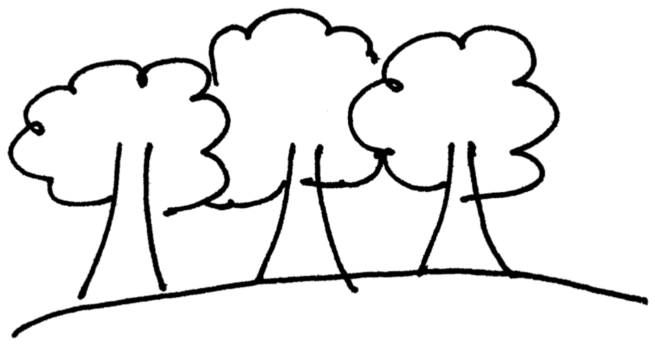
Let’s Build A Simple Interpreter. Part 7: Abstract Syntax Trees (https://ruslanspivak.com/lsbasi-part7/)

Date Tue, December 15, 2015

As I promised you last time, today I will talk about one of the central data structures that we’ll use throughout the rest of the series, so buckle up and let’s go.

Up until now, we had our interpreter and parser code mixed together and the interpreter would evaluate an expression as soon as the parser recognized a certain language construct like addition, subtraction, multiplication, or division. Such interpreters are called *syntax-directed interpreters*. They usually make a single pass over the input and are suitable for basic language applications. In order to analyze more complex Pascal programming language constructs, we need to build an *intermediate representation* (*IR*). Our parser will be responsible for building an *IR* and our interpreter will use it to interpret the input represented as the *IR*.

It turns out that a tree is a very suitable data structure for an IR.

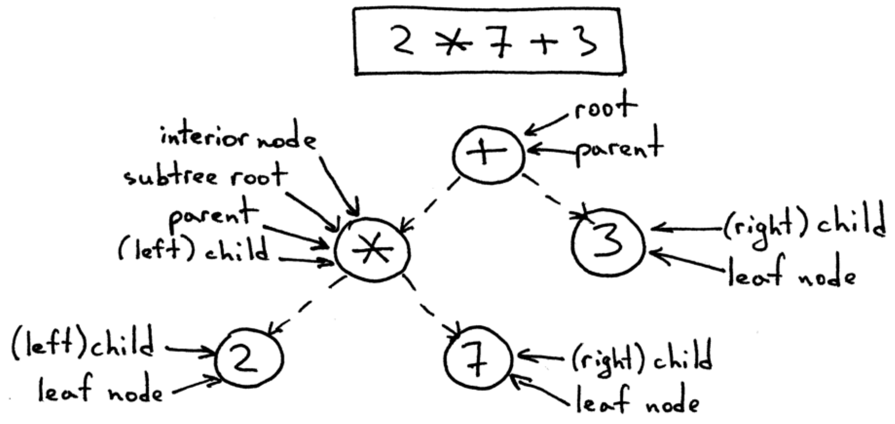


Let’s quickly talk about tree terminology.

- A *tree* is a data structure that consists of one or more nodes organized into a hierarchy.
- The tree has one *root*, which is the top node.
- All nodes except the root have a unique *parent*.
- The node labeled * in the picture below is a *parent*. Nodes labeled **2** and **7** are its *children*; children are ordered from left to right.
- A node with no children is called a *leaf* node.

- A node that has one or more children and that is not the root is called an *interior* node.
- The children can also be complete *subtrees*. In the picture below the left child (labeled `*`) of the **+** node is a complete *subtree* with its own children.
- In computer science we draw trees upside down starting with the root node at the top and branches growing downward.

Here is a tree for the expression 2 * 7 + 3 with explanations:

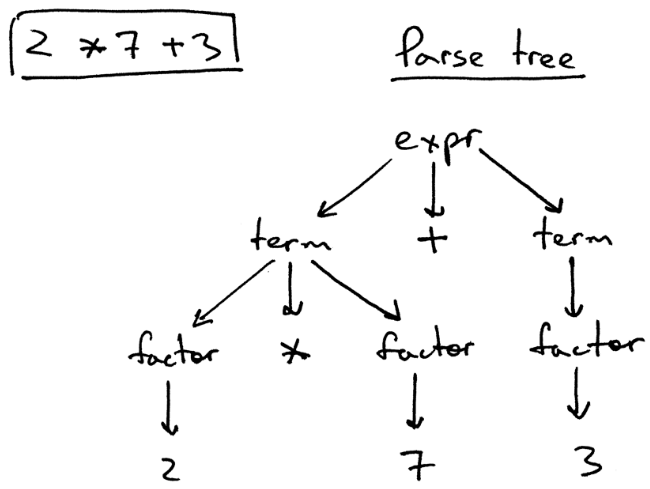


The IR we'll use throughout the series is called an *abstract-syntax tree* (*AST*). But before we dig deeper into ASTs let's talk about *parse trees* briefly. Though we're not going to use parse trees for our interpreter and compiler, they can help you understand how your parser interpreted the input by visualizing the execution trace of the parser. We'll also compare them with ASTs to see why ASTs are better suited for intermediate representation than parse trees.

So, what is a parse tree? A *parse-tree* (sometimes called a *concrete syntax tree*) is a tree that represents the syntactic structure of a language construct according to our grammar definition. It basically shows how your parser recognized the language construct or, in other words, it shows how the start symbol of your grammar derives a certain string in the programming language.

The call stack of the parser implicitly represents a parse tree and it's automatically built in memory by your parser as it is trying to recognize a certain language construct.

Let's take a look at a parse tree for the expression 2 * 7 + 3:



In the picture above you can see that:

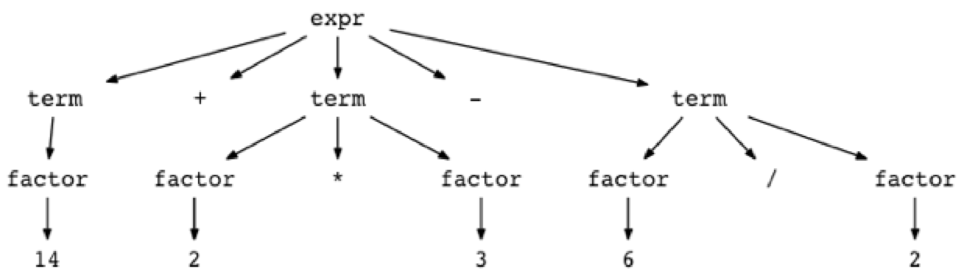
- The parse tree records a sequence of rules the parser applies to recognize the input.
- The root of the parse tree is labeled with the grammar start symbol.
- Each interior node represents a non-terminal, that is it represents a grammar rule application, like *expr*, *term*, or *factor* in our case.
- Each leaf node represents a token.

As I've already mentioned, we're not going to manually construct parser trees and use them for our interpreter but parse trees can help you understand how the parser interpreted the input by visualizing the parser call sequence.

You can see how parse trees look like for different arithmetic expressions by trying out a small utility called genptdot.py (https://github.com/rspivak/lsbasi/blob/master/part7/python/genptdot.py) that I quickly wrote to help you visualize them. To use the utility you first need to install Graphviz (http://graphviz.org) package and after you've run the following command, you can open the generated image file parsetree.png and see a parse tree for the expression you passed as a command line argument:

```
$ python genptdot.py "14 + 2 * 3 - 6 / 2" > \
  parsetree.dot && dot -Tpng -o parsetree.png parsetree.dot
```

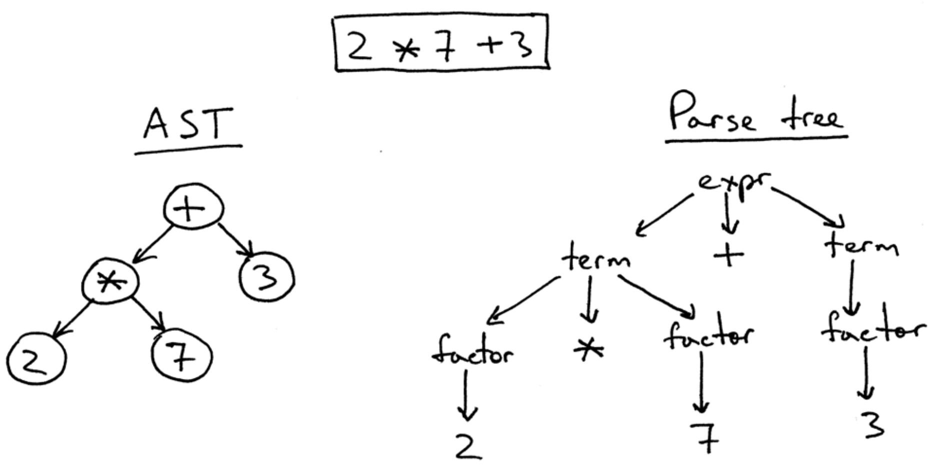
Here is the generated image parsetree.png for the expression 14 + 2 * 3 - 6 / 2:



Play with the utility a bit by passing it different arithmetic expressions and see what a parse tree looks like for a particular expression.

Now, let’s talk about *abstract-syntax trees* (AST). This is the *intermediate representation* (IR) that we’ll heavily use throughout the rest of the series. It is one of the central data structures for our interpreter and future compiler projects.

Let’s start our discussion by taking a look at both the AST and the parse tree for the expression 2 * 7 + 3:



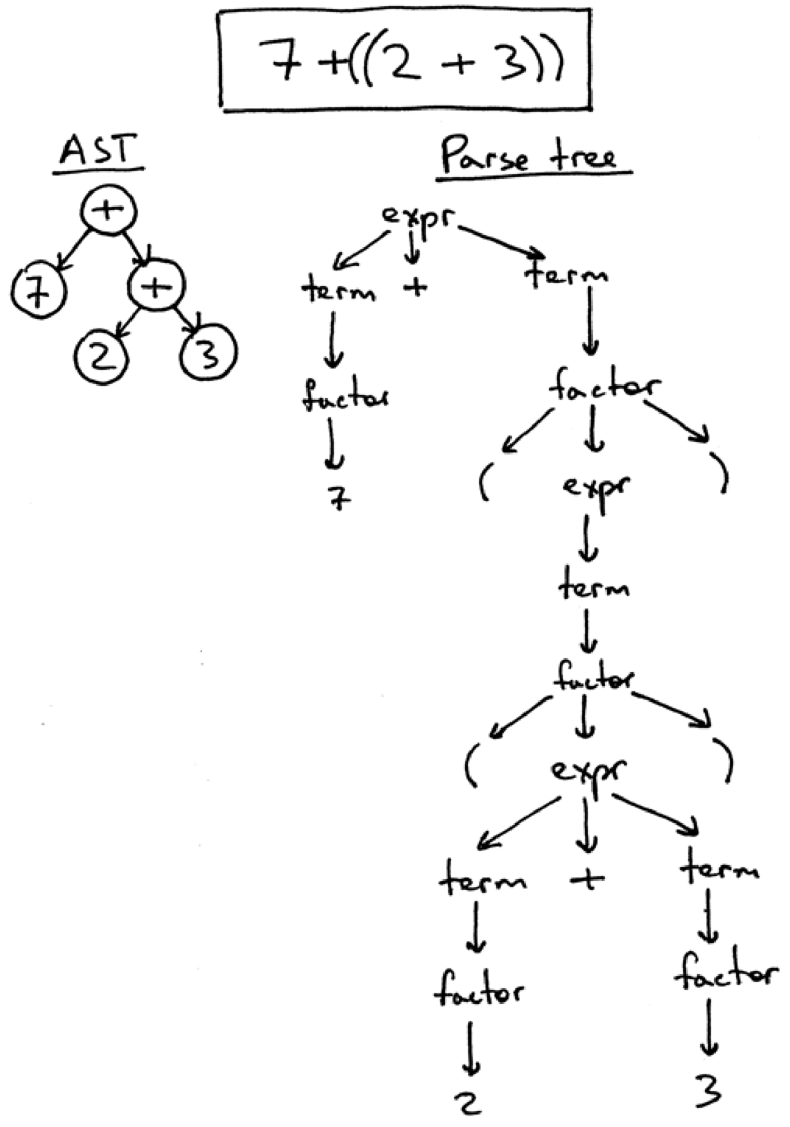
As you can see from the picture above, the AST captures the essence of the input while being smaller.

Here are the main differences between ASTs and Parse trees:

- ASTs uses operators/operations as root and interior nodes and it uses operands as their children.
- ASTs do not use interior nodes to represent a grammar rule, unlike the parse tree does.
- ASTs don’t represent every detail from the real syntax (that’s why they’re called *abstract*) - no rule nodes and no parentheses, for example.
- ASTs are dense compared to a parse tree for the same language construct.

So, what is an abstract syntax tree? An *abstract syntax tree* (*AST*) is a tree that represents the abstract syntactic structure of a language construct where each interior node and the root node represents an operator, and the children of the node represent the operands of that operator.

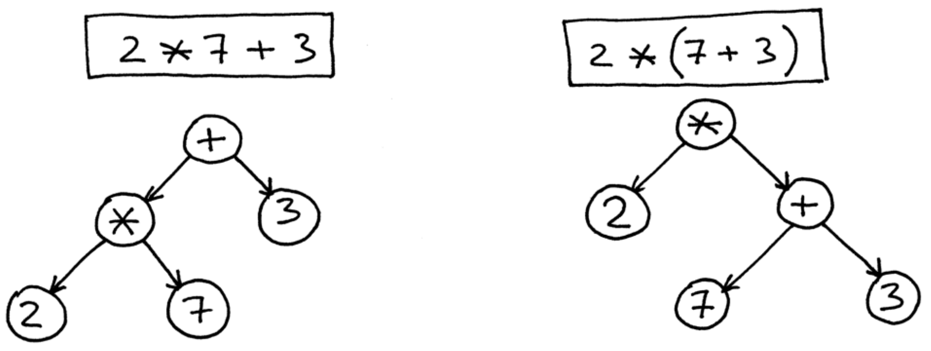
I’ve already mentioned that ASTs are more compact than parse trees. Let’s take a look at an AST and a parse tree for the expression 7 + ((2 + 3)). You can see that the following AST is much smaller than the parse tree, but still captures the essence of the input:



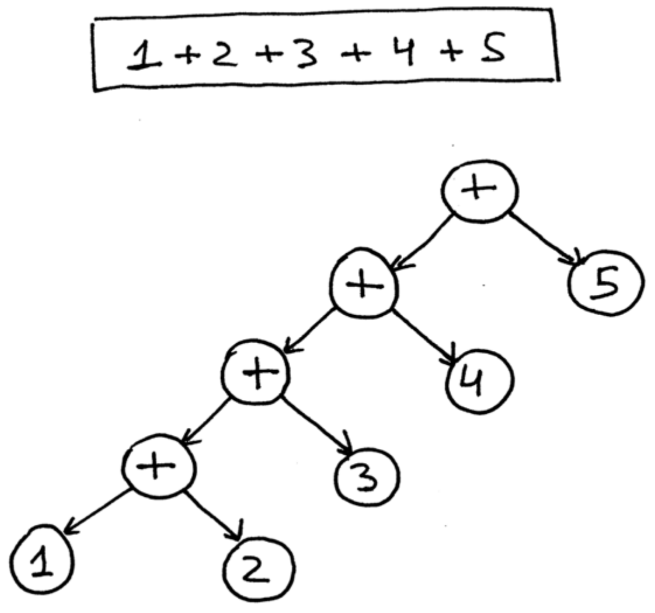
So far so good, but how do you encode operator precedence in an AST? In order to encode the operator precedence in AST, that is, to represent that “X happens before Y” you just need to put X lower in the tree than Y. And you’ve already seen that in the previous pictures.

Let’s take a look at some more examples.

In the picture below, on the left, you can see an AST for the expression 2 * 7 + 3. Let’s change the precedence by putting 7 + 3 inside the parentheses. You can see, on the right, what an AST looks like for the modified expression 2 * (7 + 3):



Here is an AST for the expression 1 + 2 + 3 + 4 + 5:



From the pictures above you can see that operators with higher precedence end up being lower in the tree.

Okay, let's write some code to implement different AST node types and modify our parser to generate an AST tree composed of those nodes.

First, we'll create a base node class called AST that other classes will inherit from:

```
class AST(object):
    pass
```

Not much there, actually. Recall that ASTs represent the operator-operand model. So far, we have four operators and integer operands. The operators are addition, subtraction, multiplication, and division. We could have created a separate class to represent each operator like AddNode, SubNode, MulNode, and DivNode, but instead we're going to have only one *BinOp* class to represent all four binary operators (a *binary operator* is an operator that operates on two operands):

```
class BinOp(AST):
    def __init__(self, left, op, right):
        self.left = left
        self.token = self.op = op
        self.right = right
```

The parameters to the constructor are *left*, *op*, and *right*, where *left* and *right* point correspondingly to the node of the left operand and to the node of the right operand. *Op* holds a token for the operator itself: Token(PLUS, ‘+’) for the plus operator, Token(MINUS, ‘-‘) for the minus operator, and so on.

To represent integers in our AST, we’ll define a class *Num* that will hold an INTEGER token and the token’s value:

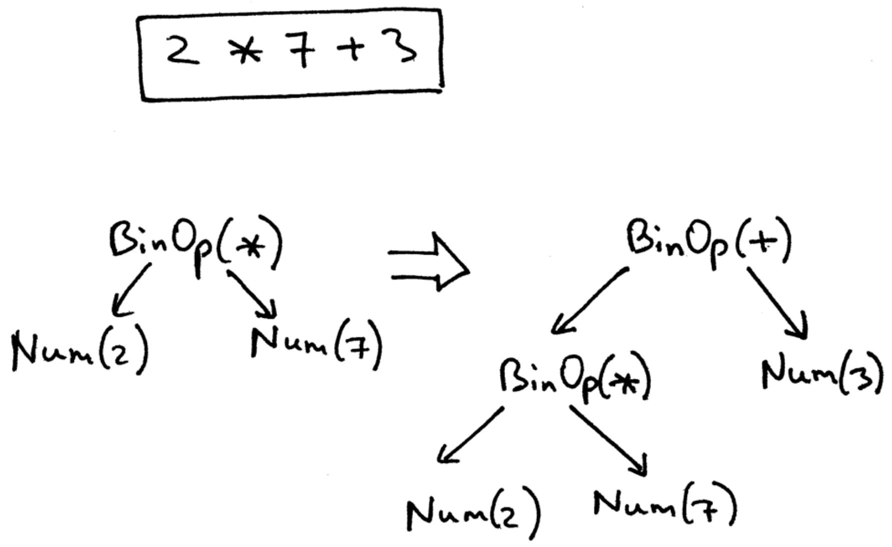
```
class Num(AST):
    def __init__(self, token):
        self.token = token
        self.value = token.value
```

As you’ve noticed, all nodes store the token used to create the node. This is mostly for convenience and it will come in handy in the future.

Recall the AST for the expression 2 * 7 + 3. We’re going to manually create it in code for that expression:

```
>>> from spi import Token, MUL, PLUS, INTEGER, Num, BinOp
>>>
>>> mul_token = Token(MUL, '*')
>>> plus_token = Token(PLUS, '+')
>>> mul_node = BinOp(
...     left=Num(Token(INTEGER, 2)),
...     op=mul_token,
...     right=Num(Token(INTEGER, 7))
... )
>>> add_node = BinOp(
...     left=mul_node,
...     op=plus_token,
...     right=Num(Token(INTEGER, 3))
... )
```

Here is how an AST will look with our new node classes defined. The picture below also follows the manual construction process above:

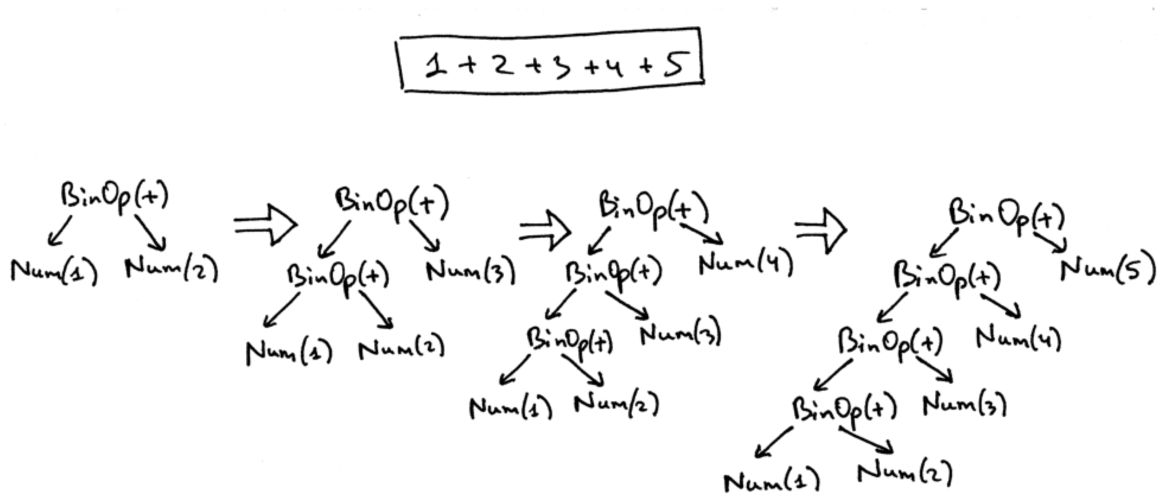


Here is our modified parser code that builds and returns an AST as a result of recognizing the input (an arithmetic expression):

```
class AST(object):
    pass


class BinOp(AST):
    def __init__(self, left, op, right):
        self.left = left
        self.token = self.op = op
        self.right = right


class Num(AST):
    def __init__(self, token):
        self.token = token
        self.value = token.value


class Parser(object):
    def __init__(self, lexer):
        self.lexer = lexer
        # set current token to the first token taken from the input
        self.current_token = self.lexer.get_next_token()

    def error(self):
        raise Exception('Invalid syntax')

    def eat(self, token_type):
        # compare the current token type with the passed token
        # type and if they match then "eat" the current token
        # and assign the next token to the self.current_token,
        # otherwise raise an exception.
        if self.current_token.type == token_type:
            self.current_token = self.lexer.get_next_token()
        else:
            self.error()

    def factor(self):
        """factor : INTEGER | LPAREN expr RPAREN"""
        token = self.current_token
        if token.type == INTEGER:
            self.eat(INTEGER)
            return Num(token)
        elif token.type == LPAREN:
            self.eat(LPAREN)
            node = self.expr()
            self.eat(RPAREN)
            return node

    def term(self):
        """term : factor ((MUL | DIV) factor)*"""
        node = self.factor()

        while self.current_token.type in (MUL, DIV):
            token = self.current_token
            if token.type == MUL:
                self.eat(MUL)
            elif token.type == DIV:
                self.eat(DIV)
            node = BinOp(left=node, op=token, right=self.factor())

        return node

    def expr(self):
        """
        expr   : term ((PLUS | MINUS) term)*
        term   : factor ((MUL | DIV) factor)*
        factor : INTEGER | LPAREN expr RPAREN
        """
        node = self.term()

        while self.current_token.type in (PLUS, MINUS):
            token = self.current_token
            if token.type == PLUS:
                self.eat(PLUS)
            elif token.type == MINUS:
                self.eat(MINUS)

            node = BinOp(left=node, op=token, right=self.term())

        return node

    def parse(self):
        return self.expr()
```

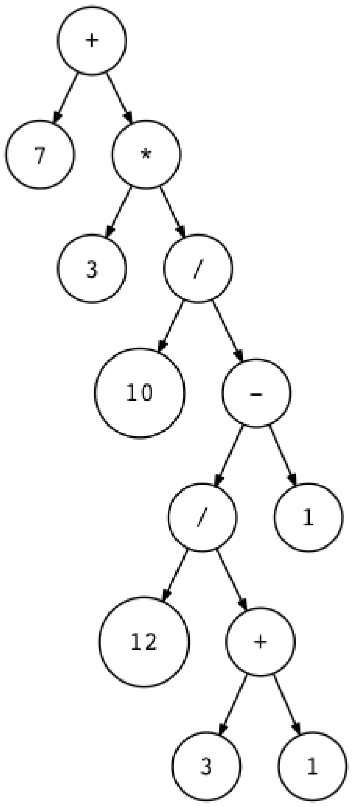
Let’s go over the process of an AST construction for some arithmetic expressions.

If you look at the parser code above you can see that the way it builds nodes of an AST is that each BinOp node adopts the current value of the *node* variable as its left child and the result of a call to a *term* or *factor* as its right child, so it’s effectively pushing down nodes to the left and the tree for the expression 1 +2 + 3 + 4 + 5 below is a good example of that. Here is a visual representation how the parser gradually builds an AST for the expression 1 + 2 + 3 + 4 + 5:



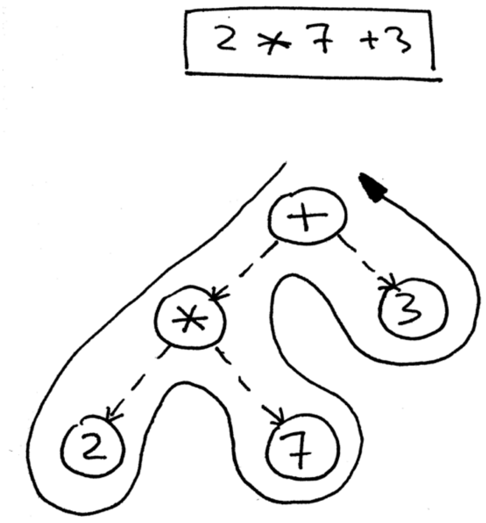
To help you visualize ASTs for different arithmetic expressions, I wrote a small utility that takes an arithmetic expression as its first argument and generates a DOT file that is then processed by the *dot* utility to actually draw an AST for you (*dot* is part of the Graphviz (http://graphviz.org) package that you need to install to run the *dot* command). Here is a command and a generated AST image for the expression 7 + 3 * (10 / (12 / (3 + 1) - 1)):

```
$ python genastdot.py "7 + 3 * (10 / (12 / (3 + 1) - 1))" > \
  ast.dot && dot -Tpng -o ast.png ast.dot
```



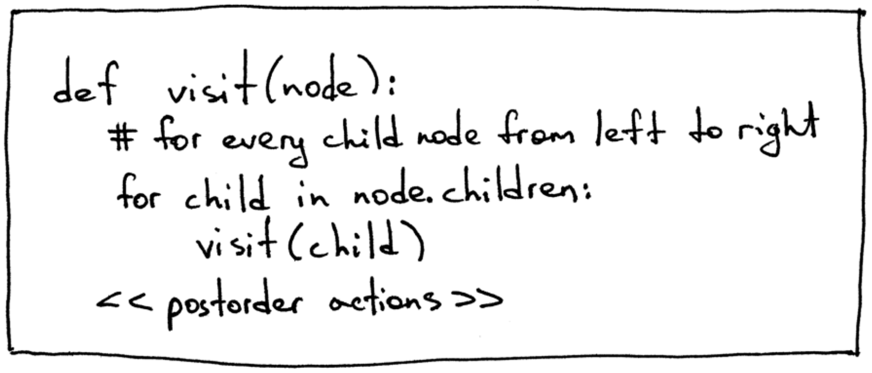
It's worth your while to write some arithmetic expressions, manually draw ASTs for the expressions, and then verify them by generating AST images for the same expressions with the genastdot.py (https://github.com/rspivak/lsbasi/blob/master/part7/python/genastdot.py) tool. That will help you better understand how ASTs are constructed by the parser for different arithmetic expressions.

Okay, here is an AST for the expression 2 * 7 + 3:



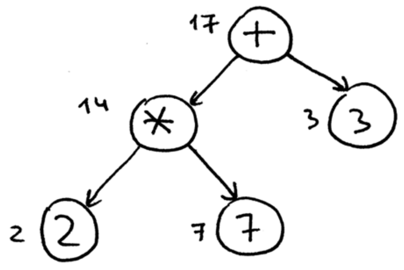
How do you navigate the tree to properly evaluate the expression represented by that tree? You do that by using a *postorder traversal* - a special case of *depth-first traversal* - which starts at the root node and recursively visits the children of each node from left to right. The postorder traversal visits nodes as far away from the root as fast as it can.

Here is a pseudo code for the postorder traversal where *<<postorder actions>>* is a placeholder for actions like addition, subtraction, multiplication, or division for a *BinOp* node or a simpler action like returning the integer value of a *Num* node:

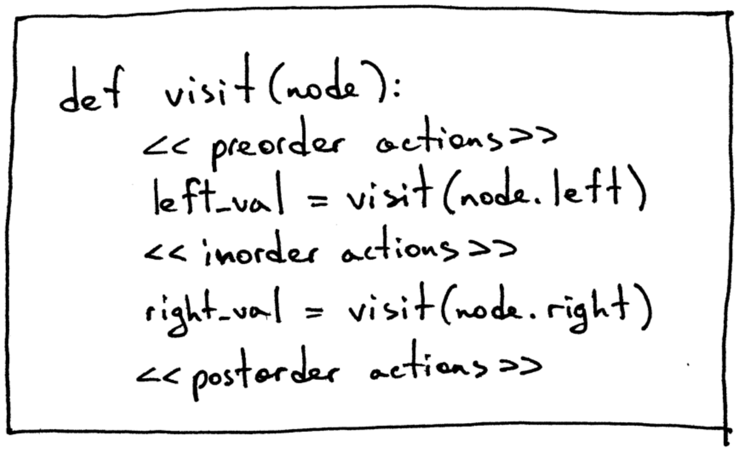


```
def visit(node):
    # for every child node from left to right
    for child in node.children:
        visit(child)
    << postorder actions >>
```

The reason we’re going to use a postorder traversal for our interpreter is that first, we need to evaluate interior nodes lower in the tree because they represent operators with higher precedence and second, we need to evaluate operands of an operator before applying the operator to those operands. In the picture below, you can see that with postorder traversal we first evaluate the expression 2 * 7 and only after that we evaluate 14 + 3, which gives us the correct result, 17:



For the sake of completeness, I'll mention that there are three types of depth-first traversal: *preorder traversal*, *inorder traversal*, and *postorder traversal*. The name of the traversal method comes from the place where you put actions in the visitation code:



```
def visit(node):
    << preorder actions>>
    left_val = visit(node.left)
    << inorder actions>>
    right_val = visit(node.right)
    << postorder actions>>
```

Sometimes you might have to execute certain actions at all those points (preorder, inorder, and postorder). You'll see some examples of that in the source code repository for this article.

Okay, let's write some code to visit and interpret the abstract syntax trees built by our parser, shall we?

Here is the source code that implements the Visitor pattern (https://en.wikipedia.org/wiki/Visitor_pattern):

```python
class NodeVisitor(object):
    def visit(self, node):
        method_name = 'visit_' + type(node).__name__
        visitor = getattr(self, method_name, self.generic_visit)
        return visitor(node)

    def generic_visit(self, node):
        raise Exception('No visit_{} method'.format(type(node).__name__))
```

And here is the source code of our *Interpreter* class that inherits from the *NodeVisitor* class and implements different methods that have the form *visit_NodeType*, where *NodeType* is replaced with the node's class name like *BinOp*, *Num* and so on:

```
class Interpreter(NodeVisitor):
    def __init__(self, parser):
        self.parser = parser

    def visit_BinOp(self, node):
        if node.op.type == PLUS:
            return self.visit(node.left) + self.visit(node.right)
        elif node.op.type == MINUS:
            return self.visit(node.left) - self.visit(node.right)
        elif node.op.type == MUL:
            return self.visit(node.left) * self.visit(node.right)
        elif node.op.type == DIV:
            return self.visit(node.left) / self.visit(node.right)

    def visit_Num(self, node):
        return node.value
```

There are two interesting things about the code that are worth mentioning here: First, the visitor code that manipulates AST nodes is decoupled from the AST nodes themselves. You can see that none of the AST node classes (BinOp and Num) provide any code to manipulate the data stored in those nodes. That logic is encapsulated in the *Interpreter* class that implements the *NodeVisitor* class.

Second, instead of a giant *if* statement in the NodeVisitor's *visit* method like this:

```
def visit(node):
    node_type = type(node).__name__
    if node_type == 'BinOp':
        return self.visit_BinOp(node)
    elif node_type == 'Num':
        return self.visit_Num(node)
    elif ...
    # ...
```

or like this:

```
def visit(node):
    if isinstance(node, BinOp):
        return self.visit_BinOp(node)
    elif isinstance(node, Num):
        return self.visit_Num(node)
    elif ...
```

the NodeVisitor's *visit* method is very generic and dispatches calls to the appropriate method based on the node type passed to it. As I've mentioned before, in order to make use of it, our interpreter inherits from the *NodeVisitor* class and implements necessary methods. So if the type of a node passed to the *visit* method is BinOp, then the *visit* method will dispatch the call to the *visit_BinOp* method, and if the type of a node is Num, then the *visit* method will dispatch the call to the *visit_Num* method, and so on.

Spend some time studying this approach (standard Python module ast (https://docs.python.org/2.7/library/ast.html#module-ast) uses the same mechanism for node traversal) as we will be extending our interpreter with many new *visit_NodeType* methods in the future.

The *generic_visit* method is a fallback that raises an exception to indicate that it encountered a node that the implementation class has no corresponding *visit_NodeType* method for.

Now, let's manually build an AST for the expression 2 * 7 + 3 and pass it to our interpreter to see the visit method in action to evaluate the expression. Here is how you can do it from the Python shell:

```
>>> from spi import Token, MUL, PLUS, INTEGER, Num, BinOp
>>>
>>> mul_token = Token(MUL, '*')
>>> plus_token = Token(PLUS, '+')
>>> mul_node = BinOp(
...     left=Num(Token(INTEGER, 2)),
...     op=mul_token,
...     right=Num(Token(INTEGER, 7))
... )
>>> add_node = BinOp(
...     left=mul_node,
...     op=plus_token,
...     right=Num(Token(INTEGER, 3))
... )
>>> from spi import Interpreter
>>> inter = Interpreter(None)
>>> inter.visit(add_node)
17
```

As you can see, I passed the root of the expression tree to the *visit* method and that triggered traversal of the tree by dispatching calls to the correct methods of the *Interpreter* class(*visit_BinOp* and *visit_Num*) and generating the result.

Okay, here is the complete code of our new interpreter for your convenience:

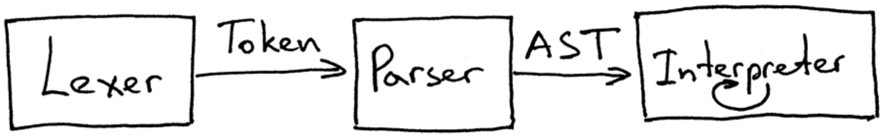
```
""" SPI - Simple Pascal Interpreter """

###############################################################################
#                                                                             #
#  LEXER                                                                      #
#                                                                             #
###############################################################################

# Token types
#
# EOF (end-of-file) token is used to indicate that
# there is no more input left for lexical analysis
INTEGER, PLUS, MINUS, MUL, DIV, LPAREN, RPAREN, EOF = (
    'INTEGER', 'PLUS', 'MINUS', 'MUL', 'DIV', '(', ')', 'EOF'
)


class Token(object):
    def __init__(self, type, value):
        self.type = type
        self.value = value

    def __str__(self):
        """String representation of the class instance.

        Examples:
            Token(INTEGER, 3)
            Token(PLUS, '+')
            Token(MUL, '*')
        """
        return 'Token({type}, {value})'.format(
            type=self.type,
            value=repr(self.value)
        )

    def __repr__(self):
        return self.__str__()


class Lexer(object):
    def __init__(self, text):
        # client string input, e.g. "4 + 2 * 3 - 6 / 2"
        self.text = text
        # self.pos is an index into self.text
        self.pos = 0
        self.current_char = self.text[self.pos]

    def error(self):
        raise Exception('Invalid character')

    def advance(self):
        """Advance the `pos` pointer and set the `current_char` variable."""
        self.pos += 1
        if self.pos > len(self.text) - 1:
            self.current_char = None  # Indicates end of input
        else:
            self.current_char = self.text[self.pos]
    def skip_whitespace(self):
        while self.current_char is not None and self.current_char.isspace():
            self.advance()

    def integer(self):
        """Return a (multidigit) integer consumed from the input."""
        result = ''
        while self.current_char is not None and self.current_char.isdigit():
            result += self.current_char
            self.advance()
        return int(result)

    def get_next_token(self):
        """Lexical analyzer (also known as scanner or tokenizer)

        This method is responsible for breaking a sentence
        apart into tokens. One token at a time.
        """
        while self.current_char is not None:

            if self.current_char.isspace():
                self.skip_whitespace()
                continue

            if self.current_char.isdigit():
                return Token(INTEGER, self.integer())

            if self.current_char == '+':
                self.advance()
                return Token(PLUS, '+')

            if self.current_char == '-':
                self.advance()
                return Token(MINUS, '-')

            if self.current_char == '*':
                self.advance()
                return Token(MUL, '*')

            if self.current_char == '/':
                self.advance()
                return Token(DIV, '/')

            if self.current_char == '(':
                self.advance()
                return Token(LPAREN, '(')

            if self.current_char == ')':
                self.advance()
                return Token(RPAREN, ')')

            self.error()

        return Token(EOF, None)


###############################################################################
#                                                                             #
#  PARSER                                                                     #
#                                                                             #
###############################################################################

class AST(object):
    pass


class BinOp(AST):
    def __init__(self, left, op, right):
        self.left = left
        self.token = self.op = op
        self.right = right


class Num(AST):
    def __init__(self, token):
        self.token = token
        self.value = token.value


class Parser(object):
    def __init__(self, lexer):
        self.lexer = lexer
        # set current token to the first token taken from the input
        self.current_token = self.lexer.get_next_token()

    def error(self):
        raise Exception('Invalid syntax')

    def eat(self, token_type):
        # compare the current token type with the passed token
        # type and if they match then "eat" the current token
        # and assign the next token to the self.current_token,
        # otherwise raise an exception.
        if self.current_token.type == token_type:
            self.current_token = self.lexer.get_next_token()
        else:
            self.error()

    def factor(self):
        """factor : INTEGER | LPAREN expr RPAREN"""
        token = self.current_token
        if token.type == INTEGER:
            self.eat(INTEGER)
            return Num(token)
        elif token.type == LPAREN:
            self.eat(LPAREN)
            node = self.expr()
            self.eat(RPAREN)
            return node

    def term(self):
        """term : factor ((MUL | DIV) factor)*"""
        node = self.factor()

        while self.current_token.type in (MUL, DIV):
            token = self.current_token
            if token.type == MUL:
                self.eat(MUL)
            elif token.type == DIV:
                self.eat(DIV)

            node = BinOp(left=node, op=token, right=self.factor())

        return node

    def expr(self):
        """
        expr   : term ((PLUS | MINUS) term)*
        term   : factor ((MUL | DIV) factor)*
        factor : INTEGER | LPAREN expr RPAREN
        """
        node = self.term()

        while self.current_token.type in (PLUS, MINUS):
            token = self.current_token
            if token.type == PLUS:
                self.eat(PLUS)
            elif token.type == MINUS:
                self.eat(MINUS)

            node = BinOp(left=node, op=token, right=self.term())

        return node

    def parse(self):
        return self.expr()


###############################################################################
#                                                                             #
#  INTERPRETER                                                                #
#                                                                             #
###############################################################################

class NodeVisitor(object):
    def visit(self, node):
        method_name = 'visit_' + type(node).__name__
        visitor = getattr(self, method_name, self.generic_visit)
        return visitor(node)

    def generic_visit(self, node):
        raise Exception('No visit_{} method'.format(type(node).__name__))


class Interpreter(NodeVisitor):
    def __init__(self, parser):
        self.parser = parser

    def visit_BinOp(self, node):
        if node.op.type == PLUS:
            return self.visit(node.left) + self.visit(node.right)
        elif node.op.type == MINUS:
            return self.visit(node.left) - self.visit(node.right)
        elif node.op.type == MUL:
            return self.visit(node.left) * self.visit(node.right)
        elif node.op.type == DIV:
            return self.visit(node.left) / self.visit(node.right)

    def visit_Num(self, node):
        return node.value

    def interpret(self):
        tree = self.parser.parse()
        return self.visit(tree)


def main():
    while True:
        try:
            try:
                text = raw_input('spi> ')
            except NameError:  # Python3
                text = input('spi> ')
        except EOFError:
            break
        if not text:
            continue

        lexer = Lexer(text)
        parser = Parser(lexer)
        interpreter = Interpreter(parser)
        result = interpreter.interpret()
        print(result)


if __name__ == '__main__':
    main()
```

Save the above code into the *spi.py* file or download it directly from GitHub (https://github.com/rspivak/lsbasi/blob/master/part7/python/spi.py). Try it out and see for yourself that your new tree-based interpreter properly evaluates arithmetic expressions.

Here is a sample session:

```
$ python spi.py
spi> 7 + 3 * (10 / (12 / (3 + 1) - 1))
22
spi> 7 + 3 * (10 / (12 / (3 + 1) - 1)) / (2 + 3) - 5 - 3 + (8)
10
spi> 7 + (((3 + 2)))
12
```

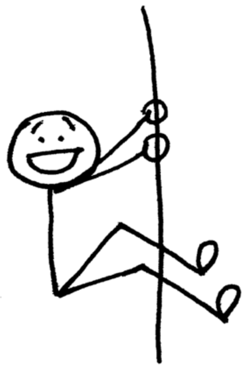
Today you've learned about parse trees, ASTs, how to construct ASTs and how to traverse them to interpret the input represented by those ASTs. You've also modified the parser and the interpreter and split them apart. The current interface between the lexer, parser, and the interpreter now looks like this:



You can read that as “The parser gets tokens from the lexer and then returns the generated AST for the interpreter to traverse and interpret the input.”

That’s it for today, but before wrapping up I’d like to talk briefly about recursive-descent parsers, namely just give them a definition because I promised last time to talk about them in more detail. So here you go: a *recursive-descent parser* is a top-down parser that uses a set of recursive procedures to process the input. Top-down reflects the fact that the parser begins by constructing the top node of the parse tree and then gradually constructs lower nodes.

And now it’s time for exercises :)



- Write a translator (hint: node visitor) that takes as input an arithmetic expression and prints it out in postfix notation, also known as Reverse Polish Notation (RPN). For example, if the input to the translator is the expression (5 + 3) * 12 / 3 than the output should be 5 3 + 12 * 3 /. See the answer here (https://github.com/rspivak/lsbasi/blob/master/part7/python/ex1.py) but try to solve it first on your own.
- Write a translator (node visitor) that takes as input an arithmetic expression and prints it out in LISP style notation, that is 2 + 3 would become (+ 2 3) and (2 + 3 * 5) would become (+ 2 (* 3 5)). You can find the answer here (https://github.com/rspivak/lsbasi/blob/master/part7/python/ex2.py) but again try to solve it first before looking at the provided solution.

In the next article, we’ll add assignment and unary operators to our growing Pascal interpreter. Until then, have fun and see you soon.