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TAKE-HOME FINAL EXAM

ME200, DYNAMICS, COOPER UNION
FALL 2025

Ground Rules READ ME *FIRST*

By submitting this quiz solution with your name, you affirm that you have abided by the code of conduct of Cooper Union. You affirm that this exam solution is your own work, and that you did not receive unauthorized assistance from any person. Collaboration with other students CURRENTLY ENROLLED IN ME200 is encouraged but must be well documented (see below).

- Direct clarifying questions about the exam to the instructor. **Check Teams** for any clarifications!
- You will submit your solutions on Teams, so give yourself some time to scan your pages in. **Please include your name in your filename when you submit.**
- Budget your time to answer all questions. Look over them all first (each has a point value) and plan your time. In most problems, a good description of how you will solve the problem is worth a significant amount of credit. Leaving a problem blank is worse than spending a little bit of time to outline your approach (even if you don't fully solve it).
- **Show all your work! Write out your equations! Be organized!** Make it easy for your instructor to follow your logic – we want to give you points!
- Use variables for most of the calculations and make it clear what value is being substituted for each variable if/when you do so.
- For numerical answers, include the units. No units will mean no credit for numerical answer. Carry units in your calculations and check units at the end.
- If you need more space, work on a spare sheet of paper and make sure to scan those pages too! Please label any extra sheets with your name and the problem you are working on.
- Please box or otherwise highlight your answers and make it clear which problem you are answering.

There are 200 total points available spread over four multi-part problems. Point values are listed for each problem.

Problem		Points Available	Total
Problem 1: Way Down Hadestown	a	15	30
	b	15	
Problem 2: Nervous Fidgeting	a	15	65
	b	15	
	c	10	
	d	25	
Problem 3: 41CS Elevator	a	5	35
	b	30	
Problem 5: The Swinging Sticks	a	15	70
	b	30	
	c	10	
	d	10	
	e	5	

Please indicate the nature and extent of all collaboration on each problem on this page. A few examples:

1. *I worked entirely alone*
2. *Checked my solution to Part 2 with ***** and revised my solution afterward (found a sign error)*
3. *Worked together extensively on the whole problem with *****
4. *Worked extensively on part 1 with ***** and checked my solution to part 2 with *****.*
5. *Developed a Python script to solve Part 4 with *****

Please also list the approximate time you spent on each problem – it will be helpful for the future to plan for timing!

Collaborations and Timing

PROBLEM 1. WAY DOWN HADESTOWN

TIME SPENT: 3 hrs

I worked extensively on part a of the problem with Bertrand and Roy. I checked my solution for part b with Bertrand and revised my solution afterward (forgot to account for normal/radial accelerations).

PROBLEM 2. NERVOUS FIDGETING

TIME SPENT: 1 hr

I worked entirely alone.

PROBLEM 3. 41CS ELEVATOR

TIME SPENT: 1 hr

I worked entirely alone.

PROBLEM 4. THE SWINGING STICKS

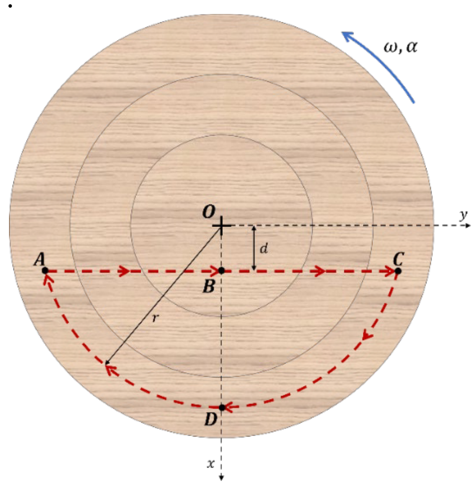
TIME SPENT: 2.5 hrs

I worked entirely alone.

PROBLEM 1. WAY DOWN HADESTOWN (30 POINTS)

The set of the Broadway play *Hadestown* (one of Prof. Rosen's favorites) uses a rotating turntable to transition the characters (and the audience) between the surface and Hades's underground city, Hadestown. You can see the turntables in action in the video here (with arguably the best verse in the show): https://www.youtube.com/watch?v=D8LTZK96LbE

Late in Act 2, the character Orpheus walks from point A to point C and then follows a circular path back to point A through point D, as he leaves Hadestown (and before something tragic happens…look it up if you don't know the story!), as shown in the figure below. His straight-line walk is $\boldsymbol{d = 0.5\ m}$ from the center and his arc has radius of $\boldsymbol{r = 2\ m}$. The turntable starts rotating when he gets to point A with a counterclockwise velocity $\boldsymbol{\omega = 0.5\ rad/s}$ and an acceleration $\boldsymbol{\alpha = 0.1\ rad/s^2}$.

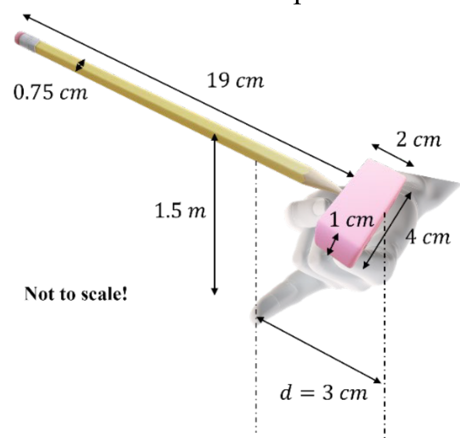


a) (15 points) Find the velocity and acceleration of Orpheus at point B if he walks with a velocity of $\boldsymbol{v = 0.5t\ m/s}$ *relative to the turntable*. He starts from rest at point A. You may assume that every ring of the turntable rotates as one.

b) (15 points) Find the velocity and acceleration of Orpheus at point D. After point B, Orpheus walks with a constant velocity of $\boldsymbol{v} = \mathbf{1.5}\ \boldsymbol{m/s}$ *relative to the turntable*. As he reaches point D, the turntable has reset to the original angular velocity and acceleration given in the problem description ($\boldsymbol{\omega} = \mathbf{0.5}\ \boldsymbol{rad/s}$ and $\boldsymbol{\alpha} = \mathbf{0.1}\ \boldsymbol{rad/s^2}$).

PROBLEM 2. NERVOUS FIDGETING (65 POINTS)

While taking this exam, you're fidgeting with your pencil and stabbing it into your eraser (just like you did in elementary school!). And then you accidentally drop it! The pencil and the eraser, now rigidly stuck together, start falling freely in a horizontal orientation. As you try to catch it, it hits your finger 3 cm from the end of the eraser. The pencil can be modeled as a cylindrical rod of a uniform material with dimensions shown below and a mass density of $\boldsymbol{\rho_p = 0.9\ g/cm^3}$. The eraser can be treated as a simple rectangular prism (not the usual eraser shape) with mass density of $\boldsymbol{\rho_e = 1.4\ g/cm^3}$. The coefficient of restitution between the pencil and your finger is $\boldsymbol{e = 0.1}$. Neglect drag. Assume that the impact is impulsive, only hits tangentially to the finger, and does not move the finger. It can be treated as a 2D problem.



a) (15 points) Find the 3D location of the center of mass of the pencil and eraser with the dimensions shown above. Indicate your axes and origin on the diagram.

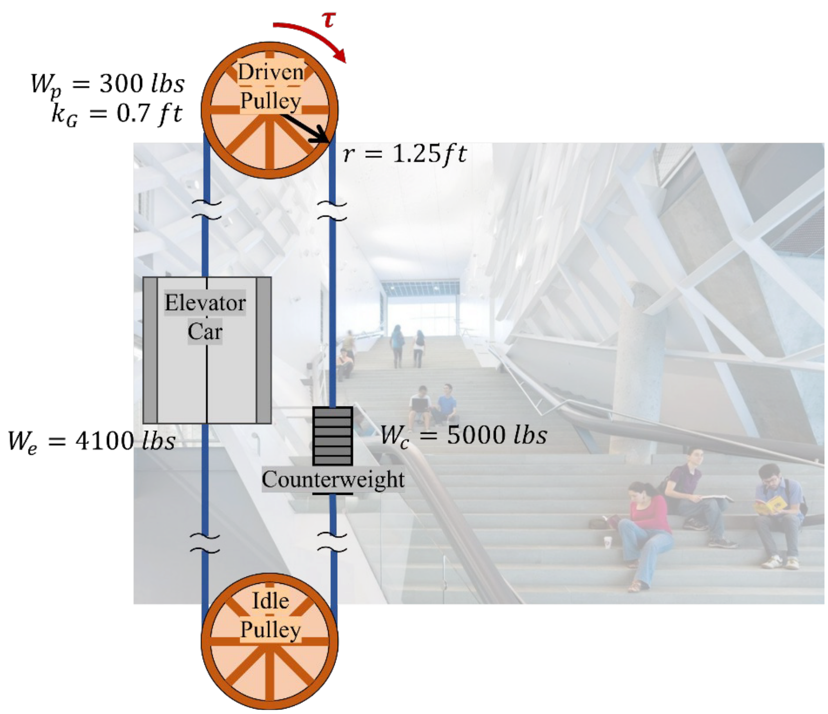
b) (15 points) Calculate the moment of inertia of the pencil and eraser system about the relevant centroidal axis. Clearly indicate which axis this is about.

c) (10 points) Draw the impulse momentum diagram for the impact.

d) (25 points) After the pencil has fallen 1.5 m, it hits your finger. Find the angular velocity and the velocity of the center of mass just before and just after the impact. You may assume that it does not rotate as it initially falls.

PROBLEM 3. 41CS ELEVATOR (35 POINTS)

You've decided to take matters into your own hands regarding the elevators in 41 Cooper Square and replace one elevator yourself. The car and counterweight are driven by a motor attached to a pulley at the top of the elevator shaft. A cable wraps around the driven pulley and an idler (non-driven) pulley at the bottom of the shaft. The cable does not slip on either pulley. The 1.25 ft radius pulleys are identical - each have a weight of 300 lbs and a radius of gyration of 0.7 ft. The elevator car weighs 4100 lbs and the counterweight is 5000 lbs.



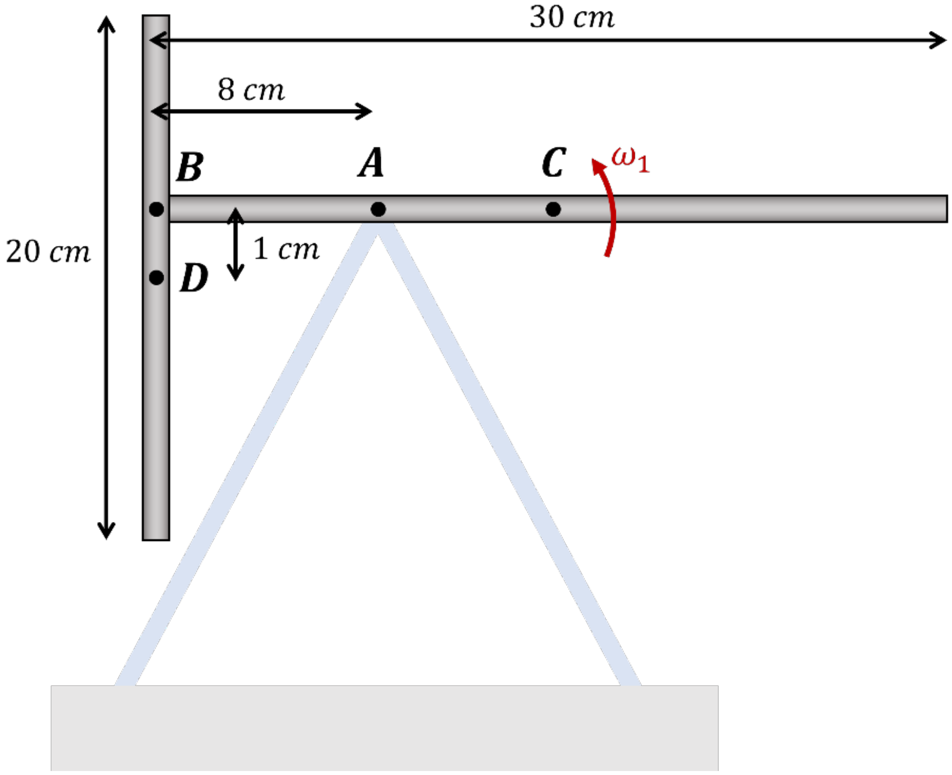
a) (5 points) Calculate the moment of inertia each of the pulleys about its axle.

b) (30 points) To speed things up during busy class times, you want the elevator car to have a velocity of 10 ft/s as it passes the second floor on its way up to the 5^{th} floor, 15 ft from the ground floor. Find the required *constant* torque of the motor to achieve this speed. Ignore the mass of the cable.

PROBLEM 4. THE SWINGING STICKS (70 POINTS)

The Swinging Sticks kinetic sculpture (video and purchase options here: https://www.the-swinging-sticks.com/en/) is a chaotic, pseudo-perpetual motion. Though the real sculptures use electricity and magnetism to continually operate, let's ignore that and analyze the kinematics and kinetics based on gravity alone. The two sticks with dimensions shown below are connected to the stand and together with frictionless pin joints at A and B. Rod BAC has a mass $\boldsymbol{m_1 = 64\ g}$ and rod BD has a mass $\boldsymbol{m_2 = 43\ g}$. The center of mass of rod BAC is located at point C and the center of mass of rod BD is at point D. The sticks can be treated as slender rods.

At the instant shown, the angular velocity of rod BAC is $\boldsymbol{\omega_1 = 2\ rad/s}$ in the counter clockwise direction. Rod BD does not have angular velocity at this point ($\boldsymbol{\omega_2 = 0\ rad/s}$).



a) (15 points) Draw the required free body and kinetic diagrams

b) (30 points) Write the kinetic equations of motion from your diagrams using variables. Please work neatly and number each of your final equations. List your unknowns (there will be a lot!).

List your unknowns here:

c) (10 points) Find the velocity and acceleration of point C at this instant. You may leave it in terms of the angular acceleration(s) $\boldsymbol{\alpha_{BD}}, \boldsymbol{\alpha_{BAC}}$

d) (10 points) Find the velocity and acceleration of point D at this instant. You may leave it in terms of the angular acceleration(s) $\boldsymbol{\alpha_{BD}}, \boldsymbol{\alpha_{BAC}}$

e) (5 points) You should hopefully have enough equations to solve now! Solve for the angular accelerations of stick BAC ($\boldsymbol{\alpha_{BAC}}$) and of stick BD ($\boldsymbol{\alpha_{BD}}$) at this point. Additionally, find the forces in the pins at A and B. You may use Python and a linear algebra approach here, but please show your work and what you plugged in. (Hint: You might want to make a matrix to solve your system of linear equations rather than trying to solve by hand)

Note that this part of the problem is worth <u>*many*</u> *fewer points than the problem set up in parts a-d. If you get stuck here, don't worry too much about it, it's just algebra! You'll get most of the credit for getting most of the way there. The actual numbers are only worth a point or two!*

Some fun pictures to make you happy that you're almost done with the exam (and the semester)!

May your break be as relaxing as Beans in the tent he made for himself, Remi wrapped up in a blanket, or, honestly, Remi anywhere.

And lest you just think Prof. Rosen is a crazy cat lady…here's a fun photo of Buckeye (remember him from the first workshop?). May you have as much fun as he has all the time!

I've enjoyed having all of you in class this semester. I hope you learned something and that it wasn't *too* painful along the way.

Have a very happy holiday! And do some recharging!