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Circular law for noncentral random matrices. (English summary)
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In this paper, the author considers random matrices of the form $X+M$, where X is the $n\times n$ upper-left corner of an infinite array of i.i.d. random variables with mean zero and variance one, and M is a deterministic matrix such that $\mathrm{Tr}(MM^*)=O(n^2)$ and $\mathrm{rank}(M)=O(n^\alpha)$ for a certain $\alpha<1$. He proves that, just as if M had not been added to X, the empirical spectral law of $(X+M)/\sqrt{n}$ converges almost surely to the uniform law on the unit disc of the complex plane. *Florent Benaych-Georges*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.