NAME: Zeshui Song **DECEMBER 13-17, 2025**

TAKE-HOME FINAL EXAM

ME200, DYNAMICS, COOPER UNION
FALL 2025

Ground Rules READ ME *FIRST*

By submitting this quiz solution with your name, you affirm that you have abided by the code of conduct of Cooper Union. You affirm that this exam solution is your own work, and that you did not receive unauthorized assistance from any person. Collaboration with other students CURRENTLY ENROLLED IN ME200 is encouraged but must be well documented (see below).

- Direct clarifying questions about the exam to the instructor. **Check Teams** for any clarifications!
- You will submit your solutions on Teams, so give yourself some time to scan your pages in. **Please include your name in your filename when you submit.**
- Budget your time to answer all questions. Look over them all first (each has a point value) and plan your time. In most problems, a good description of how you will solve the problem is worth a significant amount of credit. Leaving a problem blank is worse than spending a little bit of time to outline your approach (even if you don't fully solve it).
- **Show all your work! Write out your equations! Be organized!** Make it easy for your instructor to follow your logic – we want to give you points!
- Use variables for most of the calculations and make it clear what value is being substituted for each variable if/when you do so.
- For numerical answers, include the units. No units will mean no credit for numerical answer. Carry units in your calculations and check units at the end.
- If you need more space, work on a spare sheet of paper and make sure to scan those pages too! Please label any extra sheets with your name and the problem you are working on.
- Please box or otherwise highlight your answers and make it clear which problem you are answering.

There are 200 total points available spread over four multi-part problems. Point values are listed for each problem.

Problem		Points Available	Total
Problem 1: Way Down Hadestown	a	15	30
	b	15	
Problem 2: Nervous Fidgeting	a	15	65
	b	15	
	c	10	
	d	25	
Problem 3: 41CS Elevator	a	5	35
	b	30	
Problem 5: The Swinging Sticks	a	15	70
	b	30	
	c	10	
	d	10	
	e	5	

Please indicate the nature and extent of all collaboration on each problem on this page. A few examples:

1. *I worked entirely alone*
2. *Checked my solution to Part 2 with ***** and revised my solution afterward (found a sign error)*
3. *Worked together extensively on the whole problem with *****
4. *Worked extensively on part 1 with ***** and checked my solution to part 2 with *****.*
5. *Developed a Python script to solve Part 4 with *****

Please also list the approximate time you spent on each problem – it will be helpful for the future to plan for timing!

Collaborations and Timing

PROBLEM 1. WAY DOWN HADESTOWN

TIME SPENT: 3 hrs

I worked extensively on part a of the problem with Bertrand and Roy. I checked my solution for part b with Bertrand and revised my solution afterward (forgot to account for normal/radial accelerations).

PROBLEM 2. NERVOUS FIDGETING

TIME SPENT: 1 hr

I worked entirely alone.

PROBLEM 3. 41CS ELEVATOR

TIME SPENT: 1 hr

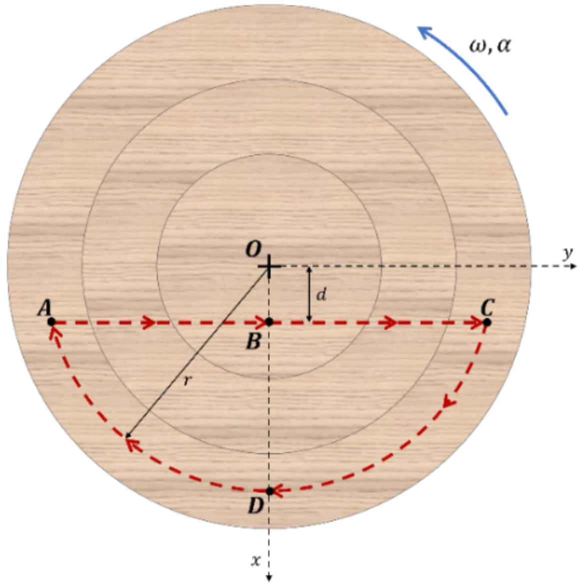
I worked entirely alone.

PROBLEM 4. THE SWINGING STICKS

TIME SPENT: 2.5 hrs

I worked entirely alone.

Problem 1. Way Down Hadestown

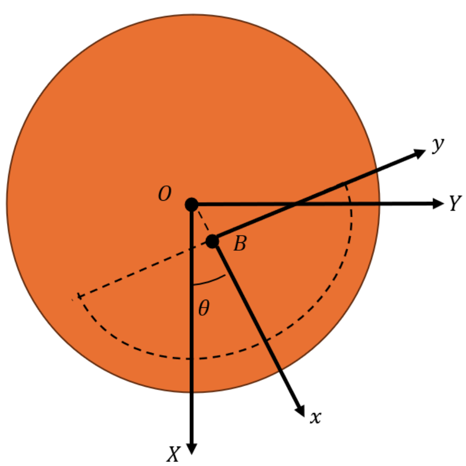


a) Given:

- Path ABC is a straight line a distance $d = 0.5\,m$ from O
- Path CDA is an arc of radius $r = 2\,m$
- When Orpheus reaches point A, the turntable has $\omega = 0.5\,rad/s$ and $\alpha = 0.1\,rad/s^2$ (counterclockwise).
- Orpheus starts from rest at point A and walks with velocity $v = 0.5t\,m/s$ relative to the turntable, where t is the time in seconds since he started walking.

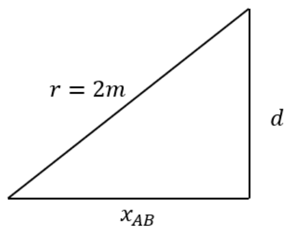
Find:

- His velocity and acceleration at point B



Kinematics

Finding time t when Orpheus reaches point B:



$$x_{AB} = \sqrt{r^2 - d^2} \implies x_{AB} = 1.9364m$$

$$\Delta x = x_{AB} = \int_0^t v dt'$$

$$\int_0^t 0.5t' dt' = \left[0.25t'^2\right]_0^t = 0.25t^2$$

$$1.9364m = 0.25t^2 \implies t = 2.783s$$

Finding θ_B when Orpheus reaches point B:

$$\theta_B = \omega t + \frac{1}{2}\alpha t^2$$

$$\theta_B = 0.5(2.783) + 0.5(0.1)(2.783)^2 = 1.7787\, rad$$

Finding ω at time t:

$$\omega = \omega_0 + \alpha t = 0.5 + 0.1(2.783) = 0.7783\, rad/s$$

Velocity

$$\mathbf{v_P} = \mathbf{v_B} + \mathbf{\Omega} \times \mathbf{r_{P/B}} + \left(\mathbf{v_{P/B}}\right)$$

Where:

- $\mathbf{v_P}$: Velocity of Orpheus relative to XYZ
- $\mathbf{v_B}$: Velocity of point B relative to XYZ

$$\mathbf{v_B} = \langle -d\omega sin\theta_B, d\omega cos\theta_B, 0\rangle = \langle -0.5(0.7783)sin(1.7787), 0.5(0.7783)cos(1.7787), 0\rangle$$

$$\mathbf{v_B} = \langle -0.3807, -0.0803, 0\rangle$$

- $\mathbf{\Omega}$: Angular velocity of turntable

$$\mathbf{\Omega} = \omega\mathbf{k}$$

- $\mathbf{r_{P/B}}$: Position of Orpheus relative to point B

$$\mathbf{r_{P/B}} = \langle 0, 0, 0\rangle$$

- $\mathbf{v_{P/B}}$: Velocity of Orpheus relative to turntable (point B)

$$\mathbf{v_{P/B}} = \langle 0, 0.5t, 0\rangle$$

Plugging in values:

$$\mathbf{v_P} = \langle -0.3807, -0.0803, 0\rangle + \langle 0, 1.3915, 0\rangle$$

$$\boxed{\mathbf{v_P} = \langle -0.3807, 1.3112, 0\rangle\ m/s}$$

Acceleration

$$\mathbf{a_P} = \mathbf{a_B} + \dot{\mathbf{\Omega}} \times \mathbf{r_{P/B}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r_{P/B}}) + 2\mathbf{\Omega} \times \left(\mathbf{v_{P/B}}\right)_{xyz} + \left(\mathbf{a_{P/B}}\right)_{xyz}$$

$$\mathbf{a_P} = \mathbf{a_B} + 2\mathbf{\Omega} \times \left(\mathbf{v_{P/B}}\right)_{xyz} + \left(\mathbf{a_{P/B}}\right)_{xyz}$$

Find $\mathbf{a_B}$
where the normal direction is $\hat{n} = \langle -cos\theta, -sin\theta \rangle$ and the tangential direction is $\hat{t} = \langle -sin\theta, cos\theta \rangle$:

$$a_{Bn} = r\omega^2$$

$$a_{Bt} = r\alpha$$

Note that at $t = 2.783s$, $\omega = 0.7783\, rad/s$ and $\alpha = 0.1\, rad/s^2$:

$$\mathbf{a_B} = (0.5)(0.7783)^2 \langle -cos\theta_B, -sin\theta_B, 0 \rangle + (0.5)(0.1) \langle -sin\theta_B, cos\theta_B, 0 \rangle$$

$$\mathbf{a_B} = 0.3028 \langle -cos(1.7787), -sin(1.7787), 0 \rangle + 0.05 \langle -sin(1.7787), cos(1.7787), 0 \rangle$$

$$\mathbf{a_B} = \langle 0.01357, -0.30659, 0 \rangle$$

Cross product:

$$2\mathbf{\Omega} \times \left(\mathbf{v_{P/B}}\right)_{xyz} = 2(\omega\hat{k}) \times (1.3915\hat{\jmath}) = -2(0.7783)(1.3915\hat{\imath}) = -2.166\hat{\imath}$$

Find $\left(\mathbf{a_{P/B}}\right)_{xyz}$:

$$\left(\mathbf{a_{P/B}}\right)_{xyz} = \frac{d}{dt}v = \frac{d}{dt}0.5t = 0.5\hat{\jmath}$$

Plugging in values:

$$\mathbf{a_P} = \langle 0.01357, -0.30659, 0 \rangle + \langle -2.166, 0, 0 \rangle + \langle 0, 0.5, 0 \rangle$$

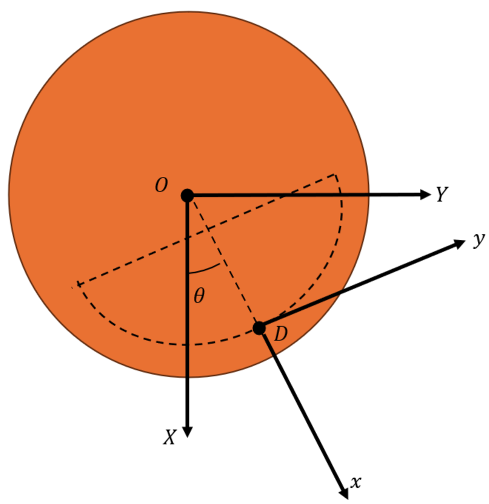
$$\boxed{\mathbf{a_P} = \langle -2.1524, 0.1934, 0 \rangle \ m/s^2}$$

b) Given:

- After point B Orpheus continues walking along path CDA at constant speed $v = 1.5\, m/s$ relative to the turntable.
- At point D, the turntable has $\omega = 0.5\, rad/s$ and $\alpha = 0.1\, rad/s^2$ (counterclockwise).

Find:

- His velocity and acceleration at point D



Kinematics from point A to point D
From part A, we know that the distance AB is 1.9364m. Thus, the distance AC is $2(1.9364) = 3.8728m$.

Also from part A, we can find the time t_{AC} it takes for Orpheus to reach point C:

$$3.8728\, m = 0.25t^2 \implies t_{AC} = 3.935\, s$$

To find the arc length s_{CD} from point C to point D, we need to find the angle in the sector AOC. Using the same triangle from part A:

$$\theta_{AOC} = 2 \times \theta_{OAB} = 2 \times cos^{-1}\left(\frac{d}{r}\right) = 2 \times cos^{-1}\left(\frac{0.5}{2}\right) = 2.63623\, rad$$

Thus the arclength s_{CD} is:

$$s_{CD} = r\theta_{AOC} = 2(2.63623) = 5.2724\, m$$

Finding the total time since Orpheus started walking from point A to point D:

- Time from A to C: $t_{AC} = 3.935\, s$

- Time from C to D:

$$s_{CD} = v_{CD}t_{CD} \implies t_{CD} = \frac{s_{CD}}{v_{CD}} = \frac{5.2724}{1.5} = 3.5149\,s$$

- Total time from A to D:

$$t_{AD} = t_{AC} + t_{CD} = 3.935 + 3.5149 = 7.4499\,s$$

Finding θ_D when Orpheus reaches point D:

- From A to D:

$$\theta_D = \omega_0 t_{AD} + \frac{1}{2}\alpha t_{AD}^2$$

$$\theta_D = 0.5(7.4499) + 0.5(0.1)(7.4499)^2 = 6.5\,rad$$

Velocity
Using the same velocity equation from part A:

$$\mathbf{v_P} = \mathbf{v_D} + \mathbf{\Omega} \times \mathbf{r_{P/D}} + \left(\mathbf{v_{P/D}}\right)$$

Where:

- $\mathbf{v_D}$: Velocity of point D relative to XYZ

$$\mathbf{v_D} = \langle -r\omega sin\theta_D, r\omega cos\theta_D \rangle$$

$$\mathbf{v_D} = \langle -(2)(0.5)sin(6.5), (2)(0.5)cos(6.5) \rangle$$

$$\mathbf{v_D} = \langle -0.21511, 0.97658, 0 \rangle$$

- $\mathbf{r_{P/D}}$: Position of Orpheus relative to point D

$$\mathbf{r_{P/D}} = \langle 0, 0, 0 \rangle$$

- $\mathbf{v_{P/D}}$: Velocity of Orpheus relative to turntable (point D)

$$\mathbf{v_{P/D}} = \langle 1.5sin\theta_D, -1.5cos\theta_D, 0 \rangle$$

$$\mathbf{v_{P/D}} = \langle 1.5sin(6.5), -1.5cos(6.5), 0 \rangle = \langle 0.3226, -1.4648, 0 \rangle$$

Plugging in values:

$$\mathbf{v_P} = \langle -0.21511, 0.97658, 0 \rangle + \langle 0.3226, -1.4648, 0 \rangle$$

$$\boxed{\mathbf{v_P} = \langle 0.10748, -0.48822, 0 \rangle \ m/s}$$

Acceleration

Using the same acceleration equation from part A:

$$\mathbf{a_P} = \mathbf{a_D} + \dot{\mathbf{\Omega}} \times \mathbf{r_{P/D}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r_{P/D}}) + 2\mathbf{\Omega} \times \left(\mathbf{v_{P/D}}\right)_{xyz} + \left(\mathbf{a_{P/D}}\right)_{xyz}$$

$$\mathbf{a_P} = \mathbf{a_D} + 2\mathbf{\Omega} \times \left(\mathbf{v_{P/D}}\right)_{xyz} + \left(\mathbf{a_{P/D}}\right)_{xyz}$$

Where:

- $\mathbf{a_D}$: Acceleration of point D relative to XYZ where the normal direction is $\hat{n} = \langle -cos\theta, -sin\theta \rangle$ and the tangential direction is $\hat{t} = \langle -sin\theta, cos\theta \rangle$:

$$a_{Dn} = r\omega^2$$

$$a_{Dt} = r\alpha$$

  Note that at D, $\omega = 0.5\, rad/s$ and $\alpha = 0.1\, rad/s^2$:

$$\mathbf{a_D} = (0.5)(0.5)^2 \langle -cos\theta_D, -sin\theta_D, 0 \rangle + (0.5)(0.1) \langle -sin\theta_D, cos\theta_D, 0 \rangle$$

$$\mathbf{a_D} = 0.125 \langle -cos(6.5), -sin(6.5), 0 \rangle + 0.05 \langle -sin(6.5), cos(6.5), 0 \rangle$$

$$\mathbf{a_D} = \langle -0.1328, 0.021939, 0 \rangle$$

- $\mathbf{v_{P/D}}$: Velocity of Orpheus relative to turntable (point D)

$$\mathbf{v_{P/D}} = \langle 0.3226, -1.4648, 0 \rangle$$

- $\mathbf{a_{P/D}}$: Acceleration of Orpheus relative to turntable (point D)

$$\mathbf{a_{P/D}} = \langle 0, 0, 0 \rangle$$

Cross product:

$$2\mathbf{\Omega} \times \left(\mathbf{v_{P/D}}\right)_{xyz} = 2(\omega\hat{k}) \times \langle 0.3226, -1.4648, 0 \rangle = (2\omega \cdot 1.4648)\hat{\imath} + (2\omega \cdot 0.3226)\hat{\jmath}$$

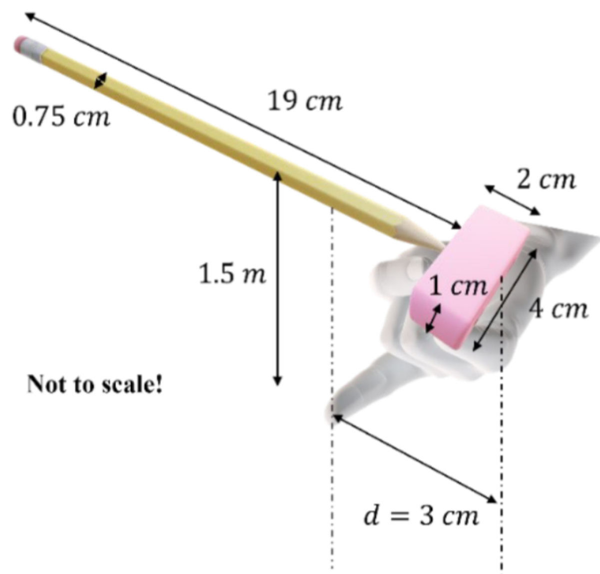
$$2\mathbf{\Omega} \times \left(\mathbf{v_{P/D}}\right)_{xyz} = \langle 1.4648, 0.3226, 0 \rangle$$

Plugging in values:

$$\mathbf{a_P} = \langle -0.1328, 0.021939, 0 \rangle + \langle 1.4648, 0.3226, 0 \rangle + \langle 0, 0, 0 \rangle$$

$$\boxed{\mathbf{a_P} = \langle 1.332, 0.3445, 0 \rangle \ m/s^2}$$

Problem 2. Nervous Fidgeting



Given:

- The pencil is a uniform rod of density $\rho_p = 0.9 g/cm^3$ and the eraser is a rectangular prism of density $\rho_e = 1.4 g/cm^3$.
- The coefficient of restitution between the pencil and finger is $e = 0.1$.
- Impact is impulsive and the finger does not move, only hitting tangentially

a) Find the 3D center of mass of the pencil-eraser system.

Finding the masses:

- Mass of pencil:

$$v_p = \pi \left(\frac{.75}{2}\right)^2 (19) = 8.3939\, cm^3$$

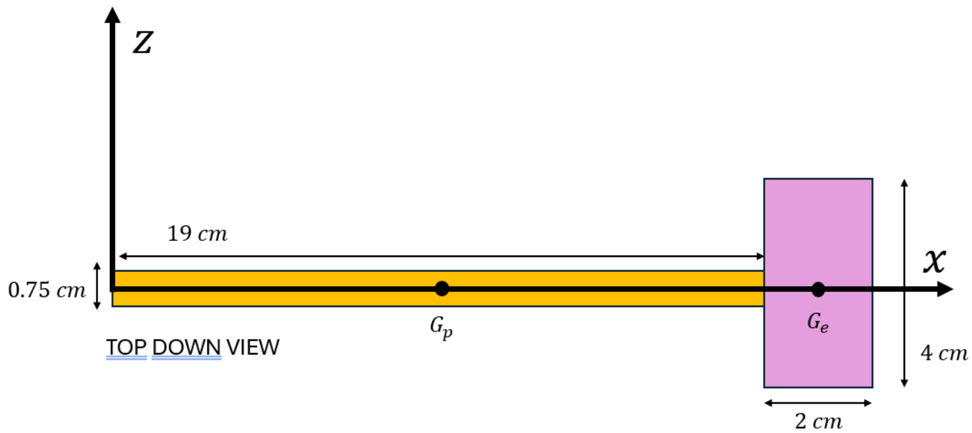
$$m_p = \rho_p v_p = (0.9)(8.3939) = 7.5545\, g$$

- Mass of eraser:

$$v_e = (1)(4)(2) = 8\, cm^3$$

$$m_e = \rho_e v_e = (1.4)(8) = 11.2\, g$$

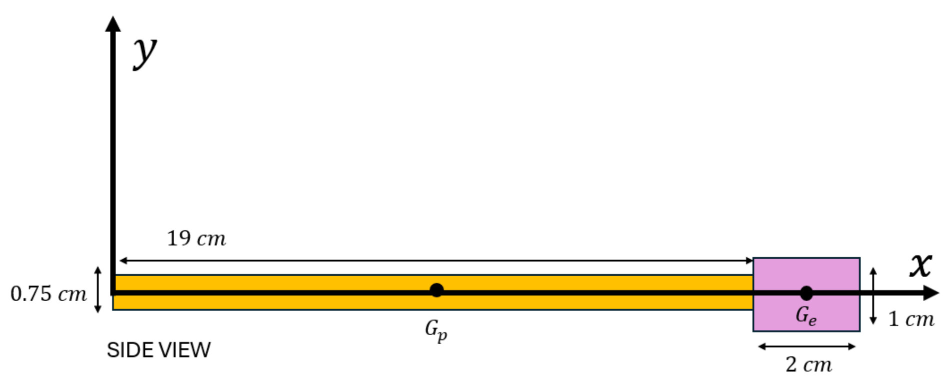
Finding the center of mass:



$$\bar{x} = \frac{m_p(9.5) + m_e(20)}{m_p + m_e}$$

$$\bar{x} = \frac{7.5545(9.5) + 11.2(20)}{7.5545 + 11.2} = 15.7704\,cm$$

$$\bar{z} = \frac{m_p(0) + m_e(0)}{m_p + m_e} = 0\,cm$$



$$\bar{y} = \frac{m_p(0) + m_e(0)}{m_p + m_e} = 0\,cm$$

$$\boxed{G_{\text{sys}} = \langle 15.7704, 0, 0\rangle\ cm}$$

b) Calculate the moment of inertia of the pencil and eraser system about the relevant centroidal axis.

Axis of rotation: z-axis through center of mass

Using the parallel axis theorem:

$$I_{G_{sys}} = I_{G_p} + m_p d_p^2 + I_{G_e} + m_e d_e^2$$

Where:

- I_{G_p}: Moment of inertia of pencil about its centroidal axis

$$I_{G_p} = \frac{1}{12} m_p L^2 = \frac{1}{12}(7.5545)(19^2) = 227.279\, g \cdot cm^2$$

- d_p: Distance from pencil centroid to system centroid

$$d_p = 15.7704 - 9.5 = 6.2704\, cm$$

- I_{G_e}: Moment of inertia of eraser about its centroidal axis

$$I_{G_e} = \frac{1}{12} m_e (b^2 + c^2) = \frac{1}{12}(11.2)(1^2 + 2^2) = 4.666\, g \cdot cm^2$$

- d_e: Distance from eraser centroid to system centroid

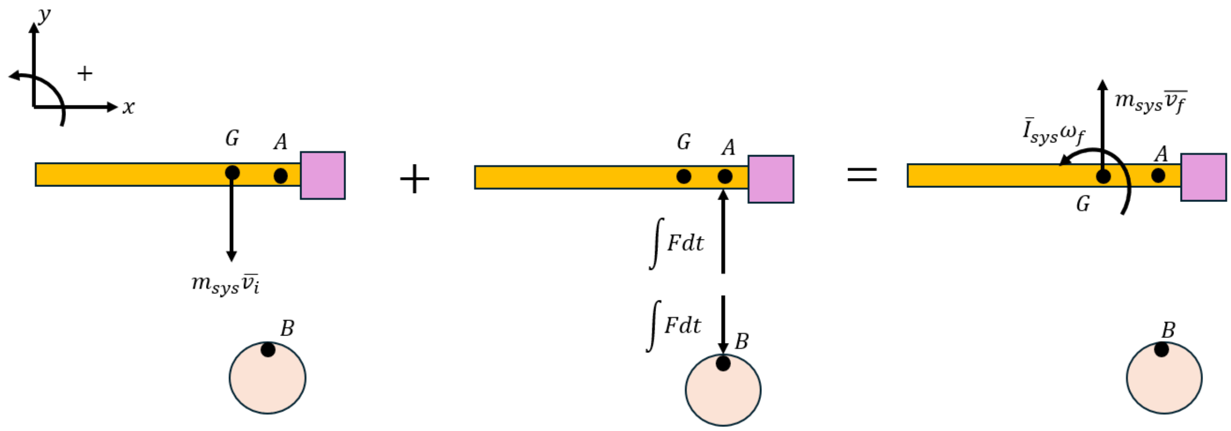
$$d_e = 20 - 15.7704 = 4.2296\, cm$$

Plugging in values:

$$I_{G_{sys}} = 227.279 + 7.5545(6.2704)^2 + 4.666 + 11.2(4.2296)^2$$

$$\boxed{I_{G_{sys}} = 729.334\, g \cdot cm^2}$$

c)



d) After the pencil has fallen 1.5 m, it hits your finger. Find the angular velocity and the velocity of the center of mass just before and just after the impact.

Finding velocity of center of mass just before impact:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.81)(1.5)} = 5.4249\, m/s = 542.49\, cm/s$$

Distance from center of mass to point of impact:

$$d = (21-3)cm - (15.7704)6cm = 2.2296\, cm$$

Conservation of angular momentum about point of impact:

$$m_{\text{sys}}\bar{v}_i(2.2296) = -m_{\text{sys}}\bar{v}_f(2.2296) + \bar{I}_{\text{sys}}\omega_f$$

$$41.815\bar{v}_i = -41.815\bar{v}_f + 729.334\omega_f$$

$$22678.79 = -41.815\bar{v}_f + 729.334\omega_f$$

Coefficient of restitution:

$$v'_B - v'_A = e[v_A - v_B]$$

$$-v'_A = (0.1)v_A$$

$$\implies v'_A = -0.1(-542.49), \quad v_A = \bar{v}$$

$$v'_A = 54.249\hat{\jmath}$$

Kinematics:

$$v'_A = \bar{v}_f + \omega\hat{k} \times r_{A/G}$$

$$v'_A = \bar{v}_f + 2.2296\omega\hat{\jmath}$$

$$\implies \bar{v}_f = 54.249\hat{\jmath} - 2.2296\omega\hat{\jmath}$$

Plugging in $\bar{v}_f$ into the angular momentum equation:

$$22678.79 = -41.815(54.249 - 2.2296\omega) + 729.334\omega$$

$$\implies \boxed{\omega_f = 30.3285\, rad/s}$$

Finding $\bar{v}_f$:

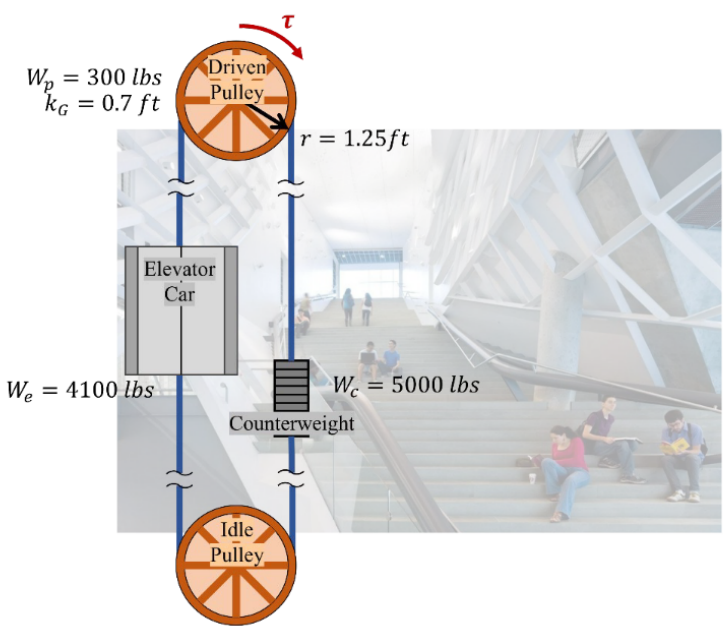
$$\bar{v}_f = 54.249\hat{\jmath} - 2.2296(30.3285)\hat{\jmath}$$

$$\boxed{\bar{v}_f = -13.3714\,(cm/s)\,\hat{\jmath}}$$

Recall that just before impact:

$$\boxed{\bar{v}_i = -542.49\,(cm/s)\,\hat{\jmath} \quad \omega_i = 0\, rad/s}$$

Problem 3: 41CS Elevator



Given:

- The cable does not slip
- Each pulley has weight $W_p = 300lbs$, radius $r_p = 1.25ft$, and radius of gyration $k_G = 0.7ft$
- Elevator car weighs $W_e = 4100lbs$
- Counterweight weighs $W_c = 5000lbs$
- One of the pulleys is driven with torque τ

a) Find the moment of inertia of each pulley about its axle.

$$\bar{I}_P = mk_G^2$$

$$\boxed{\bar{I}_P = \frac{300}{32.2}(0.7)^2 = 4.565\, lb \cdot ft^2}$$

b) Find the *constant* torque required for the elevator car to have speed 10 ft/s after moving up 15 ft from the ground floor.

Let the datum be at the ground floor and up be positive. Position 1 is at the ground floor and position 2 is at 15 ft above the ground floor. Assuming at position 1, the elevator car and the pulleys are at rest and the counterweight is at y_0 ft above the ground floor.

$$T_1 + V_{g1} + V_{e1} + U_{1\to 2}^{NC} = T_2 + V_{g2} + V_{e2}$$

$$m_c g y_0 + \tau\Delta\theta = \frac{1}{2}m_e v_f^2 + \frac{1}{2}m_c v_f^2 + 2\left(\frac{1}{2}I_p\omega_f^2\right) + m_e g(15\, ft) + m_c g(y_0 - 15\, ft)$$

$$\tau\Delta\theta = \frac{1}{2}m_e(10\, ft/s)^2 + \frac{1}{2}m_c(10\, ft/s)^2 + I_p\omega_f^2 + m_e g(15\, ft) + m_c g(-15\, ft)$$

Kinematics:

$$\Delta y_e = -\Delta y_c \implies v_e = -v_c$$

$$v = 10\, ft/s = \omega_f r \quad \text{(cable does not slip)}$$

$$\implies \omega_f = \frac{10}{1.25} = 8\, rad/s$$

Finding $\Delta\theta$:

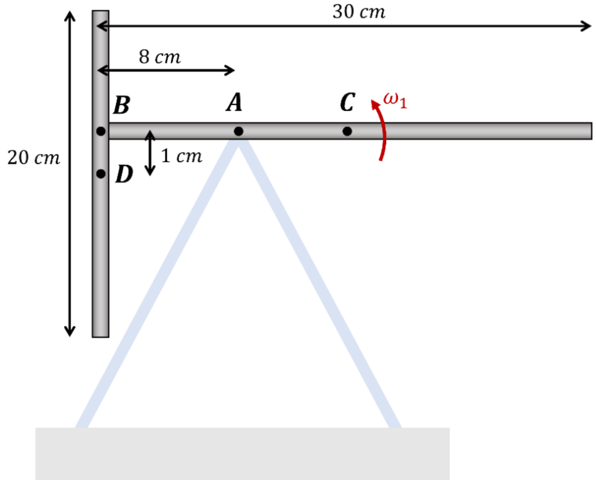
$$\Delta\theta = \frac{\Delta y}{r} = \frac{15\, ft}{1.25\, ft} = 12\, rad$$

Plugging in values:

$$\tau(12) = \frac{1}{2}\left(\frac{4100}{32.2}\right)(10)^2 + \frac{1}{2}\left(\frac{5000}{32.2}\right)(10)^2 + 4.565(8^2) + (4100)(15) + (5000)(-15)$$

$$\implies \boxed{\tau = 76.8828\, lb \cdot ft}$$

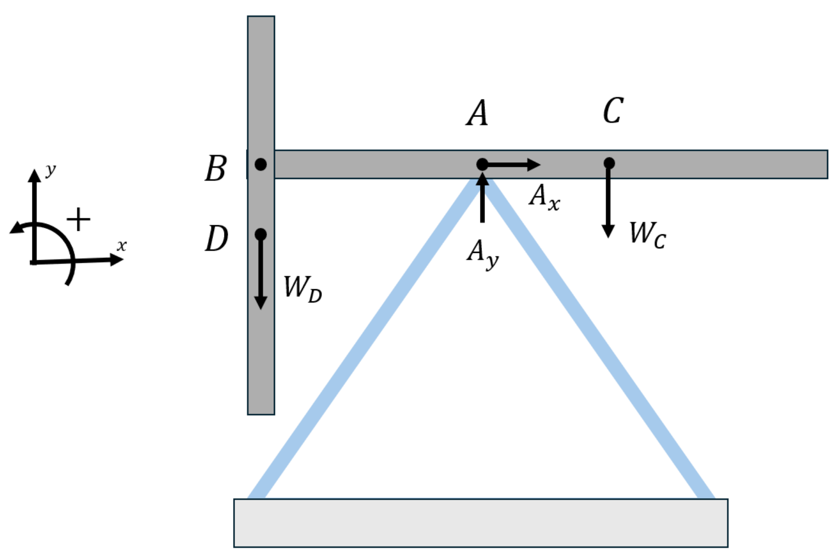
Problem 4: The Swinging Sticks



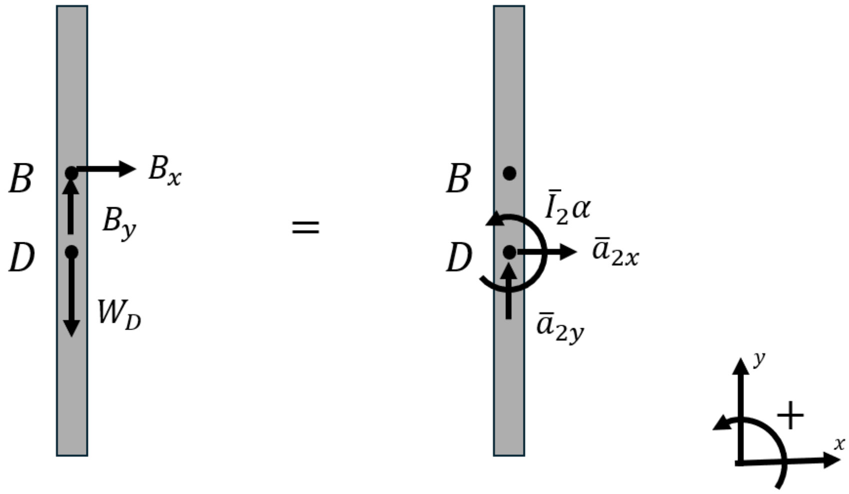
Given:

- Rod BAC has mass $m_1 = 64g = 0.064\,kg$
- Rod BD has mass $m_2 = 43g = 0.043\,kg$
- BAC has center at C, BD has center at D
- At the instant, rod BAC has angular velocity $\omega_1 = 2\,rad/s$ (CCW) and rod BD has no angular velocity $\omega_2 = 0\,rad/s$

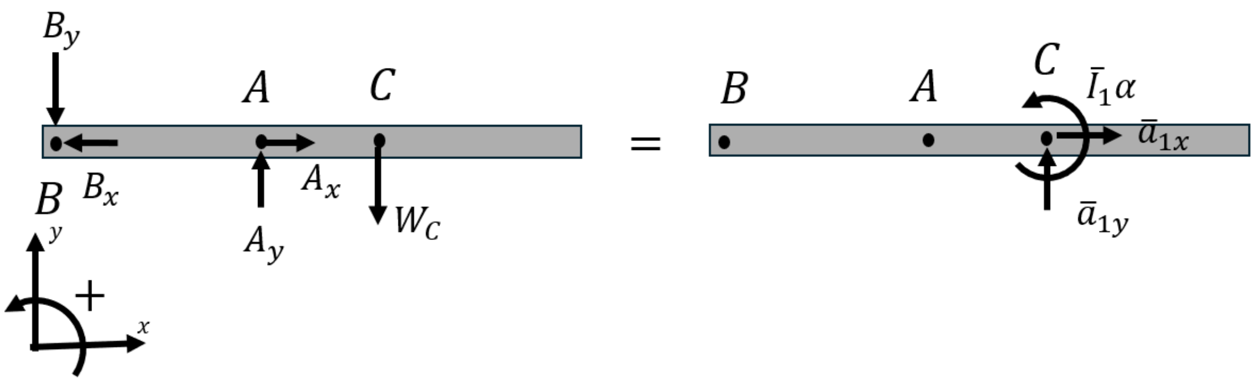
a) Full system FBD



Rod BD KD:



Rod BAC KD:



b) Moments of intertia about center of mass:

- Rod BD:
$$\bar{I}_2 = \frac{1}{12}m_2L_2^2 = \frac{1}{12}(0.043)(0.2^2) = 1.4333 \times 10^{-4}\, kg \cdot m^2$$
- Rod BAC:
$$\bar{I}_1 = \frac{1}{12}m_1L_1^2 = \frac{1}{12}(0.064)(0.3^2) = 4.8 \times 10^{-4}\, kg \cdot m^2$$

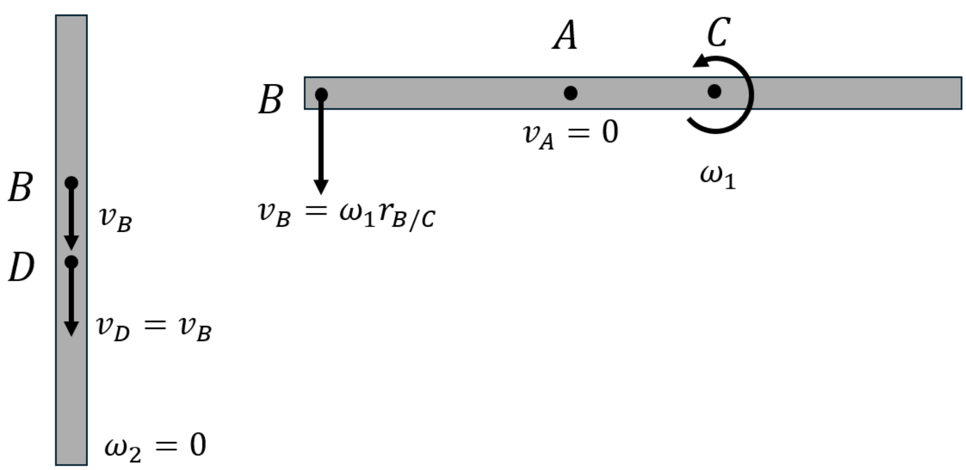
Rod BD force balances:

$$\begin{aligned}
\sum F_x &: B_x = m_2\bar{a}_{2x} \\
\sum F_y &: B_y - m_2 g = m_2\bar{a}_{2y} \\
\sum M_D &: B_x(0.01\, m) = \bar{I}_2\alpha_2
\end{aligned}$$

Rod BAC force balances:

$$\begin{aligned}
\sum F_x &: -B_x + A_x = m_1\bar{a}_{1x} \\
\sum F_y &: -B_y + A_y - m_1 g = m_1\bar{a}_{1y} \\
\sum M_C &: B_y(0.15\, m) - A_y(0.07\, m) = \bar{I}_1\alpha_1
\end{aligned}$$

Kinematics diagram:



Velocities:

$$\begin{aligned}
\mathbf{v_B} =& \mathbf{v_A} + \omega_1\hat{k} \times \mathbf{r_{B/A}} \\
=& (2)\hat{k} \times (-0.08\hat{\imath}) \\
=& -0.16\,(m/s)\,\hat{\jmath}
\end{aligned}$$

Since rod BD is not rotating at the instant, point D and point B have the same velocity:

$$\mathbf{v_D} = \mathbf{v_B} = -0.16\,(m/s)\,\hat{\jmath}$$

Accelerations:

$$\begin{aligned}\mathbf{a_B} =& \mathbf{a_A} + \alpha_1 \hat{k} \times \mathbf{r_{B/A}} - \omega_1^2 \mathbf{r_{B/A}} \\ =& \alpha_1 \hat{k} \times (-0.08\hat{\imath}) - (2)^2(-0.08\hat{\imath}) \\ =& \langle 0.32, -0.08\alpha_1, 0\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{a_C} =& \mathbf{a_A} + \alpha_1 \hat{k} \times \mathbf{r_{C/A}} - \omega_1^2 \mathbf{r_{C/A}} \\ =& \alpha_1 \hat{k} \times (0.07\hat{\imath}) - (2)^2(0.07\hat{\imath}) \\ =& \langle -0.28, 0.07\alpha_1, 0\rangle \\ \implies \mathbf{a_C} =& \mathbf{\bar{a}_1} = \langle -0.28, 0.07\alpha_1, 0\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{a_D} =& \mathbf{a_B} + \alpha_2 \hat{k} \times \mathbf{r_{D/B}} - \omega_2^2 \mathbf{r_{D/B}} \\ =& \langle 0.32, -0.08\alpha_1, 0\rangle + \alpha_2 \hat{k} \times (-0.01\hat{\jmath}) \\ =& \langle 0.32, -0.08\alpha_1, 0\rangle + \langle 0.01\alpha_2, 0, 0\rangle \\ =& \langle 0.32 + 0.01\alpha_2, -0.08\alpha_1, 0\rangle \\ \implies \mathbf{a_D} =& \mathbf{\bar{a}_2} = \langle 0.32 + 0.01\alpha_2, -0.08\alpha_1, 0\rangle\end{aligned}$$

Rewriting system of equations using kinematics

Rod BD force balances:

$$\sum F_x : B_x = m_2(0.32 + 0.01\alpha_2) \tag{1}$$

$$\sum F_y : B_y - m_2 g = m_2(-0.08\alpha_1) \tag{2}$$

$$\sum M_D : B_x(0.01\,\mathrm{m}) = \bar{I}_2 \alpha_2 \tag{3}$$

Rod BAC force balances:

$$\sum F_x : -B_x + A_x = m_1(-0.28) \tag{4}$$

$$\sum F_y : -B_y + A_y - m_1 g = m_1(0.07\alpha_1) \tag{5}$$

$$\sum M_C : B_y(0.15\,\mathrm{m}) - A_y(0.07\,\mathrm{m}) = \bar{I}_1 \alpha_1 \tag{6}$$

Unknowns: $A_x, A_y, B_x, B_y, \alpha_1, \alpha_2$ (6 unknowns and 6 equations)

c) Find the velocity and acceleration of point C at this instant. Leave in terms of $\alpha_{\mathbf{BD}}$ and $\alpha_{\mathbf{BAC}}$.

Using the kinematics derived in part b, where $\alpha_1 = \alpha_{BAC}$ and $\alpha_2 = \alpha_{BD}$:

$$\boxed{\mathbf{a_C} = \langle -0.28, 0.07\alpha_{BAC}, 0\rangle \ m/s^2}$$

Velocity of point C:

$$\begin{aligned}\mathbf{v_C} =& \mathbf{v_A} + \omega_1 \hat{k} \times \mathbf{r_{C/A}} \\ =& (2)\hat{k} \times (0.07\hat{\imath}) \\ =& 0.14\,(m/s)\,\hat{\jmath}\end{aligned}$$

$$\boxed{\mathbf{v_C} = \langle 0, 0.14, 0\rangle \ m/s}$$

d) Find the velocity and acceleration of point D at this instant. Leave in terms of $\alpha_{\mathbf{BD}}$ and $\alpha_{\mathbf{BAC}}$.

Using the kinematics derived in part b, where $\alpha_1 = \alpha_{BAC}$ and $\alpha_2 = \alpha_{BD}$:

$$\boxed{\mathbf{a_D} = \langle 0.32 + 0.01\alpha_{BD}, -0.08\alpha_{BAC}, 0\rangle \ m/s^2}$$

Velocity of point D:

$$\boxed{\mathbf{v_D} = \mathbf{v_B} = -0.16\,(m/s)\,\hat{\jmath}}$$

e) Setting up the equations in part b into matrix form to solve for the unknowns.

Rod BD force balances:

$$\sum F_x : 0A_x + 0A_y + B_x + 0B_y + 0\alpha_1 - 0.01m_2\alpha_2 = 0.32m_2 \tag{1}$$

$$\sum F_y : 0A_x + 0A_y + 0B_x + B_y + 0.08m_2\alpha_1 + 0\alpha_2 = m_2 g \tag{2}$$

$$\sum M_D : 0A_x + 0A_y + 0.01B_x + 0B_y + 0\alpha_1 - \bar{I}_2\alpha_2 = 0 \tag{3}$$

Rod BAC force balances:

$$\sum F_x : A_x + 0A_y - B_x + 0B_y + 0\alpha_1 + 0\alpha_2 = -0.28m_1 \tag{4}$$

$$\sum F_y : 0A_x + A_y + 0B_x - B_y - 0.07m_1\alpha_1 + 0\alpha_2 = m_1 g \tag{5}$$

$$\sum M_C : 0A_x - 0.07A_y + 0B_x + 0.15B_y - \bar{I}_1\alpha_1 + 0\alpha_2 = 0 \tag{6}$$

Matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & -0.01m_2 \\ 0 & 0 & 0 & 1 & 0.08m_2 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & -\bar{I}_2 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -0.07m_1 & 0 \\ 0 & -0.07 & 0 & 0.15 & -\bar{I}_1 & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0.32m_2 \\ m_2 g \\ 0 \\ -0.28m_1 \\ m_1 g \\ 0 \end{bmatrix}.$$

Python code to solve the matrix:

```
    import numpy as np
# --------------------------------------------------
# Constants
# --------------------------------------------------
m1 = 0.064 #rod BAC kg
m2 = 0.043 #rod BD kg
I1 = 4.8e-4 #rod BAC kg*m^2
I2 = 1.4333e-4 #rod BD kg*m^2
g = 9.81 #m/s^2

# Coefficient matrix
A = np.array([
    [0,     0,     1,     0,     0,         -0.01*m2],
    [0,     0,     0,     1,     0.08*m2,   0],
    [0,     0,     0.01,  0,     0,         -I2],
    [1,     0,    -1,     0,     0,          0],
    [0,     1,     0,    -1,    -0.07*m1,   0],
    [0,    -0.07,  0,     0.15, -I1,     0]
])

# Righ hand side
b = np.array([
    0.32*m2,
    m2*g,
    0.0,
    -0.28*m1,
    m1*g,
    0.0
])

# Solve the system
x = np.linalg.solve(A, b)

# Unpack results
Ax, Ay, Bx, By, alpha1, alpha2 = x

# Print results
print(f"A_x = {Ax:.4g} N")
print(f"A_y = {Ay:.4g} N")
print(f"B_x = {Bx:.4g} N")
print(f"B_y = {By:.4g} N")
print(f"alpha_1 = {alpha1:.4g} rad/s^2")
print(f"alpha_2 = {alpha2:.4g} rad/s^2")
```

Solutions:

- $A_x = -0.003734\,N$
- $A_y = 1.04\,N$
- $B_x = 0.01419\,N$
- $B_y = 0.4547\,N$
- $\alpha_{BAC} = \alpha_1 = -9.546\,rad/s^2$
- $\alpha_{BD} = \alpha_2 = 0.9897\,rad/s^2$