Dynamics Final Project Report

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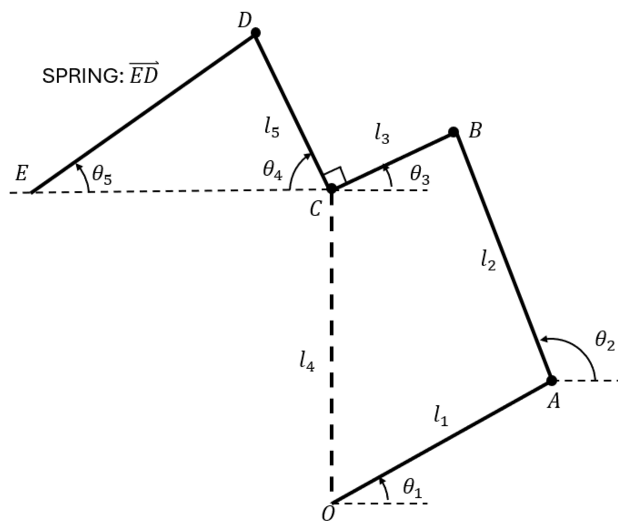
Introduction to project

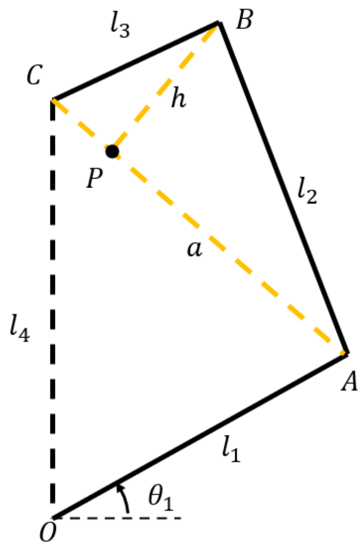
Assumptions

Constraints: Kinematics

Our goal is to find the angles θ_2, θ_3, θ_4, and θ_5 given the input angle θ_1 and the lengths l_1, l_2, l_3, l_4, and l_5. We can use two methods to solve for these angles: the vector loop method and the circle intersection method.



We found that it is faster and cleaner to use geometry to find the angles. To do that, we just need to find the coordinate of point B.



We know the coordinates of point A and point C:

$$\vec{l_1} = \langle l_1 \cos\theta_1, l_1 \sin\theta_1 \rangle, \quad \vec{l_4} = \langle 0, l_4 \rangle$$

$$A = \vec{O} + \vec{l_1}, \quad C = \vec{O} + \vec{l_4}$$

We can define the vector from point A to point C and its magnitude d:

$$\vec{d} = \vec{l_4} - \vec{l_1}, \quad d = \|\vec{d}\|$$

By the pythagorean theorem, we can find the height h from point P to point B:

$$h^2 = l_2^2 - a^2$$

By law of cosines for triangle ABP

$$l_2^2 = a^2 + h^2 - ah\cos\theta_p, \quad \theta_p = 90^\circ$$

$$l_2^2 = a^2 + h^2 \quad (1)$$

By law of cosines for triangle CPB

$$l_3^2 = (d-a)^2 + h^2 - (d-a)h\cos\theta_p, \quad \theta_p = 90^\circ$$

$$l_3^2 = (d-a)^2 + h^2 \quad (2)$$

Solving for a using (1) and (2)

$$l_3^2 - l_2^2 + a^2 = d^2 - 2ad + a^2$$

$$a = \frac{l_3^2 - l_2^2 - d^2}{-2d}$$

Knowing a and h, we can find the coordinates of point B:

$$\vec{B} = \vec{A} + a\hat{d} + h\hat{d}_\perp$$

$$\hat{d} = \frac{\vec{d}}{d}$$

$$\hat{d}_{\perp} = \left\langle -\frac{d_y}{d}, \frac{d_x}{d} \right\rangle$$

Finally, we can define vectors $\vec{l_2}$ and $\vec{l_3}$ using points A, B, and C. From there, we can find point D by numerically finding the normal vector of $\vec{l_3}$ and using the length l_5. With points D and E, we have fully defined the system and can find all the angles using trigonometry.