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Optimizing Parallel Reduction in CUDA

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Parallel Reduction

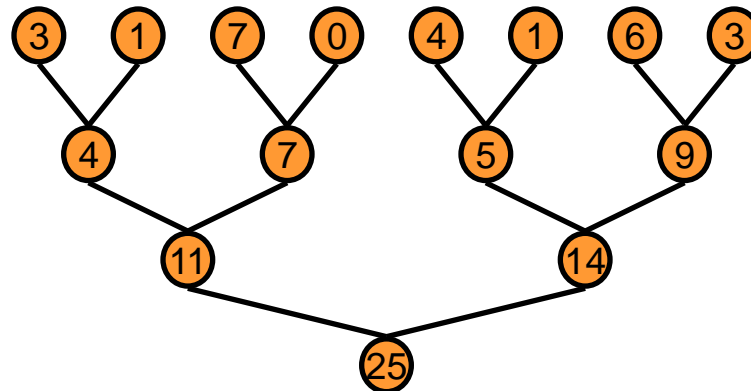


- **Common and important data parallel primitive**
- **Easy to implement in CUDA**
 - **Harder to get it right**
- **Serves as a great optimization example**
 - **We'll walk step by step through 7 different versions**
 - **Demonstrates several important optimization strategies**

Parallel Reduction



- **Tree-based approach used within each thread block**



- **Need to be able to use multiple thread blocks**
 - To process very large arrays
 - To keep all multiprocessors on the GPU busy
 - Each thread block reduces a portion of the array
- **But how do we communicate partial results between thread blocks?**

Problem: Global Synchronization

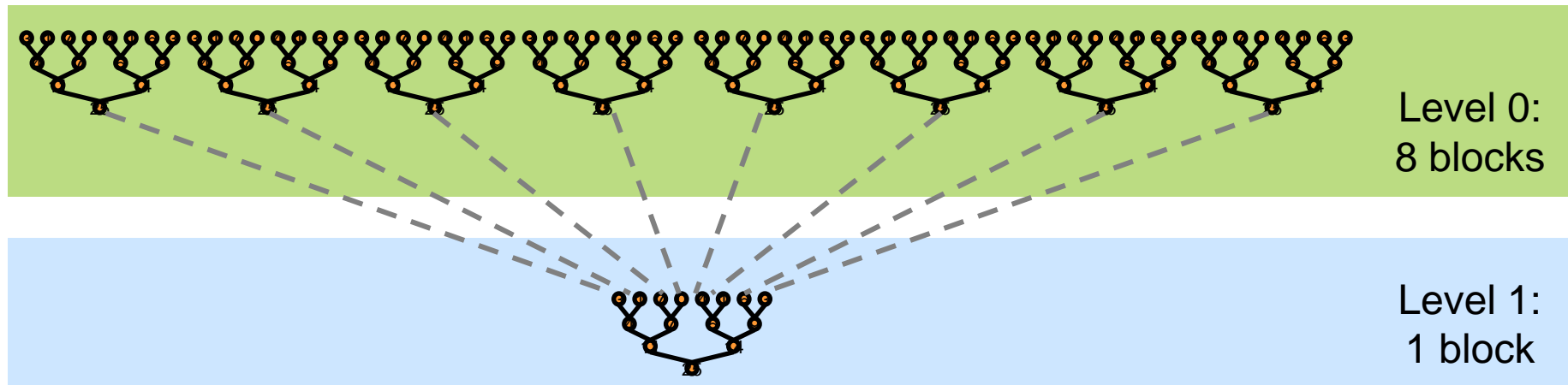


- If we could synchronize across all thread blocks, could easily reduce very large arrays, right?
 - Global sync after each block produces its result
 - Once all blocks reach sync, continue recursively
- But CUDA has no global synchronization. Why?
 - Expensive to build in hardware for GPUs with high processor count
 - Would force programmer to run fewer blocks (no more than $\frac{\text{\# multiprocessors}}{\text{\# resident blocks per multiprocessor}}$) to avoid deadlock, which may reduce overall efficiency
- Solution: decompose into multiple kernels
 - Kernel launch serves as a global synchronization point
 - Kernel launch has negligible HW overhead, low SW overhead

Solution: Kernel Decomposition



- Avoid global sync by decomposing computation into multiple kernel invocations



- In the case of reductions, code for all levels is the same
 - Recursive kernel invocation

What is Our Optimization Goal?



- We should strive to reach GPU peak performance
- Choose the right metric:
 - GFLOP/s: for compute-bound kernels
 - Bandwidth: for memory-bound kernels
- Reductions have very low arithmetic intensity
 - 1 flop per element loaded (bandwidth-optimal)
- Therefore we should strive for peak bandwidth
- Will use G80 GPU for this example
 - 384-bit memory interface, 900 MHz DDR
 - $384 * 1800 / 8 = 86.4 \text{ GB/s}$

Reduction #1: Interleaved Addressing



```
__global__ void reduce0(int *g_idata, int *g_odata) {  
    extern __shared__ int sdata[];
```

```
    // each thread loads one element from global to shared mem
```

```
    unsigned int tid = threadIdx.x;  
    unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;  
    sdata[tid] = g_idata[i];  
    __syncthreads();
```

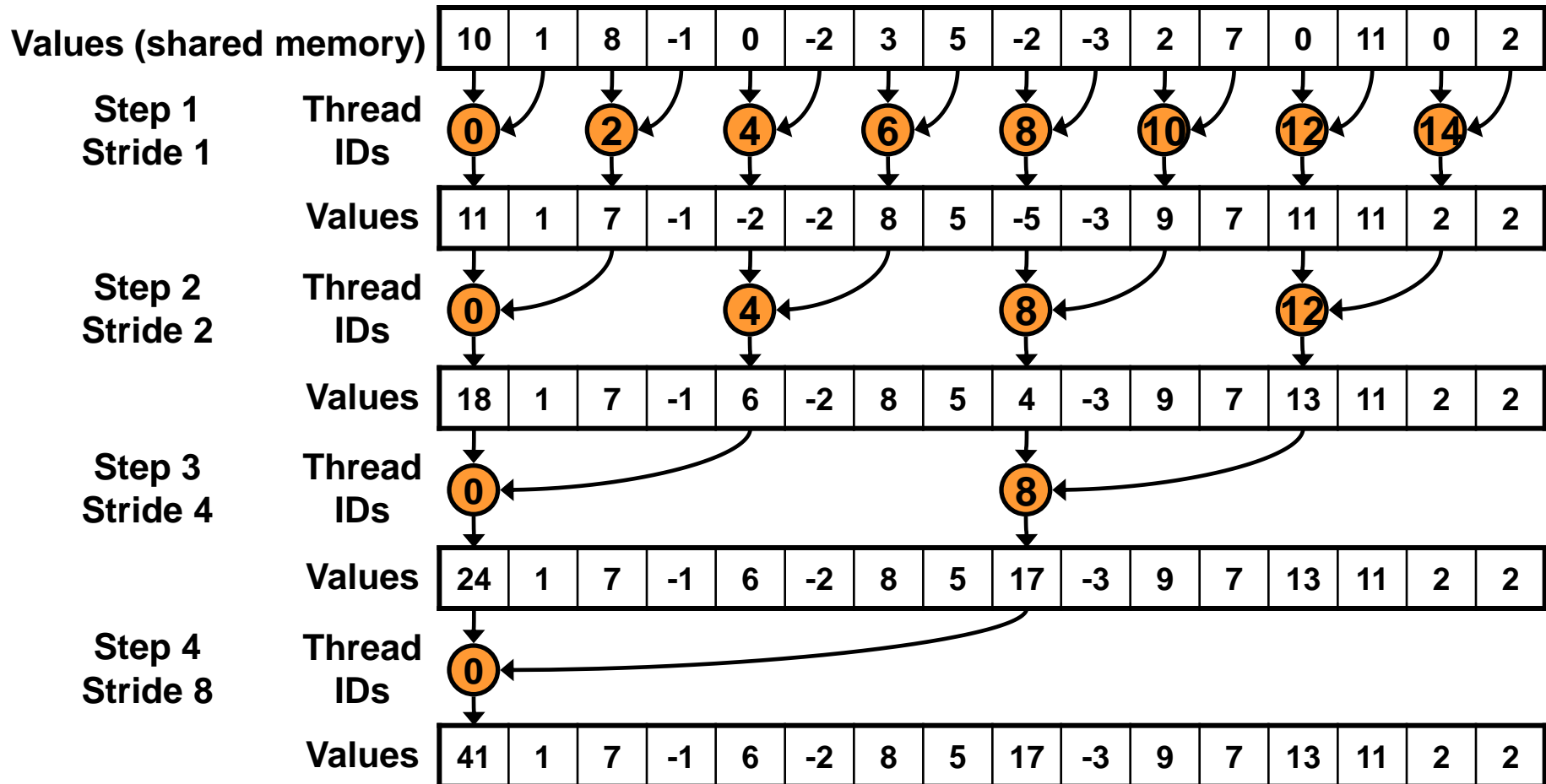
```
    // do reduction in shared mem
```

```
    for(unsigned int s=1; s < blockDim.x; s *= 2) {  
        if (tid % (2*s) == 0) {  
            sdata[tid] += sdata[tid + s];  
        }  
        __syncthreads();  
    }
```

```
    // write result for this block to global mem
```

```
    if (tid == 0) g_odata[blockIdx.x] = sdata[0];  
}
```

Parallel Reduction: Interleaved Addressing



Reduction #1: Interleaved Addressing



```
__global__ void reduce1(int *g_idata, int *g_odata) {  
    extern __shared__ int sdata[];
```

```
    // each thread loads one element from global to shared mem
```

```
    unsigned int tid = threadIdx.x;
```

```
    unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;
```

```
    sdata[tid] = g_idata[i];
```

```
    __syncthreads();
```

```
    // do reduction in shared mem
```

```
    for (unsigned int s=1; s < blockDim.x; s *= 2) {
```

```
        if (tid % (2*s) == 0) {  
            sdata[tid] += sdata[tid + s];
```

```
        }
```

```
        __syncthreads();
```

```
    }
```

Problem: highly divergent warps are very inefficient, and % operator is very slow

```
    // write result for this block to global mem
```

```
    if (tid == 0) g_odata[blockIdx.x] = sdata[0];
```

```
}
```

Performance for 4M element reduction



Time (2^{22} ints)

Bandwidth

Kernel 1:

8.054 ms

2.083 GB/s

interleaved addressing
with divergent branching

Note: Block Size = 128 threads for all tests

Reduction #2: Interleaved Addressing



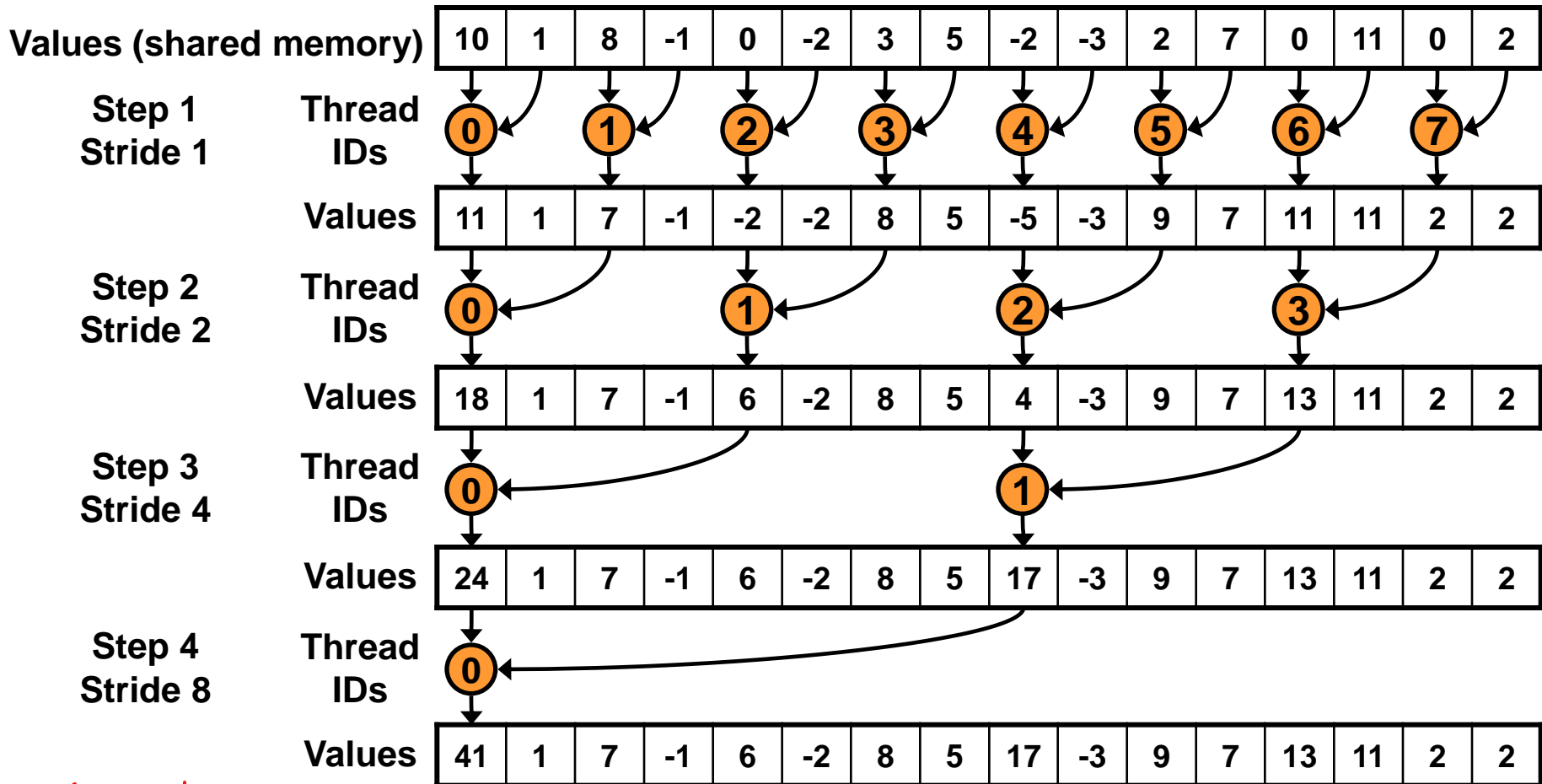
Just replace divergent branch in inner loop:

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {  
    if (tid % (2*s) == 0) {  
        sdata[tid] += sdata[tid + s];  
    }  
    __syncthreads();  
}
```

With strided index and non-divergent branch:

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {  
    int index = 2 * s * tid;  
  
    if (index < blockDim.x) {  
        sdata[index] += sdata[index + s];  
    }  
    __syncthreads();  
}
```

Parallel Reduction: Interleaved Addressing



16 threads in active

New Problem: Shared Memory Bank Conflicts

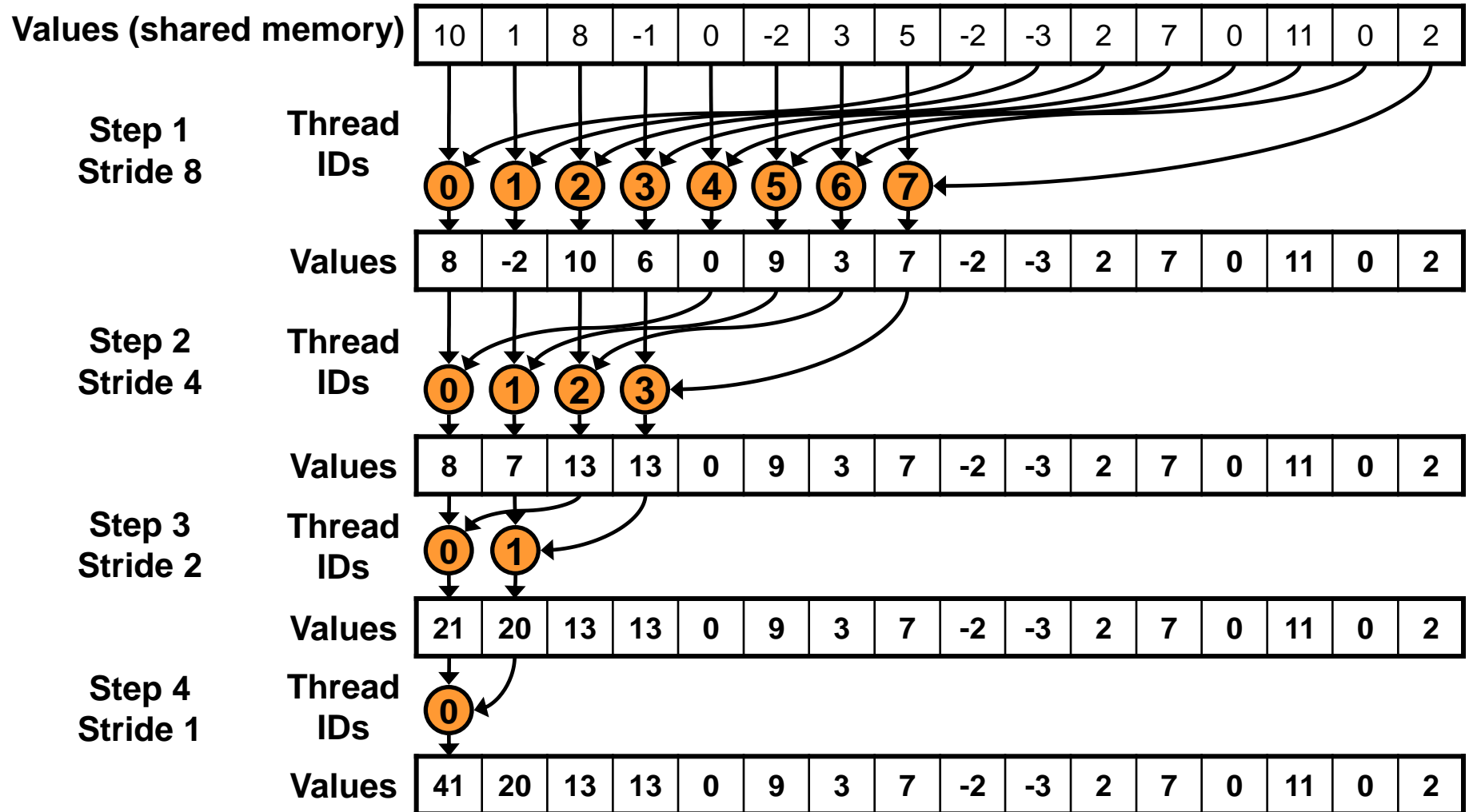


Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x

Parallel Reduction: Sequential Addressing



Sequential addressing is conflict free

Reduction #3: Sequential Addressing



Just replace strided indexing in inner loop:

```
for (unsigned int s=1; s < blockDim.x; s *= 2) {  
    int index = 2 * s * tid;  
  
    if (index < blockDim.x) {  
        sdata[index] += sdata[index + s];  
    }  
    __syncthreads();  
}
```

With reversed loop and threadIdx-based indexing:

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {  
    if (tid < s) {  
        sdata[tid] += sdata[tid + s];  
    }  
    __syncthreads();  
}
```

Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x

Idle Threads



Problem:

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {  
    if (tid < s) {  
        sdata[tid] += sdata[tid + s];  
    }  
    __syncthreads();  
}
```

Half of the threads are idle on first loop iteration!

This is wasteful...

Reduction #4: First Add During Load



Halve the number of blocks, and replace single load:

```
// each thread loads one element from global to shared mem
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;
sdata[tid] = g_idata[i];
__syncthreads();
```

With two loads and first add of the reduction:

```
// perform first level of reduction,
// reading from global memory, writing to shared memory
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];
__syncthreads();
```

Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first add during global load	0.965 ms	17.377 GB/s	1.78x	8.34x

Instruction Bottleneck



- **At 17 GB/s, we're far from bandwidth bound**
 - **And we know reduction has low arithmetic intensity**
- **Therefore a likely bottleneck is instruction overhead**
 - **Ancillary instructions that are not loads, stores, or arithmetic for the core computation**
 - **In other words: address arithmetic and loop overhead**
- **Strategy: unroll loops**

Unrolling the Last Warp



- **As reduction proceeds, # “active” threads decreases**
 - When $s \leq 32$, we have only one warp left
- **Instructions are SIMD synchronous within a warp**
- **That means when $s \leq 32$:**
 - We don't need to `__syncthreads()`
 - We don't need “if (tid < s)” because it doesn't save any work
- **Let's unroll the last 6 iterations of the inner loop**

Reduction #5: Unroll the Last Warp

```
__device__ void warpReduce(volatile int* sdata, int tid) {  
    sdata[tid] += sdata[tid + 32];  
    sdata[tid] += sdata[tid + 16];  
    sdata[tid] += sdata[tid + 8];  
    sdata[tid] += sdata[tid + 4];  
    sdata[tid] += sdata[tid + 2];  
    sdata[tid] += sdata[tid + 1];  
}
```

↑

IMPORTANT:
For this to be correct,
we must use the
“volatile” keyword!

```
// later...  
for (unsigned int s=blockDim.x/2; s>32; s>>=1) {  
    if (tid < s)  
        sdata[tid] += sdata[tid + s];  
    __syncthreads();  
}  
  
if (tid < 32) warpReduce(sdata, tid);
```

Note: This saves useless work in *all* warps, not just the last one!
Without unrolling, all warps execute every iteration of the for loop and if statement

Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing with bank conflicts	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first add during global load	0.965 ms	17.377 GB/s	1.78x	8.34x
Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x

Complete Unrolling



- **If we knew the number of iterations at compile time, we could completely unroll the reduction**
 - **Luckily, the block size is limited by the GPU to 512 threads**
 - **Also, we are sticking to power-of-2 block sizes**
- **So we can easily unroll for a fixed block size**
 - **But we need to be generic – how can we unroll for block sizes that we don't know at compile time?**
- **Templates to the rescue!**
 - **CUDA supports C++ template parameters on device and host functions**

Unrolling with Templates



- Specify block size as a function template parameter:

```
template <unsigned int blockSize>  
__global__ void reduce5(int *g_idata, int *g_odata)
```

Reduction #6: Completely Unrolled



Template <unsigned int blockSize>

```
__device__ void warpReduce(volatile int* sdata, int tid) {  
    if (blockSize >= 64) sdata[tid] += sdata[tid + 32];  
    if (blockSize >= 32) sdata[tid] += sdata[tid + 16];  
    if (blockSize >= 16) sdata[tid] += sdata[tid + 8];  
    if (blockSize >= 8) sdata[tid] += sdata[tid + 4];  
    if (blockSize >= 4) sdata[tid] += sdata[tid + 2];  
    if (blockSize >= 2) sdata[tid] += sdata[tid + 1];  
}
```

```
if (blockSize >= 512) {  
    if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }  
if (blockSize >= 256) {  
    if (tid < 128) { sdata[tid] += sdata[tid + 128]; } __syncthreads(); }  
if (blockSize >= 128) {  
    if (tid < 64) { sdata[tid] += sdata[tid + 64]; } __syncthreads(); }  
  
if (tid < 32) warpReduce<blockSize>(sdata, tid);
```

Note: all code in **RED** will be evaluated at compile time.

Results in a very efficient inner loop!

Invoking Template Kernels



- Don't we still need block size at compile time?
- Nope, just a switch statement for 10 possible block sizes:

```
switch (threads)
{
  case 512:
    reduce5<512><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 256:
    reduce5<256><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 128:
    reduce5<128><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 64:
    reduce5< 64><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 32:
    reduce5< 32><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 16:
    reduce5< 16><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 8:
    reduce5<  8><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 4:
    reduce5<  4><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 2:
    reduce5<  2><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
  case 1:
    reduce5<  1><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break;
}
```

Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
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Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: completely unrolled	0.381 ms	43.996 GB/s	1.41x	21.16x

Parallel Reduction Complexity



- **Log(N)** parallel steps, each step S does $N/2^S$ independent ops
 - **Step Complexity** is $O(\log N)$
- For $N=2^D$, performs $\sum_{S \in [1..D]} 2^{D-S} = N-1$ operations
 - **Work Complexity** is $O(N)$ – It is **work-efficient**
 - i.e. does not perform more operations than a sequential algorithm
- With P threads physically in parallel (P processors), **time complexity** is $O(N/P + \log N)$
 - Compare to $O(N)$ for sequential reduction
 - In a thread block, $N=P$, so **$O(\log N)$**

What About Cost?



- **Cost of a parallel algorithm is processors time complexity**
 - Allocate threads instead of processors: $O(N)$ threads
 - Time complexity is $O(\log N)$, so *cost* is $O(N \log N)$: **not cost efficient!**
- **Brent's theorem suggests $O(N/\log N)$ threads**
 - Each thread does $O(\log N)$ sequential work
 - Then all $O(N/\log N)$ threads cooperate for $O(\log N)$ steps
 - Cost = $O((N/\log N) * \log N) = O(N) \rightarrow$ cost efficient
- **Sometimes called *algorithm cascading***
 - Can lead to significant speedups in practice

Algorithm Cascading



- **Combine sequential and parallel reduction**
 - Each thread loads and sums multiple elements into shared memory
 - Tree-based reduction in shared memory
- **Brent's theorem says each thread should sum $O(\log n)$ elements**
 - i.e. 1024 or 2048 elements per block vs. 256
- **In my experience, beneficial to push it even further**
 - Possibly better latency hiding with more work per thread
 - More threads per block reduces levels in tree of recursive kernel invocations
 - High kernel launch overhead in last levels with few blocks
- **On G80, best perf with 64-256 blocks of 128 threads**
 - 1024-4096 elements per *thread*

Reduction #7: Multiple Adds / Thread



Replace load and add of two elements:

```
unsigned int tid = threadIdx.x;  
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;  
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];  
__syncthreads();
```

With a while loop to add as many as necessary:

```
unsigned int tid = threadIdx.x;  
unsigned int i = blockIdx.x*(blockSize*2) + threadIdx.x;  
unsigned int gridSize = blockSize*2*gridDim.x;  
sdata[tid] = 0;  
  
while (i < n) {  
    sdata[tid] += g_idata[i] + g_idata[i+blockSize];  
    i += gridSize;  
}  
__syncthreads();
```


Reduction #7: Multiple Adds / Thread



Replace load and add of two elements:

```
unsigned int tid = threadIdx.x;  
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;  
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];  
__syncthreads();
```

With a while loop to add as many as necessary:

```
unsigned int tid = threadIdx.x;  
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;  
unsigned int gridSize = blockDim.x*2;  
sdata[tid] = 0;  
  
while (i < n) {  
    sdata[tid] += g_idata[i] + g_idata[i+blockSize];  
    i += gridSize;  
}  
__syncthreads();
```

**Note: gridSize loop stride
to maintain coalescing!**

Performance for 4M element reduction



	Time (2^{22} ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
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Kernel 4: first add during global load	0.965 ms	17.377 GB/s	1.78x	8.34x
Kernel 5: unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: completely unrolled	0.381 ms	43.996 GB/s	1.41x	21.16x
Kernel 7: multiple elements per thread	0.268 ms	62.671 GB/s	1.42x	30.04x

Kernel 7 on 32M elements: 73 GB/s!

Final Optimized Kernel

```
template <unsigned int blockSize>
__device__ void warpReduce(volatile int *sdata, unsigned int tid) {
    if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
    if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
    if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
    if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
    if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
    if (blockSize >= 2) sdata[tid] += sdata[tid + 1];
}

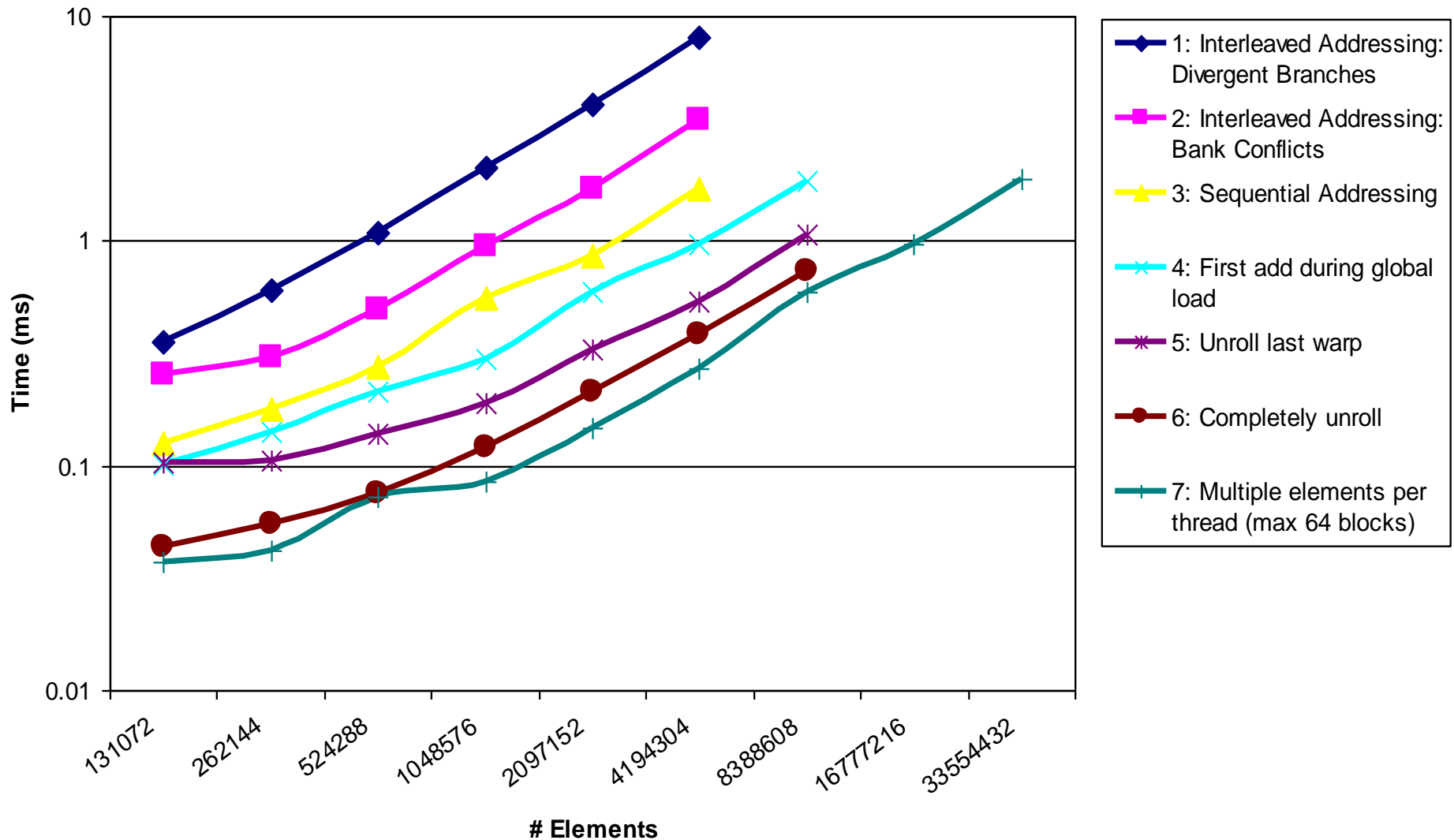
template <unsigned int blockSize>
__global__ void reduce6(int *g_idata, int *g_odata, unsigned int n) {
    extern __shared__ int sdata[];
    unsigned int tid = threadIdx.x;
    unsigned int i = blockIdx.x*(blockSize*2) + tid;
    unsigned int gridSize = blockSize*2*gridDim.x;
    sdata[tid] = 0;

    while (i < n) { sdata[tid] += g_idata[i] + g_idata[i+blockSize]; i += gridSize; }
    __syncthreads();

    if (blockSize >= 512) { if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }
    if (blockSize >= 256) { if (tid < 128) { sdata[tid] += sdata[tid + 128]; } __syncthreads(); }
    if (blockSize >= 128) { if (tid < 64) { sdata[tid] += sdata[tid + 64]; } __syncthreads(); }

    if (tid < 32) warpReduce(sdata, tid);
    if (tid == 0) g_odata[blockIdx.x] = sdata[0];
}
```

Performance Comparison



Types of optimization



- **Interesting observation:**
- **Algorithmic optimizations**
 - Changes to addressing, algorithm cascading
 - 11.84x speedup, combined!
- **Code optimizations**
 - Loop unrolling
 - 2.54x speedup, combined

Conclusion



- Understand CUDA performance characteristics
 - Memory coalescing
 - Divergent branching
 - Bank conflicts
 - Latency hiding
- Use peak performance metrics to guide optimization
- Understand parallel algorithm complexity theory
- Know how to identify type of bottleneck
 - e.g. memory, core computation, or instruction overhead
- Optimize your algorithm, *then* unroll loops
- Use template parameters to generate optimal code

- Questions: mharris@nvidia.com