

Physical Layer: Theory

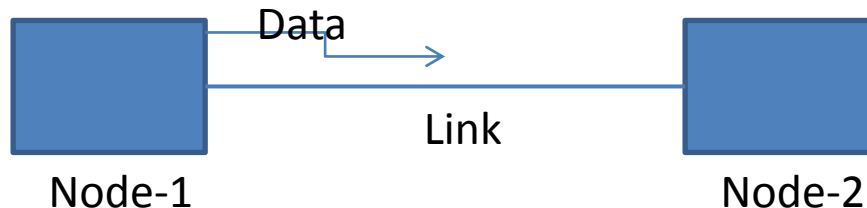
Kameswari Chebrolu

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Recap

- Nodes generate data (bits: 1's and 0's)
- Links carry signals in the form of electromagnetic waves
- Task on hand: Convert data into signals
 - Process termed: Encoding/Modulation
- First: Some Theory



Link Characteristics

- **Data Rate**: How many bits per second can be transferred on the link? (expressed in bps,kbps,Mbps,Gbps)
- **Loss rate**: What is the probability of packet error (or bit error) rate on the link?
- **Delay**: How much time does it take for the bits to reach other end?

Simple Encoding

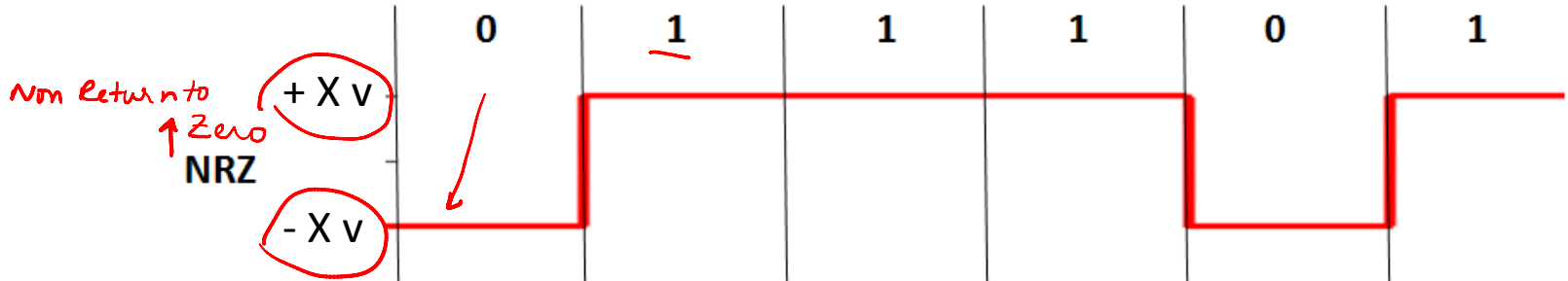
- Data: 10111011

You _____ Your Friend (Far Away)

Wire Pair

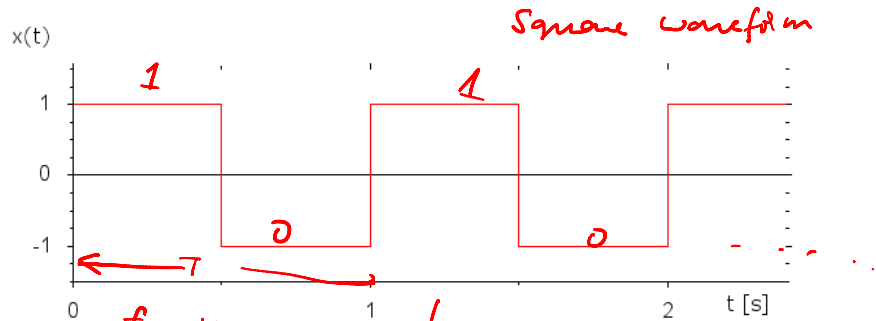
~~I will follow the wire, reach other end and convey the data in person~~

- How would you send the data over the wires?

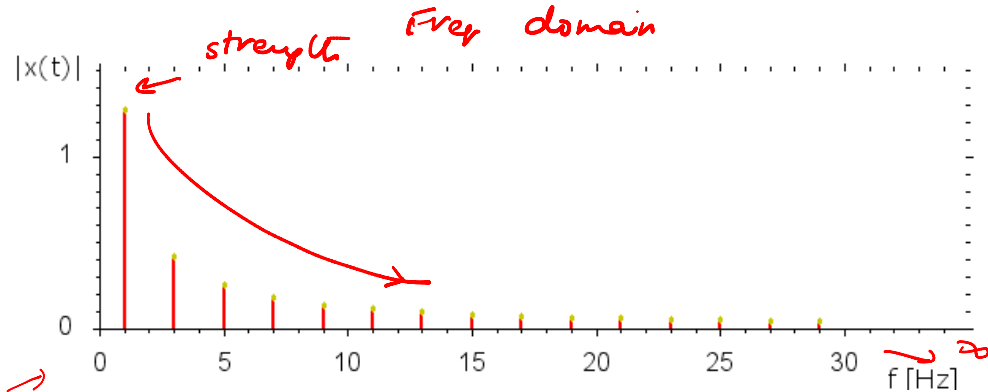


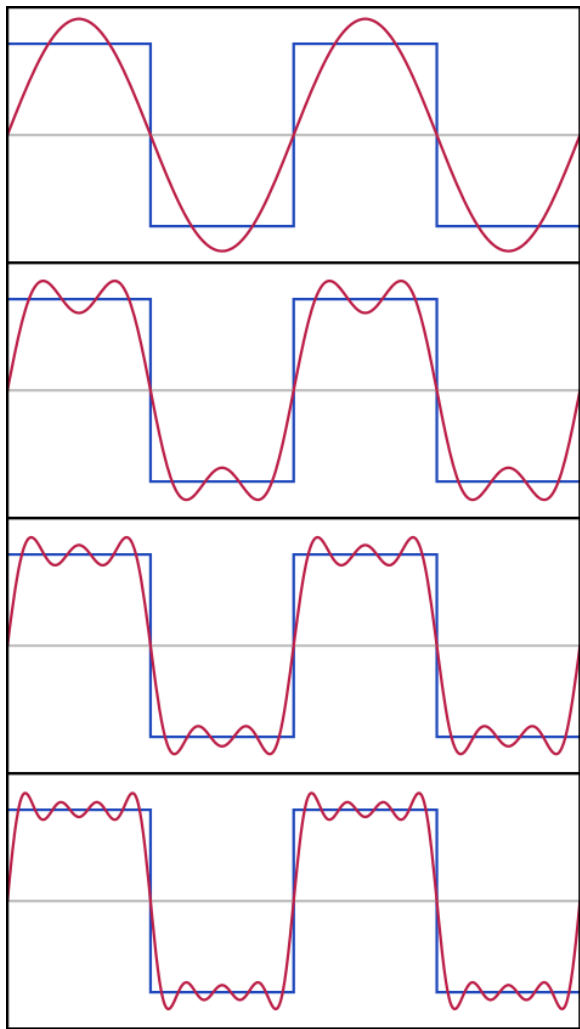
Signals and Bandwidth

- Fourier Analysis shows that any signal can be decomposed to sinusoids of different amplitude, frequency and phases



$$s(t) = A \times \frac{4}{\pi} \times \sum_{k \text{ odd}, k=1}^{\infty} \frac{\sin(2\pi k f t)}{k}$$

$$f, 3f, 5f, 7f, \dots$$




$$(4/\pi) [\sin(\underline{2\pi f t})]$$

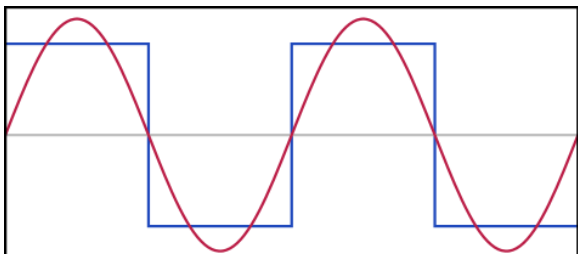
$\underline{f} \rightarrow$ fundamental freq
 $= \frac{1}{T}$

$$(4/\pi) [\sin(\underline{2\pi f t}) + (1/3)\sin(\underline{2\pi(3f)t})]$$

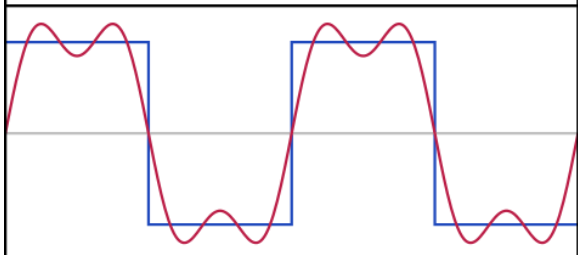
$f, 3f$

$$(4/\pi) [\sin(\underline{2\pi f t}) + (1/3)\sin(\underline{2\pi(3f)t}) + (1/5)\sin(\underline{2\pi(5f)t})]$$

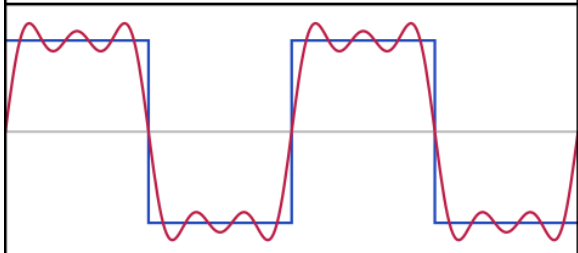
$$(4/\pi) [\sin(\underline{2\pi f t}) + (1/3)\sin(\underline{2\pi(3f)t}) + (1/5)\sin(\underline{2\pi(5f)t}) + (1/7)\sin(\underline{2\pi(7f)t})]$$



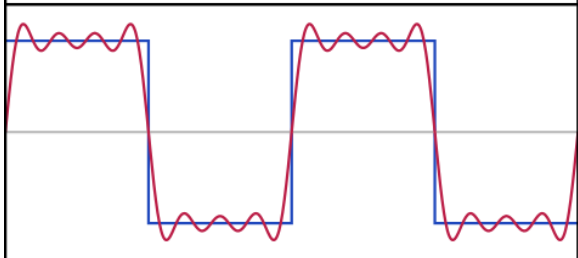
$$(4/\pi) [\sin(2\pi ft)]$$



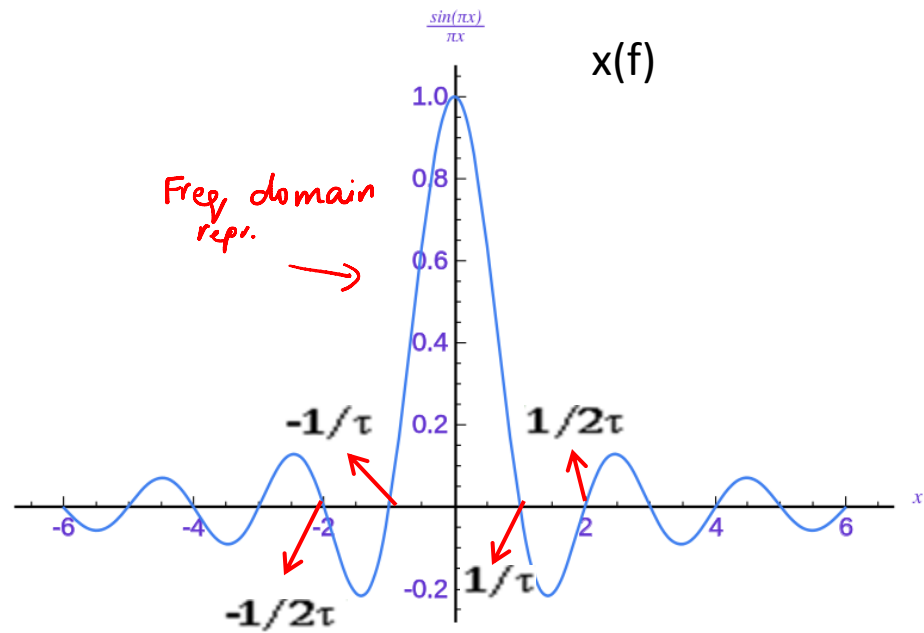
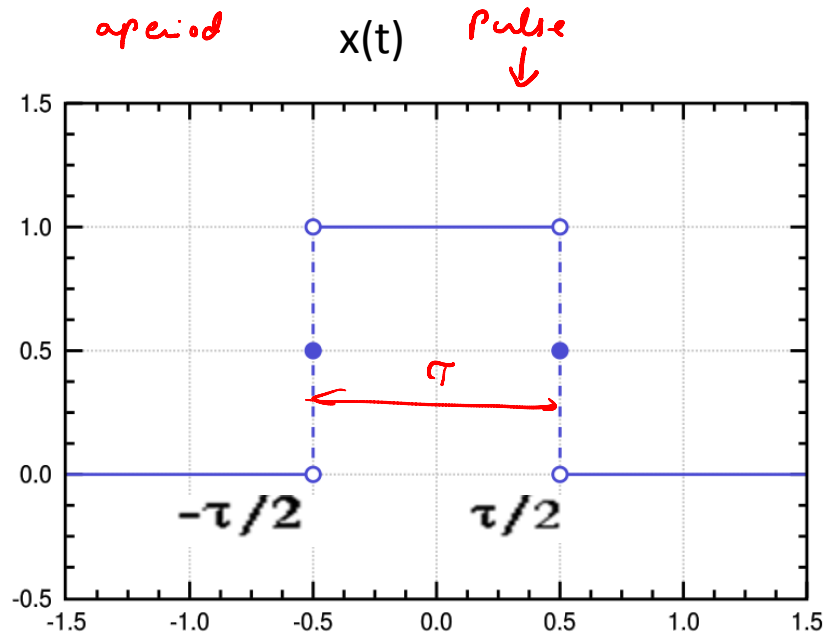
$$(4/\pi) [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$$



$$(4/\pi) [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$$



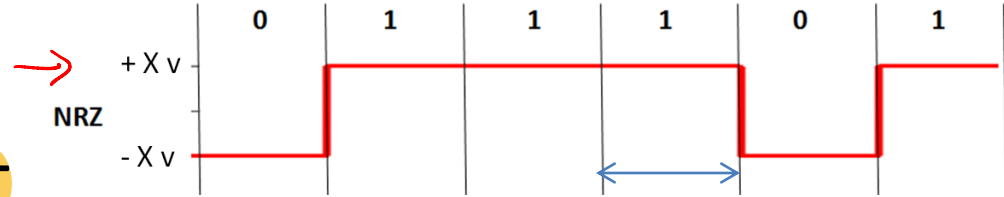
$$(4/\pi) [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$$



Fourier Transform

NRZ bandwidth

Periodic or aperiodic?



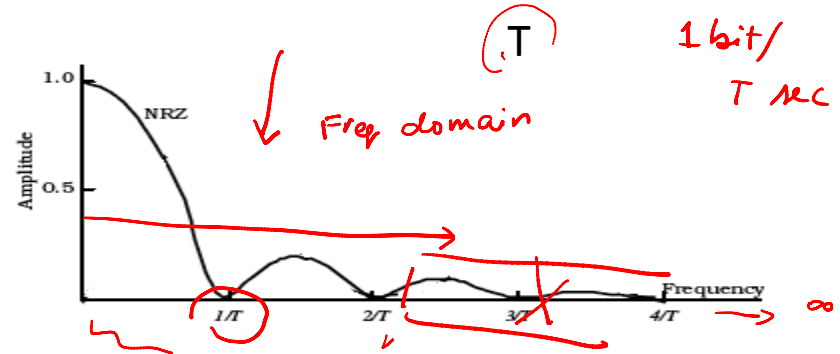
- Spectrum of a signal: Range of frequencies it contains

- Bandwidth: width of the spectrum

effective BW

– First Null Bandwidth = $1/T$

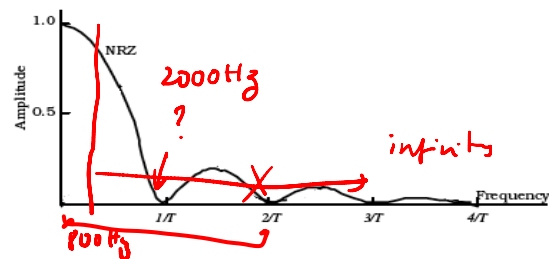
↳ Data Rate



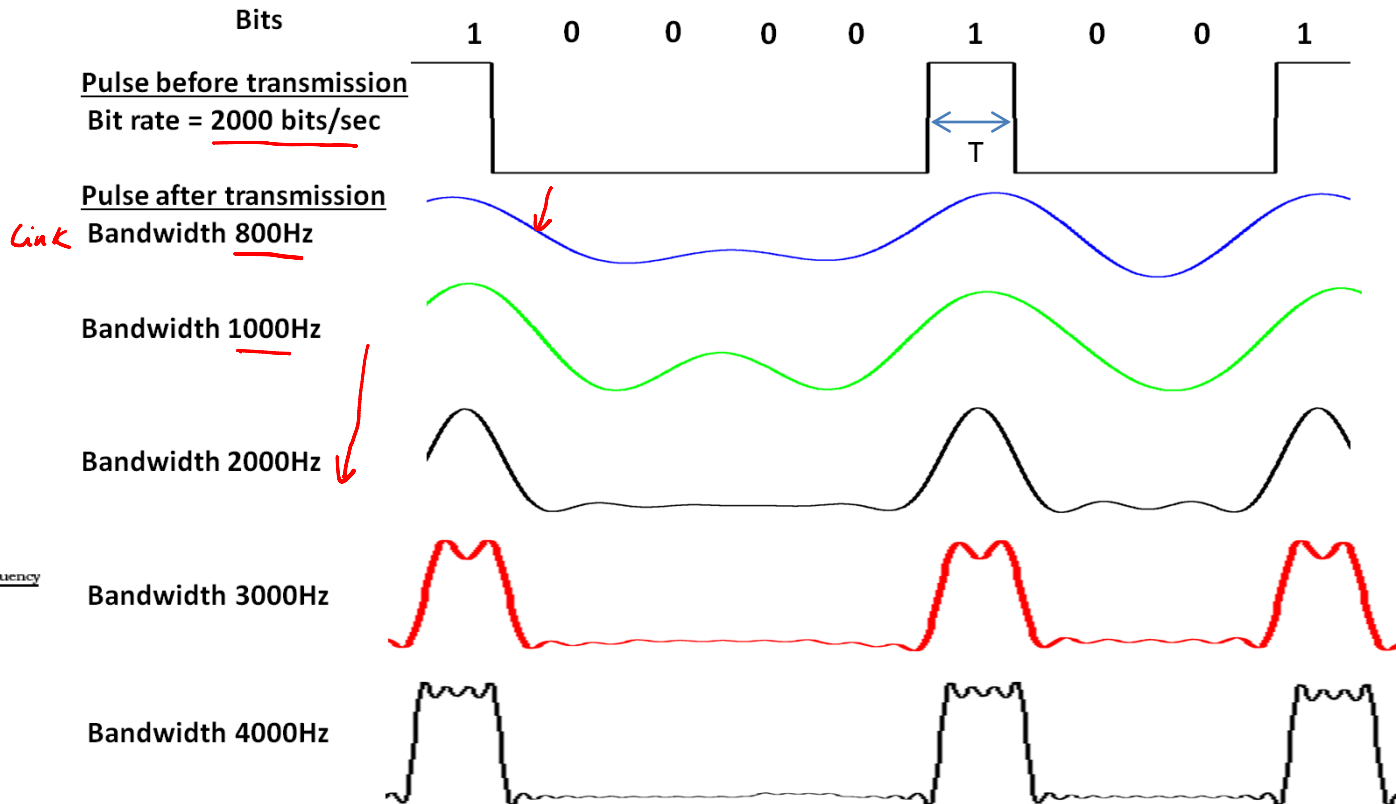
Spectrum of a random NRZ signal

Link Bandwidth

- How much link bandwidth do I need to recover signal?



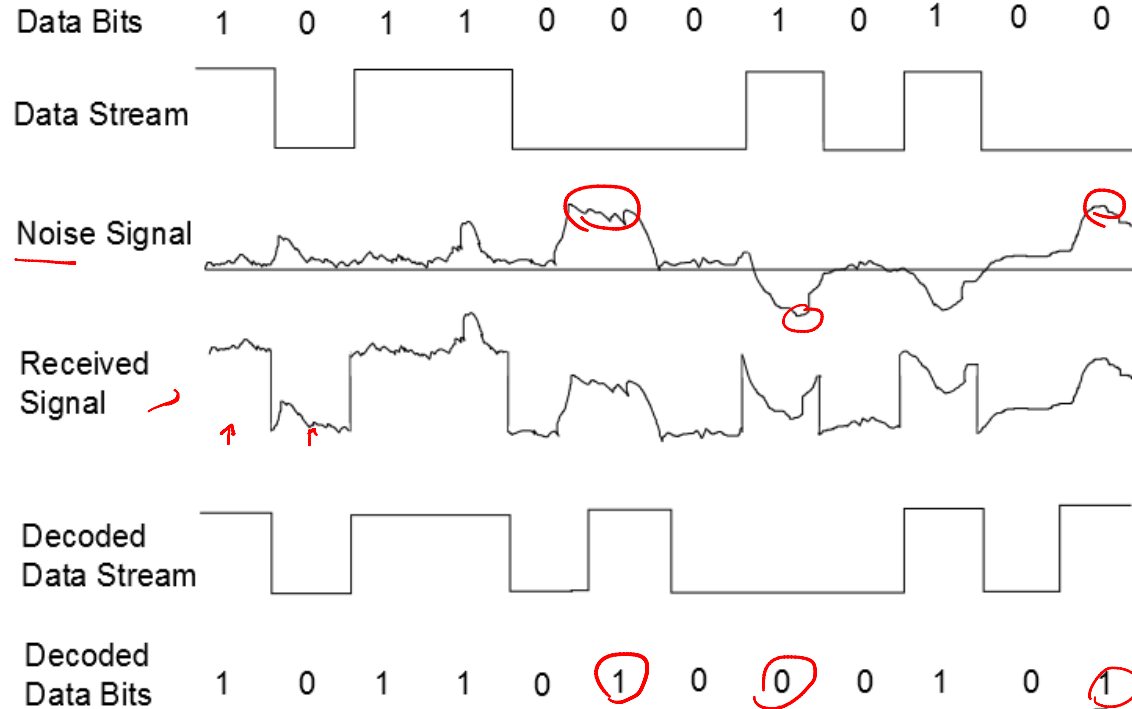
Random NRZ Signal Bandwidth



Nyquist Rate

- The number of independent pulses that could be put through a channel per unit time is limited to twice the bandwidth of the channel
- $f_p \leq 2B$
 - f_p is pulse rate (number of pulses/sec)
 - B is bandwidth of the channel
- Example: Binary pulse with rate 2000 bps needs link bandwidth of at least 1000 Hz

Effect of Noise



2000 bits/sec
1000 Hz
infinite Link BW

Bits in Error

Shannon's Theorem

- Provides an upper bound to the capacity (bps) of a link
- $C = B \log_2 (1 + S/N)$ bits/sec
 - C: capacity (bps), B: channel bandwidth (Hz), S/N: signal to noise ratio
 - S/N often expressed in dB, $10\log_{10}(S/N)$
 - E.g. 30dB corresponds to a ratio of 1000
- Example: Data over telephone line calculation
- $B = 3300\text{Hz} - 300\text{Hz} = 3000\text{Hz}$; $S/N = 1000$ (30db); $C \sim 30\text{kbps}$

Implications

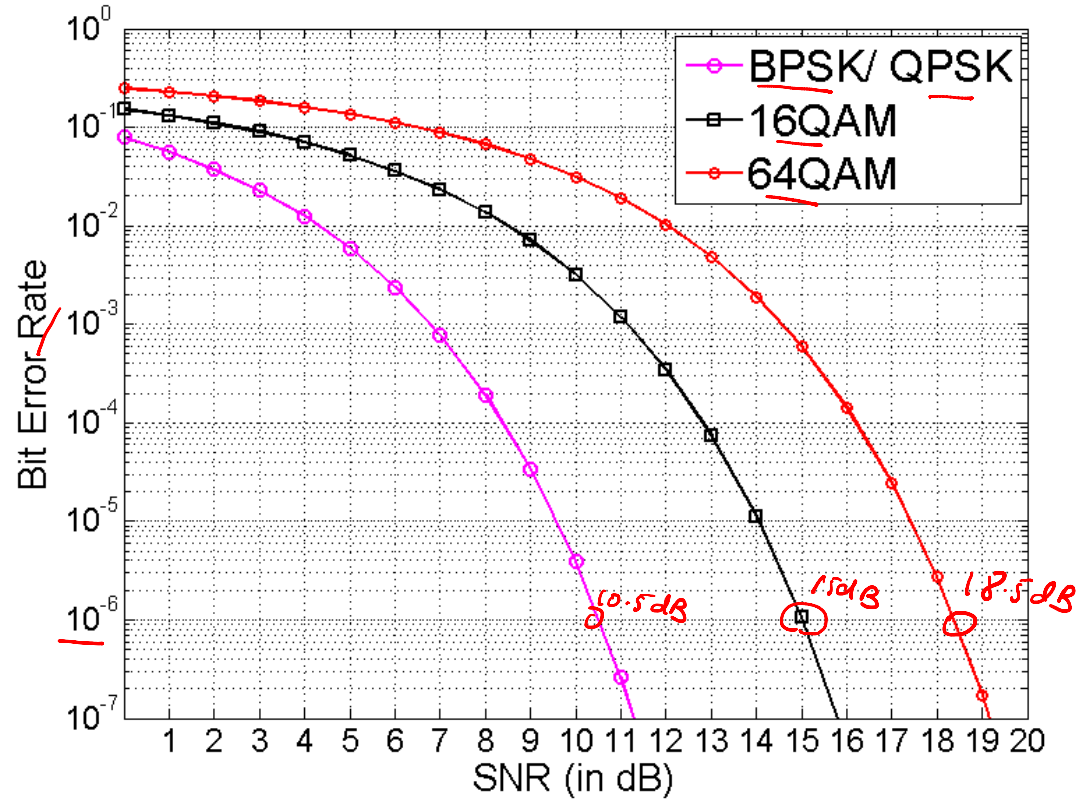
- If $R < C$, there exists a coding technique which permits transmission of data with arbitrary small error
 - The theorem does not specify how to reach this bound
 - Real systems rarely achieve this upper bound
 - E.g. WiFi: 64-QAM Modulation operating in 20Mhz bandwidth needs SNR of 27dB for packet error rate under 10% to provide 54Mbps
 - R is 54Mbps rate, B is 20Mhz band,, C is 124Mbps
- If $R > C$, the probability of error increases without bound

Error Rate

- What is the probability of bit error on a link?
- Function of received SNR and type of modulation
- For a given modulation, charts of SNR vs BER are often provided

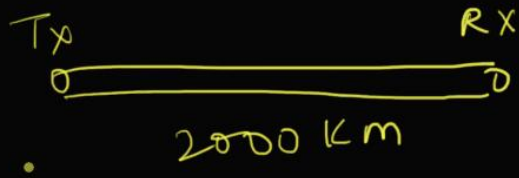
Bit Error Rate (BER)

- Packet Error Rate (PER) = $1 - (1 - \text{BER})^N$, where N is the size of packet in bits
 - BER 10^{-8} , PER = 0.008%
- Typical BERs:
 - Wireless $\sim 10^{-6}$,
 - Twisted Pair $\sim 10^{-8}$,
 - Fiber-optics $\sim 10^{-9}$ to 10^{-12}



Propagation Delay

- Time required for a bit to propagate from beginning of link to end of the link
 - Depends on speed of light in the medium (S) & distance (D)
 - speed of light: $2 * 10^8$ to $3 * 10^8$ m/s
 - Formula: D/S
- Note that Transmission delay(TD) is different from propagation delay
 - $TD(\text{sec}) = \text{Length of the packet (bits)} / \text{Data rate(bps)}$

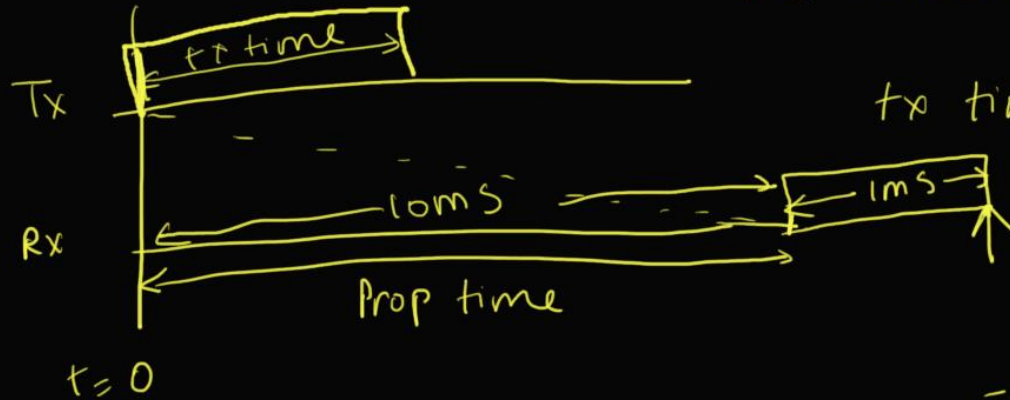


$$D = 2000 \text{ km} = 2 \times 10^6 \text{ m}$$

$$S = 2 \times 10^8 \text{ m/s}$$

$$R = 1 \text{ Mbps} = 10^6 \text{ bps}$$

$$P = 1000 \text{ bits}$$



$$tx \text{ time} = \frac{P}{R} = \frac{1000}{10^6} = 1 \text{ ms}$$

$$prop \text{ time} = D/S$$

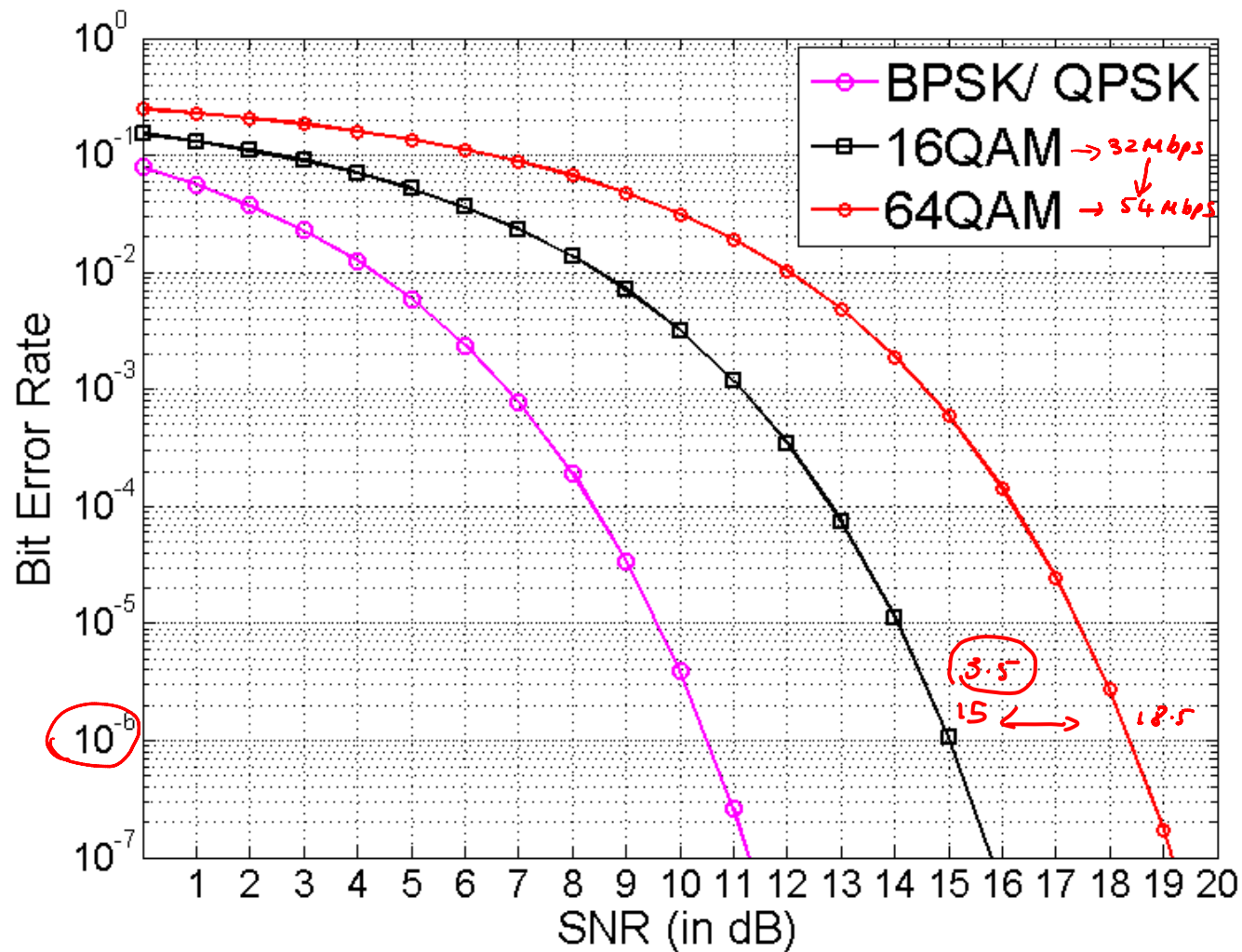
$$= \frac{2 \times 10^6}{2 \times 10^8} = 10 \text{ ms}$$

$$10 \text{ ms} + 1 \text{ ms}$$

= 11 ms ← fully receive the packet

Goals of Modulation

- Bandwidth Efficiency: Data-rate/bandwidth-required, bps/Hz
 - Goal: Try to reach Shannon limit
 - Real Systems: Ranges from 0.001 to 16
- Power Efficiency: Energy per bit/ N_o
 - N_o is noise power spectral density
 - Goal: Minimize SNR required for a given BER
- Tradeoff bandwidth efficiency and power efficiency
 - Can achieve high BW only at the expense of more energy per bit



Summary

- Signals and frequency domain representation (bandwidth they occupy)
- How many bits per sec can be sent on a link?
 - Upper bounded by Shannon theorem
 - In reality, depends on medium and modulation
- What is the packet error rate?
 - Function of BER which is determined by SNR and modulation
- Signal corresponding to a bit takes time to propagate
 - Propagation delay is function of speed of light in medium and distance
- Goals of Modulation