PH-214 Modern Physics Notes

Useful Math

- Taylor series expansions:

$$\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots$$

$$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n = 1 + mx + \frac{m(m-1)}{2}x^2 + \frac{m(m-1)(m-2)}{6}x^3 + \cdots$$

  Note: For small x, higher order terms reduce to zero

- Use complex exponentials to manipulate complicated trig functions (Euler's Identity).

$$e^{ix} = \cos x + i \sin x$$

Del Operator:

Unit 1: Electromagnetic Waves

Maxwell's Equations

1. **Gauss's Law for Electricity**
   Relates the electric flux through a closed 3D Gaussian surface to the total charge enclosed within that surface.
   $$\oint_S \vec{E} \cdot d\vec{a} = \frac{q_{\text{enc}}}{\varepsilon_0}$$
   There exists a diverging electric field if there exists a non-zero charge density at that point.
   $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

2. **Gauss's Law for Magnetism**
   Any 3D Gaussian surface will have zero net magnetic flux (no magnetic monopoles).
   $$\oint_S \vec{B} \cdot d\vec{a} = 0$$
   There exists no diverging magnetic field at any point in space because there are no magnetic monopoles.
   $$\vec{\nabla} \cdot \vec{B} = 0$$

3. **Faraday's Law of Induction**
   Relates the electric circulation around a closed Faradian loop to the changing magnetic flux through the surface bounded by the loop.
   $$\oint_P \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \oint_S \vec{B} \cdot d\vec{a}$$
   Note: For coils with N turns, multiply the flux by N.

   A changing magnetic field induces a circulating (curling) electric field.
   $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. **Ampere-Maxwell Law**
   Relates the magnetic circulation around a closed Amperian loop to the enclosed current and changing electric flux through the surface bounded by the loop.
   $$\oint_P \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}, \quad \Phi_E = \oint_S \vec{E} \cdot d\vec{a}$$
   Both a changing electric field and a non-zero current density induce a circulating (curling) magnetic field.
   $$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \quad I_{\text{enc}} = \oint_S \vec{J} \cdot d\vec{a}$$