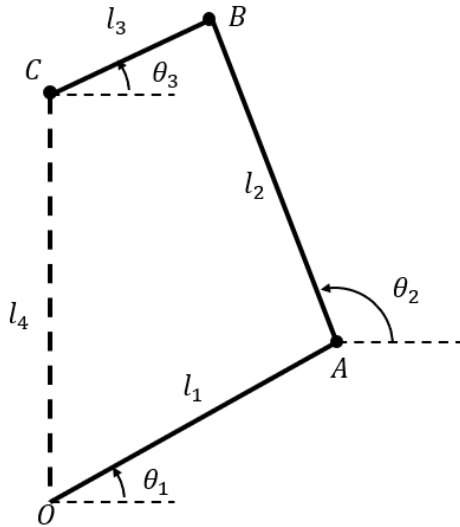


Our goal is to find the angles  $\theta_2$ ,  $\theta_3$ ,  $\theta_4$ , and  $\theta_5$  given the input angle  $\theta_1$  and the lengths  $l_1$ ,  $l_2$ ,  $l_3$ ,  $l_4$ , and  $l_5$ . We can use two methods to solve for these angles: the vector loop method and the circle intersection method.

**Using the vector loop method:**



$$\vec{r}_1 = l_1 \cos \theta_1 \hat{i} + l_1 \sin \theta_1 \hat{j}$$

$$\vec{r}_2 = l_2 \cos \theta_2 \hat{i} + l_2 \sin \theta_2 \hat{j}$$

$$\vec{r}_3 = -l_3 \cos \theta_3 \hat{i} - l_3 \sin \theta_3 \hat{j}$$

$$\vec{r}_4 = 0 \hat{i} + l_4 \hat{j}$$

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = \vec{r}_4$$

$$\hat{i} = l_1 \cos \theta_1 + l_2 \cos \theta_2 - l_3 \cos \theta_3 = 0$$

$$\hat{j} = l_1 \sin \theta_1 + l_2 \sin \theta_2 - l_3 \sin \theta_3 = l_4$$

Given:

- $l_1, l_2, l_3, l_4$
- $\theta_1$

By numerically solving the equations, we can find angles  $\theta_2$  and  $\theta_3$ . Following that, we know that  $l_5$  is always perpendicular to  $l_3$ , and thus we can find  $\theta_4$  by trigonometry. By using the vector from C to D  $\vec{l}_5$  we can find the point D. Knowing both D and E, we can find angle

$\theta_5$  using trigonometry as well.

**Using the circle intersection method:**

We found that it is faster and cleaner to use the circle intersection method to define the angles and points instead.

We know the locations of the fixed points  $O$ ,  $C$ , and  $E$ :

$$O = (0, 0) \quad C = (0, l_4) \quad E = (-6, l_4)$$

We know the Cartesian coordinates for points  $A$  and  $B$  by defining it in terms of a vector relative their respective fixed points:

$$\vec{l}_1 = \langle l_2 \cos(\theta_1), l_2 \sin(\theta_1) \rangle \implies A = \langle 0, 0 \rangle + \langle l_2 \cos(\theta_1), l_2 \sin(\theta_1) \rangle$$

$$\vec{l}_3 = \langle l_3 \cos(\theta_3), l_3 \sin(\theta_3) \rangle \implies B = \langle 0, l_4 \rangle + \langle l_3 \cos(\theta_3), l_3 \sin(\theta_3) \rangle$$