Addition & Subtraction of Interacting Nodes In A Spatio-temporal Process Evolving Over An Atmost Countable Set

ishanu chattopadhyay

1 PRELIMINARY NOTIONS & NOTATIONS

Notation 1 (A Priori Sites)**.** *Let*

$$Q = \{q_1, \cdots, q_i, \cdots, q_m\} \tag{1}$$

denote the set of nodes or sites existing a priori. Note that the location of each q_i is specified by its X and Y coordinates.

Notation 2 (Euclidean Distance)**.** *The Euclidean distance between sites q_i, q_j is denoted by $d(q_i, q_j)$. This is simply:*

$$d(q_i, q_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{2}$$

where x_i, x_j and y_i, y_j are the X and Y coordinates of q_i, q_j repectively.

Definition 1 (Scaling Of Probability distribution)**.** *For a given probability distribution $\wp$, and a scalar $c \in \mathbb{R}$, we define the scaled distribution $\wp'$ as:*

$$\wp'_i = c \odot \wp = \frac{\wp_i^c}{\sum_i \wp_i^c} \tag{3}$$

Example 1. *Let $\wp = [0.1\ 0.2\ 0.7]$ and $c = 0.2$. Then*

$$0.2 \star \wp = [0.1^{0.2}\ 0.2^{0.2}\ 0.7^{0.2}] \times \frac{1}{0.1^{0.2} + 0.2^{0.2} + 0.7^{0.2}} \tag{4}$$

Definition 2 (Addition Of Probability distribution)**.** *For a given probability distributions $\wp, \wp'$, the sum is defined as:*

$$(\wp \oplus \wp')_i = \frac{\wp_i \wp'_i}{\sum_i \wp_i \wp'_i} \tag{5}$$

Example 2. *Let $\wp = [0.1\ 0.2\ 0.7]$ and $\wp' = [0.3\ 0.3\ 0.4]$. Then,*

$$(\wp \oplus \wp')_i = [0.1 \times 0.3 \quad 0.2 \times 0.3 \quad 0.7 \times 0.4] \times \frac{1}{(0.1 \times 0.3) + (0.2 \times 0.3) + (0.7 \times 0.4)} \tag{6}$$

It follows from the above definitions that for any distribution $\wp$:

$$\wp \oplus (-1)\wp = u \tag{7}$$

where u is the uniform distribution on the same alphabet.

Definition 3 (Hybrid Scaling)**.** *Given a tuple $(c, \wp)$ where $c \in \mathbb{R}$ and $\wp$ is a probability distribution, we define hybrid scaling by θ as follows:*

$$\theta \odot (c, \wp) = (\theta c, \theta \odot \wp) \tag{8}$$

Definition 4 (Normalized Coordinates)**.** *For a site P (new or existing a priori), its normlized coordinates $\Theta(P)$ is defined as:*

$$\forall i \in \{1, \cdots, m\}, \Theta(P)_i = \frac{d(P, q_i)}{\sum_{i=1}^m d(P, q_i)} \tag{9}$$

where q_i are the a priori sites.

Definition 5 (Normalized Inverse Coordinates)**.** *We define the normalized inverse coordinates as:*

$$\forall i \in \{1, \cdots, m\}, \Theta^\star(P)_i = \frac{1}{\Theta(p)_i} \tag{10}$$

Recall the FTX files generated in the course of the *cynet* simulation. Each line in each FTX file is actually a pair $(\gamma, \wp)$, where γ is a scalar, and $\wp$ is a probability distribution.

Notation 3. *Let the FTX file for site q_i at time t be denoted as F_i^t.*

Definition 6 (FTX Scaling)**.** *Given the FTX file F_i^t, and a scalar c, the scaled FTX file $c \odot F_i^t$ is defined as:*

$$(\gamma, \wp) \in F_i^t \Rightarrow (c\gamma, c \odot \wp) \in c \odot F_i^t \tag{11}$$

Notation 4. *Given the FTX file F_i^t, we denote the regressor output as:*

$$\mathcal{R}_i(F_i^t) \tag{12}$$

This is the predicted number of visits at time t at site q_i.

Now we are ready to specify the addition calculation.

2 Addition of New Nodes

We calculate the visitors as a function of time to newly added nodes. And also the changes to the priorly eisting nodes due to the addition of one or more new nodes.

Let s be a new site added. Then we calcualte the number of *pre-visits* as:

$$\hat{u}_i(s) = \mathcal{R}_i\left(\frac{1}{\Theta(s)_i} \odot F_i^t\right) \tag{13}$$

Allowing for the possibility of adding mutiple new sites $s \in S$, we compute:

$$u_i(s) = \frac{\hat{u}_i(s)}{\mathcal{R}_i(F_i^t) + \hat{u}_i(s)} \tag{14}$$

And finally, we specify that the visit to new site s at time t is given by:

$$u(s) = \sum_{q_i \in Q} u_i(s)\mathcal{R}_i(F_i^t) \tag{15}$$

And the updated visits to site q_i is given by:

$$\forall q_i \in Q, u(q_i) = \mathcal{R}_i(F_i^t) - \sum_{s \in S} u_i(s)\mathcal{R}_i(F_i^t) \tag{16}$$

Next the subtraction algorithm.

3 Removal of Nodes

Note that the normalized coordinates can be calculated for any of the existing nodes as well. Assume that $q_j \in Q$ is to be removed. Then, the updated visits to site q_i is given by:

$$\forall q_i \neq q_j \in Q, u(q_i) = \mathcal{R}_i(F_i^t) + \frac{\mathcal{R}_j\left(\frac{1}{\Theta(q_j)_i} \odot F_j^t\right)}{\sum_i \mathcal{R}_j\left(\frac{1}{\Theta(q_j)_i} \odot F_j^t\right)}\mathcal{R}_j(F_j^t) \tag{17}$$

where we assume that $\mathcal{R}_i(x) \to 0$ as $|x| \to \infty$.