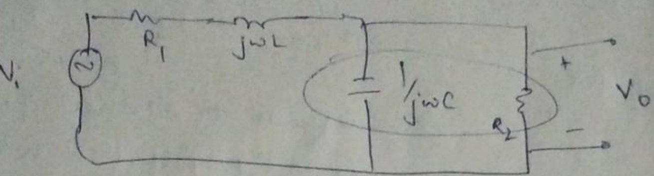
# Tutorial 4

(1)



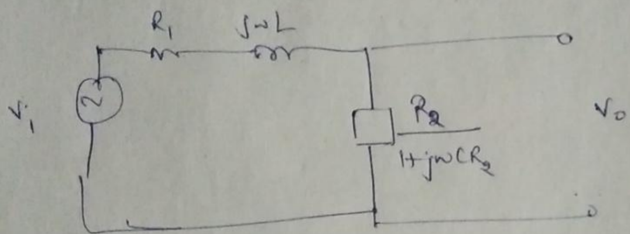

$$R_2 \parallel \frac{1}{j\omega C} = \frac{R_2 \times \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}$$

$$= \frac{R_2/j\omega C}{\frac{j\omega C R_2 + 1}{j\omega C}} = \frac{R_2}{1 + j\omega C R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$




$$\frac{V_o}{V_i} = \frac{R_2/1+j\omega C R_2}{R_1 + j\omega L + \frac{R_2}{1+j\omega C R_2}} = \frac{R_2/1+j\omega C R_2}{\frac{R_1(1+j\omega C R_2) + j\omega L(1+j\omega C R_2) + R_2}{1+j\omega C R_2}}$$

$$= \frac{R_2}{R_1 + j\omega C R_2 R_1 + j\omega L + \underline{j^2\omega^2 L C} R_2 + R_2}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

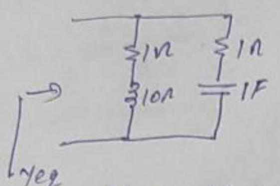
$$\sqrt{LC} = \frac{1}{\omega}$$

$$LC = 1/\omega^2$$

$$= \frac{R_2}{R_1 + j\omega C R_2 R_1 + j\omega L - \cancel{R_2} + \cancel{R_2}}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{R_2}{R_1 + j\omega L + j\omega C R_2 R_1}$$

**Q.2 (a)**




$$Y_1 = \frac{1}{1+j10\omega} \times \frac{1-j10\omega}{1-j10\omega}$$

$$Y_1 = \frac{1-j10\omega}{1+100\omega^2}$$

$$Y_{eq} = Y_1 + Y_2$$

$$Y_2 = \frac{1}{1+\frac{1}{j\omega \cdot 1}} = \frac{j\omega}{1+j\omega} \times \frac{1-j\omega}{1-j\omega} = \frac{\omega^2 + j\omega}{1+\omega^2}$$

$$Y_{eq} = Y_1 + Y_2 = \frac{1}{1+100\omega^2} - \frac{j10\omega}{1+100\omega^2} + \frac{j\omega}{1+\omega^2} + \frac{\omega^2}{1+\omega^2}$$

$$Y_{eq} = \left(\frac{1}{1+100\omega^2} + \frac{\omega^2}{1+\omega^2}\right) + j\left(\frac{\omega}{1+\omega^2} - \frac{10\omega}{1+100\omega^2}\right)$$

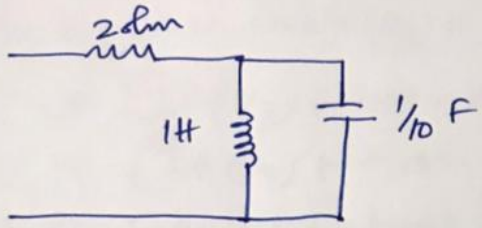
for resonance frequency put imaginary term equal to zero

$$j\left(\frac{\omega}{1+\omega^2} - \frac{10\omega}{1+100\omega^2}\right) = 0$$

$$\frac{1}{1+\omega^2} = \frac{10}{1+100\omega^2}$$

$$\omega = \frac{1}{\sqrt{10}} \text{ rad/sec}$$

## 2(b)




This is a tank circuit. Resonance will occur when $X_L = X_C$

$$\Rightarrow SL = \frac{1}{SC}$$

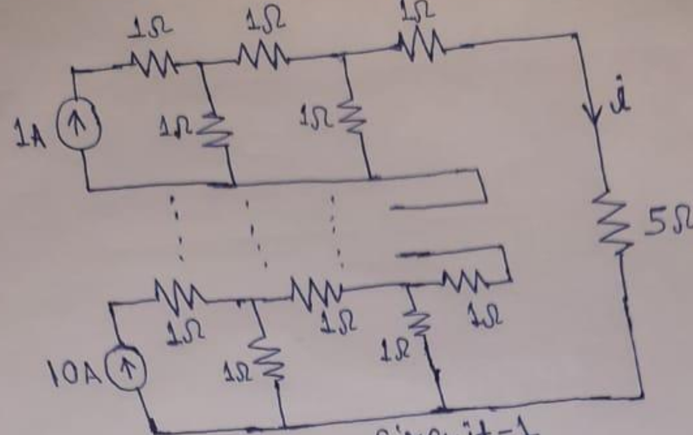
$$\Rightarrow S^2 = \frac{1}{LC} \quad \Rightarrow |\omega^2| = \frac{1}{LC}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{1 \times \frac{1}{10}}}$$

$$\Rightarrow \omega = \sqrt{10} \text{ rad}.$$

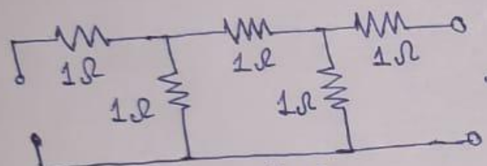
Sol-3



Circuit-1

(a) Using superposition, keeping Nth current source active and others to open circuit.

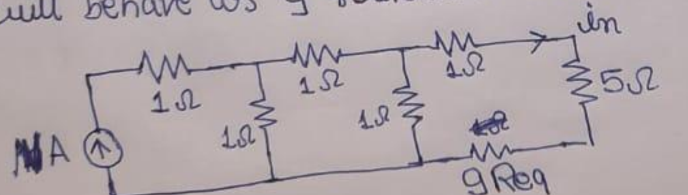
(b) When the current source is open the equivalent subcircuit look as follow:



$\Leftarrow R_{eq} = \frac{2\times 1}{3} + 1 = \frac{5}{3}\,\Omega$

Circuit-2

(c) applying super-position for Nth current source's contribution in $i$, other subcircuits will behave as 9 resistors in series of value $\frac{5}{3}\,\Omega$.



$9R_{eq} = 15\,\Omega$

$$i_n = (1/65)\,N\ \text{A}$$

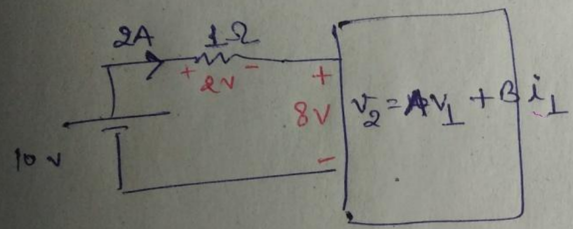
Similarly, we can find $i_1, i_2, \ldots i_{10}$

$$i = i_1 + i_2 + \ldots i_{10} = \frac{1}{65}(1+2+3\ldots+10)$$

$$i = \frac{1}{65}\left[\frac{10\times(10+1)}{2}\right] = \frac{11}{13} = 0.846\ \text{A}$$

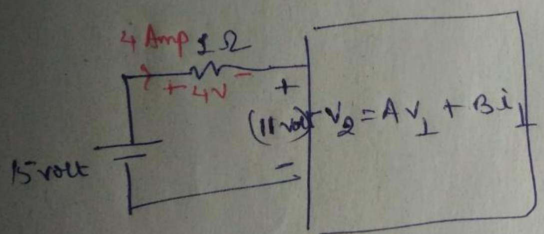
(4) when $v_1 = 10$ volt $\quad i_1 = 2A$

when $v_1 = 15$ volt $\quad v_2 = 11V$




$\Rightarrow v_2 = 10A + 2B$

$\Rightarrow 8 = 10A + 2B$



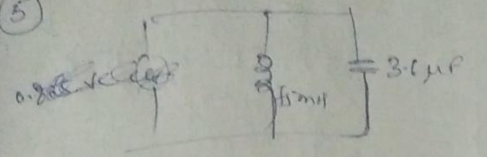

$\Rightarrow 11 = 15A + 4B$

$8 = 10A + 2B$

$11 = 15A + 4B$

$\Rightarrow A = 1$ and $B = -1$

(5)




max. charge on capacitor $= 2.9 \times 10^{-6}$ C

$\Rightarrow$ at this point capacitor is open

$\Rightarrow$ all current is flowing through inductor

$$Q = CV$$

$$2.9 \times 10^{-6} = 3.6 \times 10^{-6} \; V_{max}$$

$$\Rightarrow V_{max} = 0.805 \text{ volt}$$

at this point all energy is stored in capacitor

$$\text{energy} = \frac{1}{2} C V_{max}^2 = \frac{1}{2} \times 3.6 \times 10^{-6} \times (0.805)^2$$

$$= 1.16 \times 10^{-6}$$

$\Downarrow$

this same energy will be stored in capacitor, when current through inductor is maximum and voltage across capacitor is zero

$$\text{energy} = \frac{1}{2} L \, i_{max}^2$$

$$\Rightarrow 1.16 \times 10^{-6} = \frac{1}{2} \times 75 \times 10^{-3} \times i_{max}^2$$

$$\Rightarrow i_{max}^2 = \frac{1.16 \times 10^{-6} \times 2}{75 \times 10^{-3}} = 0.0309 \times 10^{-3} = 30 \times 10^{-6} \text{ A}$$

$$i_{max} = \sqrt{30 \times 10^{-6}} \text{ A} = 5.477 \times 10^{-3} \text{ A} = 5.477 \text{ mA}$$

Sol 6

a) True : Resistor will dessipate the total energy (in form of electric and magnetic fields) in form of heat.

b) False : $Z = R + j\omega L - \frac{j}{\omega C} = R$ at $\omega_0 = \frac{1}{\sqrt{LC}}$

Phase is $0^\circ$

c) False : $X_C = \frac{1}{j\omega C} \Rightarrow \omega \uparrow \rightarrow X_C \downarrow$

d) True : Energy get stored in form of magnetic fields in inductor and electric fields in capacitor intead of getting dessipated.

e) False : $|Z| = \sqrt{R^2 + (X_L + X_C)^2} = R$ at $\omega_0 = \frac{1}{\sqrt{LC}}$

Current amplitude is maximum.

f) ~~False~~ True : $\phi = \tan^{-1}\left(\frac{\omega L - \ldots}{R}\right)$ : ~~depends on reactances~~

g) True : $Z = R + j\omega L \left(-\frac{j}{\omega C}\right)$ : for $\omega$, very high $Z \simeq j\omega L$

h) True : $|Z| = \sqrt{R^2 + (X_L + X_C)^2} = R = Z_{min}$ at $\omega_0 = \frac{1}{\sqrt{LC}}$