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Solution: Number of Submatrices That Sum to Target

6-8 minutes

*This is part of a series of Leetcode solution explanations ([index](#)). If you liked this solution or found it useful, **please like** this post and/or **upvote** [my solution post on Leetcode's forums](#).*

[Leetcode Problem #1074 \(Hard\): Number of Submatrices That Sum to Target](#)

Description:

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Given a matrix and a target, return the number of non-empty submatrices that sum to target.

A submatrix $x1, y1, x2, y2$ is the set of all cells $matrix[x][y]$ with $x1 \leq x \leq x2$ and $y1 \leq y \leq y2$.

Two submatrices $(x1, y1, x2, y2)$ and $(x1', y1', x2', y2')$ are different if they have some coordinate that is different: for example, if $x1 \neq x1'$.

Examples:

Example 1:										
Input:	matrix = [[0,1,0],[1,1,1],[0,1,0]], target = 0									
Output:	4									
Explanation:	The four 1x1 submatrices that only contain 0.									
Visual:	<table><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr></table>	0	1	0	1	1	1	0	1	0
0	1	0								
1	1	1								
0	1	0								

Example 2:	
Input:	matrix = [[1,-1],[-1,1]], target = 0
Output:	5
Explanation:	The two 1x2 submatrices, plus the two 2x1 submatrices, plus the 2x2 submatrix.

Example 3:	
Input:	matrix = [[904]], target = 0
Output:	0

Constraints:

- $1 \leq \text{matrix.length} \leq 100$
 - $1 \leq \text{matrix}[0].\text{length} \leq 100$
 - $-1000 \leq \text{matrix}[i] \leq 1000$
 - $-10^8 \leq \text{target} \leq 10^8$
-

Idea:

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This problem is essentially a **2-dimensional** version of [#560. Subarray Sum Equals K \(S.S.E.K\)](#). By using a **prefix sum** on each row or each column, we can compress this problem down to either N^2 iterations of the $O(M)$ SSEK, or M^2 iterations of the $O(N)$ SSEK.

In the SSEK solution, we can find the number of subarrays with the target sum by utilizing a **result map (res)** to store the different values found as we iterate through the array while keeping a running sum (**csum**). Just as in the case with a prefix sum array, the sum of a subarray between **i** and **j** is equal to the sum of the subarray from **0** to **j** minus the sum of the subarray from **0** to **i-1**.

Rather than iteratively checking if $\text{sum}[0,j] - \text{sum}[0,i-1] = T$ for every pair of **i, j** values, we can flip it around to $\text{sum}[0,j] - T = \text{sum}[0,i-1]$ and since every earlier sum value has been stored in **res**, we can simply perform a lookup on $\text{sum}[0,j] - T$ to see if there are any matches.

When extrapolating this solution to our **2-dimensional** matrix (**M**),

we will need to first prefix sum the rows or columns, (which we can do **in-place** to avoid extra space, as we will not need the original values again). Then we should iterate through **M** again in the opposite order of rows/columns where the prefix sums will allow us to treat a group of columns or rows as if it were a **1-dimensional** array and apply the SSEK algorithm.

Implementation:

There are only minor differences in the code of all four languages.

Javascript Code:

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```
var numSubmatrixSumTarget = function(M, T) {
    let xlen = M[0].length, ylen = M.length,
        ans = 0, res = new Map()
    for (let i = 0, r = M[0]; i < ylen; r = M[++i])
        for (let j = 1; j < xlen; j++)
            r[j] += r[j-1]
    for (let j = 0; j < xlen; j++)
        for (let k = j; k < xlen; k++) {
            res.clear(), res.set(0,1), csum = 0
            for (let i = 0; i < ylen; i++) {
                csum += M[i][k] - (j ? M[i][j-1] : 0)
                ans += (res.get(csum - T) || 0)
                res.set(csum, (res.get(csum) || 0) +
1)
```

```
        }
    }
    return ans
};
```

```
[] []
```



Python Code:

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```
class Solution:
    def numSubmatrixSumTarget(self, M:
List[List[int]], T: int) -> int:
        xlen, ylen, ans, res = len(M[0]), len(M), 0,
defaultdict(int)
        for r in M:
            for j in range(1, xlen):
                r[j] += r[j-1]
        for j in range(xlen):
            for k in range(j, xlen):
                res.clear()
                res[0], csum = 1, 0
                for i in range(ylen):
                    csum += M[i][k] - (M[i][j-1] if j
else 0)

                ans += res[csum - T]
                res[csum] += 1

        return ans
```

Java Code:

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```
class Solution {
    public int numSubmatrixSumTarget(int[][] M, int
T) {
        int xlen = M[0].length, ylen = M.length, ans
= 0;
        Map<Integer, Integer> res = new HashMap<>();
        for (int[] r : M)
            for (int j = 1; j < xlen; j++)
                r[j] += r[j-1];
        for (int j = 0; j < xlen; j++)
            for (int k = j; k < xlen; k++) {
                res.clear();
                res.put(0,1);
                int csum = 0;
                for (int i = 0; i < ylen; i++) {
                    csum += M[i][k] - (j > 0 ?
M[i][j-1] : 0);
                    ans += res.getDefault(csum - T,
0);
                    res.put(csum,
res.getDefault(csum, 0) + 1);
                }
            }
    }
}
```

```
        return ans;
    }
}
```

```
]]]]
```

C++ Code:

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```
class Solution {
public:
    int numSubmatrixSumTarget(vector<vector<int>>& M,
int T) {
        int xlen = M[0].size(), ylen = M.size(), ans
= 0;

        unordered_map<int, int> res;
        for (int i = 0; i < ylen; i++)
            for (int j = 1; j < xlen; j++)
                M[i][j] += M[i][j-1];
        for (int j = 0; j < xlen; j++)
            for (int k = j; k < xlen; k++) {
                res.clear();
                res[0] = 1;
                int csum = 0;
                for (int i = 0; i < ylen; i++) {
                    csum += M[i][k] - (j ? M[i][j-1]
: 0);

                    ans += res.find(csum - T) !=
res.end() ? res[csum - T] : 0;
                }
            }
    }
};
```

```
        res[csum]++;  
    }  
}  
return ans;  
}  
};
```

```
[[[[]]]]
```