Ph291E Lab 4 – Bessel & Telescope

Zeshui Song
The Cooper Union

November 13, 2025

1 Purpose

Using Bessel's method to determine the focal lengths of two different converging lenses, we will construct a simple refracting telescope using these lenses and determine its magnification both experimentally and theoretically.

2 Data

Approximate Focal Lengths

$$f_1 =$$
$$f_2 =$$

Measurements for Bessel's Method Calculations for Focal Lengths

Table 1: Bessel's Method Data for Lens 1

D (cm)	d (cm)	Random Error D (mm)	Random Error d (mm)	Inst. Error (mm)
2.180				
2.190				
2.185				
2.185				
2.195				
2.180	0.005	0.002	0.002	0.0005
Mean D				**7.66 mm**
Mean d				**7.66 mm**

Table 2: Bessel's Method Data for Lens 2

D (cm)	d (cm)	Random Error D (mm)	Random Error d (mm)	Inst. Error (mm)
2.180				
2.190				
2.185				
2.185				
2.195				
2.180	0.005	0.002	0.002	0.0005
Mean D				**7.66 mm**
Mean d				**7.66 mm**

Approximate Angular Magnification

3 Calculations

Length D Sample Calculations
Mean Calculation:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{1}{6}(2.180 + 2.190 + 2.185 + 2.185 + 2.195 + 2.180)$$

$$\bar{x} = 2.1858 \text{ mm}$$

Standard Deviation:

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$S_x = \sqrt{\frac{1}{6-1}\left[(2.180 - 2.1858)^2 + (2.190 - 2.1858)^2 + \cdots + (2.180 - 2.1858)^2\right]}$$

$$S_x = 0.005845 \text{ mm}$$

Standard Deviation of the Mean (SDOM):

$$\sigma_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.005845}{\sqrt{6}}$$

$$\sigma_{\bar{x}} = 0.0023863 \text{ mm}$$

The same procedure was applied to calculate the mean, standard deviation and standard deviation of the mean for the remaining measurements.

Bessel's Method Calculations for Lens 1

$$f = \frac{D^2 - d^2}{4D}$$

$$f = \frac{(90.0)^2 - (30.0)^2}{4(90.0)}$$

$$f = 36.67 \text{ cm}$$

Error Propagation for Bessel's Method (Lens 1)
Since the measurements of D and d are independent, we can use the independent error propagation formula:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial D}\delta D\right)^2 + \left(\frac{\partial f}{\partial d}\delta d\right)^2}$$

$$\delta f = \sqrt{\left(\frac{D^2 + d^2}{4D^2}\delta D\right)^2 + \left(-\frac{d}{2D}\delta d\right)^2}$$

Chosen uncertainties: $\delta D = 0.05$ mm (random, larger than instrumental) and $\delta d = 0.005$ mm (instrumental, larger than random).

$$\delta f = \sqrt{\left(\frac{(90.0)^2 + (30.0)^2}{4(90.0)^2}(0.05)\right)^2 + \left(-\frac{30.0}{2(90.0)}(0.005)\right)^2}$$

$$\delta f = 0.0142 \text{ cm}$$

Angular Magnification

$$m_\theta = -\frac{f_{obj}}{f_{eye}}$$

$$m_\theta = -\frac{36.67}{10.0}$$

$$m_\theta = \text{-3.667}$$

Error Propagation for Angular Magnification
Since the measurements of f_{obj} and f_{eye} are independent, we can use the independent error propagation formula:

$$\delta m_\theta = \sqrt{\left(\frac{\partial m_\theta}{\partial f_{obj}}\delta f_{obj}\right)^2 + \left(\frac{\partial m_\theta}{\partial f_{eye}}\delta f_{eye}\right)^2}$$

$$\delta m_\theta = \sqrt{\left(-\frac{1}{f_{eye}}\delta f_{obj}\right)^2 + \left(\frac{f_{obj}}{f_{eye}^2}\delta f_{eye}\right)^2}$$

Chosen uncertainties: $\delta f_{obj} = 0.0142$ cm (from previous calculation) and $\delta f_{eye} = 0.05$ cm (instrumental).

$$\delta m_\theta = \sqrt{\left(-\frac{1}{10.0}(0.0142)\right)^2 + \left(\frac{36.67}{(10.0)^2}(0.05)\right)^2}$$

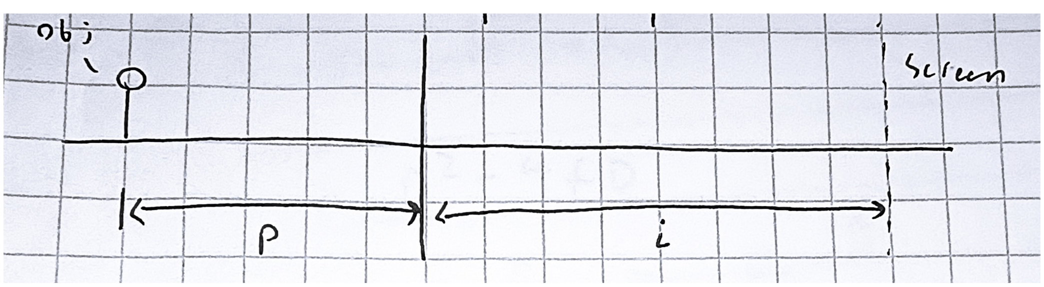
$$\delta m_\theta = 0.0191$$

4 Results

5 Conclusion

6 Answers to questions

Question 1



Thin lens equation:

$$\frac{1}{f} = \frac{1}{P} + \frac{1}{i}$$

$$\frac{1}{f} = \frac{P+i}{Pi}$$

Since $D = P + i$ and $i = D - P$,

$$\frac{1}{f} = \frac{D}{P(D-P)}$$

$$f = \frac{P(D-P)}{D} = \frac{PD - P^2}{D}$$

Putting into standard form to find the 2 P values that forms a sharp image:

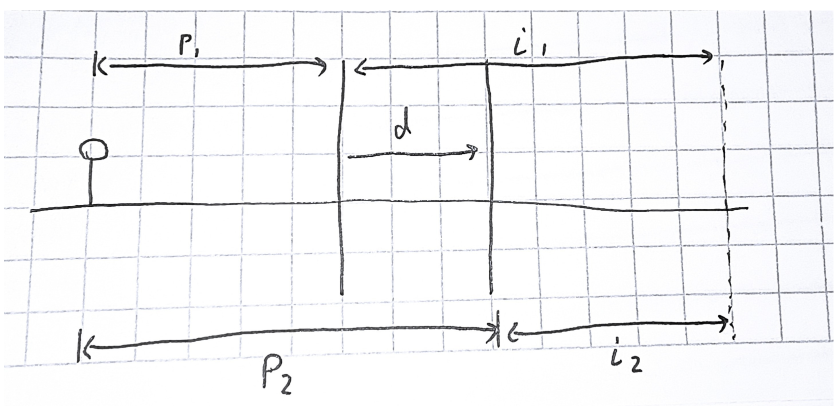
$$P^2 - PD + fD = 0$$

Using the quadratic formula:

$$P = \frac{D \pm \sqrt{D^2 - 4fD}}{2}$$

Since the two values for P are real and unique, the discriminant must be non-zero:

$$\boxed{D^2 - 4fD > 0 \implies D > 4f}$$



The distance between the two positions of the lens $P_2 - P_1 = d$ is:

$$d = P_2 - P_1 = \sqrt{D^2 - 4fD}$$

Rearranging for f:

$$d^2 = D^2 - 4fD$$

$$4fD = D^2 - d^2$$

$$\boxed{f = \frac{D^2 - d^2}{4D}}$$