Dynamics Formula Sheet

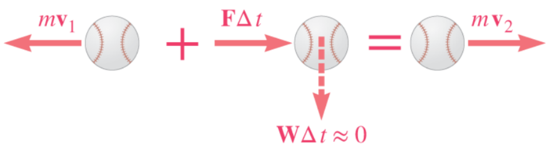
Quiz II

Impulse and Momentum

$$\vec{p} = m\vec{v} \qquad \Delta\vec{p} = \mathbf{Imp}_{1\to 2} = \int_{t_1}^{t_2} \mathbf{F}\, dt$$

Momentum changes by:

$$m\mathbf{v_1} + \mathbf{Imp}_{1\to 2} = m\mathbf{v_2}$$

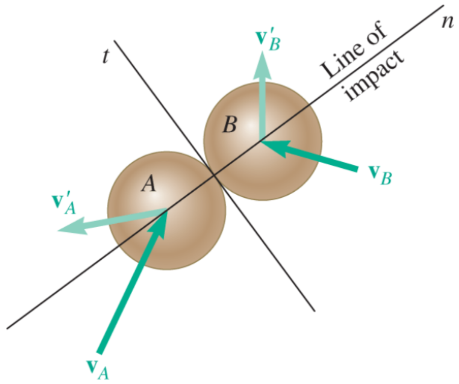


Collisions

$$e = \frac{v'_B - v'_A}{v_A - v_B}$$

- **Perfectly elastic impact:** $e = 1$
  Momentum *and* energy is conserved.
- **Perfectly plastic impact:** $e = 0$
  Particles stick together after impact. Only momentum is conserved.

Oblique Central Impact



No impulsive force in t:

$$v'_{A,t} = v_{A,t}, \quad v'_{B,t} = v_{B,t}$$

Conservation of momentum in n:

$$m_A v_{A,n} + m_B v_{B,n} = m_A v'_{A,n} + m_B v'_{B,n}$$

Relate velocities using e:

$$v'_{B,n} - v'_{A,n} = e(v_{A,n} - v_{B,n})$$

Rigid Body Rotation

Kinematics

$$\mathbf{v} = \omega \times \mathbf{r}, \quad \mathbf{a} = \alpha \times \mathbf{r} - \omega^2 \mathbf{r}$$

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \quad \alpha = \frac{d\theta}{dt}\frac{d\omega}{d\theta} = \omega\frac{d\omega}{d\theta}$$

Constant Angular Acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

General Planar Motion

$$\mathbf{v_B} = \mathbf{v_A} + \mathbf{v_{B/A}}$$

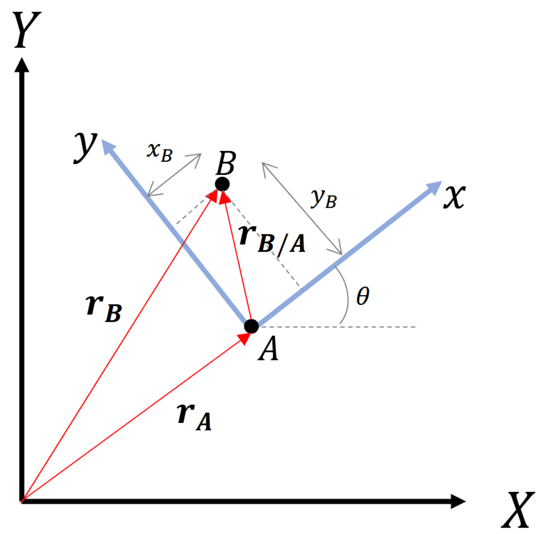
$$\boxed{\mathbf{v}_B = \mathbf{v}_A + \omega\hat{k} \times \mathbf{r}_{B/A}}$$

$$\boxed{\mathbf{a}_B = \mathbf{a}_A + \alpha\hat{k} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}}$$

Note: $\mathbf{r}_{B/A}$ is the position vector pointing from A to B

Rotating Reference Frames

Frame xyz is rotating relative to frame XYZ (the absolute frame) with angular velocity ω.



$$\boxed{\underbrace{\boldsymbol{v_B}}_{\text{Absolute velocity of B in XYZ}} = \underbrace{\underbrace{\boldsymbol{v_A}}_{\text{Velocity of origin A in XYZ}} + \underbrace{\boldsymbol{\Omega} \times \boldsymbol{r_{B/A}}}_{\text{Rotation of xyz as seen in XYZ}}}_{\text{Motion of frame xyz as seen in XYZ}} + \underbrace{(\boldsymbol{v_{B/A}})_{xyz}}_{\text{Velocity of B relative to A as seen in xyz}}}$$

$$\boxed{\underbrace{\underbrace{\boldsymbol{a_B}}_{\text{Absolute accel of B in XYZ}}}_{\text{Motion of B in XYZ}} = \underbrace{\underbrace{\boldsymbol{a_A}}_{\text{Accel of origin A in XYZ}} + \underbrace{\dot{\boldsymbol{\Omega}} \times \boldsymbol{r_{B/A}}}_{\text{Angular accel caused by rotation of xyz}} + \underbrace{\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r_{B/A}})}_{\text{Angular velocity effect caused by rotation of xyz}}}_{\text{Motion of xyz as observed from XYZ}} + \underbrace{\underbrace{2\boldsymbol{\Omega} \times (\boldsymbol{v_{B/A}})_{xyz}}_{\text{Coriolis acceleration Combined effect of B moving relative to xyz coord and the rotation of xyz frame}}}_{\text{Interacting motion}} + \underbrace{\underbrace{(\boldsymbol{a_{B/A}})_{xyz}}_{\text{Accel of B relative to A as seen in xyz}}}_{\text{Motion of B as observed in xyz}}}$$

Note: We have to use rotating frames if we are given quantities relative to a rotating body.

Center of Mass

Continuous Body:

$$\bar{x} = \frac{\int x\,dW}{W}, \quad \bar{y} = \frac{\int y\,dW}{W}$$

Discrete Masses:

$$x_c = \frac{\sum x_{ci}A_i}{\sum A_i}, \qquad \frac{\sum x_{ci}W_i}{\sum W_i}$$

$$y_c = \frac{\sum y_{ci}A_i}{\sum A_i}, \qquad \frac{\sum y_{ci}W_i}{\sum W_i}$$

Moment of Inertia

Area Moments:

$$I_x = \int y^2\,dA, \quad I_y = \int x^2\,dA$$

Polar Moment:

$$J_O = \int r^2\,dA = I_x + I_y$$

Mass Moments:

$$I_x = \int (y^2{+}z^2)\,dm,\ I_y = \int (x^2{+}z^2)\,dm,\ I_z = \int (x^2{+}y^2)\,dm$$

Radius of Gyration:

$$I_x = k_x^2 A, \quad I_y = k_y^2 A, \quad J_O = k_o^2 A$$

Rigid Body Kinetics

Sum of forces and moments about the center of mass:

$$\sum \mathbf{F} = m\bar{\mathbf{a}}, \quad \sum \mathbf{M_G} = \dot{\mathbf{H}}_\mathbf{G} = \bar{I}\alpha$$

Summing moments about a point other than **G**:

$$\sum M_P = \bar{I}\alpha + m\bar{a}d$$

Note: Parallel axis theorem only applies for fixed rotation. ($I = \bar{I} + md^2$)

Rolling Motion

There are 3 cases of rolling motion:

1. Rolling without slipping:

   $F_f \leq \mu_s N \qquad \bar{a} = r\alpha$

2. Rolling, slipping impeding

   $F_f = \mu_s N \qquad \bar{a} = r\alpha$

3. Rolling and slipping:

$F_f = \mu_k N \qquad \bar{a}$ and α becomes decoupled and independent

Note: If unsure about which case, assume rolling without slipping first and calculate F_f. If $F_f > \mu_s N$, then it is slipping, and we need to adjust our equation for F_f.

Normal and Tangential

$$\vec{v} = v\hat{u}_t$$

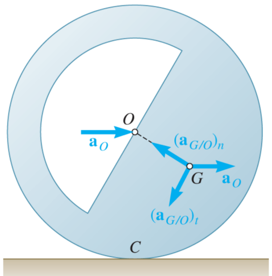
$$\vec{a} = \frac{dv}{dt}\hat{u}_t + \frac{v^2}{\rho}\hat{u}_n$$

Cylindrical (Radial and Transverse)

$$\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

$$\vec{a} = \left[\ddot{r} - r\dot{\theta}^2\right]\hat{u}_r + \left[r\ddot{\theta} + 2\dot{r}\dot{\theta}\right]\hat{u}_\theta$$

Asymmetric Rolling

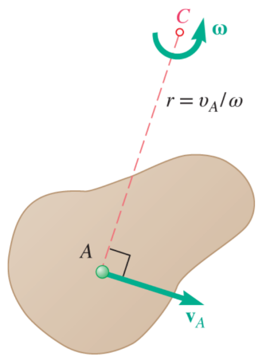


$$\bar{\boldsymbol{a}} = \boldsymbol{a_G} = \boldsymbol{a_O} + \boldsymbol{a_{G/O}}$$

$$\boldsymbol{a_G} = \boldsymbol{a_O} + (\boldsymbol{a_{G/O}})_t + (\boldsymbol{a_{G/O}})_n$$

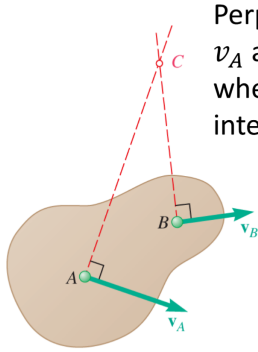
$$\boldsymbol{a_G} = \boldsymbol{a_O} + \boldsymbol{\alpha} \times \boldsymbol{r_{G/O}} - \omega^2 \boldsymbol{r_{G/O}}$$

Instantaneous Center

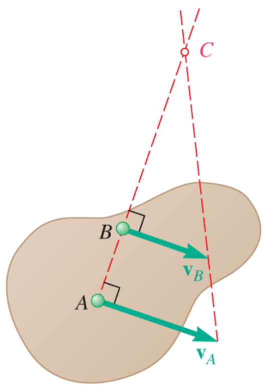


Perpendicular to v_A, at distance r from A

$$r = \frac{v_A}{\omega}$$



Perpendicular to v_A and v_B, where lines intersect



Perpendicular to v_A and v_B and line that joins ends of vectors, where lines intersect