ECE408/CS483/CSE408 Spring 2020

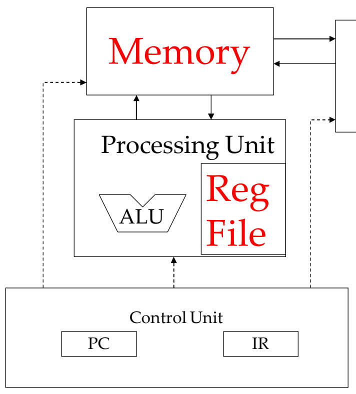
Applied Parallel Programming

Lecture 4: Memory Model

Objective

- To learn the basic features of the memories accessible by CUDA threads
- To prepare for MP-2 - basic matrix multiplication
- To learn to evaluate the performance implications of global memory accesses

The Von-Neumann Model



Instructions are Stored in Memory

- Every instruction needs to be fetched from memory, decoded, then executed.
- Instruction processing breaks into steps:

  Fetch | Decode | Execute | Memory

- Instructions come in three flavors: Operate, Data Transfer, and Control Flow.

Example: Processing an Add Instruction

- Example of an (LC-3) operate instruction:

  ADD R1, R2, R3
- meaning:
  - read R2 and R3
  - add them as unsigned/2's complement
  - write sum to R1
- Instruction processing for an operate instruction:

  Fetch | Decode | Execute | Memory

Example: Processing a Load Instruction

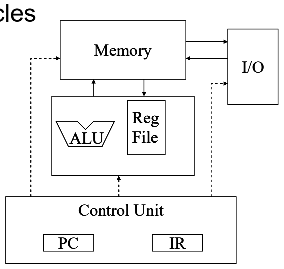
- Example of an (LC-3) data transfer instruction:

  LDR R4, R6, #3 ; a load
- meaning:
  - read R6
  - add the number 3 to it
  - load the contents of memory at the resulting address
  - write the bits to R4
- Instruction processing for a load instruction:

  Fetch | Decode | Execute | Memory

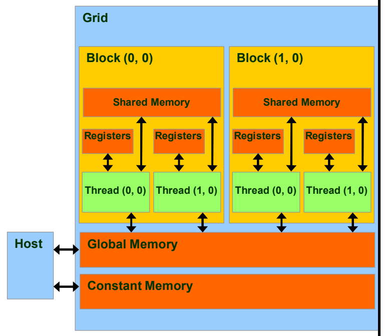
Registers vs Memory

- Registers
  - Fast: 1 cycle; no memory access required
  - Few: hundreds for CPU, O(10k) for GPU SM
- Memory
  - Slow: hundreds of cycles
  - Huge: GB or more



Programmer View of CUDA Memories

- Each thread can:
  - read/write per-thread **registers (~1 cycle)**
  - read/write per-block **shared memory (~5 cycles)**
  - read/write per-grid **global memory (~500 cycles)**
  - read/only per-grid **constant memory (~5 cycles with caching)**



CUDA Variable Type Qualifiers

Variable declaration	Memory	Scope	Lifetime
`int LocalVar;`	register	thread	thread
`__device__ __shared__ int SharedVar;`	shared	block	block
`__device__ int GlobalVar;`	global	app.	application
`__device__ __constant__ int ConstantVar;`	constant	app.	application

- `__device__`
  - optional with `__shared__` or `__constant__`
  - not allowed by itself within functions
- Automatic variables with no qualifiers
  - in registers for primitive types and structures
  - in global memory for per-thread arrays

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Next Application: Matrix Multiplication

- Given two Width × Width matrices, M and N,
  - we can multiply M by N
  - to compute a third Width × Width matrix, P:
  - P = MN

In terms of the elements of P, matrix multiplication implies computing…

$$P_{ij} = \sum_{k=1}^{Width} M_{ik}N_{kj}$$

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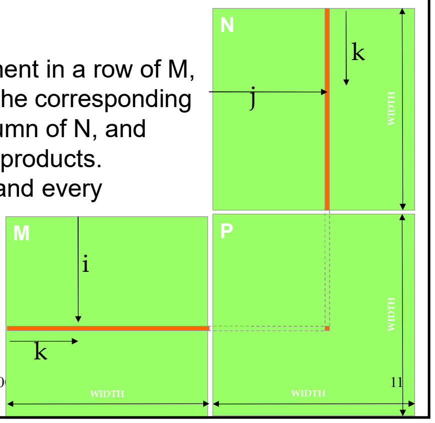
Matrix Multiplication

$P_{ij} = \sum_{k=1}^{Width} M_{ik}N_{kj}$

- Graphically, imagine
  - taking each element in a row of M,
  - multiplying it by the corresponding element in a column of N, and
  - summing up the products.
- Do that for every row and every column to produce P.



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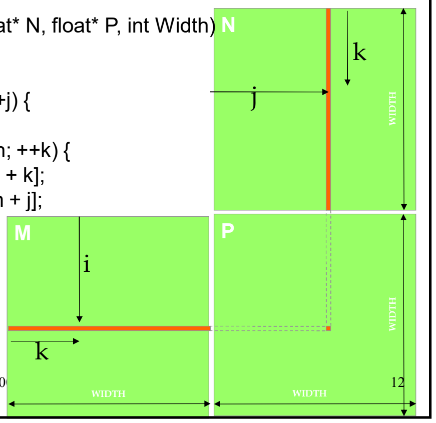
Matrix Multiplication Example
A Simple Host Version in C

```
// Matrix multiplication on the (CPU) host in single
precision
void CPUMatMul(float* M, float* N, float* P, int Width)
{
   for (int i = 0; i < Width; ++i)
      for (int j = 0; j < Width; ++j) {
         float sum = 0;
         for (int k = 0; k < Width; ++k) {
            float a = M[i * Width + k];
            float b = N[k * Width + j];
            sum += a * b;
         }
         P[i * Width + j] = sum;
      }
}
```



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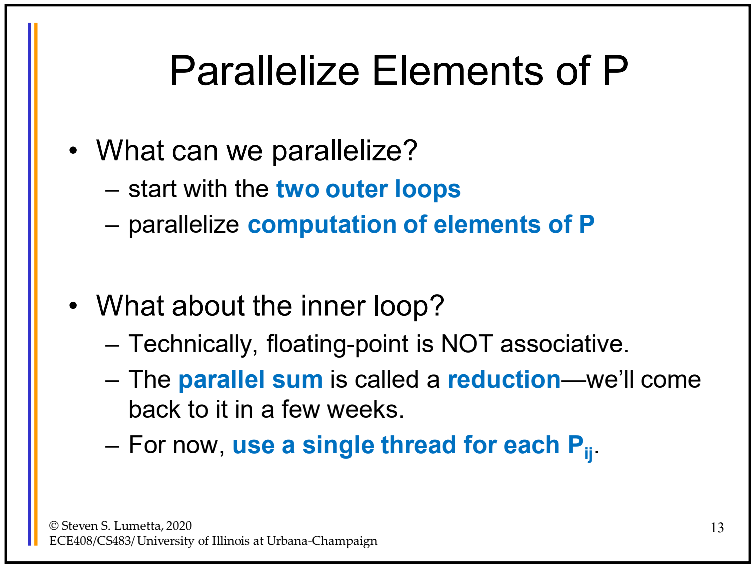
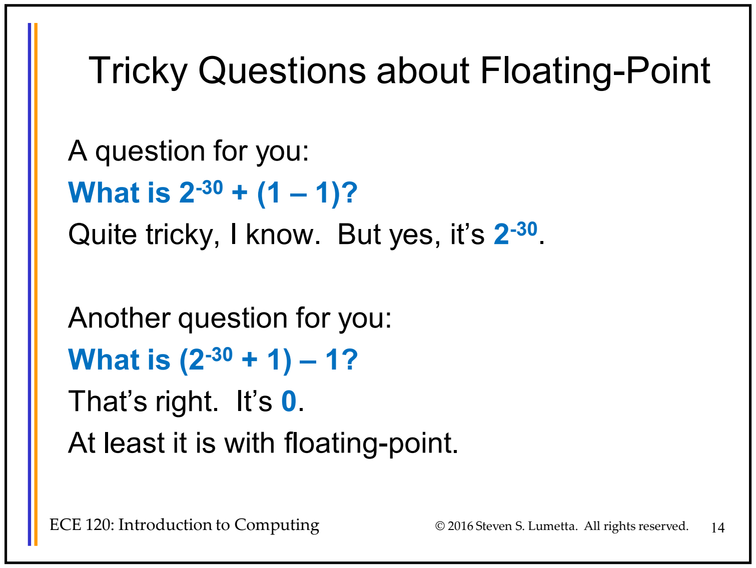


Parallelize Elements of P

- What can we parallelize?
  - start with the **two outer loops**
  - parallelize **computation of elements of P**
- What about the inner loop?
  - Technically, floating-point is NOT associative.
  - The **parallel sum** is called a **reduction**—we'll come back to it in a few weeks.
  - For now, **use a single thread for each P_{ij}**.



Tricky Questions about Floating-Point

A question for you:

What is 2^{-30} + (1 – 1)?

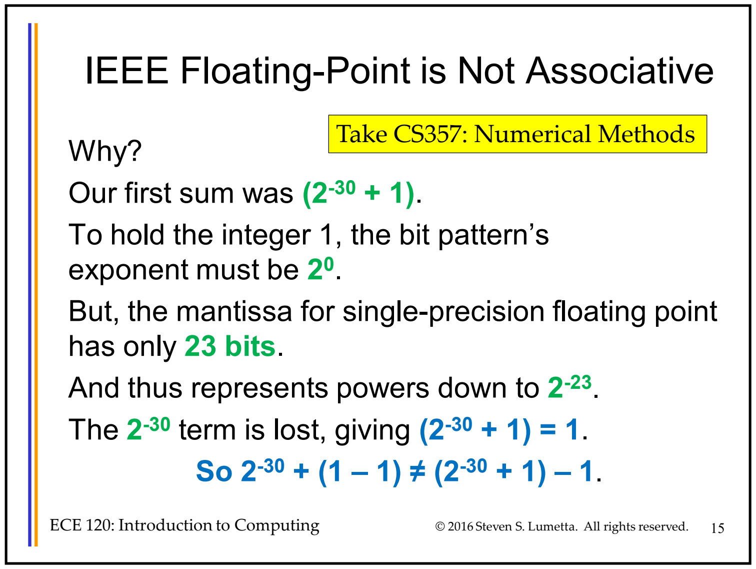
Quite tricky, I know. But yes, it's **2^{-30}**.

Another question for you:

What is (2^{-30} + 1) – 1?

That's right. It's **0**.

At least it is with floating-point.



IEEE Floating-Point is Not Associative

Take CS357: Numerical Methods

Why?

Our first sum was **(2^{-30} + 1)**.

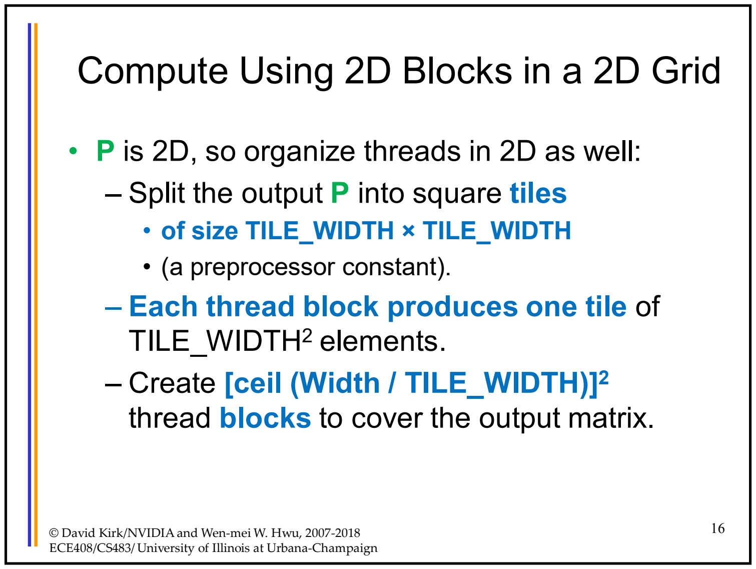
To hold the integer 1, the bit pattern's exponent must be **2^0**.

But, the mantissa for single-precision floating point has only **23 bits**.

And thus represents powers down to **2^{-23}**.

The **2^{-30}** term is lost, giving **(2^{-30} + 1) = 1**.

So 2^{-30} + (1 – 1) ≠ (2^{-30} + 1) – 1.



Compute Using 2D Blocks in a 2D Grid

- **P** is 2D, so organize threads in 2D as well:
  - Split the output **P** into square **tiles**
    - **of size TILE_WIDTH × TILE_WIDTH**
    - (a preprocessor constant).
  - **Each thread block produces one tile** of TILE_WIDTH^2 elements.
  - Create **[ceil (Width / TILE_WIDTH)]^2** thread **blocks** to cover the output matrix.

Example: Width 8, TILE_WIDTH 2

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$	$P_{0,4}$	$P_{0,5}$	$P_{0,6}$	$P_{0,7}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	$P_{1,5}$	$P_{1,6}$	$P_{1,7}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$	$P_{2,4}$	$P_{2,5}$	$P_{2,6}$	$P_{2,7}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$	$P_{3,4}$	$P_{3,5}$	$P_{3,6}$	$P_{3,7}$
$P_{4,0}$	$P_{4,1}$	$P_{4,2}$	$P_{4,3}$	$P_{4,4}$	$P_{4,5}$	$P_{4,6}$	$P_{4,7}$
$P_{5,0}$	$P_{5,1}$	$P_{5,2}$	$P_{5,3}$	$P_{5,4}$	$P_{5,5}$	$P_{5,6}$	$P_{5,7}$
$P_{6,0}$	$P_{6,1}$	$P_{6,2}$	$P_{6,3}$	$P_{6,4}$	$P_{6,5}$	$P_{6,6}$	$P_{6,7}$
$P_{7,0}$	$P_{7,1}$	$P_{7,2}$	$P_{7,3}$	$P_{7,4}$	$P_{7,5}$	$P_{7,6}$	$P_{7,7}$

Each block has
2*2 = 4 threads.

WIDTH/TILE_WIDTH = 4
Use 4×4 = 16 blocks.

Example: Same Matrix, Larger Tiles
(Width 8, TILE_WIDTH 4)

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$	$P_{0,4}$	$P_{0,5}$	$P_{0,6}$	$P_{0,7}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	$P_{1,5}$	$P_{1,6}$	$P_{1,7}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$	$P_{2,4}$	$P_{2,5}$	$P_{2,6}$	$P_{2,7}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$	$P_{3,4}$	$P_{3,5}$	$P_{3,6}$	$P_{3,7}$
$P_{4,0}$	$P_{4,1}$	$P_{4,2}$	$P_{4,3}$	$P_{4,4}$	$P_{4,5}$	$P_{4,6}$	$P_{4,7}$
$P_{5,0}$	$P_{5,1}$	$P_{5,2}$	$P_{5,3}$	$P_{5,4}$	$P_{5,5}$	$P_{5,6}$	$P_{5,7}$
$P_{6,0}$	$P_{6,1}$	$P_{6,2}$	$P_{6,3}$	$P_{6,4}$	$P_{6,5}$	$P_{6,6}$	$P_{6,7}$
$P_{7,0}$	$P_{7,1}$	$P_{7,2}$	$P_{7,3}$	$P_{7,4}$	$P_{7,5}$	$P_{7,6}$	$P_{7,7}$

Each block has
4*4 =16 threads.

WIDTH/TILE_WIDTH = 2
Use 2* 2 = 4 blocks.

Kernel Invocation (Host-side Code)

```
// TILE_WIDTH is a #define constant
dim3 dimGrid(ceil((1.0*Width)/TILE_WIDTH),
       ceil((1.0*Width)/TILE_WIDTH), 1);
dim3 dimBlock(TILE_WIDTH, TILE_WIDTH, 1);


// Launch the device computation threads!
MatrixMulKernel<<<dimGrid, dimBlock>>>
        (Md, Nd, Pd, Width);
```

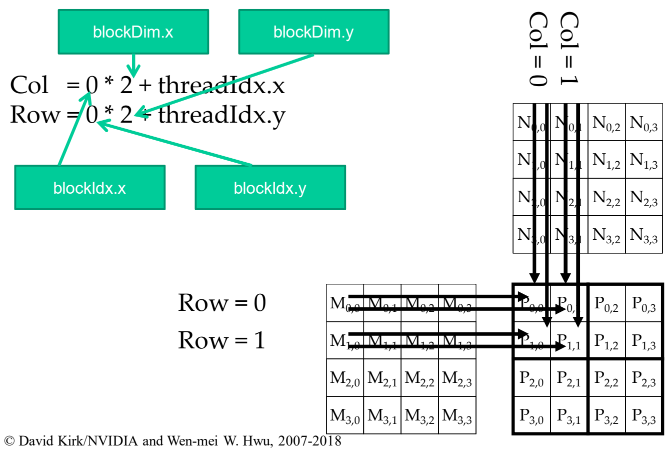
Kernel Function

```
// Matrix multiplication kernel – per thread code

__global__ void MatrixMulKernel(float* d_M, float* d_N, float* d_P, int Width)
{

  // Pvalue is used to store the element of the matrix
  // that is computed by the thread
  float Pvalue = 0;
```

Work for Block (0,0) with TILE_WIDTH 2



blockDim.x
blockDim.y
Col = 0 * 2 + threadIdx.x
Row = 0 * 2 + threadIdx.y
blockIdx.x
blockIdx.y
Col = 0
Col = 1
Row = 0
Row = 1
$N_{0,0}$ $N_{0,1}$ $N_{0,2}$ $N_{0,3}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$ $N_{1,3}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$ $N_{2,3}$
$N_{3,0}$ $N_{3,1}$ $N_{3,2}$ $N_{3,3}$
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$M_{1,0}$ $M_{1,1}$ $M_{1,2}$ $M_{1,3}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$ $M_{2,3}$
$M_{3,0}$ $M_{3,1}$ $M_{3,2}$ $M_{3,3}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$ $P_{0,3}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$ $P_{1,3}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$ $P_{2,3}$
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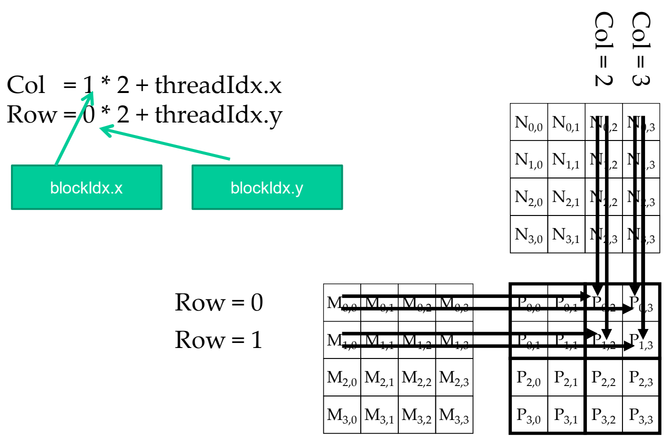
Work for Block (0,1)



Col = 1 * 2 + threadIdx.x
Row = 0 * 2 + threadIdx.y
blockIdx.x
blockIdx.y
Col = 2
Col = 3
Row = 0
Row = 1
$N_{0,0}$ $N_{0,1}$ $N_{0,2}$ $N_{0,3}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$ $N_{1,3}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$ $N_{2,3}$
$N_{3,0}$ $N_{3,1}$ $N_{3,2}$ $N_{3,3}$
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$P_{2,0}$ $P_{2,1}$ $P_{2,2}$ $P_{2,3}$
$P_{3,0}$ $P_{3,1}$ $P_{3,2}$ $P_{3,3}$

A Simple Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* d_M, float* d_N, float* d_P, int Width)
{
// Calculate the row index of the d_P element and d_M
  int Row = blockIdx.y*blockDim.y+threadIdx.y;
// Calculate the column idenx of d_P and d_N
  int Col = blockIdx.x*blockDim.x+threadIdx.x;

  if ((Row < Width) && (Col < Width)) {
    float Pvalue = 0;
// each thread computes one element of the block sub-matrix
    for (int k = 0; k < Width; ++k)
      Pvalue += d_M[Row*Width+k] *
                d_N[k*Width+Col];
     d_P[Row*Width+Col] = Pvalue;
  }
}
```

Memory Bandwidth is Overloaded!

- That's a **simple implementation**:
  - –GPU kernel is the **CPU code**
  - –with the **outer loops replaced**
  - **–with** per-thread **index calculations**!
- Unfortunately, performance is quite bad.
- Why?
- With the given approach,
  - –global **memory bandwidth**
  - **–can't** supply enough data
  - –to **keep the SMs busy**!

Where Do We Access Global Memory?

```
__global__ void MatrixMulKernel(float* d_M, float* d_N, float* d_P, int Width)
{
// Calculate the row index of the d_P element and d_M
int Row = blockIdx.y*blockDim.y+threadIdx.y;
// Calculate the column idenx of d_P and d_N
int Col = blockIdx.x*blockDim.x+threadIdx.x;

if ((Row < Width) && (Col < Width)) {
  float Pvalue = 0;
// each thread computes one element of the block sub-matrix
  for (int k = 0; k < Width; ++k)
    Pvalue += d_M[Row*Width+k] *
              d_N[k*Width+Col];
  d_P[Row*Width+Col] = Pvalue;
 }
}
```

accesses to global memory

Each Thread Requires 4B of Data per FLOP

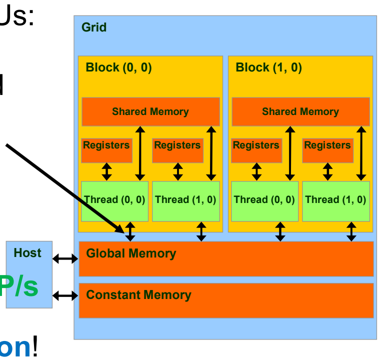
- Each threads access global memory
  - for elements of **M** and **N**:
  - **4B each**, or **8B per pair**.
  - (And once TOTAL to **P** per thread—ignore it.)
- With each pair of elements,
  - a thread does a single multiply-add,
  - **2 FLOP**—floating-point operations.
- So for every FLOP,
  - **a thread needs** 4B from memory:
  - **4B / FLOP**.

150 GB/s Bandwidth Implies 37.5 GFLOPs

- One generation of GPUs:
  - **1,000 GFLOP/s** of compute power, and
  - **150 GB/s** of memory bandwidth.
- Dividing bandwidth by memory requirements:

$$\frac{150\ GB/s}{4\ B/FLOP} = 37.5\ \text{GFLOP/s}$$

which **limits computation**!

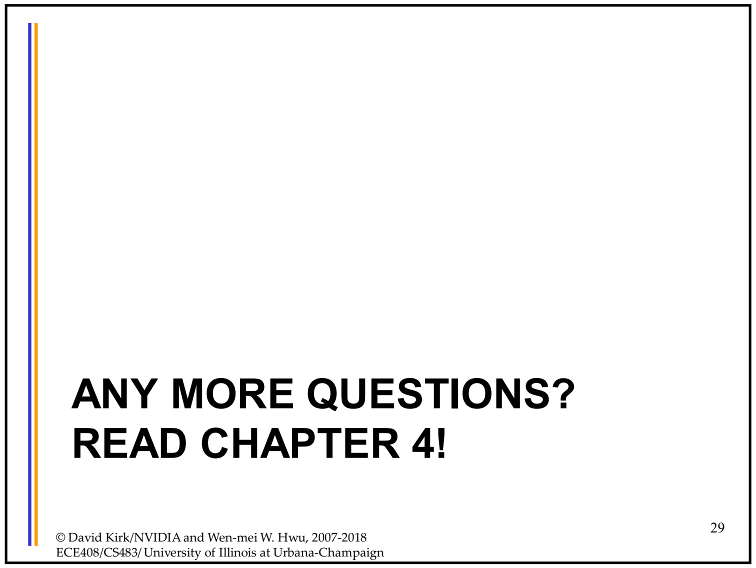


What to Do? Reuse Memory Accesses!

But **37.5 GFLOPs is a limit**.

In an **actual execution**,
- memory is not busy all the time, and
- the code **runs at** about **25 GFLOPs**.

To get closer to 1,000 GFLOPs
- we **need to** drastically **cut down**
- **accesses to global memory**.



ANY MORE QUESTIONS? READ CHAPTER 4!