

Let A_n be the set $[0, \frac{1}{2^n})$, then A_{n+1} will be $[0, \frac{1}{2^{n+1}})$.

Because $\frac{1}{2^n} > \frac{1}{2^{n+1}}$, it is proved that $A_{n+1} \subset A_n$.

Since $\forall n, \frac{1}{2^n} > 0$, so A_n is not empty and 0 is always an element.

So set $C = \bigcap_{n=1}^{\infty} A_n = \emptyset$ at least includes 0, which means $c \neq \emptyset$.

And next we should prove that $C = \{0\}$.

Assume there is another element $t \in C$ and $t \neq 0$.

It's easy to find that $0 < t < \frac{1}{2}$ (because $t \in A_1$).

Let $m = -\lceil \log_2 t \rceil + 1$, where $\lceil x \rceil$ means the smallest integer larger than x .

Considering the value of t , we can know that m is an positive integer.

Thus, we get

$$\begin{aligned} \frac{1}{2^m} &= (2^{-\lceil \log_2 t \rceil + 1})^{-1} \\ &= 2^{\lceil \log_2 t \rceil - 1} \\ &< 2^{\log_2 t - 1} \\ &= 2^{\log_2 t} * \frac{1}{2} \\ &= \frac{t}{2} \\ &< t \end{aligned}$$

Since $A_m = [0, \frac{1}{2^m})$, and we have $\frac{1}{2^m} < t$.

So $t \notin A_m$, and by definition, $t \notin C$.

Hence, no element besides 0 is in C , which means $C = \{0\}$.

Hence, it's proved true.