Probability I (SI 427)
Department of Mathematics, IIT Bombay
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Problem set 5

1. Suppose $X \sim U(0,1)$. Find the density of $Y = -\lambda^{-1}\log(1-X)$ for $\lambda > 0$.

2. Log-normal distribution: Let $Y = e^X$ where X has the $N(0,1)$ distribution. Find the density function of Y.

3. Find the value of c so that the following ϕ is a density.
$$f(x) = ce^{-x^2/2}, \quad -\infty < x < \infty.$$

4. Let X is a continuous random variable with density f. Find a formula for density function of the random variable $Y = |X|$.

5. Let X be a positive continuous random variable with density f. Find a formula for density function of the random variable $Y = 1/(1+X)$.

6. Suppose X is a continuous random variable with strictly increasing distribution function F. Show that the random variable $Y = F(X)$ is uniformly distributed on $(0,1)$.

7. Suppose X is a discrete random variable with distribution function F. Is $F(X)$ uniformly distributed on $(0,1)$?

8. Suppose (X,Y) has joint density
$$f(x,y) = ce^{-(x^2-xy+4y^2)/2}, \quad -\infty < x, y < \infty.$$
Find the value of c. Find marginal densities $f_X(x), f_Y(y)$.

9. Suppose (X,Y) is uniformly distributed over the area bounded by $y^2 = x$ and $x = 4$. Find the joint distribution of X and Y, and $\mathrm{P}(X < 3, Y < 0)$.

10. The joint density function of a random variables X, Y is given by
$$f_{X,Y}(x,y) = \begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$
Find the density of $X+Y$.

11. Suppose (X,Y) has the following joint density function
$$f(x,y) = \begin{cases} \frac{6-x-y}{8} & 0<x<2, 2<y<4 \\ 0 & \text{else where} \end{cases}$$
Find $P(X<1, Y<3)$, $P(X+Y \le 3)$.

12. Let X, Y i.i.d. continuous random variables. Find $\mathrm{P}(X > Y)$.