabbcbcbccacacacacacaaaaabbccbbcbbaa  
caccaccaabbcbbcbccacccacaabbabaaccacacacacaaaabbcbcbccbc**c**bccaccaccaacbcbbcbbababbababacbbcbccaccac  
bababbcbcb**c**accac**c**acabbcbcbcccccacccaaaabb**b**bb**c**bcbbcc**b**bcacaccacccaaaabbabbcbbbcbcbcbbbbcb  
bcbcbcbcbcbcb**b**acc**b**bbcbcbccbbccaccacaccacaaaabbababbabaaccaccaabababbcbcbccacababbabbabacc  
bababbcbcbcc**c**ac**a**babbacaccacaaabb**b**acaccacccaa**a**babbbabbababab**c**bbcbcbcc**c**ccaccacaca...  
bababbcbcbcc**c**ac**a**babbacaccacaaabb**b**acaccacccaa**a**babbbabbababab**c**bbcbcbcc**c**ccaccacaca...



## Coefficient Of Causality

---

Given two ergodic stationary quantized stochastic processes  $\mathcal{H}_A, \mathcal{H}_B$  the coefficient of causal dependence of  $\mathcal{H}_B \rightarrow \mathcal{H}_A$  is defined as:

$$\gamma_B^A = \frac{\text{expected change in entropy of the next symbol distribution in } \mathcal{H}_B \\ \text{due to observations in } \mathcal{H}_A}{\text{entropy of the next symbol distribution} \\ \text{in } \mathcal{H}_B \text{ in the absence of observations in } \mathcal{H}_A}$$



## Coefficient Of Causality

---

Given two ergodic stationary quantized stochastic processes  $\mathcal{H}_A, \mathcal{H}_B$  the coefficient of causal dependence of  $\mathcal{H}_B \rightarrow \mathcal{H}_A$  is defined as:

$$\gamma_B^A = \frac{\text{expected change in entropy of the next symbol distribution in } \mathcal{H}_B \\ \text{due to observations in } \mathcal{H}_A}{\text{entropy of the next symbol distribution} \\ \text{in } \mathcal{H}_B \text{ in the absence of observations in } \mathcal{H}_A}$$

Using symbolic cross derivatives:

$$\gamma_B^A = 1 - \frac{\mathbb{E}_{x \in \Sigma_A^*} \left( h \left( \phi_x^{\mathcal{H}_A, \mathcal{H}_B} \right) \right)}{h \left( \phi_{\lambda}^{\mathcal{H}_A, \mathcal{H}_B} \right)}$$

where  $h(\cdot)$  is the discrete Shannon entropy.



# The Predictability Decay

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Non-parametric Non-linear “Auto-correlation” Using  $\gamma$

Noisy Duffing Oscillator:

$$\ddot{x} + c_0\dot{x} + c_1x + c_2x^3 = c_3\cos(\omega t) + \eta$$

Use  $x(t)$  and  $x(t + \Delta)$  as two data streams

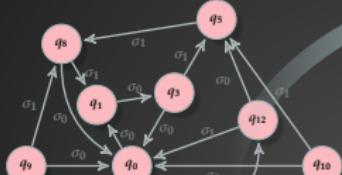




# Lots of Models Generated

Self-models: PFSAs, Cross-models: XPFSA

|          | $\sigma_0$ | $\sigma_1$ |
|----------|------------|------------|
| $q_0$    | 0.8        | 0.2        |
| $q_1$    | 0.8        | 0.2        |
| $q_2$    | 0.8        | 0.2        |
| $q_3$    | 0.8        | 0.2        |
| $q_4$    | 0.6        | 0.4        |
| $q_5$    | 0.8        | 0.2        |
| $q_6$    | 0.7        | 0.3        |
| $q_7$    | 0.7        | 0.3        |
| $q_8$    | 0.7        | 0.3        |
| $q_9$    | 0.8        | 0.2        |
| $q_{10}$ | 0.7        | 0.3        |
| $q_{11}$ | 0.9        | 0.1        |
| $q_{12}$ | 0.9        | 0.1        |

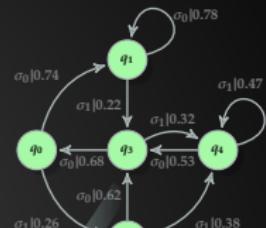


|          | $\sigma_0$ | $\sigma_1$ |
|----------|------------|------------|
| $q_0$    | 0.1        | 0.9        |
| $q_1$    | 0.1        | 0.9        |
| $q_2$    | 0.2        | 0.8        |
| $q_3$    | 0.2        | 0.8        |
| $q_4$    | 0.2        | 0.8        |
| $q_5$    | 0.1        | 0.9        |
| $q_6$    | 0.2        | 0.8        |
| $q_7$    | 0.1        | 0.9        |
| $q_8$    | 0.3        | 0.7        |
| $q_9$    | 0.2        | 0.8        |
| $q_{10}$ | 0.1        | 0.9        |

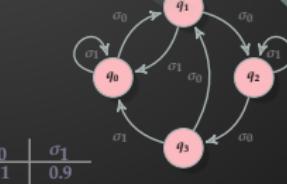
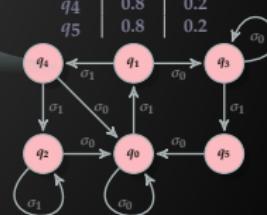


|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |
| $q_5$ | 0.2        | 0.8        |
| $q_6$ | 0.1        | 0.9        |
| $q_7$ | 0.2        | 0.8        |
| $q_8$ | 0.1        | 0.9        |
| $q_9$ | 0.2        | 0.8        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |
| $q_5$ | 0.2        | 0.8        |



|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.7        | 0.3        |
| $q_1$ | 0.7        | 0.3        |
| $q_2$ | 0.8        | 0.2        |
| $q_3$ | 0.8        | 0.2        |
| $q_4$ | 0.8        | 0.2        |



|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|       | $\sigma_0$ | $\sigma_1$ |
|-------|------------|------------|
| $q_0$ | 0.1        | 0.9        |
| $q_1$ | 0.2        | 0.8        |
| $q_2$ | 0.1        | 0.9        |
| $q_3$ | 0.2        | 0.8        |
| $q_4$ | 0.1        | 0.9        |

|  | $\sigma_0$ | $\sigma_1$ |
| --- | --- | --- |




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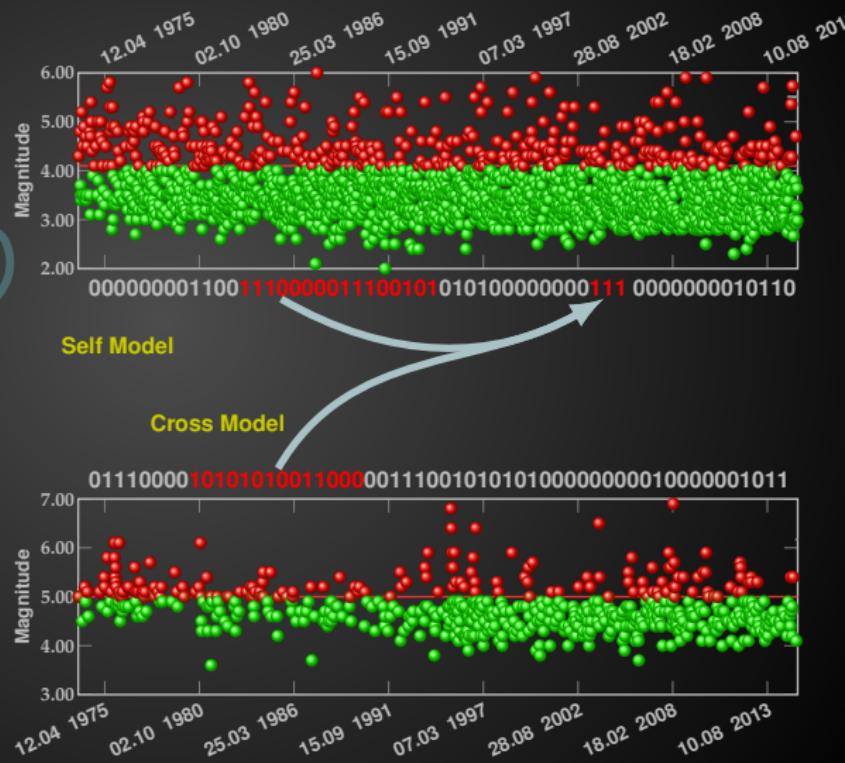
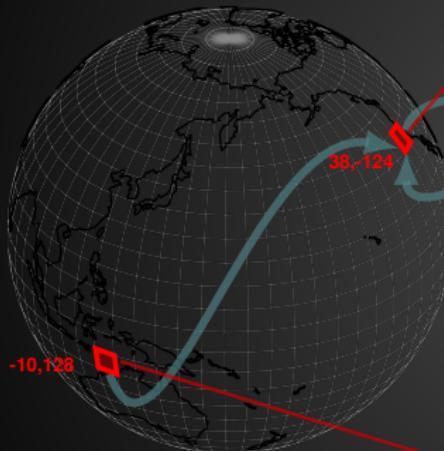
Models tested: 3,465,600

Compute time:  $\approx$  30,000 core hours.

Models with  $\gamma > 0.01$ : 195, 884



# Seismic Causality Network

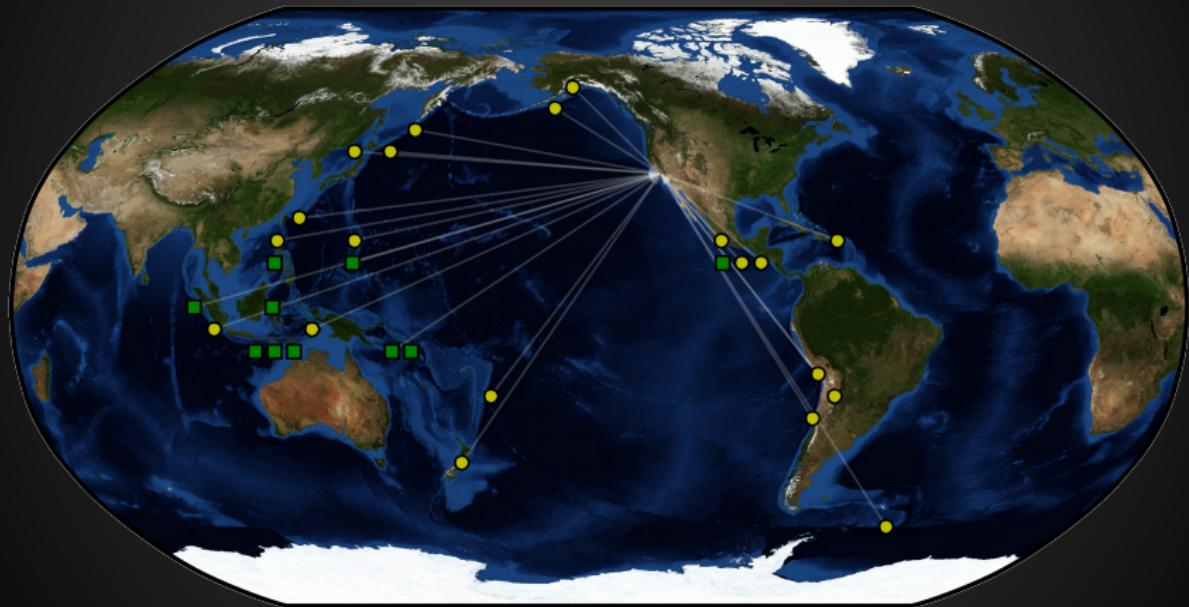




# Inferred Cross-Interactions For Earthquake Prediction California

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Infering The Pacific Ring of Fire

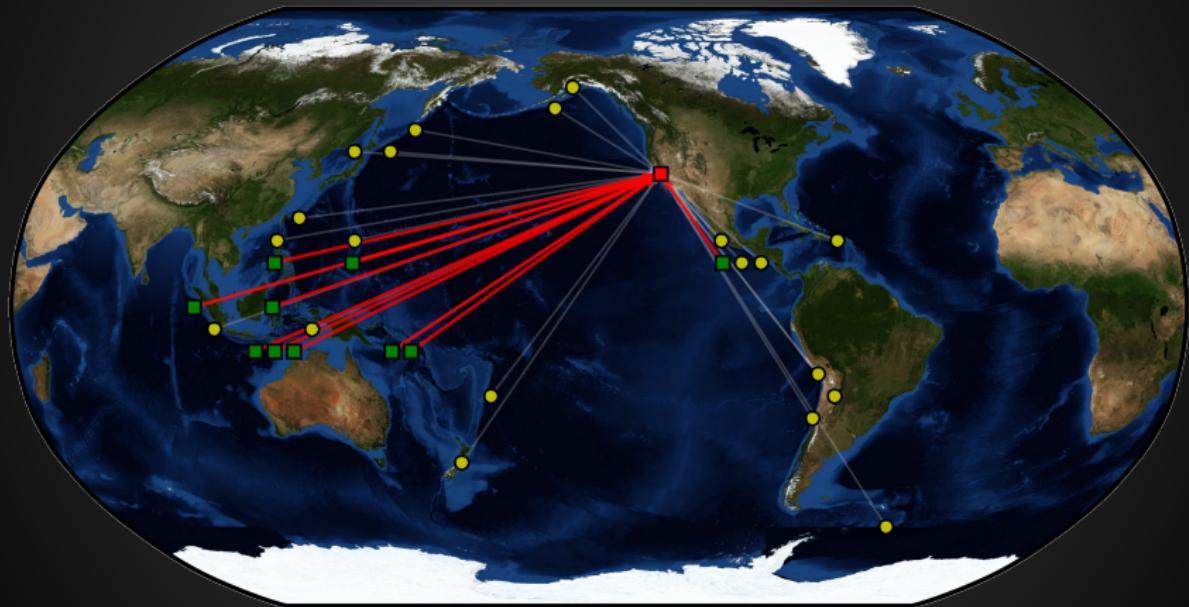




# Inferred Cross-Interactions For Earthquake Prediction California

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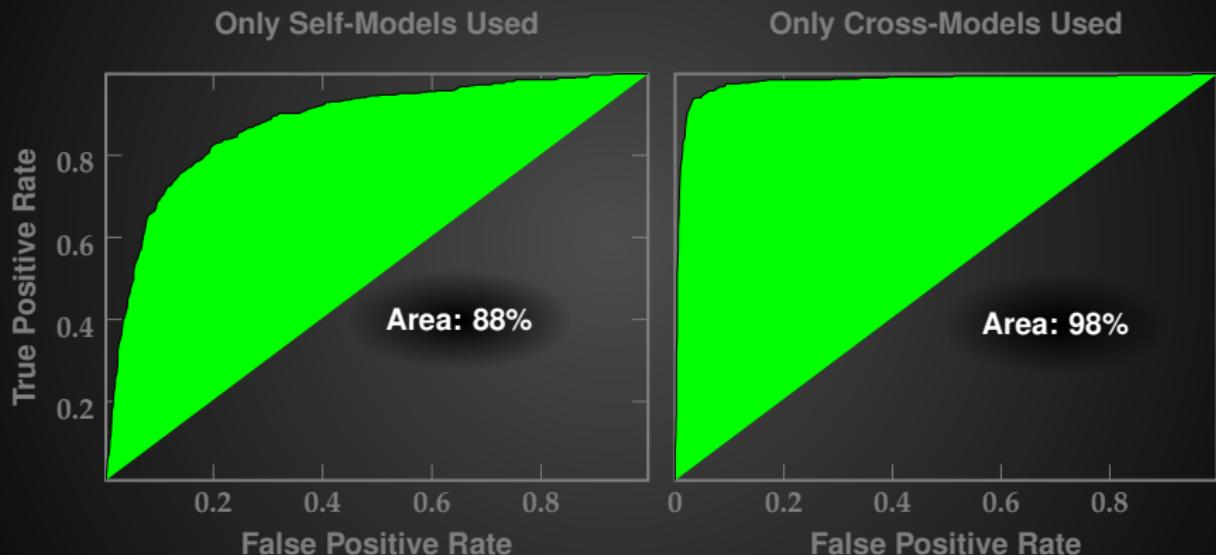
Infering The Pacific Ring of Fire





# Prediction Performance: California Seismicity

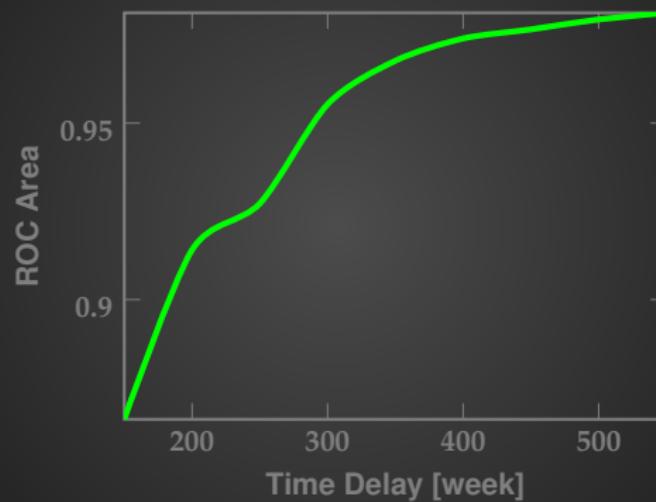
Prediction Horizon: 300 weeks





# Prediction Performance Vs Time Delay

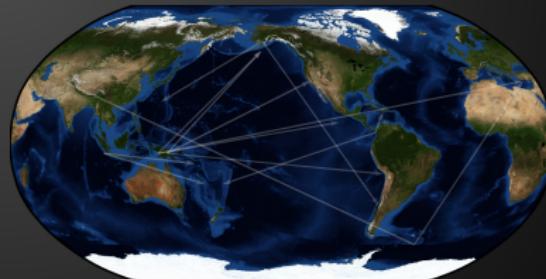
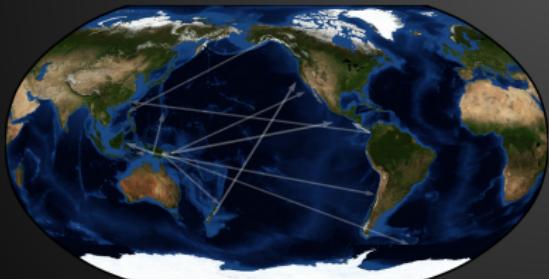
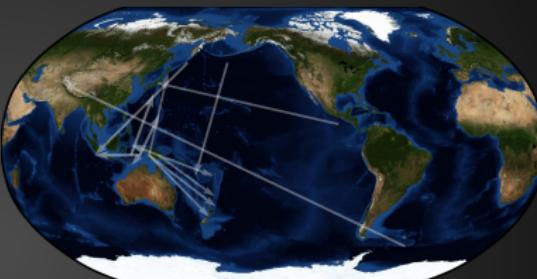
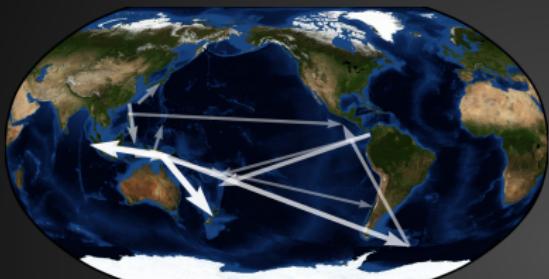
Postseismic Effects Last For A Decade





# The Global Network

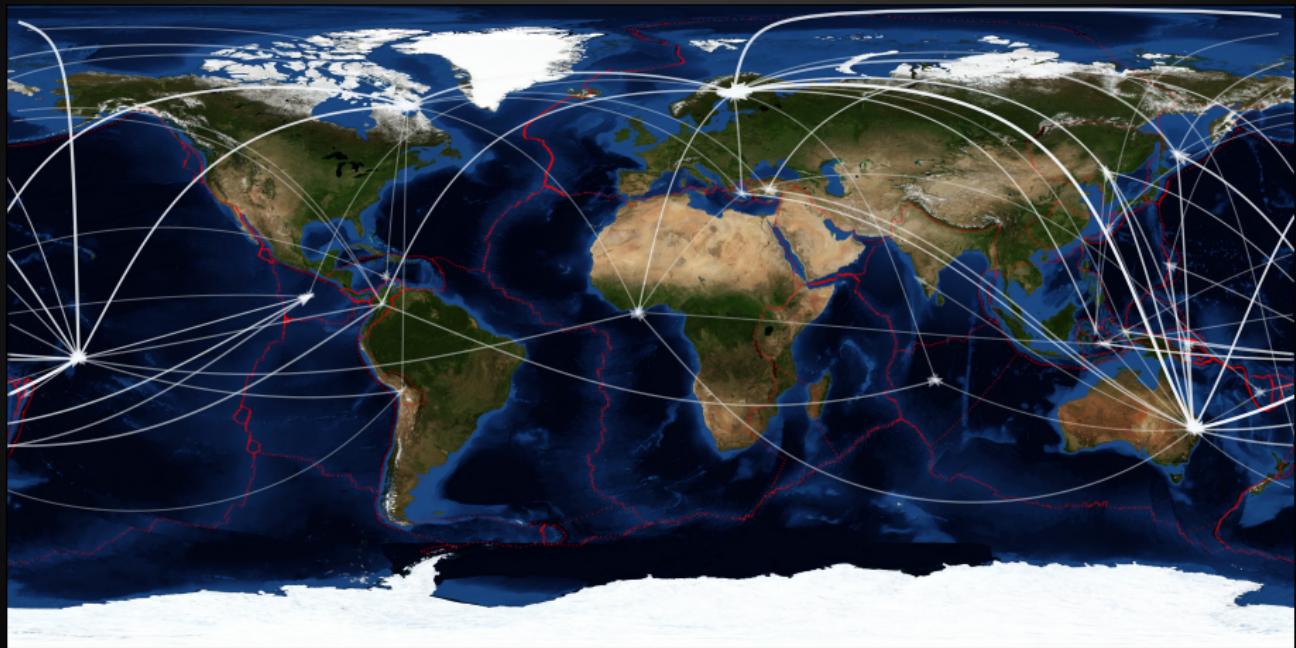
Top 20% of Inferred Connections





# The Inferred Global Network

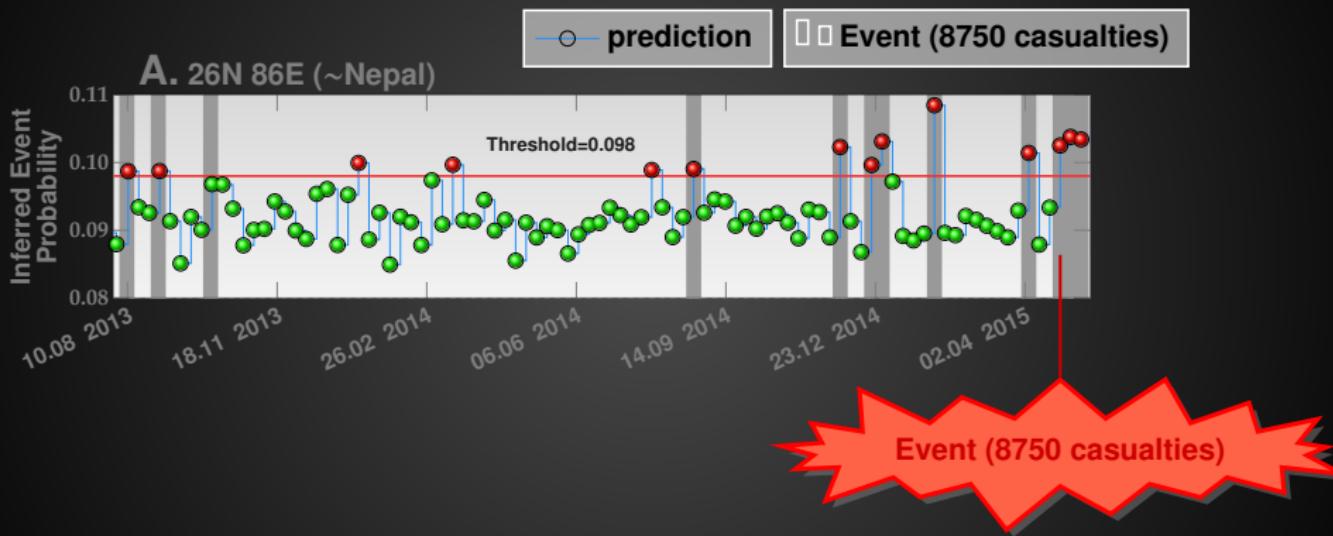
Normalized to Tectonic plates





## Prediction Performance

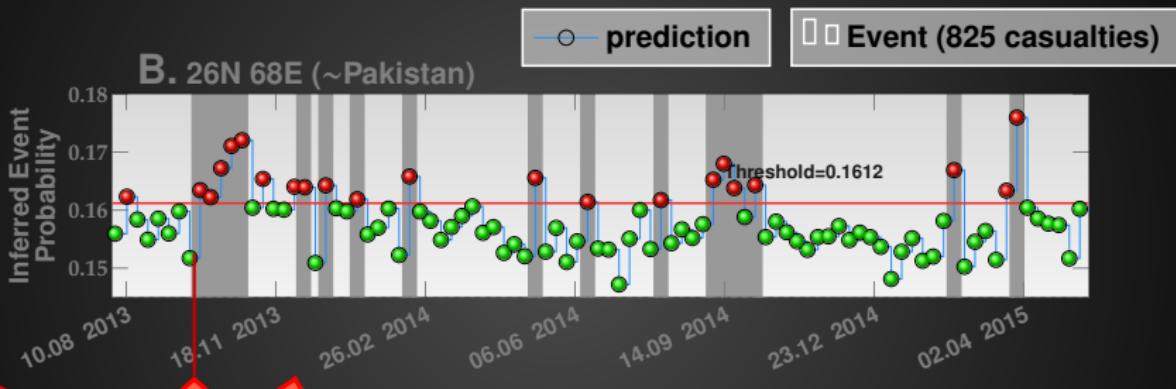
Out of Sample





## Prediction Performance

Out of Sample

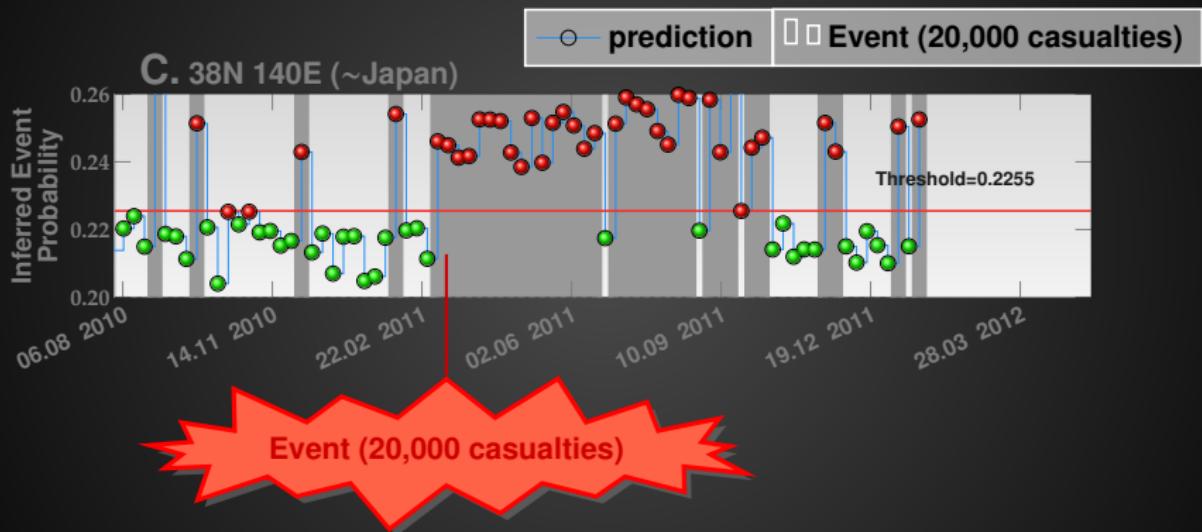


Event (825 casualties)



## Prediction Performance

Out of Sample





## Future To Do

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- Online version
- Investigate extension to higher magnitudes, and finer spatial quantizations
- Meaning & Insight

I. Chattopadhyay and H. Lipson, *Abductive learning of quantized stochastic processes with probabilistic finite automata.*, Phil. Trans. of The Royal Soc. A, Vol. 371(1984), 2013, pp 20110543.

I. Chattopadhyay , *Causality Networks* <http://arxiv.org/abs/1406.6651>

I. Chattopadhyay and H. Lipson , *Data Smashing: Uncovering Lurking Order In Data*, Royal Society Interface, 11: 20140826