Dynamics Formula Sheet

The rest of Dynamics

Work and Energy of Rigid Bodies

Review from energy of particles:
Work-Energy Principle:

$$T_1 + U_{1\to 2} = T_2$$

where T is kinetic energy and U is work done by all forces from state 1 to state 2.

Conservation of Mechanical Energy:

$$T_1 + V_{g1} + V_{e1} + U_{1\to 2}^{NC} = T_2 + V_{g2} + V_{e2}$$

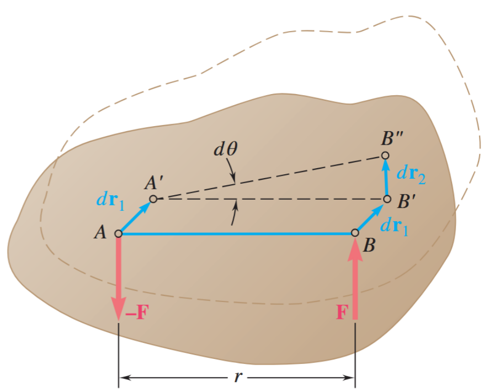
$$U_{1\to 2} = \int_{A1}^{A2} \mathbf{F} \cdot d\mathbf{r}$$

Note: Internal forces do not do work on a rigid body.

Forces that do zero work

- Force applied at a fixed point (ex: reaction forces at a pin or surface)
- Forces acting perpendicular to motion (ex: weight and normal force when horizontal motion only)
- When rolling without slipping, the friction at the contact point does no work

Work of a moment couple



$$\boxed{U_{1\to 2} = \int_{\theta_1}^{\theta_2} M \, d\theta}$$

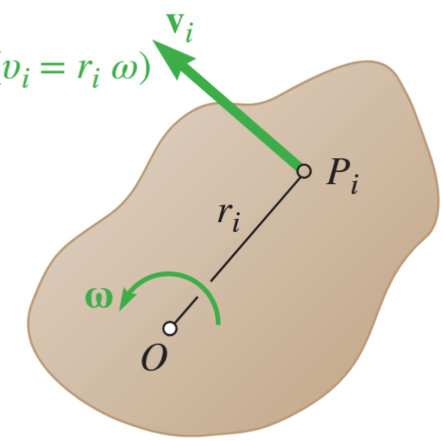
For a constant moment M:

$$U_{1\to 2} = M(\theta_2 - \theta_1)$$

Kinetic Energy of a Rigid Body
The total kinetic energy of a rigid body can be expressed as the sum of the kinetic energy of its center of mass and the kinetic energy due to rotation about the center of mass.

$$\boxed{T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2}$$

Non-centroidal Rotation



For rotation about a FIXED axis o:

$$\boxed{T = \frac{1}{2}I_o\omega^2}$$

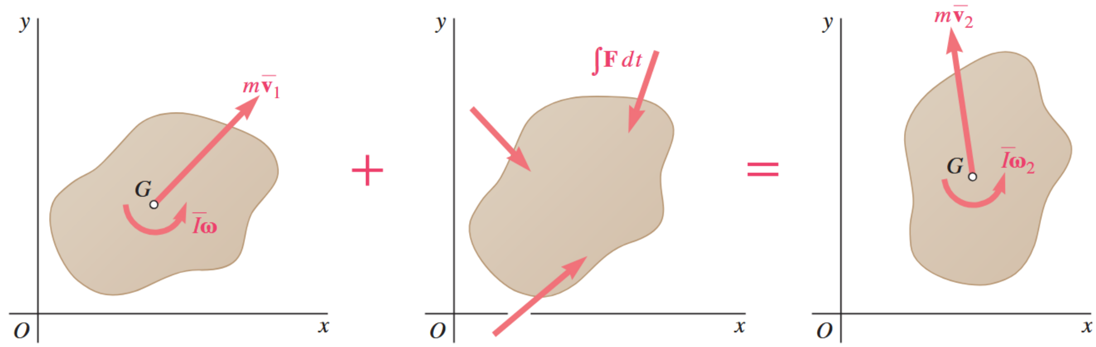
Power

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v}$$

$$P = \frac{dU}{dt} = M\frac{d\theta}{dt} = M\omega$$

Impulse and Momentum of Rigid Bodies

Like energy, total impulse and momentum of a rigid body is equal to a linear and rotational component at the center of mass.



From the impulse momentum diagram, we can write 3 equations for rigid body motion:

$$\text{X-direction:} \quad m\bar{v}_{x,1} + \sum \int_{t_1}^{t_2} \bar{\mathbf{F}}_x \, dt = m\bar{v}_{x,2}$$

$$\text{Y-direction:} \quad m\bar{v}_{y,1} + \sum \int_{t_1}^{t_2} \bar{\mathbf{F}}_y \, dt = m\bar{v}_{y,2}$$

$$\text{Moment about point } G: \quad \bar{I}\omega_1 + \sum \int_{t_1}^{t_2} \mathbf{M_G} \, dt = \bar{I}\omega_2$$

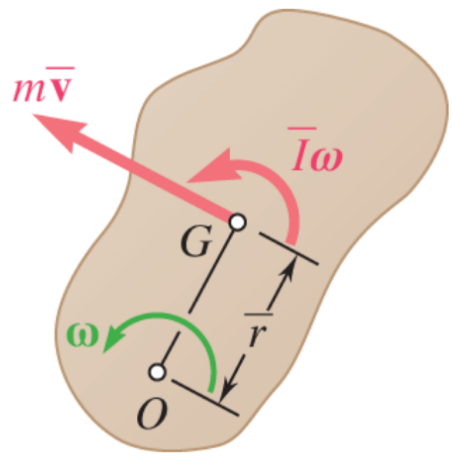
We can sum moments about any point. For a point P not at the center of mass, we have:

$$\text{Moment about point } P: \quad \bar{I}\omega_1 + m\bar{v}_1 d_{\perp 1} + \sum \int_{t_1}^{t_2} \mathbf{M_P} \, dt = \bar{I}\omega_2 + m\bar{v}_2 d_{\perp 2}$$

Where $d_\perp$ is the perpendicular distance from point P to the line of action of the velocity vector $\bar{v}$, basically the "moment" of the linear momentum about point P.

Note: The angular impulse for summing moments uses MOMENTS. Take care to convert forces to moments about the point of interest.

Non-centroidal Rotation



Impulse/momentum about fixed point o:

$$I_o\omega_1 + \sum \int_{t_1}^{t_2} \mathbf{M_o}\, dt = I_o\omega_2$$

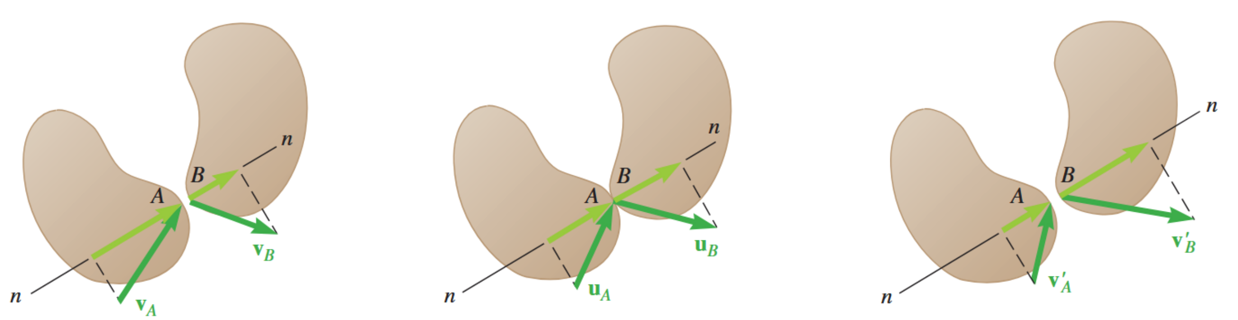
Systems of Rigid Bodies

For a system of rigid bodies, we can apply the impulse-momentum equations to the entire system as a whole or to each body individually. When applying to the entire system, internal forces and moments cancel out.

Conservation of Angular Momentum

If no external applied moments act on a rigid body or system of rigid bodies, the angular momentum remains constant.

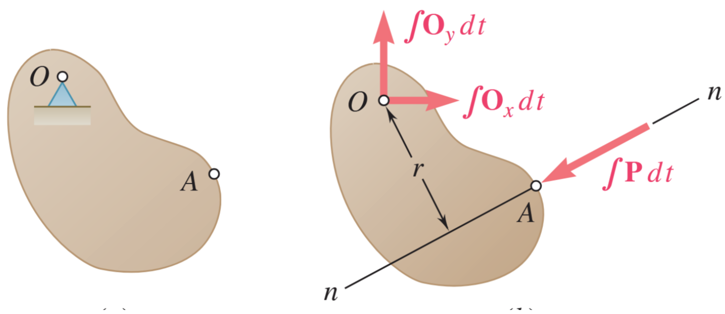
Eccentric Impact



Similar to eccentric collisions in particles, we can use the coefficient of restitution e to relate the relative velocities before and after impact.

$$\boxed{(v'_B)_n - (v'_A)_n = e[(v_a)_n - (v_B)_n]}$$

Even for constrained motion like below, the same equation for coefficient of restitution applies.



Problem approach:

- Draw impulse-momentum diagram
- Write impulse-momentum equations. If there is no external impulse, use conservation of momentum. If no external moment impulse, use conservation of angular momentum.
- Write coefficient of restitution equation
- Solve the system of equations