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2. Homework

Total score: 32 points

Topics: System properties, z-transform, pole-zero diagram, minimum phase and all-pass systems.

Submission

Submit the homework by **Sunday 11. January 2026 (23:55)** via the ISIS portal. Late submissions or submissions that are not made via the ISIS portal will not be considered. Only one submission must be uploaded per group.

The homework must be submitted as a single **iPython Notebook** without any attachments. The notebook can include code, plots, text, images, and equations. Equations can typeset with LaTex or handwritten. In the latter case, they should be included as in image. Please **run the Notebook** before the submission to make sure all plots and outputs are included.

Include the **names and matriculation numbers** of all group members in the iPython notebook, comment your code, and submit the notebook as a **single file** named for example **homework_01_group_A.ipynb**.

The following Python packages might help you to solve the tasks: `numpy`, `scipy`, `matplotlib`, `sounddevice`, `timeit`, `pyfar`. pyfar can **only** be used if it is explicitly allowed.

1 Plot system properties

a) Write the function `axes = plot_system(system, fs=2*np.pi, N=2048, color='k')` that receives a description of a system, and plots the pole-zero diagram, magnitude response, phase response, and group delay of the system similar to Fig. 1.

The function takes the following parameters:

> `system`: A tuple defining the system specified either by the `(zeros, poles, gain)` or by the `(b-coefficients, a-coefficients)`. You can use `scipy.signal` to convert between the two system representations. Note that the z-transfer function always has the same number of poles and zeros.

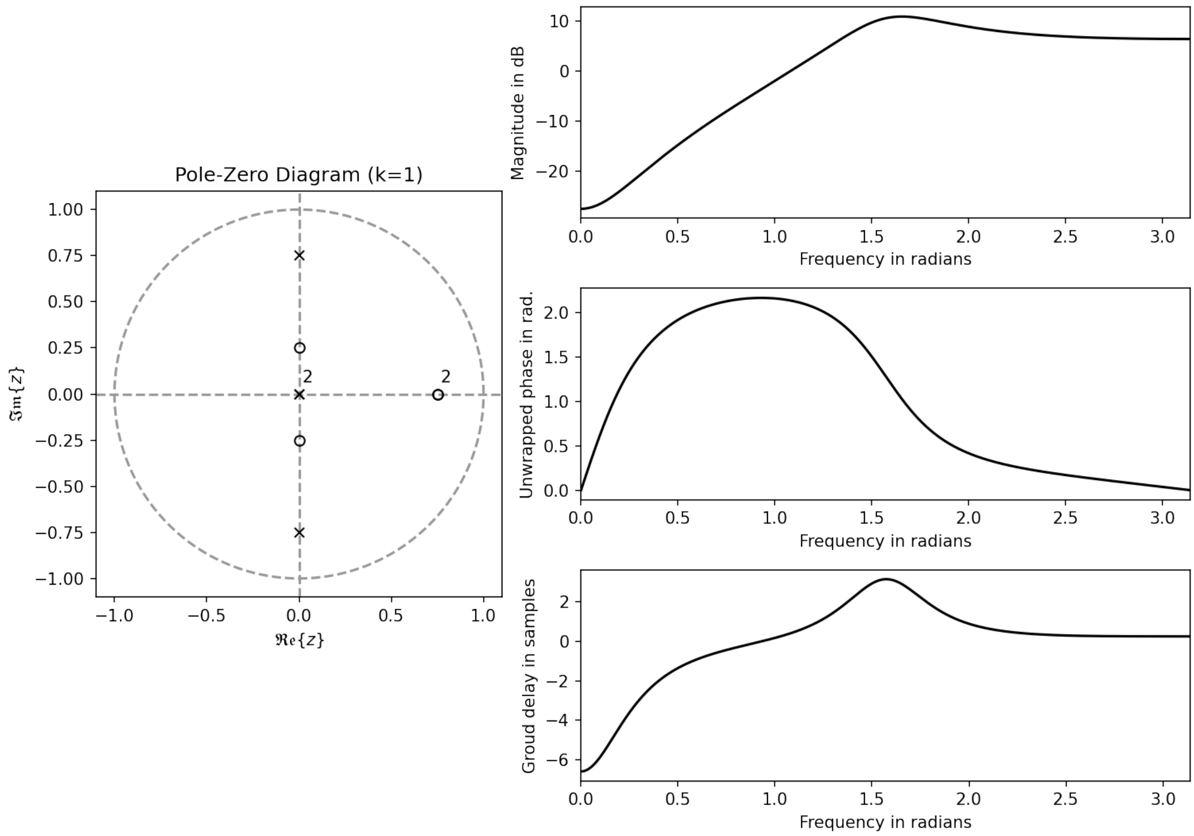

Figure 1: Example for system properties plot

`fs`: the sampling rate. Note that the notion `fs=2*np.pi` sets the sampling rate to 2π by default but that other values can be passed to the function as well.

`N`: The number of frequencies for which the magnitude/phase response and group delay are calculated

`color`: The color for plotting the data in a format accepted by `matplotlib`.

The function returns the axes objects of all four subplots and has the following features:

A docstring explaining the parameters, default values, and return values

If there are multiple poles/zeros at the same position, the number of poles/zeros should be plotted next to them.

Make sure that the number of poles equals the number of zeros.

b) Generate plots for the following two systems:

System 1: `zeros = [0.5, -0.5]`, `poles = [0.75j, -0.75j, 0.75j, -0.75j]`, `gain = 2`
System 2: `b = 0.1 * numpy.ones(10)`, `a = 1`

8 Points: Plot function (6), Plots (2)

2 z-Transfer functions

Let the poles, zero, and gains below define three systems in the z-domain

System 1: double zero at $z_n = 0.8$, double pole at $z_p = 0.95$, gain $k = 0.063$.
System 2: double zero at $z_n = 0.95$, double pole at $z_p = 0.8$, gain $k = 1$.
System 3: zeros at $z_n = \pm 0.25\text{j}$, poles at $z_p = \pm 0.75\text{j}$, gain $k = 0.45$.

a) Determine the transfer functions $H(z)$ and the filter coefficients a and b for each of the three systems by hand.

b) Plot the pole-zero diagram, the frequency response, phase response and group delay for each system using a sampling rate of $f_s = 44.1$ kHz. How would you describe the frequency responses of the systems?

c) Get the drum signal from `pyfar.signals.files` at the same sampling rate, filter it with all three systems using `scipy.signal.lfilter`, and play back the results. Do they sound as expected?

6 Points: Transfer functions and filter coefficients (3), Plots (1.5), Filter and playback (1.5)

3 Incomplete z-Transfer functions

Let four causal Systems be represented by their poles and zeros
System 1: zeros at $z_n = \{2 + 2\text{j}, 2 - 2\text{j}\}$, a pole at $z_p = 0.25 - 0.25\text{j}$.
System 2: zeros at $z_n = \{1, -1, \text{j}, -\text{j}\}$, poles at $z_p = \{-0.5, 0, 0.5, 1\}$.
System 3: zeros at $z_n = \{-0.75\text{j}, 0.75\text{j}, 1.25\text{j}\}$, poles at $z_p = \{0.5\text{j}, -0.5\text{j}\}$.
System 4: zeros at $z_n = \{0.25 + 0.25\text{j}, 0.25 - 0.25\text{j}\}$, a pole at $z_p = 0.5\text{j}$.

Add, remove, or move a single pole or zero to convert
System 1 in to an all-pass,
System 2 in to a stable system,
System 3 in to a minimum phase system,
System 4 in to a system with a real valued impulse response.

Explain, what you did and plot the resulting pole zero diagrams. Note that it is OK, if adding and removing poles and zeros changes $H(z)$.

8 Points: 2 points per system (The plots don't give points—reuse code from the previous task).

4 Minimum Phase systems

Let the pole-zero diagram given in Fig. 2 define the causal system $H(z)$.

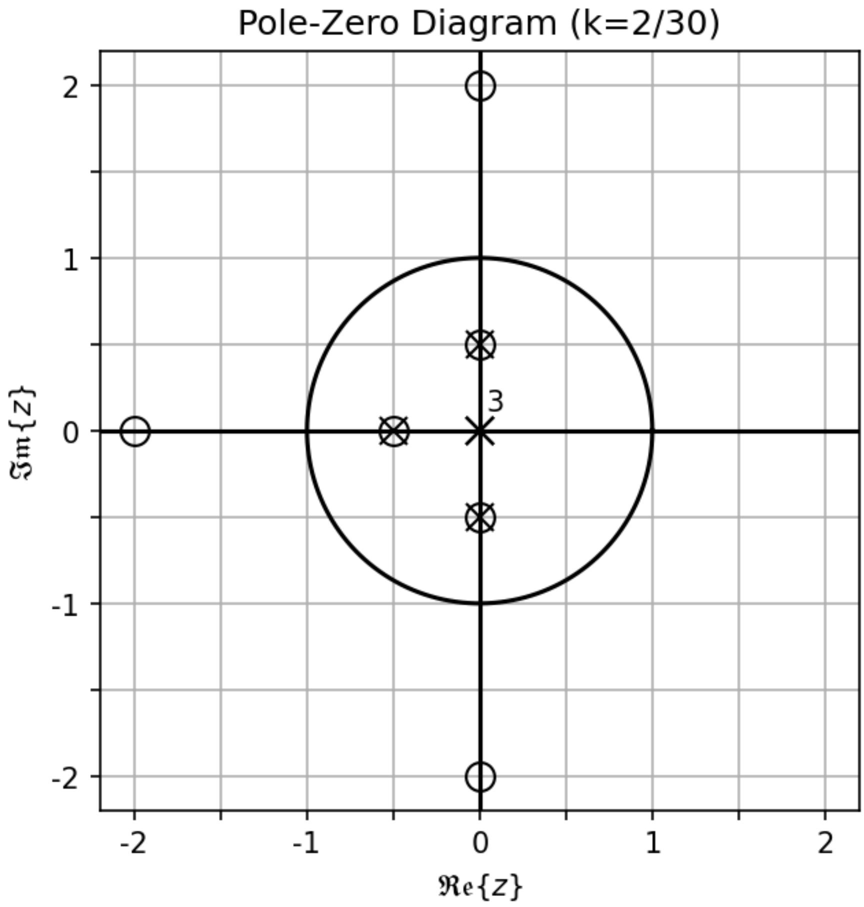

Figure 2: Pole-zero diagram of a system in the z-domain

a) Which poles and zeros from Fig. 2 belong to the minimum phase system $H_{\min}(z)$ and which the all-pass system $H_{\text{ap}}(z)$ that satisfy $H(z) = H_{\min}(z)\, H_{\text{ap}}(z)$? Distribute the poles and zeros to $H_{\min}(z)$ and $H_{\text{ap}}(z)$ and plot the resulting pole-zero diagrams.

b) Plot $|H(z)|$, $|H_{\min}(z)|$, and $|H_{\text{ap}}(z)|$ in dB in the range $-2.5 \leq \mathfrak{Re}\{z\}, \mathfrak{Im}\{z\} \leq 2.5$ (the magnitude must be color-coded to achieve this). Use

$$H(z) = k \frac{\prod_{k=1}^{M} (1 - {c_k}/{z})}{\prod_{k=1}^{N} (1 - {d_k}/{z})} .$$

with the zeros c_k and the poles d_k. Determine k_{min} and k_{ap} to assure $|H(z = \mathrm{e}^{\mathrm{j}\omega})| = |H_{\text{min}}(z = \mathrm{e}^{\mathrm{j}\omega})|$ and $|H_{\text{ap}}(z = \mathrm{e}^{\mathrm{j}\omega})| = 1$ (0 dB). You can do this analytically or numerically by evaluating the computed transfer functions at $z = \mathrm{e}^{\mathrm{j}\omega}$.

c) Plot $|H_{\text{min}}(z)|/|H(z)|$ in dB to check $|H(z = \mathrm{e}^{\mathrm{j}\omega})| = |H_{\text{min}}(z = \mathrm{e}^{\mathrm{j}\omega})|$.

10 Points: 3 points for (a), 5 points for (b), 2 points for (c).