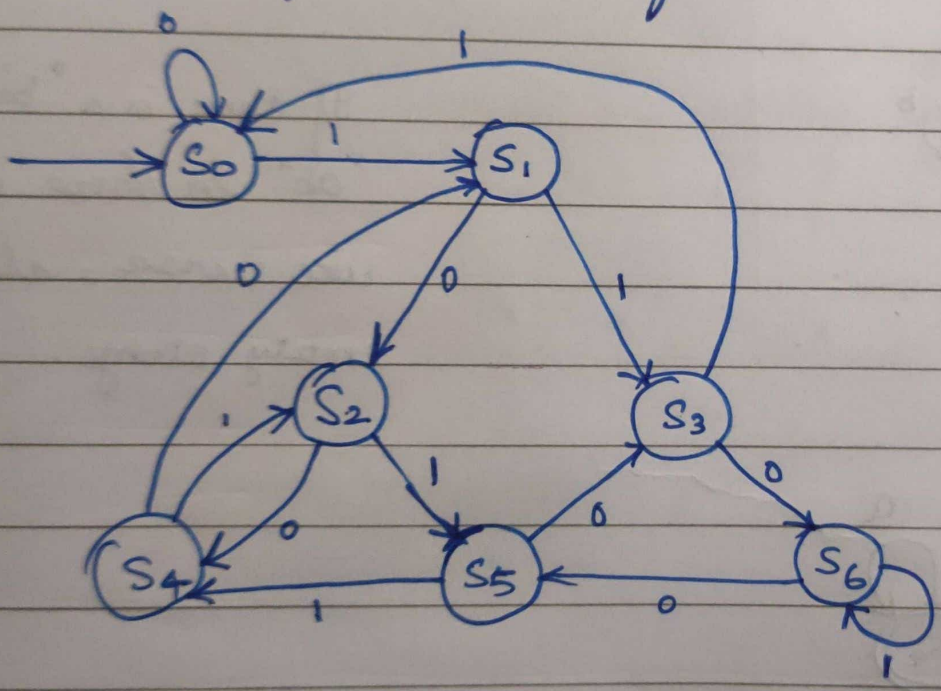
HW 20005010 3 P.Balasubramanian

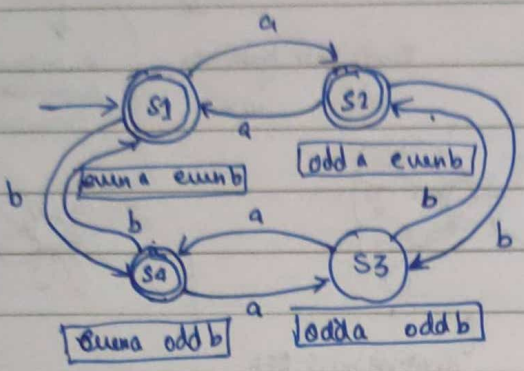
- DFA

$L_1 = \{ w = \{0,1\}^* \mid decimal(w) = 2 \pmod 7 \}$

7 states, each for remainder from 0-6 on dividing by 7

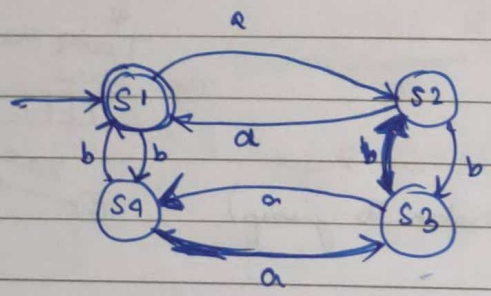


2) $L_2 = \{ w \in \{a,b\}^* \mid n_a(w) \equiv 0 \pmod 2 \text{ (even)} \quad (OR) \quad n_b(w) \equiv 0 \bmod 2 \}$



4 states, each for possible parity (odd/even) combination of 2 alphabets

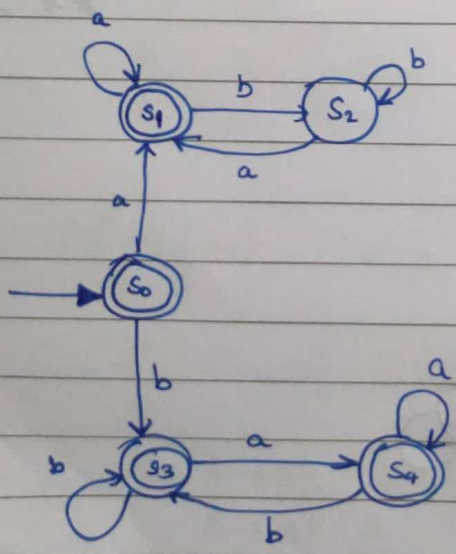
3) $L_3 = \{ w = \{a,b\}^* \mid n_a(w) \equiv 0 \pmod 2 \text{ AND } n_b(w) \equiv 0 \pmod 2 \}$



Same as before only the accepting states changed.

4) $L_4 = \{ w = \{a,b\}^* \mid n_{ab}(w) = n_{ba}(w) \}$ ("ab" and "ba" occur same number of times

(substrings "ab" and "ba" occur the same no. of times



If there is a "ba", waiting for "ab" to show up and vice versa, also considering empty strings.

CFG

1) $L_5 = \{ w \in \{ab\}^* \mid n_a(w) > n_b(w) \}$

~~$S \rightarrow Sb \mid bS \mid b \mid aSb \mid bSa \mid \epsilon$~~

$S \rightarrow Sa \mid a \mid aS_1$

$S_1 \rightarrow \epsilon$ ~~$\mid Sb$~~

$S_1 \rightarrow aS_1b \mid bS_1a$

2) $L_6 = \{ ubabw \mid u, w \in \{a,b\}^* ; |u| = |w| \}$

~~$S \rightarrow bA \mid Bb$~~

$S \rightarrow aSa \mid bSb \mid aSb \mid bSa \mid bab$

Keeping number of alphabets same at both ends of S ; bab at last.

3) $L_8 = \{ a^i b^j c^k \mid i, j, k \geq 0 ; i = 2k \}$

$S \rightarrow aaSc \mid A$ # of a's = 2x # of c's always followed by b^*

$A \rightarrow \epsilon \mid bA$ { Grammar for b^* }

4) $L_9 = \{ a^i b^j c^k \mid i, j, k \geq 0 ; i + 2j = 3k \}$

~~$S \rightarrow aSc \mid a$~~

~~$S \rightarrow aaaSc \mid aAbAc$~~ ~~$A \rightarrow bAc \mid \epsilon$~~

$S \rightarrow aaaSc \mid aabbAcc \mid abAc \mid \epsilon$ (for starting a's and corresponding c's

$A \rightarrow bbbAcc \mid \epsilon$ ↳ middle b's and corresponding c's

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HW-1

Unrestricted Grammar

3.1 $L_{10} = \{ a^{2^n} \mid n \geq 0 \}$

$S \rightarrow BXaE \mid a$ Beginning as B, end as E and multiplier as X going from B to E

$BX \rightarrow BXX$ Increasing X's

$Xa \rightarrow aaX$ Doubling no. of a's as X passes through

$XE \rightarrow E$ Removing X if reached E

$Baa \rightarrow aBa$ B passes through string

$BaE \rightarrow a$ Removing B and E if no X remains and B is near E

3.2 $L_{11} = \{ a^i b^j \mid i \geq 1 ; j \geq 0 ; j \equiv 0 \pmod{i} \}$

$S \rightarrow BXAE \mid A$ Beginning as B, end as E with A as aa^* and X as multiplier

$A \rightarrow Aa \mid a$ Grammar for aa^*

$B \rightarrow BX \mid \epsilon$ Increasing number of X's (multiplicative factor) at the beginning

$Xa \rightarrow aXY$

$Ya \rightarrow aY$ Y's passing through till E or b

$YE \rightarrow b$ if Y at end, change it to b and vanish E

$Yb \rightarrow bb$ If Y see b, then only b's are after this - change Y to b

$Xb \rightarrow b$ If X see b, then only b's are after this, change X to b

3.3) $L_{12} = \{ a^i b^j c^k \mid i, j, k \geq 0 \; ; \; i < j < 2k \}$

didn't figure out completely. written till what I could find

$S \rightarrow PQR$

$P \rightarrow Pa \mid A$ } generate a's

$Aa \rightarrow aAT$ }

$Ta \rightarrow aT$ } generate b's

$TQ \rightarrow Qb$

$AQ \rightarrow Bb$

3.4) $L_6 = \{ ww \mid w = \{a,b\}^* \}$

$S \rightarrow PR$

$P \rightarrow PaA \mid PbB \mid R$ } Transfer ww to a w aw or bw bw

$Aa \rightarrow aA$

$BAb \rightarrow bA$

$Ba \rightarrow aB$

$Bb \rightarrow bB$

$AR \rightarrow Ra$

$BQ \rightarrow Ab$

$Ra \rightarrow aR$ } end the generation

$Rb \rightarrow bR$

$RR \rightarrow \epsilon$