

Addition & Subtraction of Interacting Nodes In A Spatio-temporal Process Evolving Over An Atmost Countable Set

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1 PRELIMINARY NOTIONS & NOTATIONS

1a. Distance

Notation 1 (A Priori Sites). Let

$$Q = \{q_1, \dots, q_i, \dots, q_m\} \quad (1)$$

denote the set of nodes or sites existing a priori. Note that the location of each q_i is specified by its X and Y coordinates.

Notation 2 (Euclidean Distance). The Euclidean distance between sites q_i, q_j is denoted by $d(q_i, q_j)$. This is simply:

$$d(q_i, q_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (2)$$

where x_i, x_j and y_i, y_j are the X and Y coordinates of q_i, q_j respectively.

We can use actual travel distance if available.

1b. Probability SCaling and Summation

We define operations on probability vectors somewhat differently:

Definition 1 (Scaling Of Probability distribution). For a given probability distribution \wp , and a scalar $c \in \mathbb{R}$, we define the scaled distribution \wp' as:

$$\wp'_i = c \odot \wp = \frac{\wp_i^c}{\sum_i \wp_i^c} \quad (3)$$

Example 1. Let $\wp = [0.1 \ 0.2 \ 0.7]$ and $c = 0.2$. Then

$$0.2 \star \wp = [0.1^{0.2} \ 0.2^{0.2} \ 0.7^{0.2}] \times \frac{1}{0.1^{0.2} + 0.2^{0.2} + 0.7^{0.2}} \quad (4)$$

Definition 2 (Addition Of Probability distribution). For a given probability distributions \wp, \wp' , the sum is defined as:

$$(\wp \oplus \wp')_i = \frac{\wp_i \wp'_i}{\sum_i \wp_i \wp'_i} \quad (5)$$

Example 2. Let $\wp = [0.1 \ 0.2 \ 0.7]$ and $\wp' = [0.3 \ 0.3 \ 0.4]$. Then,

$$(\wp \oplus \wp')_i = [0.1 \times 0.3 \ 0.2 \times 0.3 \ 0.7 \times 0.4] \times \frac{1}{(0.1 \times 0.3) + (0.2 \times 0.3) + (0.7 \times 0.4)} \quad (6)$$

It follows from the above definitions that for any distribution \wp :

$$\wp \oplus (-1)\wp = u \quad (7)$$

where u is the uniform distribution on the same alphabet.

1c. Hybrid Scaling

Definition 3 (Hybrid Scaling). Given a tuple (c, \wp) where $c \in \mathbb{R}$ and \wp is a probability distribution, we define hybrid scaling by θ as follows:

$$\theta \odot (c, \wp) = (\theta c, \theta \odot \wp) \quad (8)$$

Definition 4 (Normalized Coordinates). For a site P (new or existing a priori), its normlized coordinates $\Theta(P)$ is defined as:

$$\forall i \in \{1, \dots, m\}, \Theta(P)_i = \frac{d(P, q_i)}{\sum_{i=1}^m d(P, q_i)} \quad (9)$$

where q_i are the a priori sites.

Definition 5 (Normalized Inverse Coordinates). *We define the normalized inverse coordinates as:*

$$\forall i \in \{1, \dots, m\}, \Theta^*(P)_i = \frac{1}{\Theta(p)_i} \quad (10)$$

Recall the FTX files generated in the course of the *cynet* simulation. Each line in each FTX file is actually a pair (γ, \wp) , where γ is a scalar, and \wp is a probability distribution.

Notation 3. *Let the FTX file for site q_i at time t be denoted as F_i^t .*

Definition 6 (FTX Scaling). *Given the FTX file F_i^t , and a scalar c , the scaled FTX file $c \odot F_i^t$ is defined as:*

$$(\gamma, \wp) \in F_i^t \Rightarrow (c\gamma, c \odot \wp) \in c \odot F_i^t \quad (11)$$

Notation 4. *Given the FTX file F_i^t , we denote the regressor output as:*

$$\mathcal{R}_i(F_i^t) \quad (12)$$

This is the predicted number of visits at time t at site q_i .

Now we are ready to specify the addition calculation.

2 ADDITION OF NEW NODES

We calculate the visitors as a function of time to newly added nodes. And also the changes to the priorly existing nodes due to the addition of one or more new nodes.

Let s be a new site added. Then we calculate the number of *pre-visits* as:

$$\hat{u}_i(s) = \mathcal{R}_i\left(\frac{1}{\Theta(s)_i} \odot F_i^t\right) \quad (13)$$

Allowing for the possibility of adding multiple new sites $s \in S$, we compute:

$$u_i(s) = \frac{\hat{u}_i(s)}{\mathcal{R}_i(F_i^t) + \hat{u}_i(s)} \quad (14)$$

And finally, we specify that the visit to new site s at time t is given by:

$$u(s) = \sum_{q_i \in Q} u_i(s) \mathcal{R}_i(F_i^t) \quad (15)$$

And the updated visits to site q_i is given by:

$$\forall q_i \in Q, u(q_i) = \mathcal{R}_i(F_i^t) - \sum_{s \in S} u_i(s) \mathcal{R}_i(F_i^t) \quad (16)$$

Next the subtraction algorithm.

3 REMOVAL OF NODES

Note that the normalized coordinates can be calculated for any of the existing nodes as well. Assume that $q_j \in Q$ is to be removed. Then, the updated visits to site q_i is given by:

$$\forall q_i \neq q_j \in Q, u(q_i) = \mathcal{R}_i(F_i^t) + \frac{\mathcal{R}_j\left(\frac{1}{\Theta(q_j)_i} \odot F_j^t\right)}{\sum_i \mathcal{R}_j\left(\frac{1}{\Theta(q_j)_i} \odot F_j^t\right)} \mathcal{R}_j(F_j^t) \quad (17)$$

where we assume that $\mathcal{R}_i(x) \rightarrow 0$ as $|x| \rightarrow \infty$.