Ph291E Lab 5 – Diffraction & Interference

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1 Purpose

Using a diffraction grating, we will measure the wavelength of light from a laser diode. We will then confirm this wavelength by measuring the single slit diffraction pattern produced by the same laser diode through a known aperture width. Additionally, we will measure the width of a human hair using Babinet's principle.

2 Data

Part A: Determination of laser diode emission wavelength using a diffraction grating

Set up:

- Slit separation: $d = 0.100$ mm
- Distance from grating to screen: $D = 500$ mm

x_- **(mm)**	x_+ **(mm)**	**Random Error** x_- **(mm)**	**Random Error** x_+ **(mm)**	**Vernier Caliper Inst. Error (mm)**
43.00	9.85			
43.00	9.85			
43.00	9.25			
43.00	9.20			
43.00	9.60			
43.00	9.75	0	0.12	0.02
Mean x_-				**43.00 mm**
Mean x_+				**9.58 mm**

Table 1: Diffraction Grating Maxima (First Order)

Part B: Determination of laser diode emission wavelength using single slit diffraction

Set up:

- Aperture width: a
- Distance from aperture to screen: $D = 500$ mm

a **(mm)**	x_- **(mm)**	x_+ **(mm)**	x_{-2} **(mm)**	x_{+2} **(mm)**
0.2	43.00	9.75	0	0.12
0.3	43.00	9.75	0	0.12
0.4	43.00	9.75	0	0.12

Table 2: Single Slit Diffraction Minima (First & Second Order)

Part C: Measuring the width of a human hair

Set up:

- Laser wavelength: $\lambda = 650$ nm
- Distance from hair to screen: $D = 500$ mm

x_- **(mm)**	x_+ **(mm)**	x_{-2} **(mm)**	x_{+2} **(mm)**	x_{-3} **(mm)**	x_{+3} **(mm)**
0.2	43.00	9.75	0	0.12	1

Table 3: Hair Diffraction Minima (First, Second & Third Order)

3 Calculations

Sample Calculations for maximum x_+
Mean Calculation:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{1}{6}(9.85 + 9.85 + 9.25 + 9.20 + 9.60 + 9.75)$$

$$\bar{x} = 9.5833 \text{ cm}$$

Standard Deviation:

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$S_x = \sqrt{\frac{1}{6-1}\left[(9.85 - 9.5833)^2 + (9.85 - 9.5833)^2 + \cdots + (9.75 - 9.5833)^2\right]}$$

$$S_x = 0.29268 \text{ cm}$$

Standard Deviation of the Mean (SDOM):

$$\sigma_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.29268}{\sqrt{6}}$$

$$\sigma_{\bar{x}} = 0.11948 \text{ cm}$$

The same procedure was applied to calculate the mean, standard deviation and standard deviation of the mean for the remaining measurements.

Part A Calculations for x_+
Wavelength:

$$sin\theta = \frac{x}{\sqrt{x^2 + D^2}}$$

$$\lambda = \frac{d}{m} sin\theta, \quad m = 1$$

$$\lambda = \frac{d}{m} \frac{x}{\sqrt{x^2 + D^2}}$$

$$\lambda = \frac{(0.11)(0.1)}{(1)\sqrt{(0.0958)^2 + (0.5)^2}} = 0.001916$$

The same procedure was applied to calculate the wavelength using x_-. The average of the two wavelengths is the best result, and the uncertainty is half the difference between the two results.

$$\delta\lambda = \frac{|0.001916 - 0.001916|}{2} = 0$$

Error propagation:
Since the measurements of x and D are independent, we can use the independent error propagation formula:

$$\delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial x}\delta x\right)^2 + \left(\frac{\partial\lambda}{\partial D}\delta D\right)^2}$$

Calculating the partial derivatives:

$$\frac{\partial\lambda}{\partial x} = \frac{D^2\, d}{m\,(D^2 + x^2)^{3/2}}$$

$$\frac{\partial\lambda}{\partial D} = -\frac{D\, d\, x}{m\,(D^2 + x^2)^{3/2}}$$

Thus,

$$\delta\lambda = \sqrt{\left(\frac{D^2\, d}{m\,(D^2 + x^2)^{3/2}}\delta x\right)^2 + \left(-\frac{D\, d\, x}{m\,(D^2 + x^2)^{3/2}}\delta D\right)^2}$$

Chosen uncertainties: $\delta x = 0.036$ cm (Instrumental uncertainty, larger than random) and $\delta D = 0.095$ cm (Instrumental uncertainty, larger than random).

$$\delta\lambda = \sqrt{\left(\frac{(0.11)^2\,(0.22)}{m\,\left((0.11)^2 + (0.01)^2\right)^{3/2}}(0.05)\right)^2 + \left(-\frac{(0.11)\,(0.22)\,(0.01)}{m\,\left((0.11)^2 + (0.01)^2\right)^{3/2}}(0.55)\right)^2} = 0.111$$

The same procedure was applied to calculate the error propagation using x_-.

Part B Calculations for x_+
Wavelength:

$$sin\theta = \frac{x}{\sqrt{x^2 + D^2}}$$

$$\lambda = \frac{a}{p} sin\theta, \quad p = 1,$$

$$\lambda = \frac{a}{p}\frac{x}{\sqrt{x^2 + D^2}}$$

$$\lambda = \frac{(0.11)(0.21)}{(0.11)\sqrt{(0.21)^2 + (0.02)^2}} = 0.12$$

The same procedure was applied to calculate the wavelength using first and second order minima on both sides, for varying aperture widths. The SDOM of all calculated wavelengths is taken as the uncertainty.

Part C Calculations for x_+
Hair Width:

$$sin\theta = \frac{x}{\sqrt{x^2 + D^2}}$$

$$a = \frac{p\lambda}{sin\theta}, \quad p = 1$$

$$a = \frac{p\lambda\sqrt{x^2 + D^2}}{x}$$

$$a = \frac{(1)(0.001)\sqrt{(0.21)^2 + (0.5)^2}}{0.21} = 0.0017$$

The same procedure was applied to calculate the wavelength using first, second, and third order minima on both sides. The SDOM of all calculated hair widths is taken as the uncertainty.

4 Results

5 Conclusion

6 Answers to questions

Question 1
N-Slit intensity:

$$I(\delta) = I_0 \left[\frac{sin\left(\frac{N\delta}{2}\right)}{sin\left(\frac{\delta}{2}\right)} \right]^2, \quad \delta = \frac{2\pi}{\lambda} dsin\theta$$

The principal maxima occur when both the numerator and denominator are zero. Thus,

$$\frac{\delta}{2} = m\pi, \quad m = 0, \pm 1, \pm 2, \ldots$$

Since N is an integer, the numerator will also be zero at these points. Therefore, the principal maxima occur at:

$$\frac{1}{2}\left(\frac{2\pi}{\lambda} dsin\theta\right) = m\pi$$

$$\frac{d}{\lambda} sin\theta = m$$

$$\implies sin\theta = \frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2, \ldots$$

Question 2
Single slit diffraction intensity:

$$I(\beta) = I_0 \left(\frac{sin\beta}{\beta}\right)^2, \quad \beta = \frac{\pi}{\lambda} asin\theta$$

Minima occur when the numerator is zero (excluding the central maximum at $\beta = 0$):

$$sin\beta = 0 \implies \beta = p\pi, \quad p = \pm 1, \pm 2, \ldots$$

Thus,

$$\frac{\pi}{\lambda} asin\theta = p\pi$$

$$\implies sin\theta = \frac{p\lambda}{a}, \quad p = \pm 1, \pm 2, \ldots$$