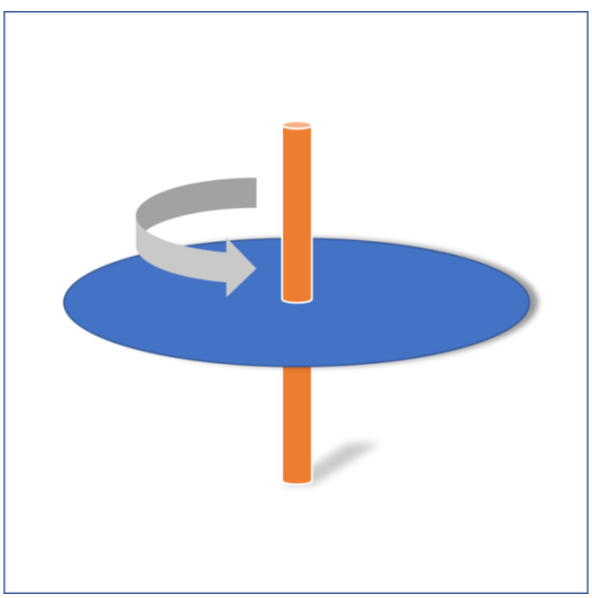
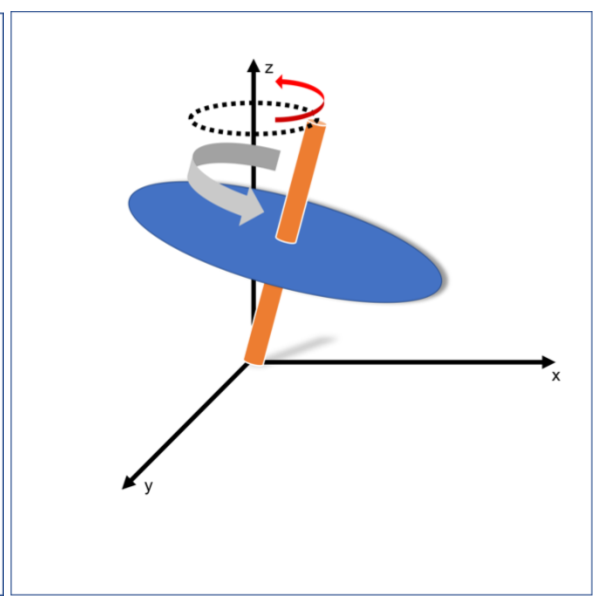
Proposal: A series of experiments based on gyroscope

Yifeng Chen & Zeshun Zong

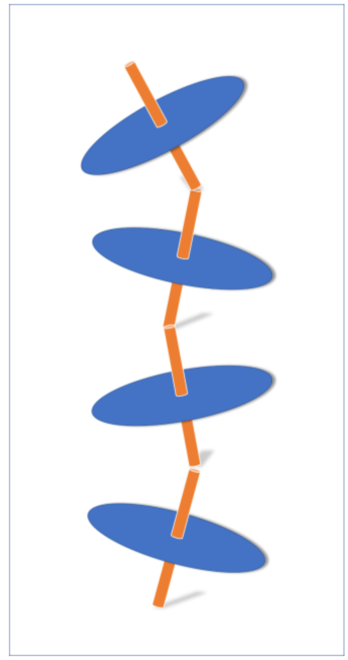
The first part of our project is to study how a gyroscope's mass and the velocity at which it rotates relates to its stability. As shown in the graph, the basic gyroscope model consists of one axis (two endpoints) and a wheel. At perfect equilibrium, the gyroscope should be rotating and standing vertically upward. It is expected that, with slight perturbation, the gyroscope would still be able to stand. We will start with modeling this behavior. The gyroscope is placed on a horizontal surface (ground) that provides normal force. We then tilt the gyroscope so that the axis of the gyroscope is not perpendicular to the ground. The normal force applied to the bottom endpoint of the axis then generates a torque that keeps the axis of the gyroscope rotating around the z-axis. (as shown in the second diagram)

The mass distribution and the angular velocity of the gyroscope will be altered, and the resulting behavior shall be observed and compared.





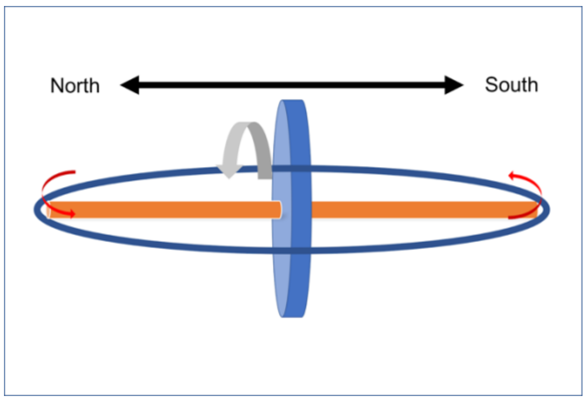
The second part of our project is to model a stack of gyroscopes. The bottom gyroscope is placed on a fixed point. Then we add a second gyroscope on top of the bottom one. This can be achieved by connecting the top of the axis of the first gyroscope and the bottom of that of the second one with a “spring and dashpot” structure. We repeat this process and get a stack of n gyroscopes. Each gyroscope in the system rotates at a different velocity. Note that all the gyroscopes except the bottom one are not only rotating, but also moving in terms of the position of the endpoint of the axis.



The main goal is to study if this system can achieve an equilibrium (equilibrium means the entire system has a periodic behavior). The system is obviously stable when every gyroscope is standing perfectly upward. But whether or not the system will collapse when gyroscopes are slightly tilted is unclear. We can then again change the mass and velocity of each gyroscope to see its effect on the equilibrium.

The last part of our project is to study the use of gyroscope in an inertial guidance system, i.e. gyrocompass. The gyroscope used in an inertial guidance system is different from the previous setup. It is mounted side-ward on a frame which is tangent to the surface of the earth. That is, the axis can only move freely in the horizontal plane (see diagram below). Since the gyroscope has the property to maintain the current axis of rotation, when we move the frame across earth’s surface and keep it tangent to earth’s surface, the frame exerts a force on the gyroscope to keep it in the horizontal plane and thus generate a torque (the direction of which is in the plane) to achieve the effect of a compass.

The direction that the axis of the gyroscope points to will be first studied. We will then try to figure out a mechanism, such as adding weights, so that the gyroscope points to the true north. Our original model does not include friction when the axis is rotating within the frame. This might cause oscillation given there is nothing to slow down the horizontal rotation. If this occurs, we will add friction to the system and study the relation between friction and the oscillating periods.



Methodology

In this project, gyroscopes will be modeled with rigid body motion. The position and velocity of the points taken on the wheel and axis will be updated using forward Euler method. In the first model, we place the gyroscope on the surface by setting a horizontal plane (z=0). The gyroscope receives a normal force when it sinks below the surface level. This method does not yield the fixed point effect. That is, the bottom endpoint may move freely on the surface. If time permits, we may also replace the normal force by connecting the endpoint to a fixed point with a spring and a dashpot to relatively fix the position of the endpoint. And we will compare these two methods.

In the second model, the first gyroscope is again placed on a surface equipped with the normal force. And the stack of gyroscopes are held together by springs and dashpots.