

Dynamics Formula Sheet

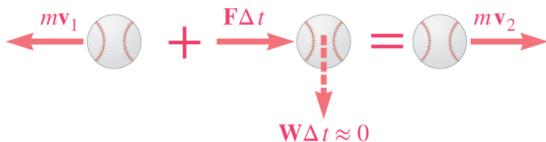
Quiz II

Impulse and Momentum

$$\vec{p} = m\vec{v} \quad \Delta\vec{p} = \mathbf{Imp}_{1 \rightarrow 2} = \int_{t_1}^{t_2} \mathbf{F} dt$$

Momentum changes by:

$$m\vec{v}_1 + \mathbf{Imp}_{1 \rightarrow 2} = m\vec{v}_2$$

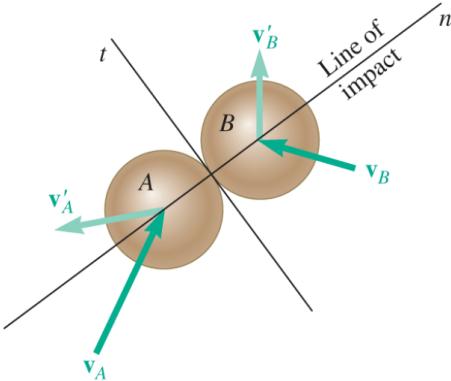


Collisions

$$e = \frac{v'_B - v'_A}{v_A - v_B}$$

- Perfectly elastic impact:** $e = 1$
Momentum and energy is conserved.
- Perfectly plastic impact:** $e = 0$
Particles stick together after impact. Only momentum is conserved.

Oblique Central Impact



No impulsive force in t :

$$v'_{A,t} = v_{A,t}, \quad v'_{B,t} = v_{B,t}$$

Conservation of momentum in n :

$$m_A v_{A,n} + m_B v_{B,n} = m_A v'_{A,n} + m_B v'_{B,n}$$

Relate velocities using e :

$$v'_{B,n} - v'_{A,n} = e(v_{A,n} - v_{B,n})$$

Rigid Body Rotation

Kinematics

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}, \quad \mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} - \boldsymbol{\omega}^2 \mathbf{r}$$

$$\boldsymbol{\omega} = \frac{d\theta}{dt}, \quad \boldsymbol{\alpha} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \quad \boldsymbol{\alpha} = \frac{d\theta}{dt} \frac{d\omega}{d\theta} = \boldsymbol{\omega} \frac{d\omega}{d\theta}$$

Constant Angular Acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

General Planar Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

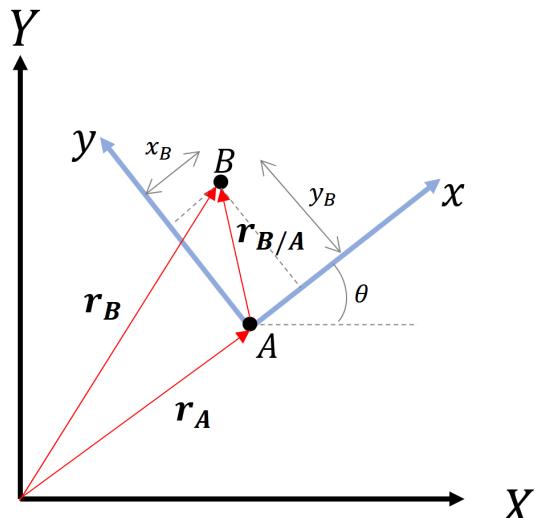
$$\boxed{\mathbf{v}_B = \mathbf{v}_A + \hat{\omega} \times \mathbf{r}_{B/A}}$$

$$\boxed{\mathbf{a}_B = \mathbf{a}_A + \hat{\omega} \times \mathbf{v}_{B/A} + \hat{\omega} \times (\hat{\omega} \times \mathbf{r}_{B/A}) - \omega^2 \mathbf{r}_{B/A}}$$

Note: $\mathbf{r}_{B/A}$ is the position vector pointing from A to B

Rotating Reference Frames

Frame xyz is rotating relative to frame XYZ (the absolute frame) with angular velocity $\boldsymbol{\omega}$.



$$\boxed{\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}}$$

Absolute velocity of origin A in XYZ
Velocity of origin A in XYZ
Rotation of xyz as seen in XYZ
Velocity of B relative to A as seen in xyz

Motion of frame xyz as seen in XYZ

$$\boxed{\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}}$$

Absolute accel of B in XYZ
Accel of origin A in XYZ
Angular accel caused by rotation of xyz
Angular velocity effect caused by rotation of xyz
Coriolis acceleration
Combined effect of B moving relative to xyz coord and the rotation of xyz frame
Motion of B in XYZ
Motion of xyz as observed from XYZ
Interacting motion
Motion of B as observed in xyz

Note: We have to use rotating frames if we are given quantities relative to a rotating body.

Center of Mass

Continuous Body:

$$\bar{x} = \frac{\int x dW}{W}, \quad \bar{y} = \frac{\int y dW}{W}$$

Discrete Masses:

$$x_c = \frac{\sum x_{ci} A_i}{\sum A_i}, \quad \bar{x} = \frac{\sum x_{ci} W_i}{\sum W_i}$$

$$y_c = \frac{\sum y_{ci} A_i}{\sum A_i}, \quad \bar{y} = \frac{\sum y_{ci} W_i}{\sum W_i}$$

Moment of Inertia

Area Moments:

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA$$

Polar Moment:

$$J_O = \int r^2 dA = I_x + I_y$$

Mass Moments:

$$I_x = \int (y^2 + z^2) dm, \quad I_y = \int (x^2 + z^2) dm, \quad I_z = \int (x^2 + y^2) dm$$

Radius of Gyration:

$$I_x = k_x^2 A, \quad I_y = k_y^2 A, \quad J_O = k_o^2 A$$

Rigid Body Kinetics

Sum of forces and moments about the center of mass:

$$\sum \mathbf{F} = m\bar{\mathbf{a}}, \quad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G = \bar{I}\alpha$$

Summing moments about a point other than \mathbf{G} :

$$\sum M_P = \bar{I}\alpha + m\bar{a}d$$

Note: Parallel axis theorem only applies for fixed rotation.
($I = \bar{I} + md^2$)

Rolling Motion

There are 3 cases of rolling motion:

1. Rolling without slipping:

$$F_f \leq \mu_s N \quad \bar{a} = r\alpha$$

2. Rolling, slipping impeding

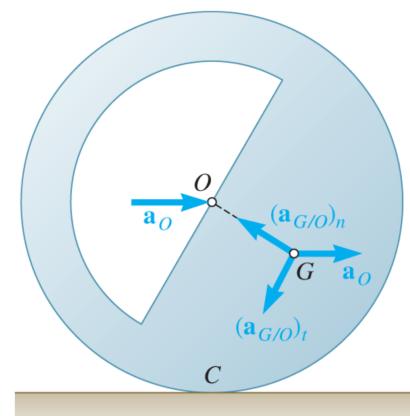
$$F_f = \mu_s N \quad \bar{a} = r\alpha$$

3. Rolling and slipping:

$$F_f = \mu_k N \quad \bar{a} \text{ and } \alpha \text{ becomes decoupled and independent}$$

Note: If unsure about which case, assume rolling without slipping first and calculate F_f . If $F_f > \mu_s N$, then it is slipping, and we need to adjust our equation for F_f .

Asymmetric Rolling

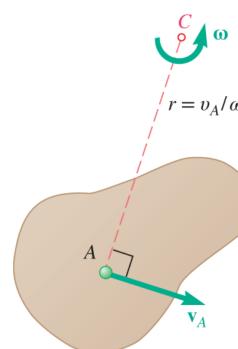


$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$\mathbf{a}_G = \mathbf{a}_O + (\mathbf{a}_{G/O})_t + (\mathbf{a}_{G/O})_n$$

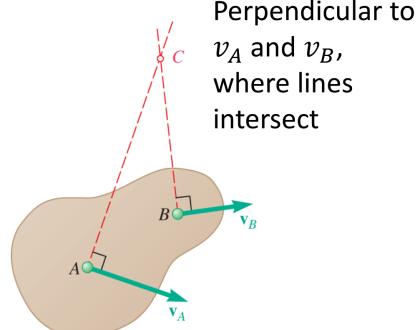
$$\mathbf{a}_G = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{G/O} - \omega^2 \mathbf{r}_{G/O}$$

Instantaneous Center

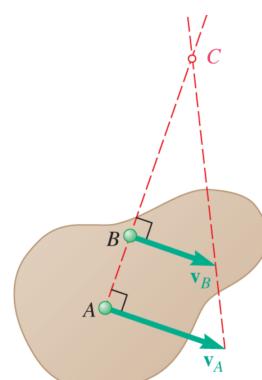


Perpendicular to v_A , at distance r from A

$$r = \frac{v_A}{\omega}$$



Perpendicular to v_A and v_B , where lines intersect



Perpendicular to v_A and v_B and line that joins ends of vectors, where lines intersect