

# Unsigned Addition

```
unsigned char a = 255;  
unsigned char b = 1;  
  
unsigned char c = a + b;  
printf("c=%d", c)
```

运行结果:

**c = 0**

## Unsigned Addition

九曲阑干

For  $x$  and  $y$   $0 \leq x < 2^w$ ,  $0 \leq y < 2^w$

$$x +_w y = \begin{cases} x + y, & x + y < 2^w \\ x + y - 2^w, & 2^w \leq x + y < 2^{w+1} \end{cases}$$

## Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)
{
    unsigned sum = x + y;

    if(sum >= x)
        return 1;
    else
        return 0;
}
```

## Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)     $0 \leq x < 2^w, 0 \leq y < 2^w$ 
{
    unsigned sum = x + y;
    if(sum >= x)
        return 1;
    else
        return 0;
}
```

$x + y \geq x, \quad x + y \geq y$   
 溢出:  $x + y - 2^w$   
 $y - 2^w < 0$   
 $x + y - 2^w < x$

溢出结果  
 溢出:  $x + y - 2^w$   
 溢出:  $x + y - 2^w$   
 溢出:  $x + y - 2^w$

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 溢出:  $x + y - 2^w$   
 溢出:  $x + y - 2^w$

# Two's Complement Addition

For  $x$  and  $y$   $-2^{w-1} \leq x < 2^{w-1} - 1$ ,  $-2^{w-1} \leq y < 2^{w-1} - 1$

正数相加  $\Rightarrow$  溢出  $127+1=128(OF) \Rightarrow -128(128-256)$

$$x + {}^t_w y = \begin{cases} \underline{x + y - 2^w}, & 2^{w-1} \leq x + y \quad \underline{\text{Positive overflow}} \\ \underline{x + y}, & -2^{w-1} \leq x + y < 2^{w-1} \\ \underline{x + y + 2^w}, & x + y < -2^{w-1} \quad \underline{\text{Negative overflow}} \end{cases}$$

$x+y$  是 (cpu 能识别) (按位异或)  $\leftarrow$  2 个正数相加变正数  $-128+1=-127 \rightarrow 127 (-128+256)$

## Overflow

$$(1) x \geq 0, \quad y \geq 0$$

$$x + y < 0 \quad \text{Positive Overflow}$$

$$(2) x \leq 0, \quad y \leq 0$$

$$x + y > 0 \quad \text{Negative Overflow}$$

Python中对ADD字节码指令的实现中，判断溢出代码如下：

```
sum = x + y;
if ((sum ^ x) < 0 && (sum ^ y) < 0)
    overflow
```

溢出时，sum与(x和y)的符号不一样。上面if判断cover了正溢出和负溢出。

## Additive Inverse

For  $x$ ,  $0 \leq x < 2^w$

$$x + x' = x' + x = 0$$

$$y - x \longrightarrow y + x'$$

For  $x$ ,  $x'$   $0 \leq x < 2^w, 0 \leq x' < 2^w$

$$x + x' = 2^w = 0$$

$$-\frac{u}{w}x = \begin{cases} x, & x = 0 \\ 2^w - x, & x \geq 0 \end{cases}$$

# Two's-Complement Negation

For  $x$ ,  $-2^{w-1} \leq x < 2^{w-1} - 1$

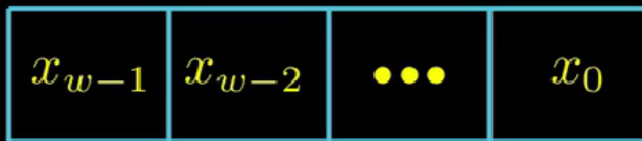
$$-\overset{t}{w}x = \begin{cases} -x, & x > TMin_w \\ TMin_w, & x = TMin_w \end{cases}$$

$$Tmin_w + Tmin_w = -2^{w-1} + (-2^{w-1}) = -2^w$$

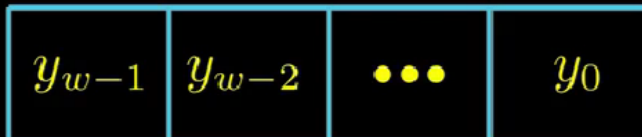
$$Tmin_w + \overset{t}{w}Tmin_w = -2^w + 2^w = 0$$

## Unsigned Multiplication

$x$

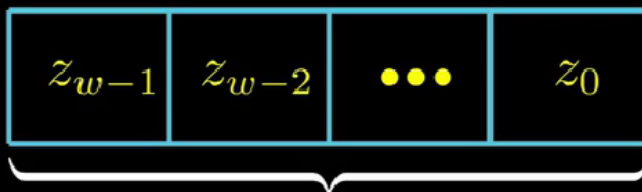


$y$



$\nwarrow \underline{x * y}$   
 $2w$  位 (max)  
 然后截断为  $w$  位

$z = x \cdot y$



$w$