Unsigned Addition

```
unsigned char a = 255;
unsigned char b = 1;

unsigned char c = a + b;
printf("c=%d", c)
```

运行结果:

c = 0

Unsigned Addition

九曲阑干

For x and y $0 \leq x < 2^w,\ 0 \leq y < 2^w$

$$x +_w^u y = \begin{cases} x + y, & x + y < 2^w \\ x + y - 2^w, & 2^w \leq x + y < 2^{w+1} \end{cases}$$

Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)
{
    unsigned sum = x + y;

    if(sum >= x)
        return 1;
    else
        return 0;
}
```

Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)
{
    unsigned sum = x + y;

    if(sum >= x)
        return 1;
    else
        return 0;
}
```

return sum >= x;

$0 \le x < 2^w, 0 \le y < 2^w$

$x + y \ge x, \quad x + y \ge y$

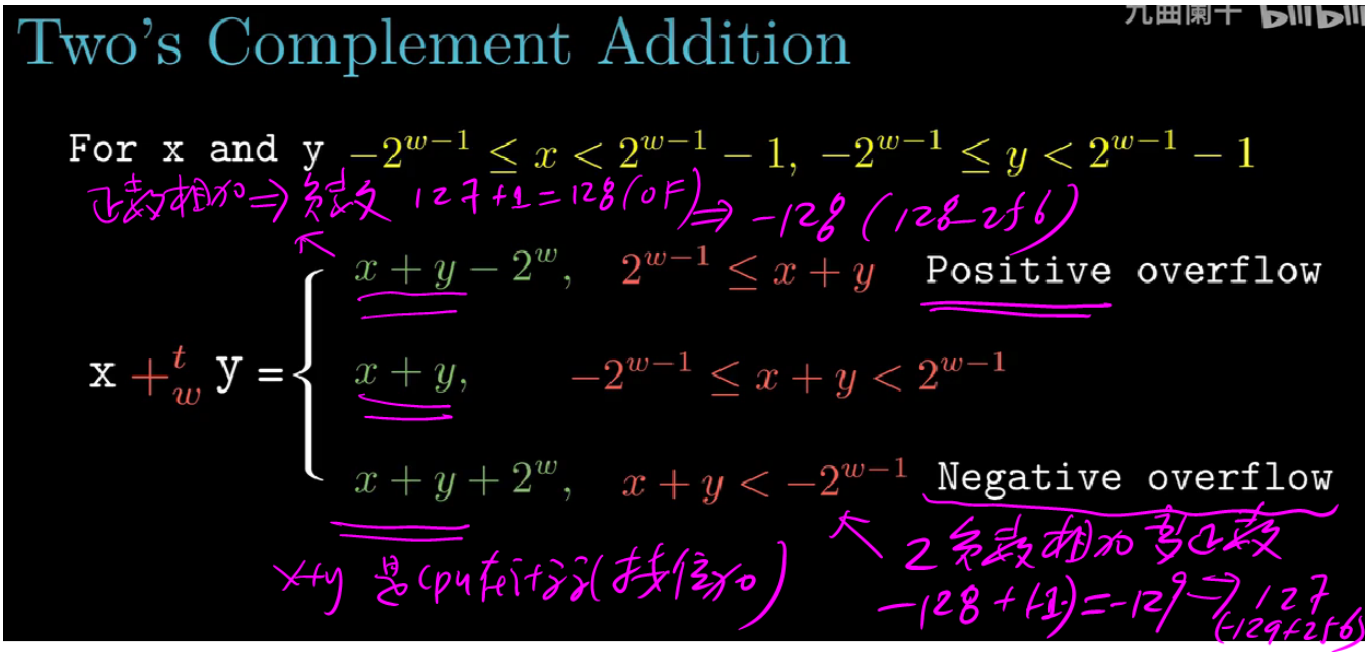
equivalent

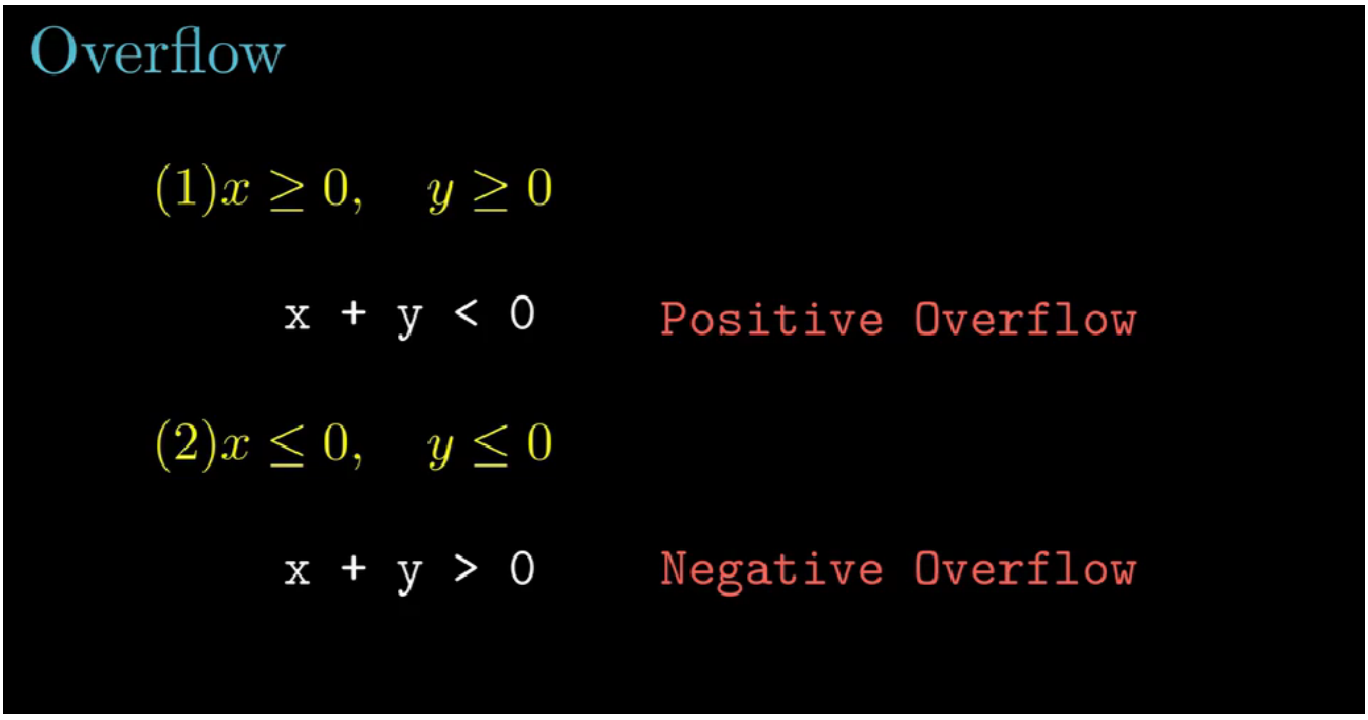
sum

overflow: x + y -2^w

$y - 2^w < 0$

$x + y - 2^w < x$

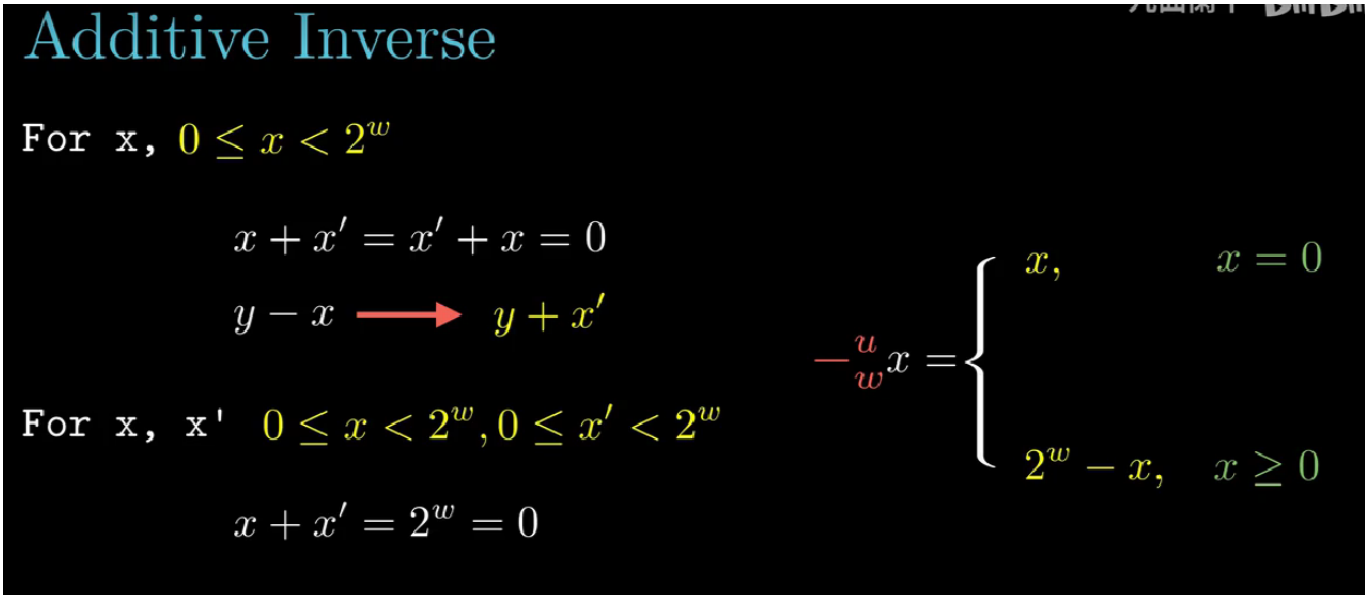




Python中对ADD字节码指令的实现中，判断溢出代码如下：

```
sum = x + y;
if ((sum ^ x ) < 0 && (sum ^ y) < 0)
    overflow
```

溢出时，sum与(x和y)的符号不一样。上面if判断cover了正溢出和负溢出。



Two's-Complement Negation

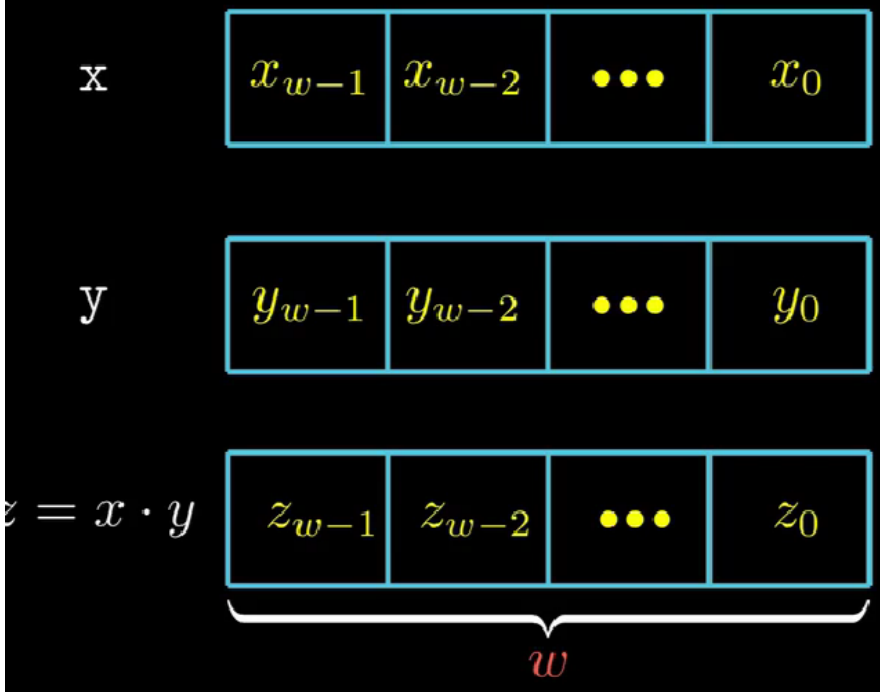
For x, $-2^{w-1} \leq x < 2^{w-1} - 1$

$$-_w^t x = \begin{cases} -x, & x > TMin_w \\ TMin_w, & x = TMin_w \end{cases}$$

$$Tmin_w + Tmin_w = -2^{w-1} + (-2^{w-1}) = -2^w$$

$$Tmin_w +_w^t Tmin_w = -2^w + 2^w = 0$$

Unsigned Multiplication



x * y

2w位 (max)

然后截断为w位