

# Dynamics Formula Sheet

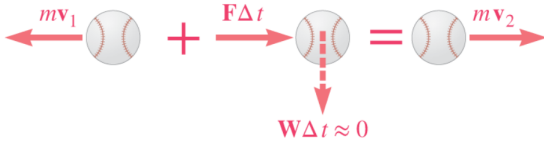
## Quiz II

### Impulse and Momentum

$$\vec{p} = m\vec{v} \quad \Delta\vec{p} = \text{Imp}_{1 \rightarrow 2} = \int_{t_1}^{t_2} \mathbf{F} dt$$

Momentum changes by:

$$m\mathbf{v}_1 + \text{Imp}_{1 \rightarrow 2} = m\mathbf{v}_2$$

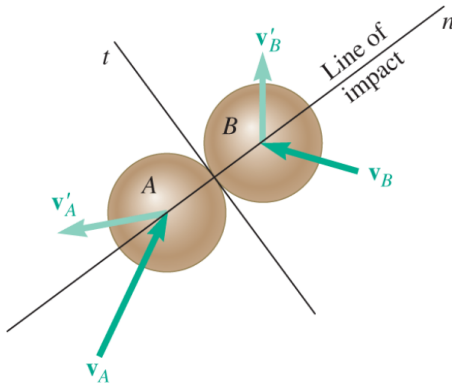


### Collisions

$$e = \frac{v'_B - v'_A}{v_A - v_B}$$

- **Perfectly elastic impact:**  $e = 1$   
Momentum *and* energy is conserved.
- **Perfectly plastic impact:**  $e = 0$   
Particles stick together after impact. Only momentum is conserved.

### Oblique Central Impact



No impulsive force in  $t$ :

$$v'_{A,t} = v_{A,t}, \quad v'_{B,t} = v_{B,t}$$

Conservation of momentum in  $n$ :

$$m_A v_{A,n} + m_B v_{B,n} = m_A v'_{A,n} + m_B v'_{B,n}$$

Relate velocities using  $e$ :

$$v'_{B,n} - v'_{A,n} = e(v_{A,n} - v_{B,n})$$

### Rigid Body Rotation

#### Kinematics

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}, \quad \mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}$$

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \quad \alpha = \frac{d\theta}{dt} \frac{d\omega}{d\theta} = \omega \frac{d\omega}{d\theta}$$

### Constant Angular Acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

### General Planar Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

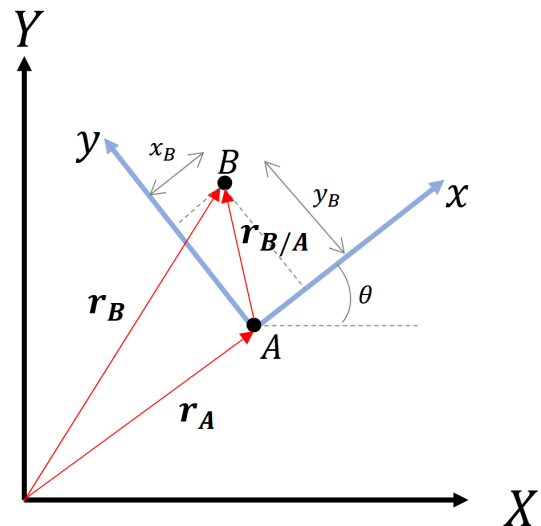
$$\mathbf{v}_B = \mathbf{v}_A + \omega \hat{k} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \hat{k} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

Note:  $\mathbf{r}_{B/A}$  is the position vector pointing from A to B

### Rotating Reference Frames

Frame  $xyz$  is rotating relative to frame  $XYZ$  (the absolute frame) with angular velocity  $\omega$ .



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

Absolute velocity of B in XYZ      Velocity of origin A in XYZ      Rotation of xyz as seen in XYZ      Velocity of B relative to A as seen in xyz

Motion of frame  $xyz$  as seen in XYZ

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

Absolute accel of B in XYZ      Accel of origin A in XYZ      Angular accel caused by rotation of xyz      Angular velocity effect caused by rotation of xyz      Coriolis acceleration Combined effect of B moving relative to xyz coord and the rotation of xyz frame      Accel of B relative to A as seen in xyz

Motion of B in XYZ      Motion of xyz as observed from XYZ      Interacting motion      Motion of B as observed in xyz

Note: We have to use rotating frames if we are given quantities relative to a rotating body.

## Center of Mass

Continuous Body:

$$\bar{x} = \frac{\int x dW}{W}, \quad \bar{y} = \frac{\int y dW}{W}$$

Discrete Masses:

$$x_c = \frac{\sum x_{ci} A_i}{\sum A_i}, \quad \frac{\sum x_{ci} W_i}{\sum W_i}$$

$$y_c = \frac{\sum y_{ci} A_i}{\sum A_i}, \quad \frac{\sum y_{ci} W_i}{\sum W_i}$$

## Moment of Inertia

Area Moments:

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA$$

Polar Moment:

$$J_O = \int r^2 dA = I_x + I_y$$

Mass Moments:

$$I_x = \int (y^2 + z^2) dm, \quad I_y = \int (x^2 + z^2) dm, \quad I_z = \int (x^2 + y^2) dm$$

Radius of Gyration:

$$I_x = k_x^2 A, \quad I_y = k_y^2 A, \quad J_O = k_o^2 A$$

## Rigid Body Kinetics

Sum of forces and moments about the center of mass:

$$\sum \mathbf{F} = m\bar{\mathbf{a}}, \quad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G = \bar{I}\alpha$$

Summing moments about a point other than  $\mathbf{G}$ :

$$\sum M_P = \bar{I}\alpha + m\bar{a}d$$

Note: Parallel axis theorem only applies for fixed rotation.  
( $I = \bar{I} + md^2$ )

## Rolling Motion

There are 3 cases of rolling motion:

1. Rolling without slipping:

$$F_f \leq \mu_s N \quad \bar{a} = r\alpha$$

2. Rolling, slipping impending

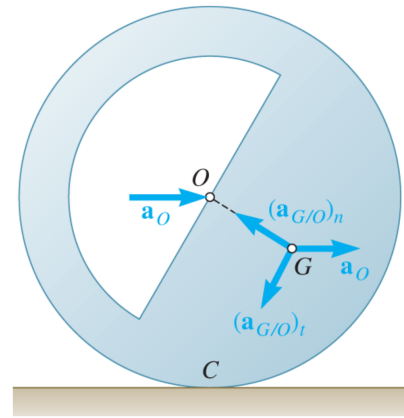
$$F_f = \mu_s N \quad \bar{a} = r\alpha$$

3. Rolling and slipping:

$$F_f = \mu_k N \quad \bar{a} \text{ and } \alpha \text{ becomes decoupled and independent}$$

Note: If unsure about which case, assume rolling without slipping first and calculate  $F_f$ . If  $F_f > \mu_s N$ , then it is slipping, and we need to adjust our equation for  $F_f$ .

## Asymmetric Rolling



$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$\mathbf{a}_G = \mathbf{a}_O + (\mathbf{a}_{G/O})_t + (\mathbf{a}_{G/O})_n$$

$$\mathbf{a}_G = \mathbf{a}_O + \alpha \times \mathbf{r}_{G/O} - \omega^2 \mathbf{r}_{G/O}$$

## Instantaneous Center

