



Our goal is to find the angles θ_2 , θ_3 , θ_4 , and θ_5 given the input angle θ_1 and the lengths l_1 , l_2 , l_3 , l_4 , and l_5 . We can use two methods to solve for these angles: the vector loop method and the circle intersection method.

Using the vector loop method:

$$\begin{aligned}
 \vec{r}_1 &= l_1 \cos \theta_1 \hat{i} + l_1 \sin \theta_1 \hat{j} \\
 \vec{r}_2 &= l_2 \cos \theta_2 \hat{i} + l_2 \sin \theta_2 \hat{j} \\
 \vec{r}_3 &= -l_3 \cos \theta_3 \hat{i} - l_3 \sin \theta_3 \hat{j} \\
 \vec{r}_4 &= 0\hat{i} + l_4 \hat{j} \\
 \vec{r}_1 + \vec{r}_2 + \vec{r}_3 &= \vec{r}_4
 \end{aligned}$$

Given:
 • l_1, l_2, l_3, l_4
 • θ_1

By numerically solving the equations, we can find angles θ_2 and θ_3 . Following that, we know that l_5 is always perpendicular to l_3 , and thus we can find θ_4 by trigonometry. By using the vector from C to D \vec{l}_5 we can find the point D. Knowing both D and E, we can find angle

θ_5 using trigonometry as well.

Using the circle intersection method:

We found that it is faster and cleaner to use the circle intersection method to define the angles and points instead.

We know the locations of the fixed points O , C , and E :

$$O = (0, 0) \quad C = (0, l_4) \quad E = (-6, l_4)$$

We know the Cartesian coordinates for points A and B by defining it in terms of a vector relative their respective fixed points:

$$\vec{l}_1 = \langle l_2 \cos(\theta_1), l_2 \sin(\theta_1) \rangle \implies A = \langle 0, 0 \rangle + \langle l_2 \cos(\theta_1), l_2 \sin(\theta_1) \rangle$$

$$\vec{l}_3 = \langle l_3 \cos(\theta_3), l_3 \sin(\theta_3) \rangle \implies B = \langle 0, l_4 \rangle + \langle l_3 \cos(\theta_3), l_3 \sin(\theta_3) \rangle$$