

Unsigned Addition

```
unsigned char a = 255;  
unsigned char b = 1;  
  
unsigned char c = a + b;  
printf("c=%d", c)
```

运行结果:

c = 0

Unsigned Addition

九曲阑干

For x and y $0 \leq x < 2^w$, $0 \leq y < 2^w$

$$x +_w y = \begin{cases} x + y, & x + y < 2^w \\ x + y - 2^w, & 2^w \leq x + y < 2^{w+1} \end{cases}$$

Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)
{
    unsigned sum = x + y;

    if(sum >= x)
        return 1;
    else
        return 0;
}
```

Detecting Overflow of Unsigned Addition

```
int uadd_ok(unsigned x, unsigned y)     $0 \leq x < 2^w, 0 \leq y < 2^w$ 
{
    unsigned sum = x + y;
    if(sum >= x)
        return 1;
    else
        return 0;
}
```

$x + y \geq x, \quad x + y \geq y$
 溢出: $x + y - 2^w$
 $y - 2^w < 0$
 $x + y - 2^w < x$

溢出结果
 溢出: $x + y - 2^w$
 溢出: $x + y - 2^w$
 溢出: $x + y - 2^w$

溢出: $x + y - 2^w$
 溢出: $x + y - 2^w$
 溢出: $x + y - 2^w$

Two's Complement Addition

For x and y $-2^{w-1} \leq x < 2^{w-1} - 1$, $-2^{w-1} \leq y < 2^{w-1} - 1$

正数相加 \Rightarrow 溢出 $127 + 1 = 128 (OF) \Rightarrow -128 (128 - 256)$

$$x +_w y = \begin{cases} \underline{x + y - 2^w}, & 2^{w-1} \leq x + y \quad \text{Positive overflow} \\ \underline{x + y}, & -2^{w-1} \leq x + y < 2^{w-1} \\ \underline{x + y + 2^w}, & x + y < -2^{w-1} \quad \text{Negative overflow} \end{cases}$$

$x+y$ 是 (positive) 溢出的 \leftarrow 2 个正数相加变正数
 $-128 + 1 = -127 \rightarrow 127$
 $(-128 + 256)$

Overflow

$$(1) x \geq 0, \quad y \geq 0$$

$$x + y < 0 \quad \text{Positive Overflow}$$

$$(2) x \leq 0, \quad y \leq 0$$

$$x + y > 0 \quad \text{Negative Overflow}$$

Additive Inverse

For x , $0 \leq x < 2^w$

$$x + x' = x' + x = 0$$

$$y - x \longrightarrow y + x'$$

For x , x' $0 \leq x < 2^w, 0 \leq x' < 2^w$

$$x + x' = 2^w = 0$$

$$-_w x = \begin{cases} x, & x = 0 \\ 2^w - x, & x \geq 0 \end{cases}$$

Two's-Complement Negation

For x , $-2^{w-1} \leq x < 2^{w-1} - 1$

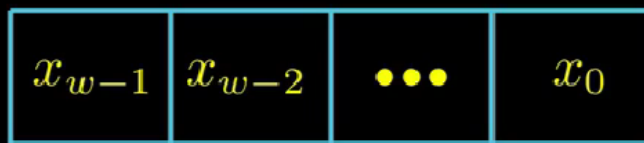
$$-\overset{t}{w}x = \begin{cases} -x, & x > TMin_w \\ TMin_w, & x = TMin_w \end{cases}$$

$$Tmin_w + Tmin_w = -2^{w-1} + (-2^{w-1}) = -2^w$$

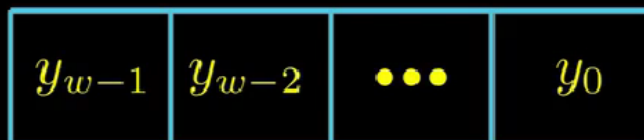
$$Tmin_w + \overset{t}{w}Tmin_w = -2^w + 2^w = 0$$

Unsigned Multiplication

x

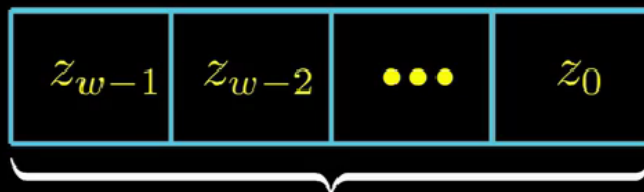


y



$\nwarrow \underline{x * y}$
 $2w$ 位 (max)
 然后截断为 w 位

$z = x \cdot y$



w