ECE408 Spring 2020

Applied Parallel Programming

Lecture 19: Parallel Sparse Methods

Objective

- To learn to regularize irregular data with
  - Limiting variations with clamping
  - Sorting
  - Transposition
- To learn to write a high-performance SpMV kernel based on JDS transposed format

Coordinate (COO) format

- Explicitly list the column and row indices for every non-zero element

		Row 0		Row 2			Row 3	
Nonzero values	data[7]	{ 3,	1,	2,	4,	1,	1,	1 }
Column indices	col_index[7]	{ 0,	2,	1,	2,	3,	0,	3 }
Row indices	row_index[7]	{ 0,	0,	2,	2,	2,	3,	3 }

COO Allows Reordering of Elements

		Row 0		Row 2			Row 3	
Nonzero values	data[7]	{ 3,	1,	2,	4,	1,	1,	1 }
Column indices	col_index[7]	{ 0,	2,	1,	2,	3,	0,	3 }
Row indices	row_index[7]	{ 0,	0,	2,	2,	2,	3,	3 }

Nonzero values	data[7]	{ 1	1,	2,	4,	3,	1	1 }
Column indices	col_index[7]	{ 0	2,	1,	2,	0,	3,	3 }
Row indices	row_index[7]	{ 3	0,	2,	2,	0,	2,	3 }

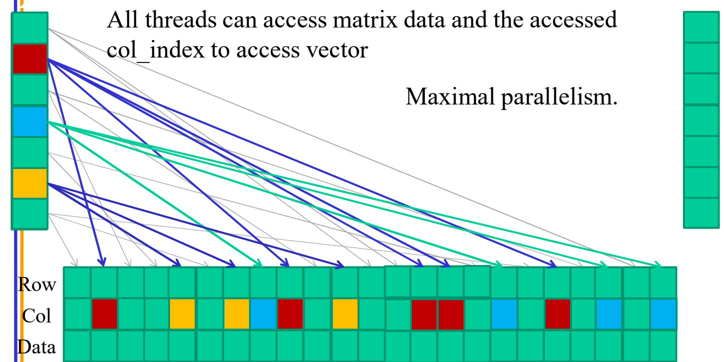
```
for (int i = 0; i < num_elem; row++)
  y[row_index[i]] += data[i] * x[col_index[i]];
```

a sequential loop that implements SpMV/COO

COO Kernel Design Accessing Input Matrix and Vector

All threads can access matrix data and the accessed col_index to access vector

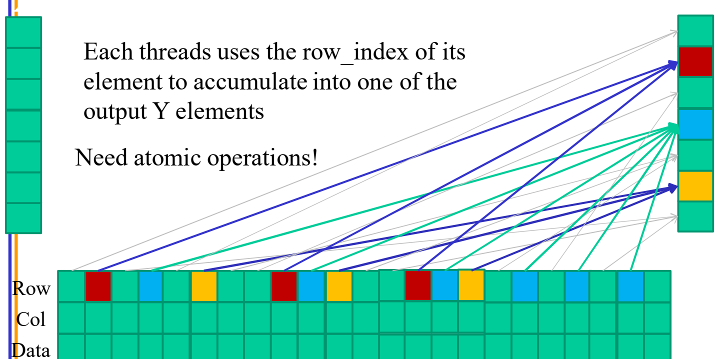
Maximal parallelism.



COO kernel Design Accumulating into Output Vector

Each threads uses the row_index of its element to accumulate into one of the output Y elements

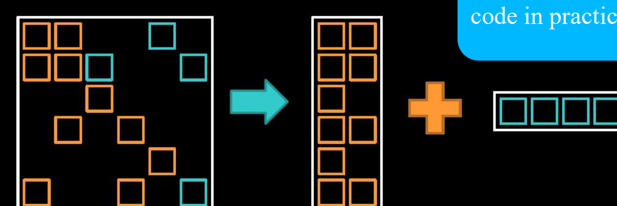
Need atomic operations!



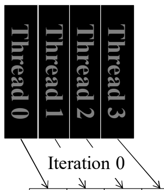
Hybrid Format

- ELL handles *typical* entries
- COO handles *exceptional* entries
  - Implemented with segmented reduction

Often implemented in sequential host code in practice



Reduced Padding with Hybrid Format



	data		col_index	
Thread 0	3	1	0	2
Thread 1	*	*	*	*
Thread 2	2	4	1	2
Thread 3	1	1	0	3

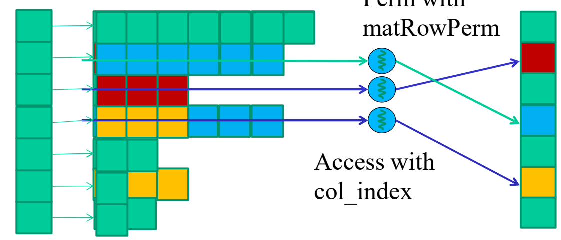
data	3	*	2	1	1	*	4	1
index	0	*	1	0	2	*	2	3

ELL

data	1
col_index	3
row_index	2

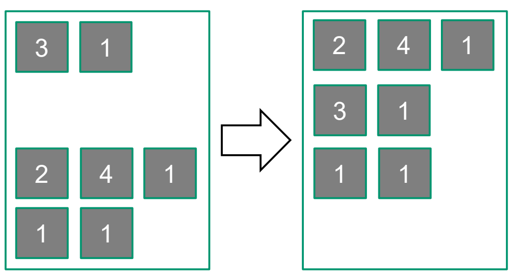
COO

JDS (Jagged Diagonal Sparse) Kernel Design for Load Balancing



Sort rows into descending order according to number of non-zero. Keep track of the original row numbers so that the output vector can be generated correctly.

Sorting Rows According to Length (Regularization)



CSR   JDS

CSR to JDS Conversion

		Row 0	Row 2	Row 3
Nonzero values	data[7]	{ 3, 1,	2, 4, 1,	1, 1 }
Column indices	col_index[7]	{ 0, 2,	1, 2, 3,	0, 3 }
Row Pointers	row_ptr[5]	{ 0, 2,	2,	5, 7 }

		Row 2	Row 0	Row 3	
Nonzero values	data[7]	{ 2, 4, 1,	3, 1,	1 1	}
Column indices	col_index[7]	{ 1, 2, 3,	0, 2,	0, 3	}
JDS Row Pointers	jds_row_ptr[5]	{0,	3,	5,	7,7 }
JDS Row Indices	jds_row_perm[4]	{2,	0,	3,	1 }

JDS Summary

Nonzero values	data[7]	{ 2, 4, 1, 3, 1, 1, 1 }
Column indices	Jds_col_index[7]	{ 1, 2, 3, 0, 2, 0, 3 }
JDS row indices	Jds_row_perm[4]	{ 2, 0, 3, 1 }
JDS Row Ptrs	Jds_row_ptr[5]	{ 0, 3, 5, 7, 7 }

2	4	1	3	1	1	1
1	2	3	0	0	2	3

A Parallel SpMV/JDS Kernel

```
__global__ void SpMV_JDS(int num_rows, float *data,
    int *col_index, int *jds_row_ptr,int *jds_row_perm,
    float *x, float *y) {
  int row = blockIdx.x * blockDim.x + threadIdx.x;
  if (row < num_rows) {
    float dot = 0;
    int row_start = jds_row_ptr[row];
    int row_end =   jds_row_ptr[row+1];
    for (int elem = row_start; elem < row_end; elem++) {
      dot += data[elem] * x[col_index[elem]];
    }
    y[jds_row_perm[row]] = dot;
  }
}
```

		Row 2	Row 0	Row 3
Nonzero values	data[7]	{ 2, 4, 1,	3, 1,	1 1 }
Column indices	col_index[7]	{ 1, 2, 3,	0, 2,	0, 3 }
JDS Row Pointers	jds_row_ptr[5]	{0,	3,	5,	7,7
JDS Row Indices	jds_row_perm[4]	{2,	0,	3,	1 }

JDS vs. CSR - Control Divergence

- Threads still execute different number of iterations in the JDS kernel for-loop
  - However, neighboring threads tend to execute similar number of iterations because of sorting.
  - Better thread utilization, less control divergence

Nonzero values	data[7]	{ 2, 4, 1, 3, 1, 1, 1 }
Column indices	col_index[7]	{ 1, 2, 3, 0, 2, 0, 3 }
JDS row indices	Jds_row_perm[4]	{ 2, 0, 3, 1 }
JDS Row Ptrs	Jds_row_ptr[5]	{ 0, 3, 5, 7, 7 }

data	2	4	1	3	1	1	1
col_index	1	2	3	0	2	0	3

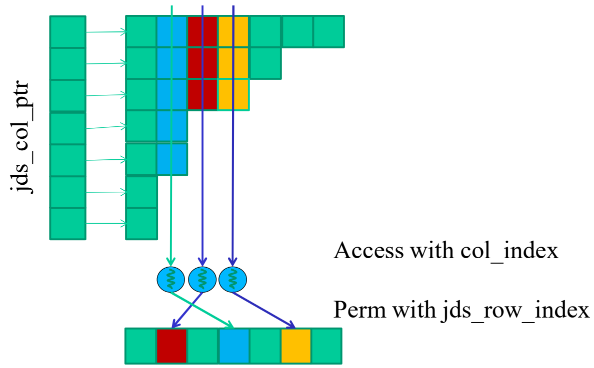
JDS vs. CSR Memory Divergence

- Adjacent threads still access non-adjacent memory locations

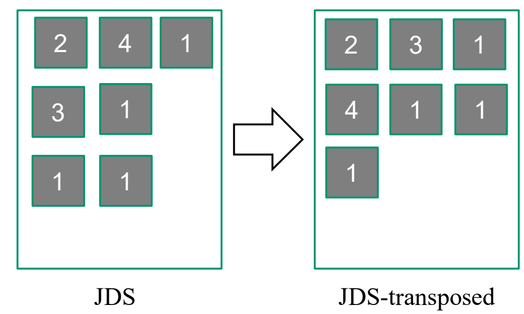
Nonzero values	data[7]	{ 2, 4, 1, 3, 1, 1, 1 }
Column indices	col_index[7]	{ 1, 2, 3, 0, 2, 0, 3 }
JDS row indices	jds_row_perm[4]	{ 2, 0, 3, 1 }
JDS Row Ptrs	jds_row_ptr[5]	{ 0, 3, 5, 7, 7 }

data	2	4	1	3	1	1	1
col_index	1	2	3	0	2	0	3

JDS with Trasposition



Transposition for Memory Coalescing

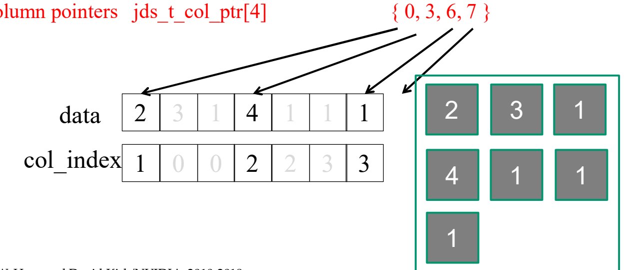


JDS Format with Transposed Layout

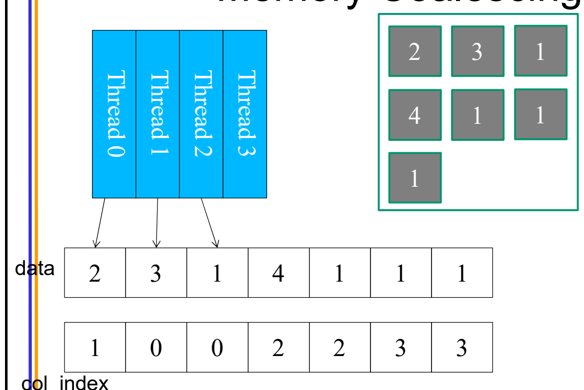
Row 0	3	0	1	0	Thread 0
Row 1	0	0	0	0	Thread 1
Row 2	0	2	4	1	Thread 2
Row 3	1	0	0	1	Thread 3

JDS row indices Jds_row_perm[4] { 2, 0, 3, 1 }

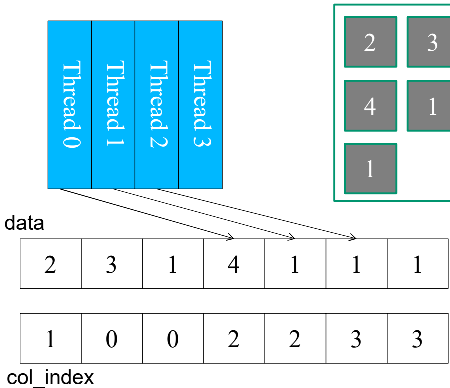
JDS column pointers jds_t_col_ptr[4] { 0, 3, 6, 7 }



JDS with Transposition Memory Coalescing

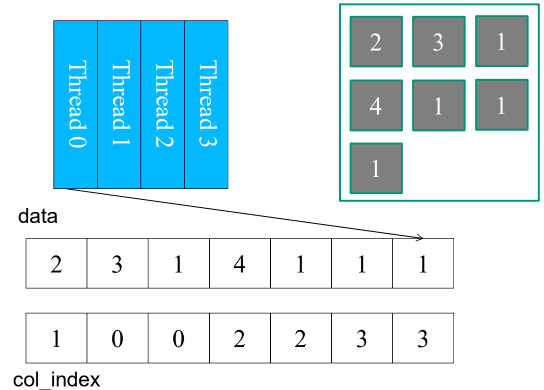


JDS with Transposition Memory Coalescing



Not aligned with DRAM bursts but OK with recent GPUS

JDS with Transposition Memory Coalescing



A Parallel SpMV/JDS_T Kernel

```
1. __global__ void SpMV_JDS_T(int num_rows, float *data,
       int *col_index, int *jds_t_col_ptr, int *jds_row_perm,
       float *x, float *y) {
2.     int row = blockIdx.x * blockDim.x + threadIdx.x;
3.     if (row < num_rows) {
4.       float dot = 0;
         unsigned int sec = 0;
5.       while (jds_t_col_ptr[sec+1]-jds_t_col_ptr[sec] > row){
6.         dot += data[jds_t_col_ptr[sec]+row] *
                  x[col_index[jds_t_col_ptr[sec]+row]];
7.         sec++;
         }
8.       y[jds_row_perm[row]] = dot;
       }
     }
```

Column indices col_index[7] {1, 0, 3, 2, 2, 3 3 }

JDS_T Column Pointers jds_t_col_ptr[5] {0, 3, 6, 7,7 }

JDS Row Indices jds_row_perm[4] {2, 0, 3, 1}

MP7 Variable Names

JDS_T Length of Cols matRows[4] {3, 2, 2, 0 }

	Sec 0	Sec 1	Sec 2
Nonzero values matData[7]	{2, 3, 1,	4, 1, 1	1 }
Column indices matCols[7]	{1, 0, 3,	2, 2, 3	3 }

JDS_T Column Pointers matColStart[4] {0, 3, 6, 7 }

JDS Row Indices matRowPerm[4] {2, 0, 3, 1 }

Sparse Matrices as Foundation for Advanced Algorithm Techniques

- Graphs are often represented as sparse adjacency matrices
  - Used extensively in social network analytics, natural language processing, etc.
- Binning techniques often use sparse matrices for data compaction
  - Used extensively in ray tracing, particle-based fluid dynamics methods, and games
- These will be covered in ECE508/CS508

ANY QUESTIONS?