Ph291E Lab 4 – Bessel & Telescope

Zeshui Song
The Cooper Union

November 13, 2025

1 Purpose

Using Bessel's method to determine the focal lengths of two different converging lenses, we will construct a simple refracting telescope using these lenses and determine its magnification both experimentally and theoretically.

2 Data

Approximate Focal Lengths

$$f_1 = 10 \pm 0.3 \text{ cm}$$
$$f_2 = 4.6 \pm 0.6 \text{ cm}$$

Measurements for Bessel's Method Calculations for Focal Lengths

Table 1: Bessel's Method Data for Lens 1

D (cm)	**d (cm)**	**Random Error D (cm)**	**Random Error d (cm)**	**Optical Bench Inst. Error (cm)**
43.00	9.85			
43.00	9.85			
43.00	9.25			
43.00	9.20			
43.00	9.60			
43.00	9.75	0	0.12	0.05
Mean D				**43.00 cm**
Mean d				**9.58 cm**

Image sharpness uncertainty (for position d):

- Position 1: 0.30 cm
- Position 2: 0.20 cm

Table 2: Bessel's Method Data for Lens 2

D (cm)	**d (cm)**	**Random Error D (cm)**	**Random Error d (cm)**	**Optical Bench Inst. Error (cm)**
24.95	7.45			
24.95	8.25			
24.95	7.55			
24.95	7.95			
24.95	8.20			
24.95	7.30	0	0.16	0.05
Mean D				**24.95 cm**
Mean d				**7.78 cm**

Image sharpness uncertainty (for position d):

- Position 1: 0.05 cm
- Position 2: 0.60 cm

3 Calculations

Sample Calculations for Distance d (Lens 1)
Mean Calculation:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{1}{6}(9.85 + 9.85 + 9.25 + 9.20 + 9.60 + 9.75)$$

$$\bar{x} = 9.5833 \text{ cm}$$

Standard Deviation:

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$S_x = \sqrt{\frac{1}{6-1}\left[(9.85 - 9.5833)^2 + (9.85 - 9.5833)^2 + \cdots + (9.75 - 9.5833)^2\right]}$$

$$S_x = 0.29268 \text{ cm}$$

Standard Deviation of the Mean (SDOM):

$$\sigma_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.29268}{\sqrt{6}}$$

$$\sigma_{\bar{x}} = 0.11948 \text{ cm}$$

The same procedure was applied to calculate the mean, standard deviation and standard deviation of the mean for the remaining measurements.

Bessel's Method Calculations for Lens 1

$$f = \frac{D^2 - d^2}{4D}$$

$$f = \frac{(43.00)^2 - (9.58)^2}{4(43.00)}$$

$$f = 10.22 \text{ cm}$$

Error Propagation for Bessel's Method (Lens 1)
Since the measurements of D and d are independent, we can use the independent error propagation formula:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial D}\delta D\right)^2 + \left(\frac{\partial f}{\partial d}\delta d\right)^2}$$

$$\delta f = \sqrt{\left(\frac{D^2 + d^2}{4D^2}\delta D\right)^2 + \left(-\frac{d}{2D}\delta d\right)^2}$$

Chosen uncertainties: $\delta D = 0.05$ cm (Instrumental uncertainty, larger than random) and $\delta d = 0.30$ cm (Image sharpness uncertainty, larger than random and instrumental).

$$\delta f = \sqrt{\left(\frac{(43.00)^2 + (9.58)^2}{4(43.00)^2}(0.05)\right)^2 + \left(-\frac{9.58}{2(43.00)}(0.30)\right)^2}$$

$$\delta f = 0.036 \text{ cm}$$

Angular Magnification
Where f_{obj} is the focal length of the lens 1 and f_{eye} is the focal length of the lens 2.

$$m_\theta = -\frac{f_{obj}}{f_{eye}}$$

$$m_\theta = -\frac{10.216}{5.630}$$

$$m_\theta = \text{-1.814}$$

Error Propagation for Angular Magnification
Since the measurements of f_{obj} and f_{eye} are independent, we can use the independent error propagation formula:

$$\delta m_\theta = \sqrt{\left(\frac{\partial m_\theta}{\partial f_{obj}}\delta f_{obj}\right)^2 + \left(\frac{\partial m_\theta}{\partial f_{eye}}\delta f_{eye}\right)^2}$$

$$\delta m_\theta = \sqrt{\left(-\frac{1}{f_{eye}}\delta f_{obj}\right)^2 + \left(\frac{f_{obj}}{f_{eye}^2}\delta f_{eye}\right)^2}$$

Chosen uncertainties: $\delta f_{obj} = 0.036$ cm (calculated) and $\delta f_{eye} = 0.095$ cm (calculated).

$$\delta m_\theta = \sqrt{\left(-\frac{1}{5.630}(0.036)\right)^2 + \left(\frac{10.216}{(5.630)^2}(0.095)\right)^2}$$

$$\delta m_\theta = 0.031$$

4 Results

Focal Lengths:

- Lens 1: $f_{obj} = 10.22 \pm 0.04$ cm
- Lens 2: $f_{eye} = 5.63 \pm 0.10$ cm

Angular Magnification:

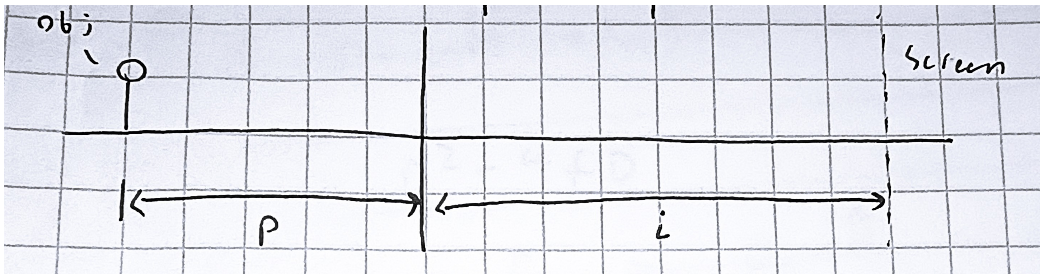
- Theoretical: $m_\theta = -1.81 \pm 0.03$
- Experimental approximation: $m_\theta \approx -1.36$

5 Conclusion

The focal lengths of the two lenses were determined using Bessel's method to be 10.22 ± 0.04 cm for lens 1 and 5.63 ± 0.10 cm for lens 2. The approximate focal length of lens 1 (10 ± 0.3 cm) is within the uncertainty range of the calculated focal length, while the approximate focal length of lens 2 (4.6 ± 0.6 cm) is slightly outside the uncertainty range of the calculated focal length. The theoretical angular magnification of the telescope was calculated to be -1.81 ± 0.03, while the experimental approximation yielded a value of -1.36. This discrepancy could be due to systemic errors such as the lenses not being set up exactly $f_{obj} + f_{eye}$ apart, and bias in visually comparing the lengths of tape seen through the telescope.

6 Answers to questions

Question 1



Thin lens equation:

$$\frac{1}{f} = \frac{1}{P} + \frac{1}{i}$$

$$\frac{1}{f} = \frac{P+i}{Pi}$$

Since $D = P + i$ and $i = D - P$,

$$\frac{1}{f} = \frac{D}{P(D-P)}$$

$$f = \frac{P(D-P)}{D} = \frac{PD - P^2}{D}$$

Putting into standard form to find the 2 P values that forms a sharp image:

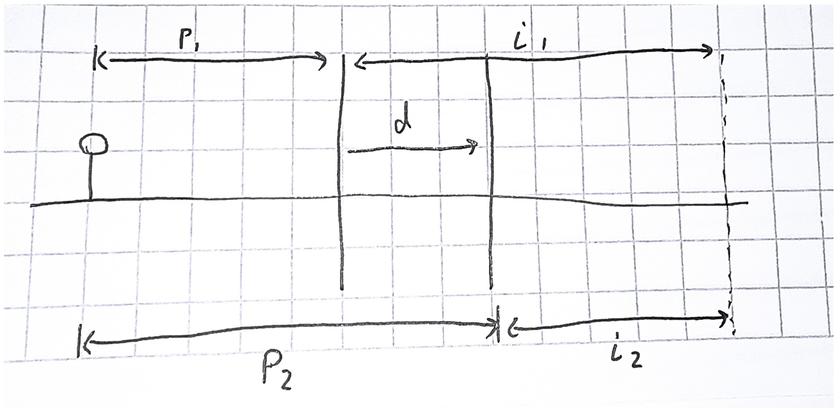
$$P^2 - PD + fD = 0$$

Using the quadratic formula:

$$P = \frac{D \pm \sqrt{D^2 - 4fD}}{2}$$

Since the two values for P are real and unique, the discriminant must be non-zero:

$$\boxed{D^2 - 4fD > 0 \implies D > 4f}$$



The distance between the two positions of the lens $P_2 - P_1 = d$ is:

$$d = P_2 - P_1 = \sqrt{D^2 - 4fD}$$

Rearranging for f:

$$d^2 = D^2 - 4fD$$

$$4fD = D^2 - d^2$$

$$\boxed{f = \frac{D^2 - d^2}{4D}}$$

Question 2

Thin lens equation:

$$\frac{1}{f} = \frac{1}{P} + \frac{1}{i}$$

$$\frac{1}{f} = \frac{P+i}{Pi}$$

$$f = \frac{Pi}{P+i} = \frac{i}{1+i/P}$$

For the approximate focal length we assumed $P \to \infty$, so $i/P \to 0$:

$$f_{approx} = i$$

However, P is finite but large, so

$$1 + i/P > 1$$

$$\implies f = \frac{i}{1+i/P} < i = f_{approx}$$