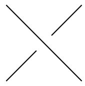
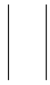
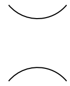
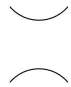
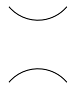
Something goes wrong with these braid matrices. Let me illustrate. If

$R \equiv$  $\equiv \pm i \Big(q^{\pm\frac{1}{2}}$  $- q^{\mp\frac{1}{2}}$  $\Big)$,

then we can clearly drop the $\pm i$ when calculating $RR^\dagger$, since they will eliminate to 1 in that product. Let $\widetilde{R} \equiv \mathfrak{Im}(R)$. The coefficients in $\widetilde{R}$ are real, so it is self adjoint. This means we can invoke the Binomial theorem to calculate $RR^\dagger$, and we obtain

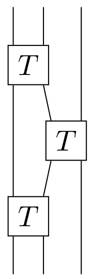
$RR^\dagger = \widetilde{R}^2 = q^{\pm 1}$  $- 2$  $+ q^{\mp 1+1}$ 

The $+1$ in the last term (in the exponent) comes from eliminating a loop.

For the two possible choices of signs, the coefficients of  are given by -1 and $-(2-q^2)$, the latter being equal to 0 iff $q = 2\cos\frac{\pi}{4}$.

Therefore, R is, in general, not perfect. So either there is a major misunderstanding here, or (this is what I believe) R is somehow defined incorrectly.

I believe that because I too have verified that a tangle of the form

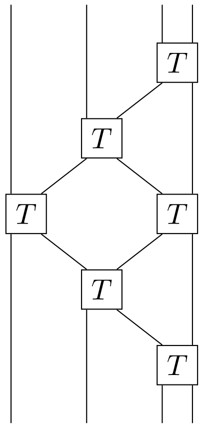


is perfect if T is perfect.

> By the way: Last time I have said that there's no harm in defining *perfectness* by saying: all the 'rotated multiplications' are equal to id_k, instead of proportional. That is not quite true, because: if we partitioned the legs of e.g. a 3-tangle into sets of size 2 and 4, then for these bipartitions the product is $q \cdot \mathrm{id}_2$, i.e. this notion of *perfectness* would not be downward compatible, in a sense. So we should probably stick with proportionality.
>
> But I guess we could get sort of a "generating set" S for perfect tangles (the generating operation being rotation, if we require that the coefficient of id_k of $T \in S$ be of modulus 1. Does that make sense to you? After a bit of thinking I suppose that this is not in fact generating all perfect tangles.

I do not see how the Reidemeister moves force this tangle to be perfect, though – I guess I might need to carefully work through section **2.11** in *Planar Algebras, I.*

While it is not entirely clear to me how the generalization at the end of your notes works, I interpreted it so that I get, for example, the following 4-tangle:



And now buckle up: This is perfect (given that both T is perfect and I did not make a mistake). This would be the first example of a perfect 4 tangle! Quite exciting. But I don't understand why this construction works. Did Vaughan make that up on the spot, or has he thought about it before?

I have to prove that this construction yields perfect k-tangles for all k, but I can check whether this is true for $k = 6$, which is apparently a case Vaughan is interested in. In general such a k-tangle would consist of $\frac{k(k-1)}{2}$ internal boxes. So on 6 strands there would be 15 boxes, which is tedious but doable.

The nice thing about tangles M of this form, though, is that one only has to check the cases $\mathrm{rot}^l M \cdot \mathrm{rot}^{-l} M^\dagger$ for $0 \leq l < k$, since they are highly symmetric.

Addendum: I have tried generalizing this to T being a 3-box, but it was not successful, I tried about 5 different ways, didn't work.