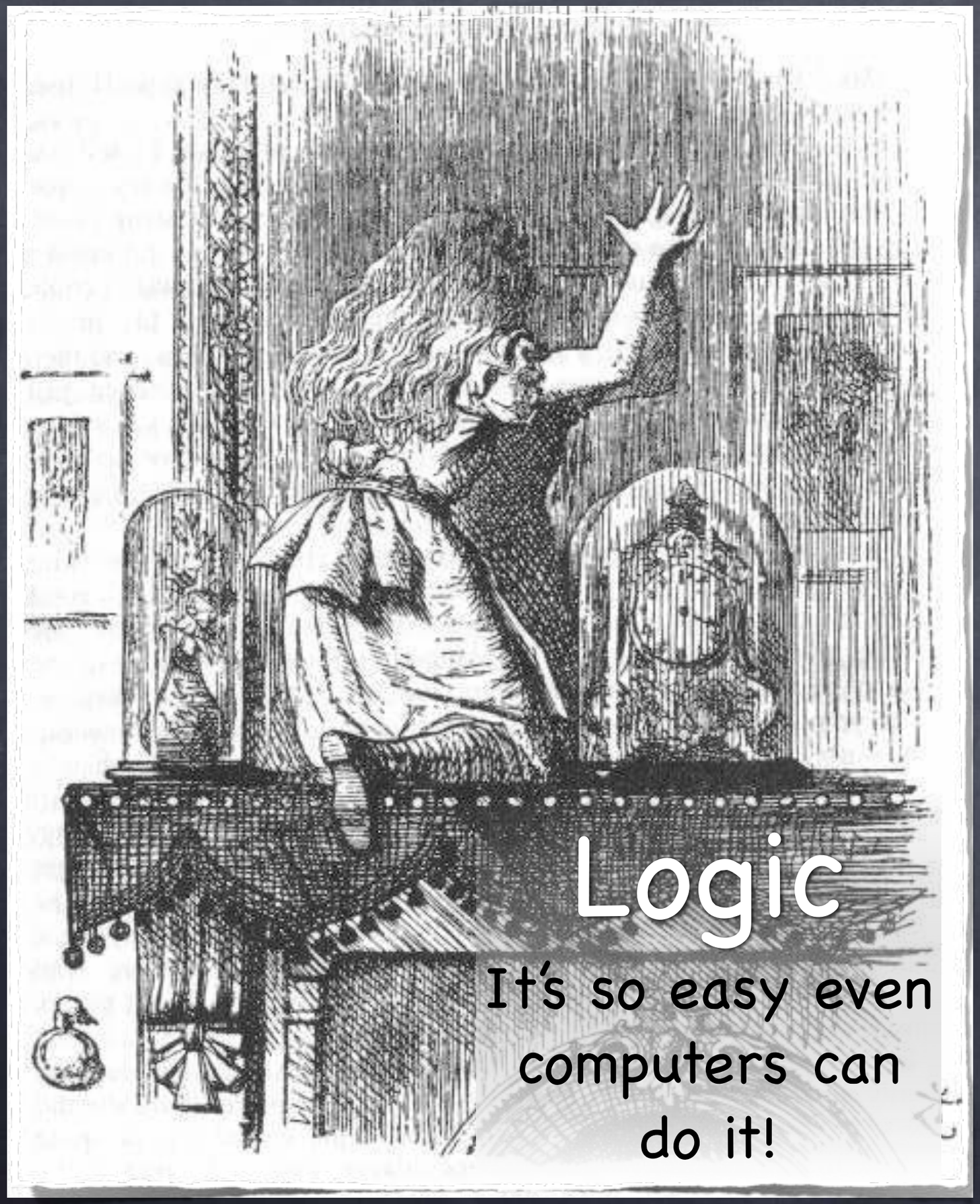


Through the Looking Glass

The Looking Glass

- A mirror which shows the negation of every proposition
- Reflection changes T & F to F & T (resp.)
  - ∨ & ∧ are reflected as ∧ & ∨ (resp.)

Flies(Alice) ∨ Flies(J'wock) is True

∨	T	F
T	T	T
F	T	F

∧	F	T
F	F	F
T	F	T

¬ Flies(Alice) is True

?	F	T
F	F	F
T	F	T

∨	T	F
T	T	T
F	T	F

¬Flies(Alice) ?
¬Flies(J'wock) is False

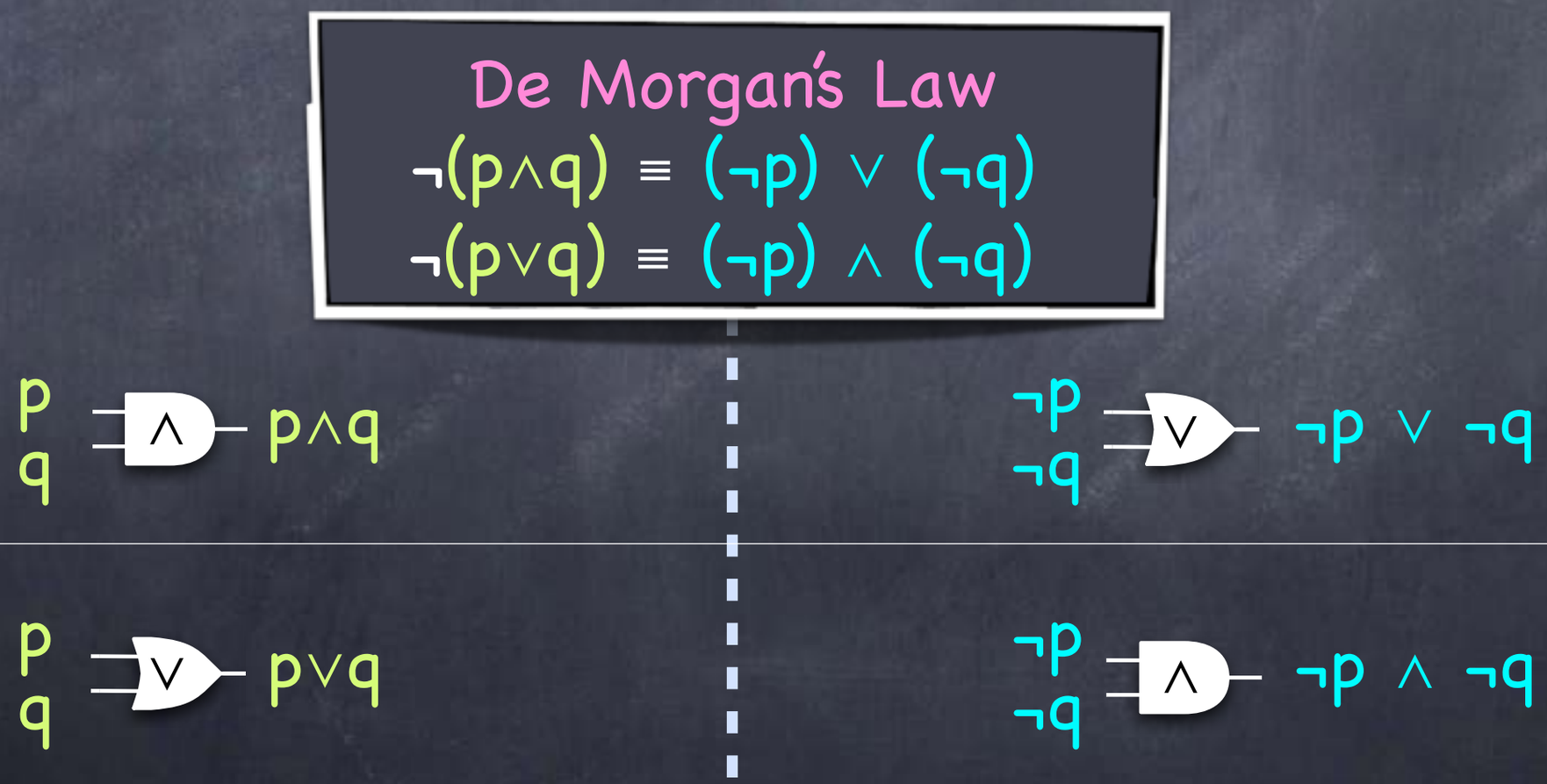
The Looking Glass

- A mirror which shows the negation of every ~~proposition~~ wire
- Reflection changes T & F to F & T (resp.)
  - ∨ & ∧ are reflected as ∧ & ∨ (resp.)

De Morgan's Law

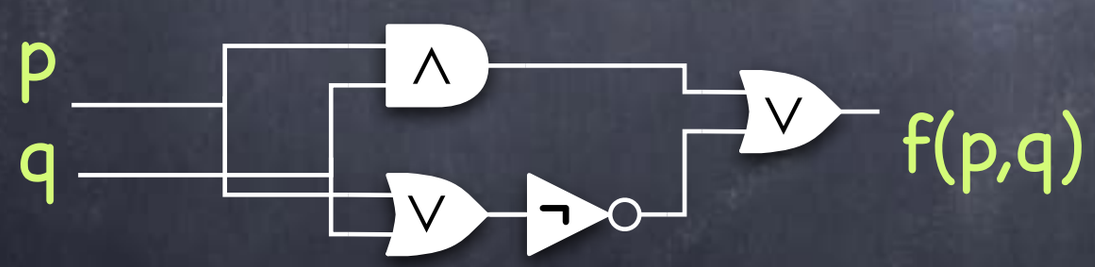
$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$

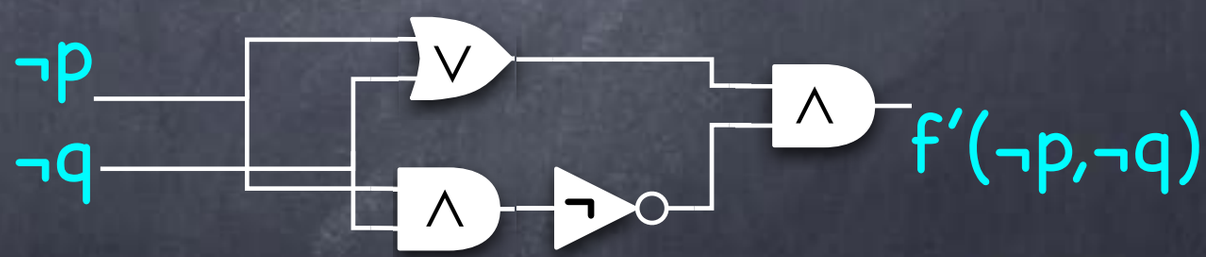
$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$



The Looking Glass

- A mirror which shows the negation of every ~~proposition~~ wire
- Reflection changes T & F to F & T (resp.)
  - ∨ & ∧ are reflected as ∧ & ∨ (resp.)

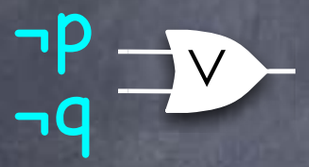


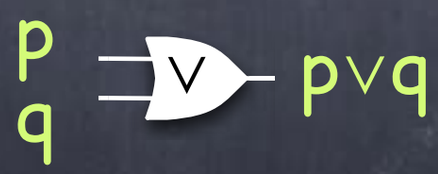


$$\neg\ f(p,q) \equiv f'(\neg p, \neg q)$$

The Looking Glass

- Reflection changes T & F to F & T (resp.)
  - ∨ & ∧ are reflected as ∧ & ∨ (resp.)
  - ∀ & ∃ are reflected as ∃ & ∀ (resp.)





∀x Pred(x)

∃x ¬Pred(x)

∃x Pred(x)

∀x ¬Pred(x)

Two quantifiers

x	y	Likes(x,y)	∃y Likes(x,y) i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

- ∀x ∃y Likes(x,y)
  - Everyone likes someone
  - ∀x LikesSomeone(x)
  - True

Two quantifiers

x	y	Likes(x,y)	∃y Likes(x,y) i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

- ∀x ∃y Likes(x,y)
  - Everyone likes someone
  - ∀x LikesSomeone(x)
  - True

- ∃x ¬(∃y Likes(x,y))

Two quantifiers

x	y	Likes(x,y)	∃y Likes(x,y) i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

- ∀x ∃y Likes(x,y)
  - Everyone likes someone
  - ∀x LikesSomeone(x)
  - True

- ∃x ∀y ¬Likes(x,y)
  - Someone doesn't like anyone
  - ∃x DoesntLikeAnyone(x)
  - False

Two quantifiers

x	y	Likes(x,y)
Alice	Alice	TRUE
	Jabberwock	FALSE
	Flamingo	TRUE
Jabberwock	Alice	FALSE
	Jabberwock	TRUE
	Flamingo	FALSE
Flamingo	Alice	FALSE
	Jabberwock	FALSE
	Flamingo	TRUE

- ∃y ∀x Likes(x,y)

Two quantifiers

x	y	Likes(x,y)	∀x Likes(x,y) i.e., EveryoneLikes(y)
Alice	Alice	TRUE	FALSE
Jabberwock		FALSE	
Flamingo		FALSE	
Alice	Jabberwock	FALSE	FALSE
Jabberwock		TRUE	
Flamingo		FALSE	
Alice	Flamingo	TRUE	FALSE
Jabberwock		FALSE	
Flamingo		TRUE	

- ∃y ∀x Likes(x,y)
  - Someone is liked by everyone
  - False

- ∀y ∃x ¬Likes(x,y)
  - Everyone is disliked by someone
  - True