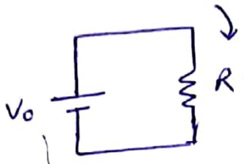
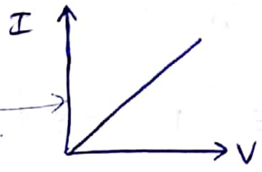
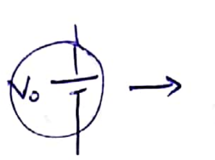
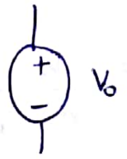
$i = \frac{V_0}{R}$ (Ohm's law)







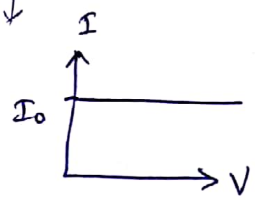


(Battery) Ideal voltage source

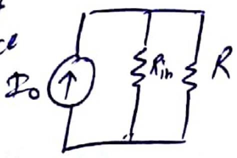
$i = \frac{V_0}{R+R_{in}} \simeq \frac{V_0}{R}$ ($R_{in} << R$)

Ideal current source.

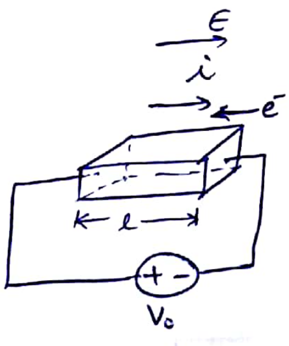
$V = I_0 R$.



current source



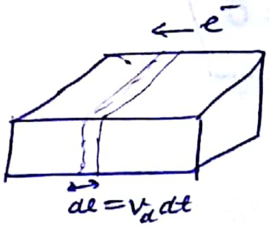
($R_{in} >> R$).



$E = \frac{V_0}{l}$

$F = (-e)E$

$v_d \to$ drift



dt

electron density $= n$.

$n A v_d dt (-e)$

$i = \frac{Q}{dt}$

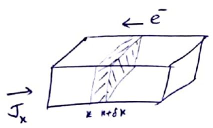
$\Rightarrow \boxed{i = neAv_d}$

current density : $J = \frac{i}{A} = nev_d$.

Resistivity: $\boxed{\rho = \frac{E}{J}}$ or $\boxed{J = \sigma E}$.

$\Rightarrow \rho = \frac{(V_0/l)}{nev_d} \Rightarrow V_0 = nev_d \rho l \Rightarrow \rho = \frac{V_0}{nev_d l} = \frac{E}{nev_d}$.

$R = \frac{V_0}{i} = \frac{V_0}{JA} = \frac{El}{JA} = \frac{\rho l}{A} \Rightarrow \boxed{R = \frac{\rho l}{A}}$.



$n(\vec{r}, t)$

$\frac{J_x(x)\, dA}{(-e)}$ = no. of $\bar{e}$s leaving the volume per unit time.

$\frac{J_x(x+\delta x)\, dA}{(-e)}$ = no. of $\bar{e}$s entering the vol./time

change in no. of $\bar{e}$s $= \frac{(J_x(x+\delta x) - J_x(x))\, \overbrace{dA}^{\delta y \delta z}}{(-e)} \cdot dt$

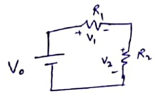
$$= \left(\frac{\partial J_x}{\partial x} \cdot \delta x\right) \frac{\delta y \delta z \cdot dt}{(-e)}$$

$$= \left(\frac{1}{-e}\right) \underbrace{\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right)}_{\vec{\nabla} \cdot \vec{J}} \cdot \delta V \cdot \delta t$$

$$\frac{\partial n(\vec{r}, t)}{\partial t} \cancel{\delta t}\, \delta V = \left(\frac{-1}{e}\right) (\vec{\nabla} \cdot \vec{J}) \cdot \cancel{\delta V \cdot \delta t}$$

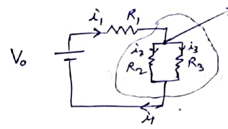
$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{\vec{\nabla} \cdot \vec{J}}{e} \Rightarrow \boxed{\frac{\partial n}{\partial t} + \frac{1}{e} \vec{\nabla} \cdot \vec{J} = 0}$$

continillity eqn.



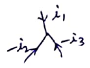
$V_0 = V_1 + V_2$ (Kirchaff's Voltage law) **KVL**. | $\vec{\nabla} \times \vec{E} = 0$, $\oint \vec{E} \cdot \vec{dl} = 0$

$\int \vec{E} \cdot \vec{dl}$ = voltage.



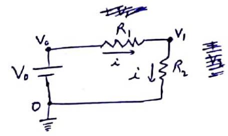
$\frac{\partial n}{\partial t} = 0$.

$i_1 = i_2 + i_3$ (Kirchoff's Current law) **KCL**



$i_1 + (-i_2) + (-i_3) = 0$.

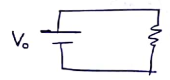
series connection:



$i = \frac{V_0 - V_1}{R_1} = \frac{V_1 - 0}{R_2} \Rightarrow V_0 R_2 = V_1(R_1 + R_2)$

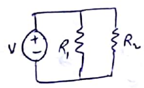
$\Rightarrow V_1 = \frac{R_2 V_0}{R_1 + R_2}$

$\Rightarrow i = \frac{V_0 - \frac{V_0 R_2}{R_1 + R_2}}{R_1} \Rightarrow \boxed{i = \frac{V_0}{R_1 + R_2}}$.



$R = R_1 + R_2$.

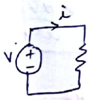
Parallel connection:-



$i_1 = \frac{V}{R_1}$, $i_2 = \frac{V}{R_2}$

$i = i_1 + i_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\Rightarrow \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$, $i = \frac{V}{R}$.



$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

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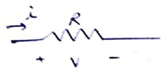
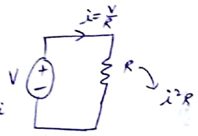
Instantaneous power:- (absorbed)

~~= P(t)~~ $p = v(t)\, i(t)$



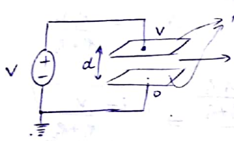
$v = iR$

$$\boxed{p = \frac{v^2}{R} = i^2 R = vi} = +ve.$$

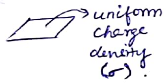


$P = -Vi$

Capacitor:-



$E = \frac{V}{d}$.



$\int \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon}$ (Gauss law)

ε dielectric constant

$$E = \frac{\sigma}{\varepsilon} = \frac{V}{d} = \frac{Q}{A\varepsilon} \Rightarrow Q = \frac{A\varepsilon}{d} \cdot V$$

$\frac{A\varepsilon}{d}$ → capacitance (C)

$\cancel{\Rightarrow V = \frac{Qd}{A\varepsilon} \Rightarrow Q =} \Rightarrow Q = CV$.

$Q(t) = CV(t)$

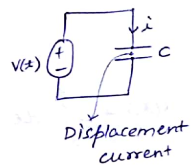
$$\boxed{i = \frac{dQ}{dt} = \frac{c\,dV}{dt}}.$$

$$\rightarrow V(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt.$$

$$V(t) = \frac{1}{C}\int_{-\infty}^{t_0} i(t)\, dt + \frac{1}{C}\int_{t_0}^{t} i(t)\, dt = \left[V(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t)\, dt\right].$$

$P_{abs}(t) = V(t)\, i(t)$

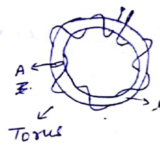
$$E = \int V \underbrace{i}_{\frac{c\,dV}{dt}} dt = \int c V dV = \boxed{\frac{1}{2} C V^2}.$$



$i = \frac{c\,dV}{dt}$ $\frac{\partial n}{\partial t} + \frac{1}{e} \nabla\cdot\vec{J} = 0$.

$$\boxed{J_d = \varepsilon \frac{\partial E}{\partial t}}.$$

Inductor :-



No of turns = n.

l..

$B = \mu H$

B: magnetic induction; H: magnetic field

$\nabla \times \vec{H} = \vec{J}$

$\int H\, dl = i$

$\Rightarrow Hl = ni$. (n loops)

$\Rightarrow H = \frac{ni}{l} \Rightarrow B = \frac{\mu n i}{l}$

Magnetic flux $\phi = \int \vec{B}\cdot\vec{ds}$

$$= \frac{\mu_0 n i}{l} \cdot nA \Rightarrow \phi = \frac{\mu n^2 A}{l} i$$

$\frac{\mu n^2 A}{l}$ → Inductance (L)

$\Rightarrow \phi = Li$.

$\phi = Li$ (inductor) $\longrightarrow v(t) = L\dfrac{di(t)}{dt}$

$Q = CV$ (capacitor) $\longrightarrow i(t) = C\dfrac{dV}{dt}$

$V = \dfrac{d\phi}{dt}$ $\left(\vec{\nabla}\times\vec{E} = \dfrac{-\partial\vec{B}}{\partial t}\right)$

$\hookrightarrow \int(\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\dfrac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$

$\Rightarrow \oint \vec{E}\cdot d\vec{l} = -\dfrac{\partial\phi}{\partial t}$.

$i(t) = \dfrac{1}{L}\int_{-\infty}^{t} V dt = i(t_0) + \dfrac{1}{L}\int_{t_0}^{t} V dt$

$E = \int \overset{P}{V i}\, dt$

$= \int\left(L\dfrac{di}{dt}\right) i\, dt = \int L i\, di$

$\boxed{E = \dfrac{1}{2}Li^2}$

$E = \dfrac{1}{2}Li^2$ (inductor) $\frac{Q^2}{2C}$

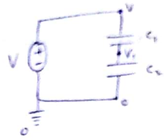
$E = \dfrac{1}{2}CV^2$ (capacitor) $\frac{\phi^2}{2L}$.

SI units : C (farad)

L (Henry)

R (Ohm, Ω)

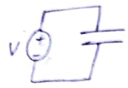
Conductance, $G = \frac{1}{R}$ (mho, ℧)



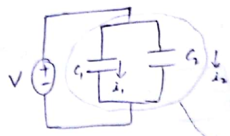
$i_1 = C_1\dfrac{d}{dt}(V - V_1)$

$i_2 = C_2\dfrac{d}{dt}(V_1 - 0)$

$\dfrac{i_1}{C_1} + \dfrac{i_2}{C_2} = \dfrac{dV}{dt} \Rightarrow i\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) = \dfrac{dV}{dt}$



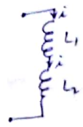
$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$



$i_1 = C_1\dfrac{dV_0}{dt}$, $i_2 = C_2\dfrac{dV_0}{dt}$

$i = i_1 + i_2 = (C_1 + C_2)\dfrac{dV_0}{dt}$.

$C_1 + C_2$.



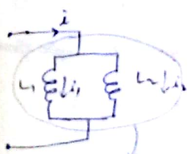
$V_L = L\dfrac{di}{dt}$

$V_{L_1} = L_1\dfrac{di}{dt}$

$V_{L_2} = L_2\dfrac{di}{dt}$

$V = V_{L_1} + V_{L_2} = (L_1 + L_2)\dfrac{di}{dt}$

$L = L_1 + L_2$.



$V = L_1\dfrac{di_1}{dt}$

$V = L_2\dfrac{di_2}{dt}$

$\dfrac{V}{L_1} + \dfrac{V}{L_2} = \dfrac{d(i_1 + i_2)}{dt} = \dfrac{di}{dt}$

$\Rightarrow V\left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right) = \dfrac{di}{dt}$ $\frac{1}{L}$

L.

$L = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}$.

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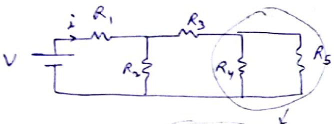
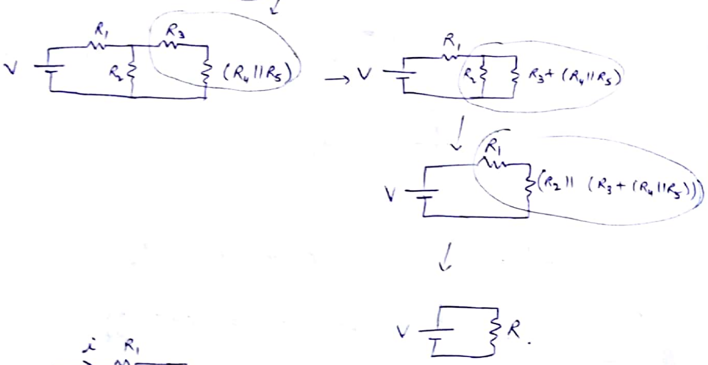
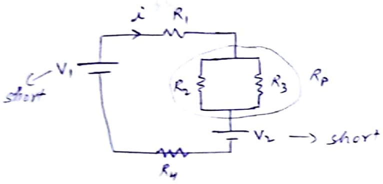


— series-parallel circuit





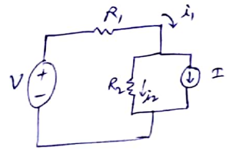
$IR_1 + IR_p + V_2 + IR_4 = V_1$

$\Rightarrow I = \dfrac{V_1 - V_2}{R_1 + R_p + R_4} = \underbrace{\dfrac{V_1}{R_1+R_p+R_4}}_{\text{by shorting } V_2} - \underbrace{\dfrac{V_2}{R_1+R_p+R_4}}_{\text{by shorting } V_1}$.

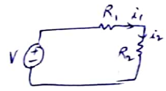
Superposition.

→ Keep one voltage source active and short all others & superpose.

→ In case of current sources, open them (remove).

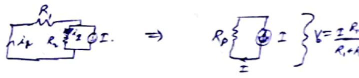


1) oppen current source.



$i_1 = i_2 = \dfrac{V}{R_1+R_2}$

2) short the voltage source.



$i_1 = \dfrac{IR_2}{R_1+R_2}$, $i_2 = \dfrac{-IR_1}{R_1+R_2}$.

Superposition :

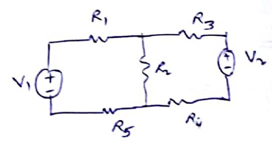
$i_1 = \dfrac{V}{R_1+R_2} + \dfrac{IR_2}{R_1+R_2}$, $i_2 = \dfrac{V}{R_1+R_2} - \dfrac{IR_1}{R_1+R_2}$.

R_1

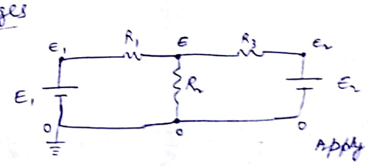
$P = i^2 R_1 = (i_1+i_2)^2 R_1$

$= \underbrace{i_1^2 R_1}_{\text{1st case}} + \underbrace{i_2^2 R_1}_{\text{2nd case}} + 2 i_1 i_2 R_1$ ✗

No superposition for P.

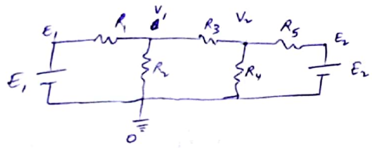


Node voltages



Apply KCL at nodes:

$E\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = \frac{E_1}{R_1} + \frac{E_2}{R_3}$. $\Leftarrow$ $\frac{E-E_1}{R_1} + \frac{E-0}{R_2} + \frac{E-E_2}{R_3} = 0$.



KCL at nodes :-

$$\frac{E_1 - V_1}{R_1} + \frac{0 - V_1}{R_2} + \frac{V_2 - V_1}{R_3} = 0 \quad \& \quad \frac{V_1 - V_2}{R_3} + \frac{0 - V_2}{R_4} + \frac{E_2 - V_2}{R_5} = 0.$$

$$G_{11} V_1 + G_{12} V_2 = I_1$$

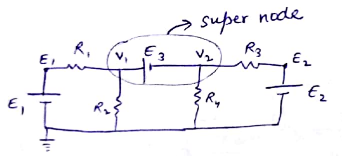
$$G_{21} V_1 + G_{22} V_2 = I_2$$

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Cramer's rule :

$$\cancel{V_1 = \frac{G_{11}}{G}} \; G \qquad V_1 = \frac{\begin{vmatrix} I_1 & G_{12} \\ I_2 & G_{22} \end{vmatrix}}{D_G}$$

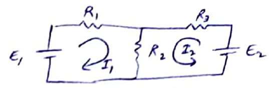
$$D_G = \begin{vmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{vmatrix} \qquad V_2 = \frac{\begin{vmatrix} G_{11} & I_1 \\ G_{12} & I_2 \end{vmatrix}}{D_G}$$



(i) — $\frac{E_1 - V_1}{R_1} + \frac{0 - V_1}{R_2} + \frac{E_2 - V_2}{R_3} + \frac{0 - V_2}{R_4} = 0 \Rightarrow V_1\left(\frac{1}{R_1}\frac{1}{R_2}\right) + V_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{E_1}{R_1} + \frac{E_2}{R_3}$.

(ii) — $V_1 = V_2 + E_3$.

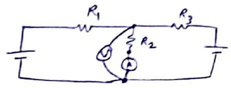
Mesh currents :-



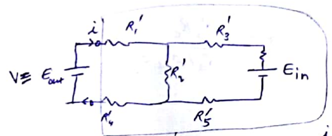
KVL : $\cancel{E_1 = (R_1 + R_2) I_1}$ $E_1 = R_1 I_1 + R_2 (I_1 + I_2) \Rightarrow I_1 \overbrace{(R_1 + R_2)}^{R_{11}} + I_2 \overbrace{R_2}^{R_{12}} = E_1$.

$E_2 = R_3 I_2 + R_2 (I_1 + I_2) \qquad \Rightarrow I_1 \underbrace{R_2}_{R_{21}} + I_2 \underbrace{(R_2 + R_3)}_{R_{22}} = E_2$.

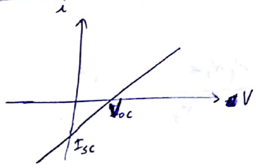
1) series-parallel
2) Superposition
3) Node ~~a~~ method
4) mesh method.

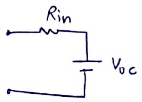


Thevenin's theorem :-



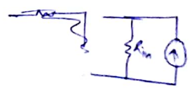
$$i = \frac{1}{R_1} E_{out} + \frac{1}{R_2} E_{in}.$$





or

Norton's theorem



$I = \frac{V_{oc}}{R_{in}}$

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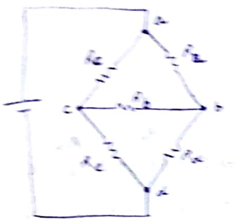
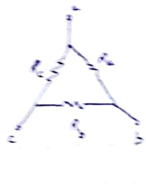
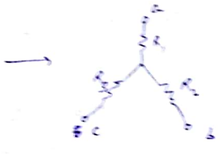
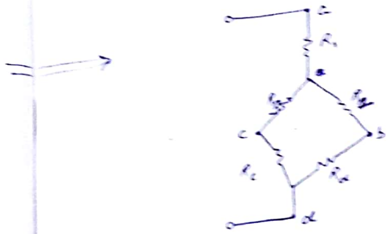




Δ



Y



$$(R_{ab})_Y = (R_{ab})_\Delta$$

$$R_1 + R_2 = R_a \parallel (R_b + R_c) = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c}.$$

$$(R_{bc})_Y = (R_{bc})_\Delta$$

$$(R_{ac})_Y = (R_{ac})_\Delta.$$

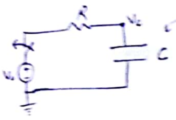
Show

$$R_1 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_3 = \frac{R_b R_c}{R_a+R_b+R_c}$$

$$R_2 = \frac{R_a R_b}{R_a+R_b+R_c}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

open circuit after long time



$$\frac{V_s - V_c}{R} = \frac{C\,dV_c}{dt}$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC}$$

$$V_c(t) = \underbrace{V_{C_H}(t)}_{\text{homogeneous}} + \underbrace{V_{C_p}(t)}_{\text{particular}}$$

$$\frac{dV_{CH}}{dt} + \frac{V_{CH}}{RC} = 0$$

try: $V_{CH}(t) = Ae^{st} = Ae^{-\frac{t}{RC}}.$

$$Ase^{st} + \frac{Ae^{st}}{RC} = 0 \Rightarrow s = \frac{-1}{RC}.$$

$$\frac{dV_{CP}}{dt} + \frac{V_p}{RC} = \frac{V_o}{RC} \Rightarrow \boxed{V_{CP} = V_o}.$$ (Assuming V_{cp} as constant).

$$\Rightarrow \boxed{V_C(t) = Ae^{-t/RC} + V_o}.$$ — use boundary condition

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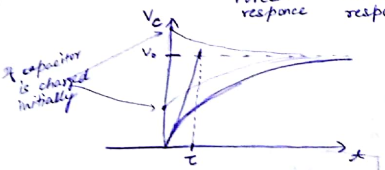
At $t=0$, $V_c = 0$ (initially capacitor is not charged)

$V_c(t=0) = 0$

$\Rightarrow A e^{-0} + V_0 = 0 \Rightarrow A = -V_0$.

so, $\boxed{V_c(t) = \underbrace{V_0}_{\text{forced responce}}\underbrace{(1 - e^{-t/RC})}_{\text{transient response}}}$.

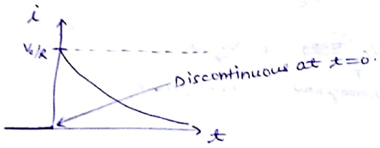


$RC =$ time constant (τ).

$e^{-5} \approx 0.0067$

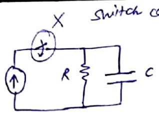
$$i(t) = \frac{V_0 - V_c(t)}{R} = \frac{V_0}{R} e^{-t/RC}.$$

$$= \frac{C\,dV_c(t)}{dt} = \frac{V_0}{R} e^{-t/RC}.$$

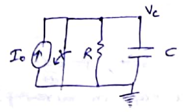


At $t = 0^+ \Rightarrow$ Voltage drop across resistor $= V_0$.

$t = \infty \Rightarrow$ ——— " ——— capacitor $= V_0$.



switch cannot be in series with current source (when switch is open, it makes problem).



switch opened at $t=0$.

$$I_0 = \frac{V_c}{R} + C\frac{dV_c}{dt}$$

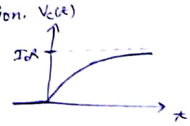
$$V_c(t) = V_{CH}(t) + V_{CP}$$

$V_{CH}(t) = A e^{-t/RC}$

$V_{CP}(t) = I_0 R$.

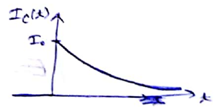
$V_c(t) = A e^{-t/RC} + I_0 R$

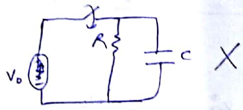
$V_c = 0$ at $t = 0$ — Boundary condition.

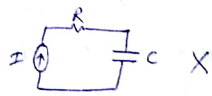


$\Rightarrow V_c(t) = I_0 R\,(1 - e^{-t/RC})$.

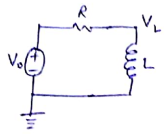
$$i_c(t) = \frac{C\,dV_c(t)}{dt} = I_0 e^{-t/RC}.$$







(current will keep on going and V_c will increase to infinity)



$$I_L R + L\frac{dI_L}{dt} = V_0.$$

$I_L(t) = I_{LH}(t) + I_{LP}$

$I_L = 0$ at $t = 0$ — Boundary condition

$\Rightarrow \boxed{I_L = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)}$.

$\left(\tau = \frac{L}{R}\right)$.

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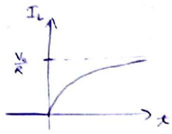
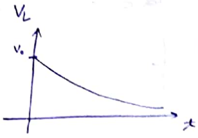
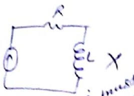
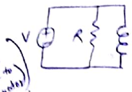




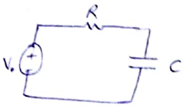


X

(At $t=0$, i must be zero (due to inductor))



X (V across L is const., $V = L\frac{di}{dt}$, i keeps on increasing)



Energy ~~Power~~ given by voltage source $= \int_0^{\infty} P\,dt$

$= \int_0^{\infty} i V_0 \, dt$

$= \int_0^{\infty} \frac{V_0^2}{R} e^{-t/RC} dt$

$= \frac{V_0^2}{R}(RC)\left[1 - e^{-t/RC}\right]_0^{\infty}$

$= CV_0^2(1)$

$\Rightarrow \boxed{E = CV_0^2}$

$\boxed{\text{Energy stored in capacitor} = \frac{1}{2}CV_0^2}$

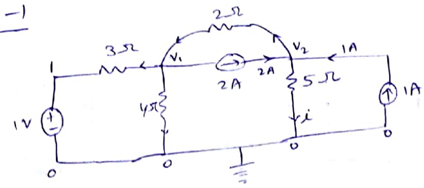
Energy spent in resistor $= \frac{1}{2}CV_0^2$.

$\hookrightarrow \int_0^{\infty} i^2(t) R\, dt = \left(\frac{V_0^2}{R}\right) R \int_0^{\infty} e^{-2t/\tau}$

$= \frac{1}{2}CV_0^2$.

Tut-1

1



$2+1 = \frac{V_2}{5} + \frac{V_2 - V_1}{2}$

$\frac{V_2 - V_1}{2} = \frac{V_1}{4} + 2 + \frac{V_1 - 1}{3}$

$3 = \frac{V_2}{5} + \frac{V_2 - V_1}{2}$

$\frac{V_2 - V_1}{2} = \frac{7V_1}{12} + \frac{V_1}{3} + \frac{5}{3}$

$6V_2 - 13V_1 = \cancel{7V_1} + 20$

$30 = 2\cancel{V_2} + 7V_2 - 5V_1$

$6V_2 - 13V_1 = 20$

$7V_2 - 5V_1 = 30$

$(30 - 91)V_2 = 100 - 3900$

$V_2 = \frac{3800}{61}$

$\frac{+280}{61 \times 5} = \frac{58}{61}$ A.

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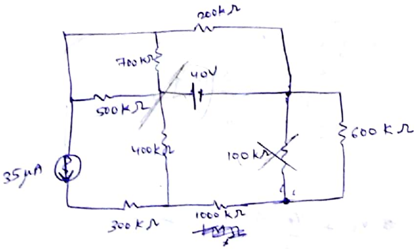
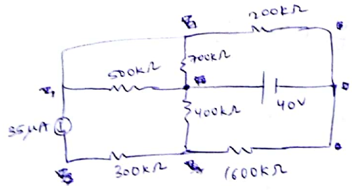
(2)



Open (I) & Short ⊣⊢

$R_{th} = 420 k\Omega$

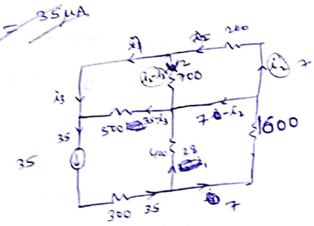
$\frac{1400 \times 600}{2000} =$



600
1400

$\frac{3000}{14}$

$\frac{V_3 - V_2}{300k} = 35\mu A$



$1600 i_1 = 400(35 - i_1)$

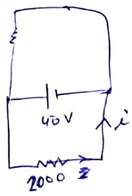
$i_1 = 7$

$200 i_2 + 700(i_2 - i_3) = 0$

$9 i_2 = 7 i_3$.

$700(i_2 - i_3) + 500(35 - i_3) = 0$.

$7 i_2 - 12 i_3 + 35 \times 5 = 0$



$i = 0.0249$ A.

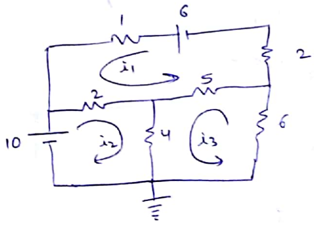
$12V + \frac{7}{1000000} \times 600 \times 1000$ = V_{th}.

$\frac{42}{10} + 12 = 16.2V$

$\frac{2000 \times 5900}{2400 + 5900}$

$\frac{40(A+B)}{AB} \times \frac{B}{A+B}$

(3)



$i_2 = \frac{19}{9}$, $i_3 = \frac{-11}{350}$

$6 = i_1 \times 1 + 2(i_1 + i_2) + 5(i_1 - i_3) + 2 i_1$

$10 = 2(i_1 + i_2) + 4(i_2 + i_3)$

$6 i_3 + 5(i_3 - i_1) + 4(i_2 + i_3) = 0$.

$10 i_1 + 2 i_2 - 5 i_3 = 6$,

$2 i_1 + 6 i_2 + 4 i_3 = 10$,

$5 i_1 - 4 i_2 - 15 i_3 = 0$.

$28 i_2 + 25 i_3 = 44$

$36 i_2 + 50 i_3 = 50$

$18 i_2 = 38$

$i_2 = \frac{38}{28}$

$1 - \frac{38}{50} \times \frac{15}{14}$

$\frac{350 - 34}{350} = \frac{-11}{350}$

$i_3 = \frac{-4}{25}$

$\frac{209}{41}$

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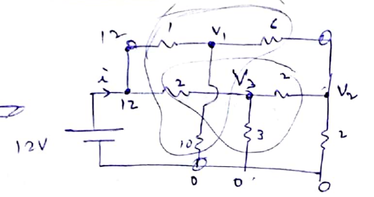
4



$$\frac{360-30V_1}{1} + \frac{0-3V_1}{\cancel{10}} + \frac{5V_2-5V_1}{2} = 0$$

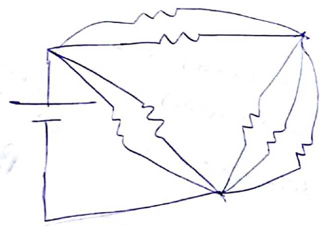
$$\frac{3V_3-36}{2} + \frac{2V_3-2V_2}{2} + \frac{2V_3-0}{3} = 0$$

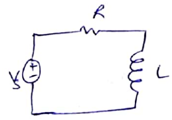
$$\frac{3V_2-3V_3}{2} + \frac{V_2-V_1}{6} + \frac{3V_2-0}{2} = 0$$

$$38V_1 - 5V_2 = 360$$

$$7V_3 - 2V_2 = 36,$$

$$7V_2 - 3V_3 - V_1 = 0$$





$V_s = V_1 + V_2$

$V_s = Ri + L\frac{di}{dt}$

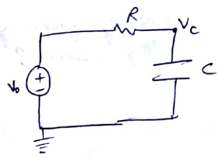
$(C_1 = \frac{1}{R})$

$i_s = i_1 + i_2$

$i_s = C_1 V + C\frac{dV}{dt}$

$1/R$

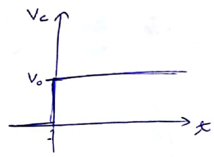
$$\boxed{\begin{aligned} x_s &= x_1 + x_2 \\ x_s &= a_1 y + a_2 \frac{dy}{dt} \end{aligned}}$$

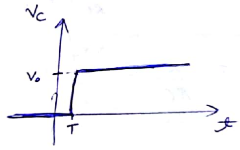


$$V_c(t) = V_0(1-e^{-t/RC}) + \underbrace{V_i e^{-t/RC}}_{\text{Natural responce}}$$

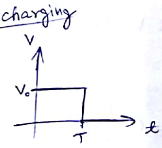
$$= V_0(1-e^{-(t-T)/RC})$$

(time invariance)





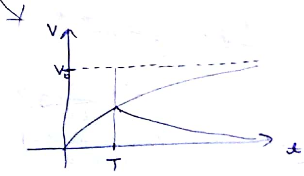
Discharging



$t<T$: $V_c(t) = V_0(1-e^{-t/RC})$

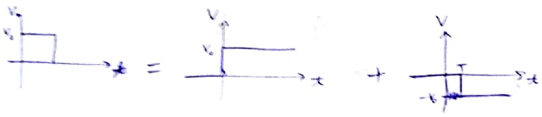
$t>T$: $V_c(t) = V_c(T)\, e^{-(t-T)/RC}$

$= V_0(1-e^{-T/RC})\, e^{-(t-T)/RC}$



$t > T$ [illegible]

$$V_c(t) = V_0\left(1 - e^{-t/RC}\right) + (-V_0)\left(1 - e^{-(t-T)/RC}\right).$$



δ-func. $\begin{cases} T \to 0, \ V_0 \to \infty \\ \text{show that } V_0 T = K. \end{cases}$

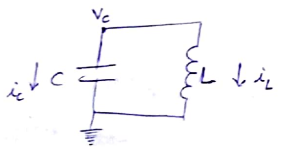
$$V_c(t) = V_0\left(1 - e^{-T/RC}\right) e^{-(t-T)/RC}$$

$$e^{-T/RC} = 1 - \left(\frac{T}{RC}\right) + \left(\frac{T}{RC}\right)^2 + \ldots$$

Energy given by Voltage source?

$$1 - e^{-T/RC} = \frac{T}{RC} - \left(\frac{T}{RC}\right)^2 + \ldots$$

Impulse response → $\boxed{V_c(t) = \frac{K}{RC} e^{-t/RC}}$ $V_c(t) = \frac{V_0 T}{RC} e^{-t/RC}$



KCL

$$C\frac{dV_c}{dt} + \frac{1}{L}\int_{-\infty}^{t} V_c(t')\,dt' = 0$$

$$\boxed{\frac{d^2 V_c}{dt^2} + \frac{V_c(t)}{CL} = 0.}$$

Try: $V_c = Ae^{st}$

$s^2 + \frac{1}{LC} = 0$ (characteristic equation) $\Rightarrow s = \pm\frac{1}{\sqrt{LC}}$

$\Rightarrow s = \pm\frac{j}{\sqrt{LC}}$ $(j^2 = -1)$

$$V_c(t) = K_1\cos(\omega_0 t) + K_2\sin(\omega_0 t)$$

$\left(\frac{d^2V_c}{dt^2} + \omega_0^2 V_c = 0\right)$

Need 2 boundary conditions

$V_c(t=0) = V_c(0) \Rightarrow K_1 = V_c(0)$

$i_L(t=0) = i_L(0)$

$i_L(t) = -C\frac{dV_c}{dt}$ $(-i_c)$

$= -C\omega_0(-K_1\omega_0\cos(\omega_0 t) + K_2\omega_0\sin(\omega_0 t))$

$\Rightarrow i_L(0) = -K_2 C\omega_0$

$\Rightarrow K_2 = -\frac{i_L(0)}{\omega_0 C}$

$\Rightarrow V_c(t) = V_c(0)\cos(\omega_0 t) - i_L(0)\sqrt{\frac{L}{C}}\sin(\omega_0 t)$; $\omega_0 = \frac{1}{\sqrt{LC}}$

$i_L(t) = \sqrt{\frac{C}{L}}V_c(0)\sin(\omega_0 t) - i_L(0)\cos(\omega_0 t)$.

$\left[a\cos x - b\sin x = \sqrt{a^2+b^2}\cos(x+\phi)\right.$; $\left.\tan\phi = b/a.\right]$

$$V_c(t) = \left[V_c(0)^2 + \frac{L}{C}i_L(0)^2\right]^{1/2}\cos\left(\omega_0 t + \tan^{-1}\left(\sqrt{\frac{L}{C}}\frac{i_L(0)}{V_c(0)}\right)\right)$$

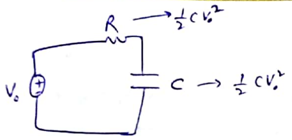
$$i_L(t) = \sqrt{\frac{C}{L}}\left[\quad\right]^{1/2}\sin\left(\quad\right)$$

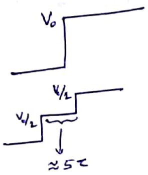
90° phase apart

If $i_L(0) = 0$

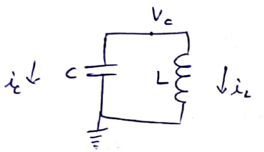
$\Rightarrow V_c(t) = V_c(0)\cos(\omega_0 t)$

$i_L(t) = \sqrt{\frac{C}{L}}V_c(0)\sin(\omega_0 t)$





$$E \text{ dissipated} = \frac{1}{2}C\left(\frac{V_0}{2}\right)^2 + \frac{1}{2}C\left(\frac{V_0}{2}\right)^2 = \frac{1}{4}CV_0^2$$



$$\text{Energy stored} = \frac{1}{2}CV^2(t) + \frac{1}{2}L(i(t))^2$$

$$= \frac{1}{2}C\left(V_c^2(0) + \frac{L}{C}i_L(0)^2\right)\cos^2\theta$$

$$+\frac{1}{2}L\left(\frac{C}{L}\right)(\,-\,-\,-\,-\,-\,)\sin^2\theta$$

$$E = \frac{1}{2}CV_c^2(0) + \frac{1}{2}Li_L^2(0) \quad \text{(conserved)}$$

$$\frac{1}{2}CV_{c(max)}^2 = \frac{1}{2}Li_{L(max)}^2 \Rightarrow \frac{V_{c(max)}}{i_{L(max)}} = \sqrt{\frac{L}{C}}. \quad \text{(characteristic impedance)}$$

$$E(t) = \frac{1}{2}CV_c^2(t) + \frac{1}{2}Li_L^2(t)$$

$$\frac{dE(t)}{dt} = CV_c\frac{dV_c(t)}{dt} + Li_L\frac{di_L}{dt} = V_c(-i_L) + V_c(i_L)$$

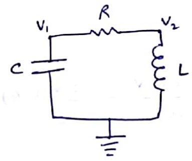
$$= 0$$

$$i_c = C\frac{dV_c}{dt} = -i_L$$

$$V_c = L\frac{di_L}{dt}$$

$\Rightarrow E(t) = \text{constant}$

Energy is conserved



$$C\frac{dV_1}{dt} + \frac{V_1 - V_2}{R} = 0. \Rightarrow V_2 = RC\frac{dV_1}{dt} + V_1$$

$$\frac{V_2 - V_1}{R} + \frac{1}{L}\int_{-\infty}^{t} V_2(t')\,dt' = 0$$

$$C\frac{dV_1}{dt} + \frac{1}{L}\int_{-\infty}^{t}\left(RC\frac{dV_1}{dt'} + V_1\right)dt' = 0.$$

$$\frac{d^2V_1}{dt^2} + \frac{R}{L}\frac{dV_1}{dt} + \frac{V_1}{CL} = 0.$$

Try $V_1 = Ae^{st}$

$$s^2 + \frac{Rs}{L} + \frac{1}{CL} = 0 \quad \text{(characteristic eqn.)}$$

$$s = \frac{-(R/L) \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}}{2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} < \begin{matrix} s_1 \\ s_2 \end{matrix}$$

Define $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

$$V_1(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

1) $\alpha < \omega_0$ — underdamped dynamics $\rightarrow \frac{R}{2L} < \frac{1}{\sqrt{LC}} \Rightarrow \frac{R}{2} < \sqrt{\frac{L}{C}}$.
2) $\alpha = \omega_0$ — Critically damped
3) $\alpha > \omega_0$ — overdamped

Define $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$$V_1(t) = A_1e^{-\alpha t}e^{j\omega_d t} + A_2e^{-\alpha t}e^{-j\omega_d t}$$

$$= e^{-\alpha t}(K_1\cos\omega_d t + K_2\sin\omega_d t)$$

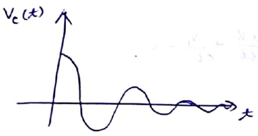
$V_c(t=0) = V_c(0)$, $i_L(t=0) = 0$.

$K_1 = V_c(0)$

$i_L = -C\frac{dV_1}{dt} \Rightarrow C(K_1\alpha - K_2\omega_d) = 0 \Rightarrow K_2 = \frac{CK_1\alpha}{\omega_d} = \frac{C\alpha}{\omega_d}V_c(0).$

$V_c(t) = V_c(0)\sqrt{1+\left(\frac{\alpha}{\omega_0}\right)^2}\, e^{-\alpha t}\cos(\omega_d t + \phi)$

$i_L(t) = \text{------}\ e^{-\alpha t}\sin(\omega_d t).$



<u>Quality factor:-</u>

(dimensionless) $Q = \frac{\omega_0}{2\alpha} = \frac{1}{R}\sqrt{\frac{L}{C}}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \alpha = \frac{R}{2L}$

If α small, Q large.
(R small)

$T = \frac{2\pi}{\omega_d}$ (period of oscillation)

Q oscillations $= \frac{2\pi Q}{\omega_d}$

Amplitude $\sim e^{-\frac{2\pi Q}{\omega_d}\alpha} \approx e^{-\pi} = 4\%.$

(If $Q >> 1, \omega_d \approx \omega_0$).

3) $\alpha > \omega_0$ (overdamped)

$\frac{R}{2} > \sqrt{\frac{L}{C}}$

$$\left.\begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{aligned}\right] \text{ real } -\text{ve}.$$

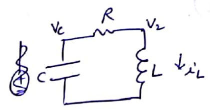
$V_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$i_L(t) = -C\frac{dV_c}{dt} = -C[s_1 A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t}]$

At $t = 0$, $V_c(0)$
$i_L(0) = 0$

$A_1 + A_2 = V_c(0)$, $s_1 A_1 + s_2 A_2 = 0 \Rightarrow A_1 = \frac{s_2 V_c(0)}{s_2 - s_1} = +\text{ve}$ $\left(\begin{matrix} s_1, s_2 < 0 \\ s_2 \text{ more } -\text{ve}\end{matrix}\right)$

$A_2 = \frac{s_1 V_c(0)}{s_1 - s_2} = -\text{ve}$



$\alpha = \frac{R}{2L}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha < \omega_0 \rightarrow$

$\boxed{\alpha = \omega_0}$

$\alpha > \omega_0$

$\boxed{\alpha = \omega_0} \rightarrow \left.\begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha \\ s_2 &= -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha \end{aligned}\right\}$ $s_1 = s_2$. only one independent solution. (but should have 2)

$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + \alpha^2 V_c = 0. \quad - e^{-\alpha t}, \; te^{-\alpha t}.$

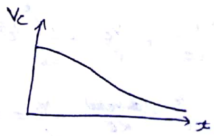
$V_c(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} = e^{-\alpha t}(A_1 + A_2 t)$

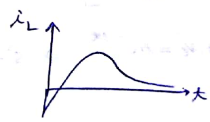
$i_L = -C\frac{dV_c}{dt} = -Ce^{-\alpha t}(-A_1\alpha - A_2\alpha t + A_2).$

$V_c(0)$, $\quad i_L(t=0) = 0$

$A_1 = V_c(0) \quad -A_1\alpha + A_2 = 0 \Rightarrow A_2 = A_1\alpha$

$\Rightarrow V_c(t) = V_c(0)[1 + \alpha t]e^{-\alpha t}.$





Energy loss :-

$$E = \frac{1}{2} C (V_c(t))^2 + \frac{1}{2} L (i_L(t))^2.$$

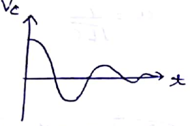
$$\frac{dE}{dt} = C V_c \frac{dV_c}{dt} + L i_L \frac{di_L}{dt}$$

$i_C = -i_L$ $\qquad V_L = V_2$

$$\frac{dE}{dt} = -V_c i_L + V_2 i_L = (V_2 - V_c) i_L = -R i_L^2.$$

$$\left(\frac{V_c - V_2}{R} = i_L\right).$$

Underdamped case :-



$T = \frac{2\pi}{\omega_d} \approx \omega_0$

(bcoz R is small).

In one cycle:

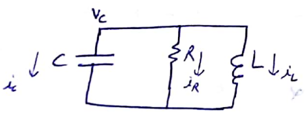
$\Delta E = R \langle i_L^2 \rangle T.$

$E = L \langle i_L^2 \rangle$

$$\frac{E}{\Delta E} = \frac{L}{RT} = \frac{L \cdot \omega_0}{R \cdot 2\pi} = \frac{1}{R}\sqrt{\frac{L}{C}} \cdot \frac{1}{2\pi}.$$

Q

$$\Rightarrow \boxed{Q = \frac{2\pi E}{\Delta E}}.$$



$$C\frac{dV_c}{dt} + \frac{V_c}{R} + \frac{1}{L}\int_{-\infty}^{t} V_c(t')dt' = 0$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{(dV_c/dt)}{RC} + \frac{V_c}{LC} = 0$$

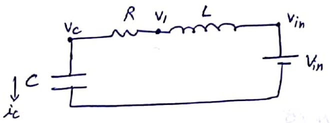
Assume $V_c = Ae^{-st}$ $\Rightarrow s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$ (characteristic eqn.)

$\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ $\qquad$ ($\alpha = \frac{R}{2L}$ in series case).

$s^2 + 2\alpha s + \omega_0^2 = 0 \Rightarrow$ $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$ $\qquad$ $\alpha < \omega_0$, $\alpha = \omega_0$, $\alpha > \omega_0$

~~$Q = RC/L$~~.

$Q = R\sqrt{\frac{C}{L}}$ $\qquad$ ($Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ in series case).



$$C\frac{dV_c}{dt} + \frac{V_c - V_1}{R} = 0 \Rightarrow V_1 = RC\frac{dV_c}{dt} + V_c.$$

$$\frac{V_1 - V_c}{R} + \frac{1}{L}\int_{-\infty}^{t}(V_1 - V_{in})\,dt' = 0 \qquad \text{Put } V_1.$$

$$\Rightarrow \frac{1}{R}\left(\frac{dV_1}{dt} - \frac{dV_c}{dt}\right) + \frac{1}{L}(V_1 - V_{in}) = 0$$

$$\Rightarrow \frac{1}{R}\left(RC\frac{d^2V_c}{dt^2} + \cancel{\frac{dV_c}{dt}} - \cancel{\frac{dV_c}{dt}}\right) + \frac{1}{L}\left(RC\frac{dV_c}{dt} + V_c - V_{in}\right) = 0$$

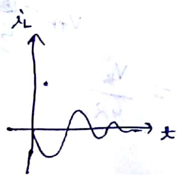
$$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{V_{in}}{LC}$$

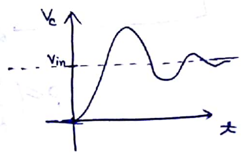
$$V_c(t) = \underbrace{V_{cH}(t)}_{\text{Homogenous}} + V_{cp}(t)$$

$V_{cp}(t) = V_{in}.$

$A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s^2 + 2\alpha s + \omega_0^2 = 0.$ $\qquad$ $V_c(0) = 0$, $i_L(0) = 0$



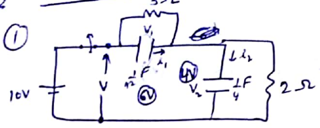


Underdamped case.

At $t=\infty \rightarrow$ voltage drop is across C.
$t=0^+ \rightarrow$ ——— " ——— L.

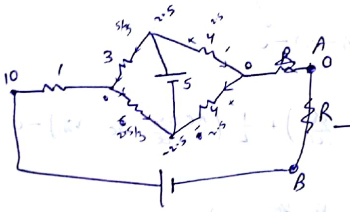
Tut-2

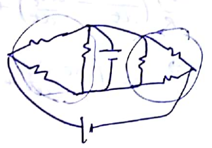
(1)



$i_1 = -2\, e^{-t/0.5}$ Amperes
$v_1 = 6e^{-t/0.5}$ volts
$v_2 = 4e^{-t/0.5}$ volts
$i_2 = -2\, e^{-t/0.5}$ Amperes.
$V = V_1 + V_2 = 6e^{-t/0.25}$

(2)



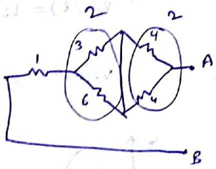


When $R_L = R_{TH} \Rightarrow$ max. power dissipation

$\left(2.5 - \frac{5}{3} - 10\right)$

$\frac{15}{2} + 5/3$

$\frac{45+10}{6}$

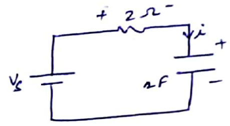


$\Rightarrow R_{TH} = 5\Omega$.

$V_{TH} = \frac{55}{6}$ V.

$P = \left(\frac{V_{th}}{R_{th}+R}\right)^2 R = \frac{V_{th}^2}{4R}$.

(3)

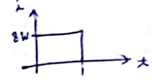


$i = C\frac{dV}{dt} = 2A \quad 0 \le t \le 1$
$= 0 \quad t \ge 1$.

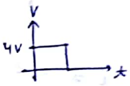
$V_s =$

$i(t) =$

$P_R(t) =$

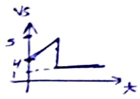


$V_R(t) =$

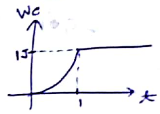


~~$V_s(t) =$~~

$V_s(t) =$

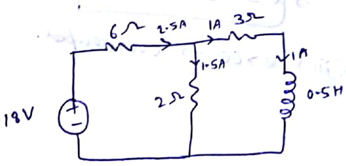


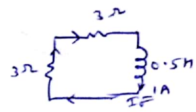
$W_C(t) =$

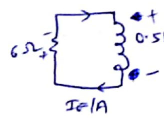


$\frac{1}{2}CV^2$

(4)

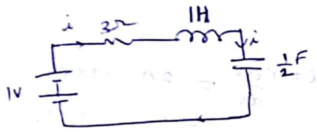






$i(t) = I_0 e^{-Rt/L}$
$i(t) = e^{-9t}$.
$V_L(t) = L\frac{di}{dt} = -4.5e^{-9t}$.

(3)



$$3i + \frac{di}{dt} + 2q = 0$$

$$\Rightarrow \cancel{3\frac{di}{dt}} \quad \Rightarrow \frac{d^2i}{dt^2} + 3\frac{di}{dt} + 2i = 0$$

$$\Rightarrow i = e^{-t},\ e^{-2t}$$

$$i = c_1 e^{-t} + c_2 e^{-2t}$$

$$\frac{di}{dt} = 1 \quad (t = 0^+)$$

$$-c_1 - 2c_2 = 1 \quad \Rightarrow \quad c_1 + 2c_2 = -1$$

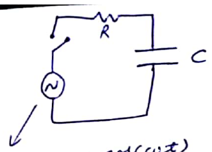
$$c_1 + c_2 = 0$$

$$\Rightarrow c_1 = 1,\ c_2 = -1.$$

$$\Rightarrow \boxed{i = e^{-t} - e^{-2t}}.$$

(iii) $\alpha > \omega \Rightarrow$ overdamped.

(4)



$V_{in}(t) = V_i \cos(\omega t)$

ω: angular frequency

Time period $(T) = 2\pi/\omega$.

$\cancel{f = 2\pi}$ $\quad f = \frac{1}{T} = \frac{\omega}{2\pi}$ (freq.)

$$\frac{V_{in} - V_c}{R} = C\frac{dV_c}{dt}.$$

$$\Rightarrow RC\frac{dV_c}{dt} + V_c = V_i \cos(\omega t)$$

$$V_c(t) = V_{cH}(t) + V_{cp}(t)$$

$V_{cH}(t) = Ae^{-t/\tau}$

$\tau = RC$.

Try: $V_{cp}(t) = k_1 \cos(\omega t) + k_2 \sin(\omega t)$

$$\Rightarrow RC\omega\,(-k_1 \sin\omega t + k_2 \cos\omega t) + (k_1 \cos\omega t + k_2 \sin\omega t) = V_i \cos\omega t$$

$\Rightarrow k_2 = k_1 RC\omega$ — equating $\sin\omega t$ terms

$RC\omega\, k_2 + k_1 = V_i$ — equating $\cos\omega t$ terms

$$\Rightarrow k_2 = \frac{V_i RC\omega}{1 + (RC\omega)^2}, \quad k_1 = \frac{V_i}{1 + (RC\omega)^2}$$

$$V_c(t) = \underbrace{\frac{V_i}{1 + (RC\omega)^2}\left[\cos(\omega t) + (RC\omega)\sin(\omega t)\right]}_{\text{sinusoidal steady state}} + \underbrace{Ae^{-t/\tau}}_{0\ (t \gg \tau)}.$$

If $V_{in} = V_i \sin\omega t = V_i \cos(\omega t - \pi/2)$

$$V_c(t) = \frac{V_i}{1 + (RC\omega)^2}\left[\sin(\omega t) - RC\omega \cos(\omega t)\right].$$

If $V_{in} = \cos\omega t + k \sin\omega t$.

$V_c(t) = V_{\text{due to } \cos\omega t} + V_{\text{due to } \sin\omega t}$ (linear circuit)

$k = j$ $(j^2 = -1)$

$$V_{in} = V_i(\cos\omega t + j\sin\omega t) = V_i e^{j\omega t}$$

(Euler's theorem)

$$V_0 e^{j(\omega t + \phi)} = (V_0 e^{j\phi}) e^{j\omega t}.$$

$V_{in} = V_i e^{j\omega t}$.

$RC \frac{dV_c}{dt} + V_c = V_{in}$

Try $V_c = V_1 e^{j\omega t}$.

$RCV_1 e^{j\omega t}(j\omega) + V_1 e^{j\omega t} = V_{in} = V_i e^{j\omega t}$

$RCV_1 (j\omega) + V_1 = V_i$

$$\boxed{V_1 = \frac{V_i}{RC\omega j + 1}} = \frac{V_i(1 - RC\omega j)}{1-(RC\omega)^2}.$$

$$V_1 = \frac{V_i}{1-(RC\omega)^2} - j\frac{RC\omega V_i}{1-(RC\omega)^2}$$

$V_{in} = V_i(\cos\omega t + j\sin\omega t)$

If $V_{in} = V_i \cos\omega t$ = real part = $\mathrm{re}(V_1 e^{j\omega t})$

$= \cancel{\frac{V_i}{1-(RC\omega)^2}} =$

$V_{in} = V_i \sin\omega t$ = imag part = $\mathrm{imag}(V_1 e^{j\omega t})$

$V_{in} = \underset{Re(V_i e^{j\omega t})}{V_i e^{j\omega t}}$, $V_{out} = \underset{Re(V_1 e^{j\omega t})}{V_1 e^{j\omega t}}$

If $V_{in} = \cos(\omega t) \Rightarrow V_i = 1$

If $V_{in} = \sin(\omega t) \Rightarrow V_i = -j$.

$v_i = a + jb = \sqrt{a^2+b^2}\, e^{j\theta}$ $\quad \theta = \tan^{-1}(b/a)$.

Resistance:-

$i(t) = Re[\bar{I} e^{j\omega t}]$

$i(t)$ $\bar{I} e^{j\omega t} = I_1 e^{j\theta}$.

$V(t) = Re[\underset{V_1 e^{j\phi}}{\bar{V}} e^{j\omega t}]$

$V(t) = i(t)R \Rightarrow V_1\cos(\omega t+\phi) = RI_1\cos(\omega t+\theta)$

$\Rightarrow V_1 = RI_1, \ \phi = \theta$

$\Rightarrow \boxed{\frac{\bar{V}}{\bar{I}} = R}$.

Capacitor:-

$i(t) = \bar{I} e^{j\omega t} = (I_1 e^{j\theta}) e^{j\omega t}$. } $i(t) = I_1\cos(\omega t+\theta)$.

$V(t) = \bar{V} e^{j\omega t} = (V_1 e^{j\phi}) e^{j\omega t}$. } $V(t) = V_1\cos(\omega t+\phi)$.

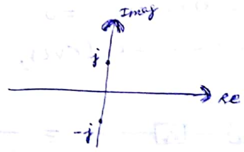
$I_1, V_1 \Rightarrow +ve$.

$i(t) = C\frac{dV_c}{dt} \Rightarrow I_1\cos(\omega t+\theta) = -CV_1\omega\sin(\omega t+\phi)$.

$= CV_1\omega\cos(\frac{\pi}{2}+\omega t+\phi)$.

$\Rightarrow I_1 = CV_1\omega, \ \theta = \frac{\pi}{2}+\phi$.

$$\frac{\bar{V}}{\bar{I}} = \frac{V_1 e^{j\phi}}{I_1 e^{j\theta}} = \frac{1}{\omega C} e^{-\pi/2 j}. = \frac{1}{j\omega C}.$$



Inductor:-

$V = L\frac{di}{dt} \Rightarrow \frac{\bar{V}}{\bar{I}} = j\omega L$

$I = \bar{I} e^{j\omega t}$

$V = \bar{V} e^{j\omega t} = \underbrace{L\bar{I}(j\omega)}_{LdI/dt} e^{j\omega t}$

$\Rightarrow \frac{\bar{V}}{\bar{I}} = j\omega L$

$\frac{\bar{V}}{\bar{I}}$ = complex = impedance = Z.

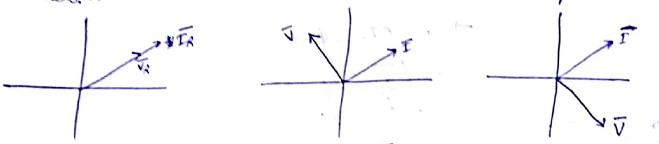
$Z = R$ — Resistor

$Z = \frac{1}{j\omega C}$ — Capacitor

$Z = j\omega L$ — Inductor

(Capacitor and Inductor:) Frequency dependent

Resistor | Inductor | Capacitor



Inductor: $\bar{V} = j\omega L \bar{I}$

* Voltage leads current.
* Current lags voltage

Capacitor: $\bar{V} = \frac{\bar{I}}{j\omega C}$

* Current leads voltage
* Voltage lags current.

Resistor:

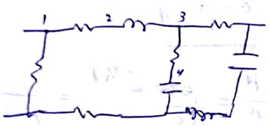
$\bar{I}$ = real

$I = I_0 \cos \omega t \Rightarrow \bar{I} = I_0 e^{j\omega t}$

$V = \mathrm{Re}(j\omega L \bar{I}_0 e^{j\omega t})$

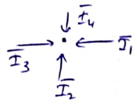
~~$= j\omega L I_0 \cos\omega t$~~

$= -I_0 \omega L \sin(\omega t)$.

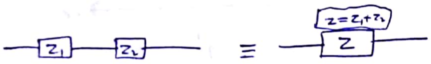


$V_1(t) + V_2(t) + V_3(t) \ldots = 0$

$\bar{V}_1 + \bar{V}_2 + \ldots = 0$ (KVL).



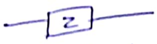
$\bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4 = 0$ (KCL).



$\bar{V}_{Z_1} = \bar{I} Z_1 \qquad \bar{V}_{Z_2} = \bar{I} Z_2$

$\bar{V}_Z = \bar{I}(Z_1 + Z_2)$

$\equiv$



$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$.

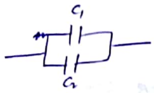
$\bar{I}_{Z_1} = \frac{\bar{V}}{Z_1}, \quad \bar{I}_{Z_2} = \frac{\bar{V}}{Z_2}$

$\bar{I} = \bar{I}_{Z_1} + \bar{I}_{Z_2} = \bar{V}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) \Rightarrow \boxed{\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}}$

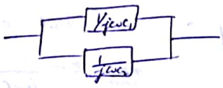
$\Rightarrow$

$Z_s = Z_1 + Z_2 = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} = \frac{1}{j\omega C}$

$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$.

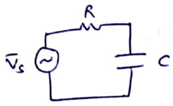


$\Rightarrow$



$j\omega C = \frac{1}{Z} = j\omega C_1 + j\omega C_2 = j\omega(C_1 + C_2)$

$\Rightarrow C = C_1 + C_2$.



$Z = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$

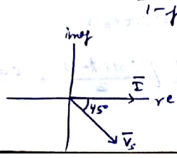
$\bar{I} = \frac{\bar{V}_s}{Z} = \frac{\bar{V}_s j\omega C}{1 + j\omega RC} = \frac{\bar{V}_s (j\omega C)(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$.

$R = 1\Omega, \quad \omega C = 1/\Omega$.

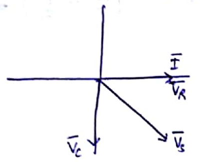
$Z = 1 + \frac{1}{j} = (1 - j)\ \Omega$.

$\bar{I} = \frac{\bar{V}_s}{1 - j} = \frac{\bar{V}_s (1 + j)}{2} \Rightarrow \bar{V}_s = \bar{I}(1 - j)$

$\Rightarrow \bar{V}_s = (\sqrt{2}\bar{I})\underbrace{\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)}_{e^{-\pi/4}}$

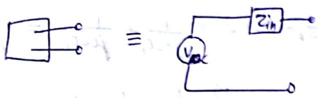


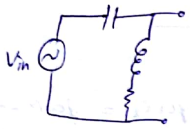
$\bar{V}_c = \bar{I} Z_c = \bar{I} \frac{1}{j\omega C} = -j\bar{I}$



$\bar{V}_R + \bar{V}_c = \bar{V}_s$.

Thevenin's theorem:-





Find out V_{oc}.

short V_{in} $\rightarrow Z = \frac{1}{j\omega C} + j\omega L + R$

$= j\left(\omega L - \frac{1}{\omega C}\right) + R$.

$\bar{V}_{oc} = \frac{\bar{V}_{in}}{Z} \cdot (j\omega L + R)$.

$Z_{in} = \frac{1}{j\omega C} \,||\, (R + j\omega L)$.

Power:-

$\rightarrow \boxed{Z} \rightarrow$ $i(t) = I\cos(\omega t + \theta) \rightarrow \bar{I} = Ie^{i\theta}$.

$v(t) = V\cos(\omega t + \phi) \rightarrow \bar{V} = Ve^{i\phi}$.

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

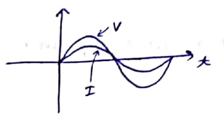
$v(t)\, i(t) = VI\cos(\omega t + \phi)\cos(\omega t + \theta)$.

$\underbrace{\cancel{=VI}}_{P(t)} = VI \cdot \frac{1}{2}\left[\cos(\phi - \theta) + \cos(2\omega t + \theta + \phi)\right]$.

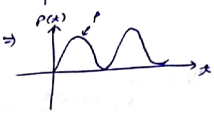
$\underbrace{\langle P(t) \rangle}_{P_{avg}} = \frac{1}{2} VI\cos(\phi - \theta) = \frac{1}{2} Re(\bar{V}\bar{I}^*) = \frac{1}{2} Re(\bar{V}^*\bar{I})$

$I^* \rightarrow$ conjugate.

$\phi = 0 \Rightarrow P_{avg} = \frac{1}{2} VI \longrightarrow$

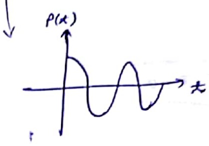


$\Rightarrow$

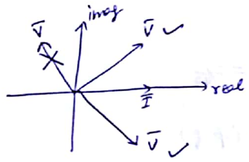


$\phi - \theta = 90° \Rightarrow P_{avg} = 0$

$\phi - \theta = -90° \Rightarrow P_{avg} = 0$

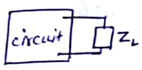


$P_{avg} = \frac{1}{2}|\bar{I}|^2 Re(z) = \frac{1}{2}\frac{|\bar{V}|^2}{Re(z)}$.

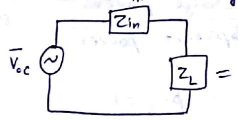


$\bar{V}$ & $\bar{I}$ angle can be at max $90°$.

Maximum Power transfer:-



$Z_{in} = R_{in} + jX_{in}$



$= R_L + jX_L$

Resistance.

Reactance.

$\bar{I} = \frac{\bar{V}_{oc}}{Z_{in} + Z_L}$

$\bar{V}_L = \frac{\bar{V}_{oc}}{Z_{in} + Z_L} \cdot Z_L$

$P_L = \frac{1}{2} Re(\bar{V}_L \bar{I}_L^*) = \frac{1}{2} Re\left(\frac{|V_{oc}|^2 Z_L}{(Z_{in} + Z_L)(Z_{in}^* + Z_L^*)}\right)$

$= \frac{1}{2}|V_{oc}|^2 \frac{R_L}{(R_{in} + R_L)^2 + \underbrace{(X_{in} + X_L)^2}_{\rightarrow 0 \text{ for max. power}}}$

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$$\frac{\partial}{\partial R_L}\left(\frac{R_L}{(R_{in}+R_L)^2}\right)=0 \Rightarrow \frac{2(\cancel{R_{in}+R_L})\cdot R_L-(R_{in}+R_L)^{\cancel{2}}}{(R_{in}+R_L)^{\cancel{4}\,3}}=0$$

$$\Rightarrow \boxed{R_{in}=R_L}.$$

For max. power along Z_L : $\boxed{R_L=R_{in}}$.

$\boxed{Z_L=Z_{in}^*}$.

$V=V_0$
$=V_0\cos\omega t$.

RMS value : $\sqrt{\dfrac{\int_0^T V_0^2\cos^2\omega t\,dt}{T}} = \sqrt{\dfrac{\frac{1}{2}V_0^2\cdot T+0}{T}}$

$= V_0/\sqrt{2}\ = V_e \leftarrow$ effective

$P_{avg}=\frac{1}{2}VI\cos(\theta)$
$=V_eI_e\cos(\theta)$. $\quad V_e=V/\sqrt{2},\ I_e=I/\sqrt{2}$.

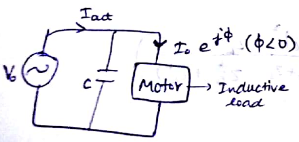
$V_eI_e \rightarrow$ Apparent power

$\cos(\theta) \rightarrow$ Power factor (p.f.)

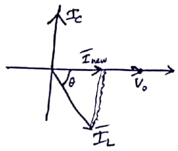
$$p.f=\frac{\text{Average power}}{\text{Apparent power}} \qquad \theta=p.f.\text{ angle}.$$

Resistance — $p.f=1$
Inductor/Capacitor — $p.f.=0$.

$\theta>0$ — Inductive load (lagging)
$\theta<0$ — capacitive load (leading)

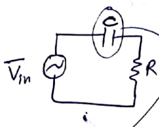


p.f lags by 0.8
want $p.f.\approx 1$.



$|I_{out}|$ less

· Power through motor remains same but I_{out} decreases. We can reduce power loss in transmission ~~lines~~ wires. ($I\downarrow, I^2R\downarrow$).



$$I=\frac{V_{in}}{R+\frac{1}{j\omega C}} \qquad V_R=\frac{V_{in}}{R+\frac{1}{j\omega C}}\cdot R$$

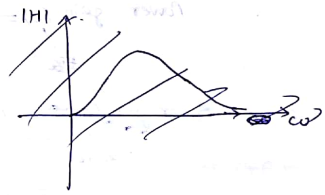
$\omega\to\infty$, $C\to$ shortcircuit ($Z\downarrow$)
$\omega\to 0$, $C\to$ opencircuit ($Z\uparrow$)

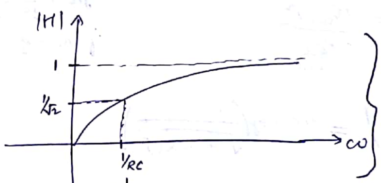
$H(\omega)$ ↓ transfer function

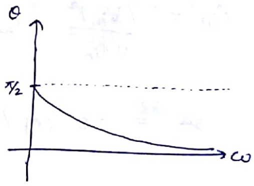
$$H(\omega)=\frac{V_R}{V_{in}}=\frac{1}{1+\frac{1}{j\omega RC}}=\frac{j\omega RC}{1+j\omega RC}.$$

$$\Rightarrow H(\omega)=\frac{j\omega RC(1-j\omega RC)}{(1+(\omega RC)^2)}=\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\cdot e^{i\theta}.$$

$\left[\tan\theta=\frac{1}{\omega RC}\right]$.





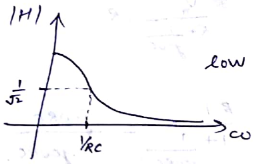


High pass filter
(ω large passes).

$\bar{V_c} = \frac{(1/j\omega c)}{R + (1/j\omega c)} \cdot V_{in}$

$H = \frac{1}{1+jR\omega C} = \left(\frac{V_c}{V_{in}}\right)$ $|\bar{H}| = \frac{1}{\sqrt{1+(R\omega C)^2}}$

$\tan\theta = \frac{Im}{Re} = -\omega RC.$



low pass filter.

$|\bar{H}| = \left|\frac{\bar{V_2}}{\bar{V_1}}\right| =$ voltage gain

P_{in} —[Amplifier]— P_{out}

Power gain $= \frac{P_{out}}{P_{in}}$.

Power gain (B) $= \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$
Bel

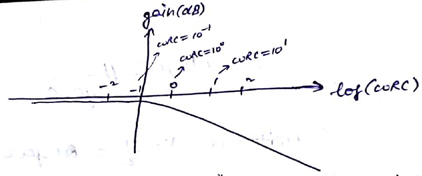
Power gain (dB) $= 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$.
decibel

$= 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

$P_{in} = \frac{|V_{in}|^2}{2R_{in}}$, $P_{out} = \frac{|V_{out}|^2}{2R_{out}}$ (Load) if $R_{in} = R_L$.

$gain^{(dB)} = 20\log\left|\frac{V_c}{V_{in}}\right| = 20\log\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)$

$= -10\log(1+(\omega RC)^2).$



If $\omega RC >> 1 \Rightarrow gain^{(dB)} = -10\log((\omega RC)^2) \Rightarrow gain^{(dB)} = \underbrace{-20\log(\omega RC)}_{\text{straight line slope } (-20)}$.

slope $= -20$ dB/decade ($\omega' = 10\omega$)

decade : freq. change by factor of 10.

If $\omega' = 10\omega$
$gain' = -20\log(10\omega)$
$= -20 - 20\log(\omega) = -20 + gain.$

slope $= 6.02$ dB / octave ($\omega' = 2\omega$)

$gain' = -20\log(2\omega)$
$= -20\log 2 - 20\log\omega = -6.02 + gain.$

Tut-3

(1) $V_e = 220$ V, $f = 60$ Hz, $I_e = 20$ A, $\cos\theta = 0.75$ (lagging)

(a) $P_{avg} = V_e I_e \cos\theta = 220 \times 20 \times 0.75 = 3300$ W.

(b) ~~Reactive part = $V_e I_e \sin\theta$~~

~~$P_{avg} = \frac{V_e^2}{Re(Z)} \Rightarrow Re$~~

$P_{avg} = \frac{V_e^2}{\cancel{Re(Z)}} Re\left(\frac{1}{Z}\right).$

$P_{avg} = \frac{V_e^2}{\cancel{Re(Z)}} Re\left(\frac{1}{Z}\right).$ $\frac{Im(Z)}{Re(Z)} = \sin\theta$

$\Rightarrow \underbrace{Im(Z)}_{\omega L} = Re(Z)\sin\theta$
$= \frac{V_e^2 \sin\theta}{P_{avg}}.$

$Im(Z) = 0 \Rightarrow Im\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = 0$

$\Rightarrow j\omega C + Im\left(\frac{1}{j L\omega + R}\right) = 0$

$\boxed{\omega C - \frac{L\omega}{R^2 + L^2\omega^2} = 0.}$

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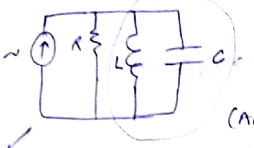


$Z = R \parallel j\omega L \parallel \frac{1}{j\omega C}$.

(Admittance) $Y = \frac{1}{Z} = \cancel{R + j\omega L + \frac{1}{j\omega C}}$

$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$.

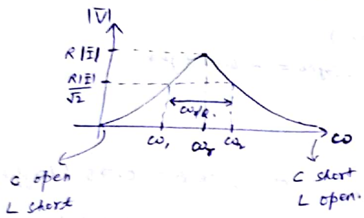
$\left(Q = R\sqrt{\frac{C}{L}}\right)$.

$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$

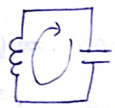
$\bar{V} = \frac{\bar{I}}{Y}$.

At $\omega = \omega_r$, $\omega C = \frac{1}{\omega L} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$.

$|Y|$ min. resonance



Tank circuit



$Z = \infty$ at $\omega = \omega_r$.

$Y = \frac{1}{R}\left(1 + jR\left(\omega C - \frac{1}{\omega L}\right)\right)$

Need $R\left(\omega C - \frac{1}{\omega L}\right) = \pm 1$. for $|Y| = \frac{\sqrt{2}}{R}$ (half power points)

$R\left(\omega C - \frac{1}{\omega L}\right) = +1 \Rightarrow \omega^2 RLC - L\omega - R = 0$.

$\Rightarrow \omega = \frac{L \pm \sqrt{L^2 + 4R^2LC}}{2RLC}$

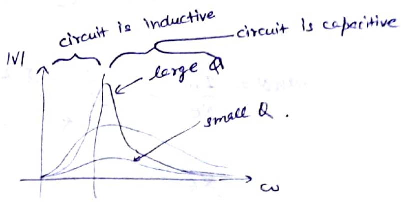
$\Rightarrow \omega = \frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$.

$\frac{\omega_r}{Q} = \frac{1}{\sqrt{LC}} \cdot \sqrt{\frac{L}{C}} \cdot \frac{1}{R} = \frac{1}{RC}$.

$\omega = \frac{\omega_r}{2Q} \pm \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$. $\Rightarrow \omega_2 = \frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$.

$\omega_1 = -\frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$.

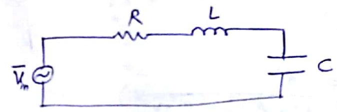
$B.W. = \omega_2 - \omega_1 = \frac{\omega_r}{Q}$.



$Q = \frac{\omega_r}{B.W.}$ = selectivity of circuit.

At $\omega = \omega_r$, $\bar{I}_L = \frac{\bar{V}}{Z_L} = \frac{\bar{I}R}{j\omega_r L}$, $\omega_r = \frac{1}{\sqrt{LC}}$.

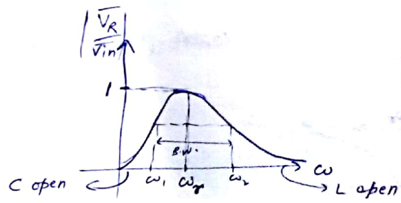
$|\bar{I}_L| = |\bar{I}|\, R\sqrt{\frac{C}{L}} = Q|\bar{I}|$.



$Z = R + j\omega L + \frac{1}{j\omega C}$, $\bar{I} = \frac{\bar{V}_{in}}{Z}$, $\bar{V}_R = \frac{\bar{V}_{in}}{Z} \cdot R$

$\Rightarrow \bar{V}_R = \frac{\bar{V}_{in} \cdot R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$

At $\omega = \omega_R = \frac{1}{\sqrt{LC}}$.



$BW = \omega_2 - \omega_1 = \omega_r / Q$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

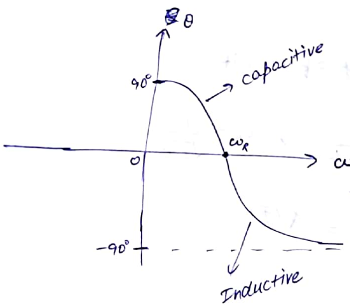
$$H = \frac{V_R}{V_{in}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

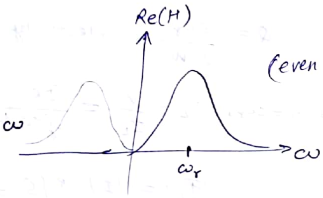
$$= \frac{R\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$$\tan\theta = \frac{Im}{Re} = \frac{\frac{1}{\omega C} - \omega L}{R}.$$

If $\omega \to 0 \Rightarrow \tan\theta = \frac{1}{\omega RC}$.

$\omega \to \infty \Rightarrow \tan\theta = -\frac{\omega L}{R}$.

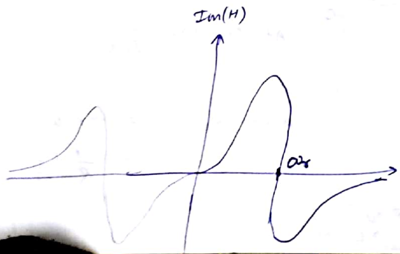




(even func.)

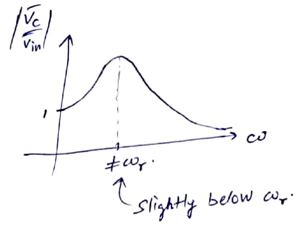
$$Re(H) = \frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$$Im(H) = \frac{R\left(\frac{1}{\omega C} - \omega L\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

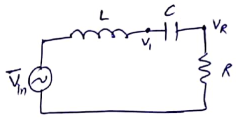


(odd func.)

$$H' = \frac{\overline{V_C}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$



at $\omega = \omega_r$
$|H| = Q$.



$$\overline{I} = \frac{\overline{V_{in}}}{R + j\left(L\omega - \frac{1}{\omega C}\right)}$$

$$\overline{V_R} = \overline{I}R = \overline{V_{in}} \cdot \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

$$\frac{V_R}{R} + C\frac{d}{dt}(V_R - V_1) = 0 \quad \text{—(i)}$$

$$\frac{1}{L}\int_{-\infty}^{t}(V_1 - V_{in})\,dt' + \underbrace{C\frac{d}{dt}(V_1 - V_R)}_{V_R/R} = 0$$

$$\Rightarrow \frac{1}{L}(V_1 - V_{in}) \cancel{+ \frac{dV_R}{dt\,R} = 0} + \frac{1}{R}\frac{d}{dt}(V_R) = 0.$$

$$\Rightarrow \cancel{\frac{1}{L}\frac{dV_1}{dt} + \frac{1}{R}\frac{d^2V_R}{dt^2} = 0}$$ diff. , use $\frac{dV_{in}}{dt} = \frac{V_R}{RC} + \frac{dV_R}{dt}$ — By (i).

$$\frac{dV_R}{dt^2} + \frac{R}{L}\frac{dV_R}{dt} + \frac{1}{LC}V_R = \frac{R}{L}\frac{dV_{in}}{dt}.$$

Assume:- $e^{j\omega t}$.

$$\left(-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}\right)\overline{V_R} = \frac{R}{L}j\omega\overline{V_{in}}$$

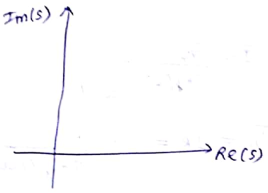
$$\overline{V_R} = \left(\frac{j\omega R}{L}\right)\overline{V_{in}} \cdot \frac{1}{-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}} = \frac{R\overline{V_{in}}}{+j\omega L + R + \frac{1}{j\omega C}}$$ H(ω)

e^{st}

$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$ (if $V_{in} = 0$)

$$S = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}.$$

$$H(s) = \frac{\left(\frac{SR}{L}\right)}{(S-S_1)(S-S_2)}.$$



$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t}\, d\omega \quad \text{(Fourier transform)}$$

$$H(t) = \frac{1}{2\pi} \int H(\omega)\, e^{j\omega t}\, d\omega$$

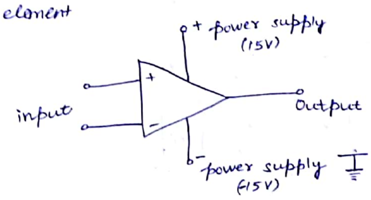
$$H(\omega) = \int_{-\infty}^{\infty} H(t)\, e^{-j\omega t}\, dt$$

← causality.

Op. Amp. (operational amplifier):-

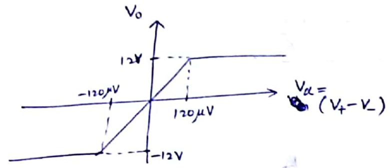
↓

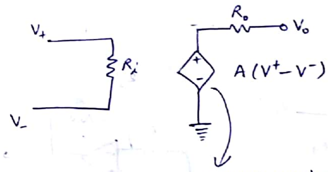
Active element



$$\boxed{V_o = A(V_+ - V_-)}$$

$A = 10^5$



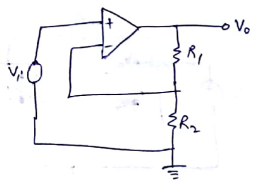


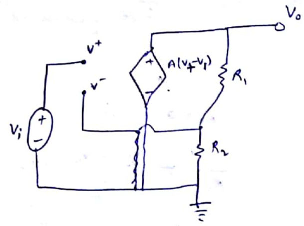
Voltage controlled Dependent voltage source

$R_i \approx 2\,M\Omega$ → almost open.

$R_o \approx 75\,\Omega$ → almost short.

$A \approx 10^5$





$V_+ = V_i$ —(i)

$\frac{V_-}{R_2} + \frac{V_- - V_o}{R_1} = 0$ —(ii)

$V_- = \frac{R_2 V_o}{R_1 + R_2}$, $V_o = A(V_+ - V_-)$ —(iii)

$$\boxed{V_o = \frac{AV_i}{1 + \frac{AR_2}{R_1 + R_2}}}$$

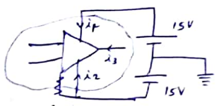
Scanned by CamScanner

A is very large $\Rightarrow$ $V_o = \dfrac{AV_i}{1 + \dfrac{AR_2}{R_1+R_2}}$

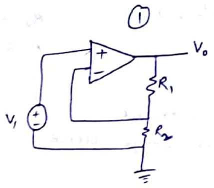
$\Rightarrow$ $V_o = \left(\dfrac{R_1+R_2}{R_2}\right) V_i$

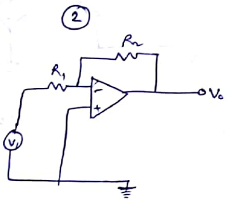
$\Rightarrow$ $\boxed{V_o = \left(1 + \dfrac{R_1}{R_2}\right) V_i}$

$\boxed{\text{Gain} = 1 + \dfrac{R_1}{R_2}}$

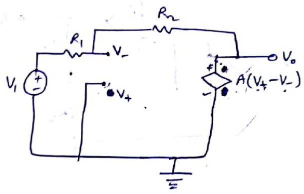


$R_{in} \to \infty$
$R_o \to 0$
$A \to \infty$





(-ve feedback)



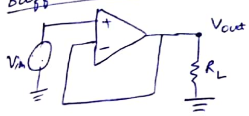
$V_+ = 0$ —(i)

$\dfrac{V_1 - V_-}{R_1} = \dfrac{V_- - V_o}{R_2}$ —(ii)

$V_o = A(V_+ - V_-)$ —(iii)

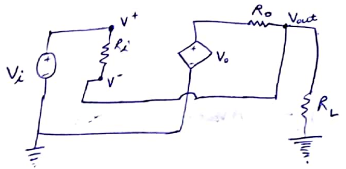
$\Rightarrow$ $V_o = \dfrac{-AR_2/(R_1+R_2)}{1 + \left(\dfrac{AR_1}{R_1+R_2}\right)}$ $\Rightarrow$ $\boxed{V_o = \dfrac{-R_2}{R_1} V_1}$

Buffer circuit :-



$V_{out} = V_{in}$

V_{in} does not supply any current. All current comes from power source.



$V_o = A(V^+ - V^-)$.

$\dfrac{V_{out}}{R_L} + \dfrac{V_{out} - V_o}{R_o} + \dfrac{\overbrace{V^- - V^+}^{V_{out} - V_{in}}}{R_i} = 0$.

$V_o = A(V^+ - V^-)$, $V^+ = V_i$, $V_- = V_{out}$.

$= A(V_{in} - V_{out})$

eliminate V_o.

$V_{out}\left(\dfrac{1}{R_L} + \dfrac{1+A}{R_o} + \dfrac{1}{R_i}\right) = V_{in}\left(\dfrac{1}{R_i} + \dfrac{A}{R_o}\right)$

$$\boxed{V_{out} = V_{in}\dfrac{(R_o + AR_i)R_L}{R_iR_o + R_LR_i(1+A) + R_LR_o}}$$

$\dfrac{R_o + AR_i}{R_iR_o + R_LR_i(1+A) + R_LR_o} = \dfrac{1}{R_{TH} + R_L}$

$R_{TH}(R_o + AR_i) = R_iR_o + R_iR_L$

$\boxed{R_{TH} = \dfrac{R_i(R_o+R_L)}{R_o + AR_i}}$

$\boxed{R_{TH} = \dfrac{R_L + R_o}{A}}$

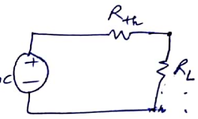
$\rightarrow$ $A \to \infty$ $\Rightarrow$ $V_{out} = \dfrac{\cancel{A}\cancel{R_i}\cancel{R_L}}{\cancel{R_L}\cancel{R_i}\cancel{A}} V_{in}$ $\Rightarrow$ $\boxed{V_{out} = V_{in}}$

$i = \dfrac{V_{in} - V_{out}}{R_i}$ $V_{out} \approx \dfrac{A}{1+A} V_{in}$

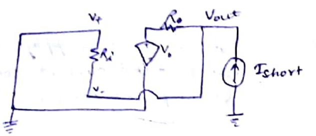
$V_{in} - V_{out} = \dfrac{1}{1+A} V_{in}$. $\Rightarrow$ $i = \dfrac{1}{(1+A)}\dfrac{V_{in}}{R_i}$.

$\Rightarrow$ $R_{in} = AR_i$.

$V_i \approx V_{oc}$



$\rightarrow$ $\boxed{V_{out} = V_{in}\dfrac{R_L}{R_{TH} + R_L}}$ =



$$\Rightarrow \frac{V_{out} - V_+^{\;\approx 0}}{R_i} + \frac{V_{out} - V_o}{R_o} = I$$

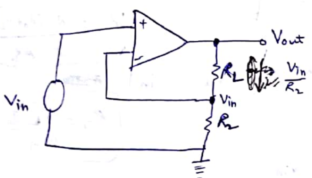
$$V_o = A\,(V_+ - V_-) = A\,(0 - V_-) = -A V_{out}.$$

$$\Rightarrow \frac{V_{out}}{R_i} + \frac{V_{out} + A V_{out}}{R_o} = I$$

$$\Rightarrow I = V_{out}\left(\frac{1}{R_i} + \frac{1+A}{R_o}\right) \quad \frac{1}{R_{TH}}$$

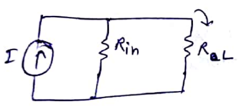
$$R_{TH} = \frac{V_{out}}{I_{short}}$$

$$\Rightarrow R_{TH} = \frac{R_o R_i}{R_o + R_i(1+A)} \approx \frac{R_o}{A}.$$

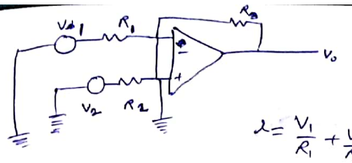


$\frac{V_{in}}{R_2}$ (independent of R_L).

current source



R_{in} large



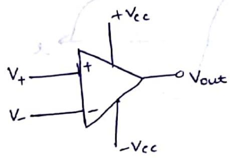
ADDER

$$i = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \cancel{R_3/V_o}$$

$$V_o = -R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right).$$

If $R_1 = R_2$

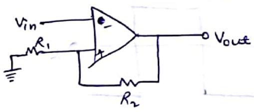
$$\Rightarrow V_o = \frac{-R_3}{R_1}(V_1 + V_2)$$



$V_+ > V_-$, $V_{out} = V_{cc}$.

$V_+ < V_-$, $V_{out} = -V_{cc}$.

Schmitt trigger

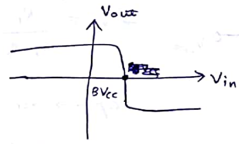


Assume $V_{out} = +V_{cc}$.

$\Rightarrow V_+ = \beta V_{cc} \quad \left(\beta = \frac{R_1}{R_1 + R_2}\right).$

$\Rightarrow$ if $V_{in} < \beta V_{cc}$ (OK)

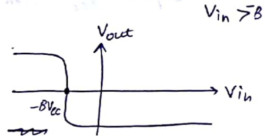
If $V_{in} > \beta V_{cc}$, $V_{out} = -V_{cc}$.



Assume $V_{out} = -V_{cc}$ $\Rightarrow V_+ = -\beta V_{cc}$

If $V_{in} < -\beta V_{cc} \Rightarrow V_{out} = +V_{cc}$.

$V_{in} > -\beta V_{cc} \Rightarrow$ (OK)



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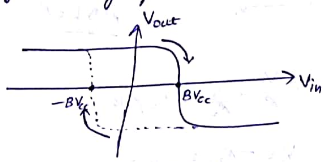
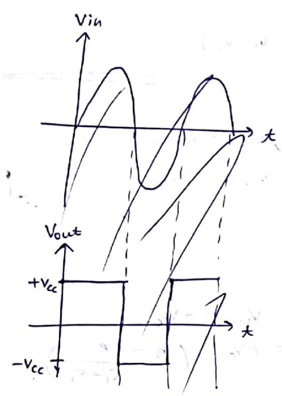
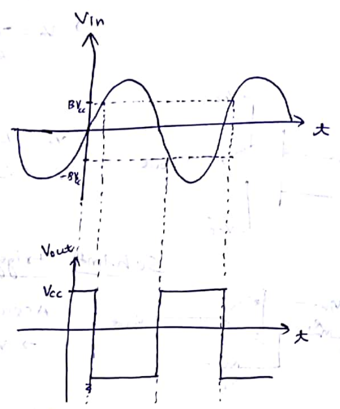
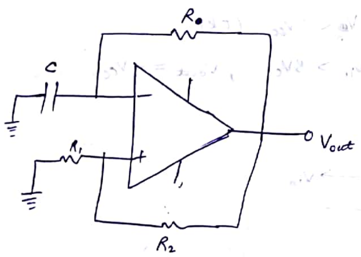
Combining the graphs,



Hysteresis loop.







A stable device (oscillator).

Assume $V_+ > V_-$, $V_{out} = +V_{cc}$

$\Rightarrow V_+ = \dfrac{R_1}{R_1+R_2} V_{out} = BV_{cc}$. $(B<1)$

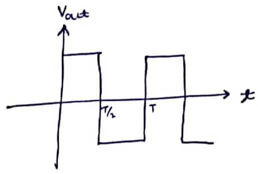
cap $\longrightarrow V_{cc}$ with, $\tau = RC$

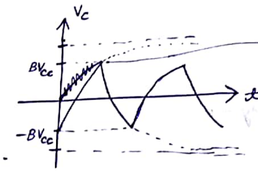
When $V_C > BV_{cc} \Rightarrow$ output $= -V_{cc}$.

$\Rightarrow V_+ = -BV_{cc}$.

cap $\rightarrow -V_{cc}$ with $\tau = RC$.

V_{out}





$T = ?$ $\quad V_C = V_-(t) = V_{cc} + Ae^{-t/RC}$

At $t=0$, $V_-(0) = V_{cc} + A$
$= -BV_{cc}$

$\Rightarrow A = -(1+B)V_{cc}$.

$$\boxed{V_-(t) = V_{cc} - (1+B)V_{cc}\, e^{-t/RC}}.$$

$V_-(t = T/2) = BV_{cc}$

$\Rightarrow V_{cc} - (1+B)V_{cc}\, e^{-T/2RC} = BV_{cc}$

$\Rightarrow e^{-T/2RC} = \dfrac{(1-B)}{1+B}$

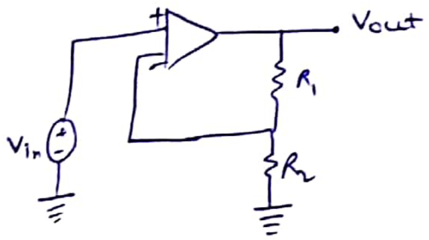
$\Rightarrow \boxed{T = 2RC \ln\left(\dfrac{1+B}{1-B}\right)}$ $\quad B = \dfrac{R_1}{R_1+R_2}$.

$\dfrac{V_{out} - V_c}{R} = i = \dfrac{c\,dV_c}{dt}$

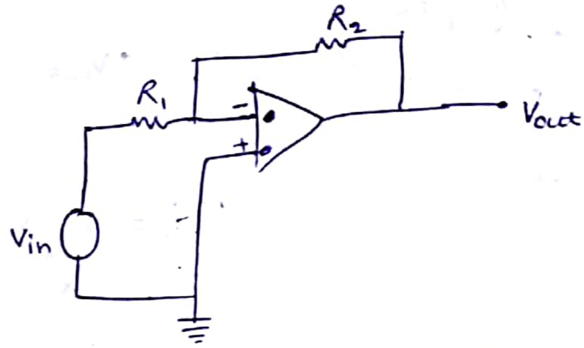
$-\dfrac{t}{RC} = \ln(V_{out} - V_c)$

$V_{out} = V_c + Ae^{-t/RC}$

op-amp

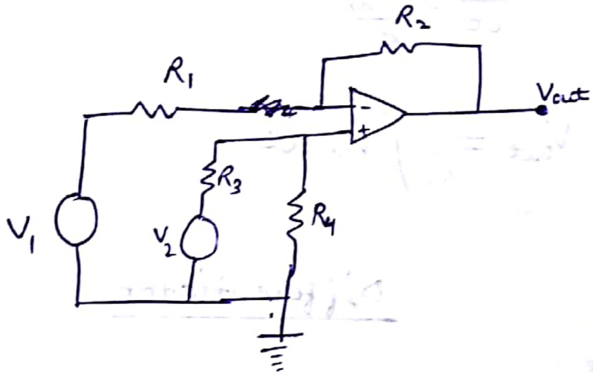


$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$



$$V_{out} = -\frac{R_2}{R_1} V_{in}.$$

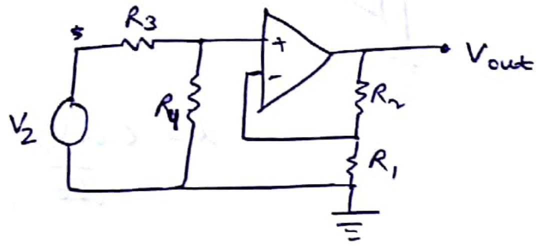
Input current = 0.
Virtual short b/w V_+ & V_-.



Put $V_2 = 0$

$$V_{out,1} = -\frac{R_2}{R_1} V_1 .$$

Put $V_1 = 0$



$$V_{out,2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_2 .$$

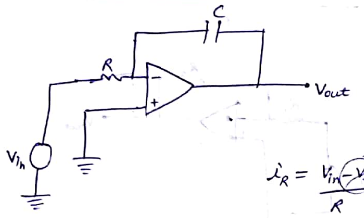
Let $R_3 = R_1$, $R_4 = R_2$

$$V_{out,2} = \frac{R_2}{R_1} V_2 .$$

$$V_{out} = V_{out,1} + V_{out,2}$$

$$\boxed{V_{out} = \frac{R_2}{R_1}(V_2 - V_1)}$$

Integrator



$$i_R = \frac{V_{in} - V_-}{R} = i_c$$

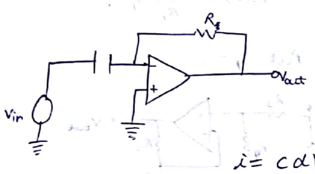
$$\Rightarrow i_c = C\frac{dV_c}{dt} = \frac{V_{in}}{R}$$

$$V_c = -V_{out}$$

$$\Rightarrow -C\frac{dV_{out}}{dt} = \frac{V_{in}}{R}$$

$$\Rightarrow V_{out} = \frac{-1}{RC}\int V_{in}\,dt$$

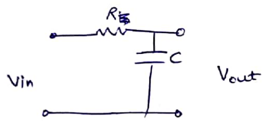
Differentiator



$$i = C\frac{dV_{in}}{dt}$$

$$V_{out} = -Ri = -RC\frac{dV_{in}}{dt}$$

Low-pass filter

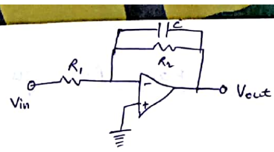


$$V_{out} = \frac{V_{in}}{R+\frac{1}{j\omega C}}\cdot\frac{1}{j\omega C}$$

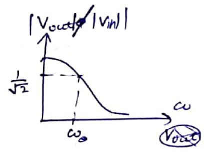
$$\Rightarrow V_{out} = \frac{V_{in}}{1+\omega RCj} = \frac{V_{in}}{1+j(\omega/\omega_0)}$$

$\omega_0 = \frac{1}{RC} =$ cutoff freq.

Low-Pass filter



$$V_{out} = -\frac{Z_2}{Z_1}\cdot V_{in} = -\frac{\left(R_2 \,\|\, \frac{1}{j\omega C}\right)}{R_1}\cdot V_{in}$$

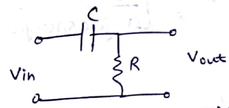


$$= \frac{-1}{R_1}\left(\frac{R_2\cdot\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}\right)V_{in}$$

$$\Rightarrow V_{out} = \frac{-R_2}{R_1}\left(\frac{1}{1+j(\omega/\omega_0)}\right)V_{in},$$

$\omega_0 = \frac{1}{R_2C}$ — cutoff freq.

High-Pass filter

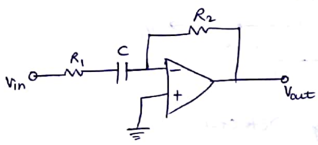


$$V_{out} = \frac{V_{in}}{R+\frac{1}{j\omega C}}\cdot R = \frac{V_{in}}{1+\frac{1}{j\omega RC}}$$

$$\Rightarrow V_{out} = \frac{V_{in}}{1-j(\omega_0/\omega)} \qquad \omega_0 = \frac{1}{RC}$$

$$\Rightarrow V_{out} = \frac{j(\omega/\omega_0)}{1+j(\omega/\omega_0)}\cdot V_{in}$$

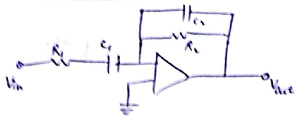
High-Pass filter



$$V_{out} = -\frac{Z_2}{Z_1}V_{in} = \frac{-R_2}{R_1+\frac{1}{j\omega C}}\cdot V_{in} = \frac{-R_2}{R_1}\left(\frac{j\omega C R_1}{1+j\omega C R_1}\right)$$

$$\Rightarrow \omega_0 = \frac{1}{R_1C}$$

Band Pass filter



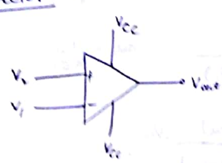
$$V_{out} = -\frac{Z_2}{Z_1} V_{in} = \frac{-\left(R_2 \parallel \frac{1}{j\omega C_2}\right)}{\left(R_1 + \frac{1}{j\omega C_1}\right)} V_{in}$$

$$V_{out} = -\frac{R_2 \cdot \frac{1}{j\omega C_2}}{\left(R_2 + \frac{1}{j\omega C_2}\right)\left(R_1 + \frac{1}{j\omega C_1}\right)} \cdot V_{in}$$

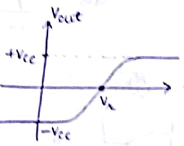
$$\Rightarrow V_{out} = \frac{-R_2 \cdot j\omega C_1}{(1+j\omega R_2 C_2)(1+j\omega R_1 C_1)} \cdot V_{in} = -\frac{R_2}{R_1}\left[\frac{j\omega C_1 R_1}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_2)}\right] V_{in}$$

Assume $R_2 > R_1$, $\omega_L = \frac{1}{R_1 C_1}$, $\omega_H = \frac{1}{R_2 C_2}$.

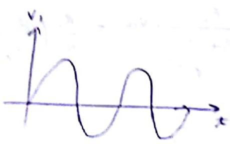
Comparator

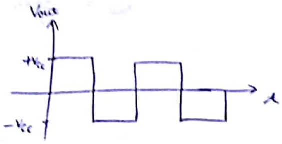


$V_2 = V_R$ (reference pt.)

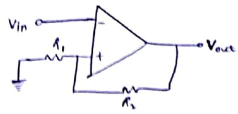


If $V_R = 0 \Rightarrow$ Zero crossing Detector





(+ve feedback)

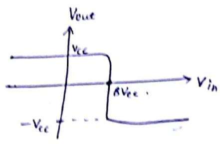


Assume $V_{out} = +V_{cc}$.

$$\Rightarrow V_+ = \left(\frac{R_1}{R_1+R_2}\right) V_{cc} = BV_{cc}.$$

If $V_{in} < BV_{cc} \Rightarrow$ OK

If $V_{in} > BV_{cc} \Rightarrow$ output should be $-$ve.



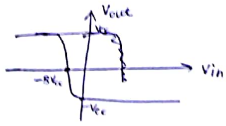
Assume $V_{out} = -V_{cc} \Rightarrow V_+ = -V_{cc}$.

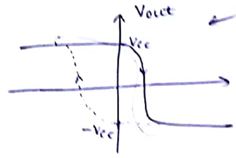
$$\Rightarrow V_+ = \left(\frac{R_1}{R_1+R_2}\right)(-V_{cc})$$

$$\Rightarrow V_+ = -BV_{cc}$$

If $V_{in} > -BV_{cc} \Rightarrow$ OK

If $V_{in} < -BV_{cc} \Rightarrow$ Output = +ve.





Semi-conductors

Metals-

Insulators-

Metal: Cu - (29 electrons)

$(Ar)\,3d^{10}\underbrace{4s^1}_{\text{Free electron}}$

$\xrightarrow{E}$ Cu^+ (-e) Cu^+ (-e) Cu^+ (-e) Cu^+ (-e) Cu^+ (-e) Cu^+ (-e) } Average velocity = 0 of electrons

$$\frac{d\bar{p}}{dt} = -eE - \frac{\bar{p}}{\tau}$$

$\bar{p}$ = avg. momentum.

τ ← Time constant.

momentum lost due to collision, etc.

Steady state ⇒ $\frac{d\bar{p}}{dt} = 0$.

$$\Rightarrow \boxed{\bar{p} = -eE\tau}$$

$v_d = -\frac{eE\tau}{m}$. ⇒ $\left|\frac{v_d}{E}\right| = \frac{e\tau}{m} = \mu$ (mobility)

drift

Current density :-

J = (Charge density) x velocity

$J = -nev_d$

n = density

$J = \left(\frac{ne^2\tau}{m}\right)E$ | $J = \sigma E$

Conductivity = σ.

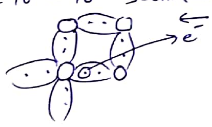
$\sigma = ne\mu$.

Units :-

$\mu = \frac{m}{s}\cdot\frac{m}{V} = m^2/Vs$. common unit = cm^2/Vs.

$\sigma = \frac{1}{\Omega m}$ $\rho = \frac{1}{\sigma} = \Omega m$. ($\Omega cm$).

$\rho = 10^{-4} - 10^{-6}\ \Omega cm$ (metals).



Si $(Ne)\,3s^2 3p^2$.

Ge $(Ar)\,3s^2 4p^2$.

$\xrightarrow{}$ Holes $\vec{E}$

Density of conduction $\bar{e}s$ = $n = 1.5\times10^{10}/cm^3$ (Si) → $n_i = 1.5\times10^{10}/cm^3$

$= 2.6\times10^{13}/cm^3$ (Ge)

Density of Si = $5\times10^{23}/cm^3$ (Si) $2\times10^{23}/cm^3$ (Ge)?.

$J = nq\mu E$ $[v_d = \mu E]$.

$= nq\mu_n E + pq\mu_p E$

Density of e^-. Density of h!

$$\Rightarrow \sigma = \underbrace{q(n\mu_n + p\mu_p)}_{\sigma}E \qquad \boxed{n=p}$$

Bipolar conductivity

Law of mass action ⇒ $n\cdot p = n_i^2$.

↳ intrinsic (pure semiconductor)

$\underbrace{N_D + p}_{+ve\ charge} = \underbrace{n + N_A}_{-ve\ charge}$.

N_D: Donor donates e^- & becomes +vely charged.

Assume n-type semiconductor :-

$N_A = 0$

$\Rightarrow N_D + p = n$

$p = \frac{n_i^2}{n}$ $\Rightarrow N_D + \frac{n_i^2}{n} = n$.

$\Rightarrow n^2 - N_D n - n_i^2 = 0$

$$\Rightarrow n = \frac{N_D \pm \sqrt{N_D^2 + 4n_i^2}}{2}.$$

$$\Rightarrow n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2}.$$

$N_D >> n_i$

$\boxed{n \simeq N_D}$. $\boxed{p \approx \frac{n_i^2}{N_D}}$.

P-type semiconductor:-

$N_D = 0$ $N_A >> n_i$

$\Rightarrow \boxed{p \simeq N_A}$ $\boxed{n = \frac{n_i^2}{N_A}}$.

$N_D = 10^{15}/cm^3$, $N_A = 0$ in si semiconductor

$(n_i = 1.5 \times 10^{10}/cm^3)$.

$n \approx 10^{15}/cm^3$ $(f(si)) = 5 \times 10^{22}/cm^3)$.

$$p \approx \frac{(1.5\times10^{10})^2}{10^{15}} \Rightarrow p \approx 2.25 \times 10^5/cm^3.$$

$$\sigma = q(n\mu_n + p\mu_p) \simeq qn\mu_n$$
$$\simeq q N_D \quad (\sigma \text{ increased } 10^5 \text{ times})$$

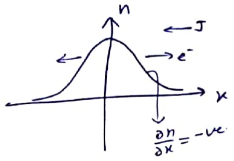
$\longrightarrow E$

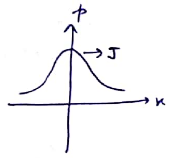
$\leftarrow e^-$ $J = \underset{\downarrow\ +ve}{\sigma} E$

$\longrightarrow h^+$

$$J = J_n + J_p \quad \Big| \quad J_n = q\mu_n nE \cancel{+ q\mu_p pE} + q \underset{\text{Diffusion coefficient}}{D_n} \frac{\partial n}{\partial x}.$$

$$J_p = \underbrace{q\mu_p E}_{\text{Drift}} - \underbrace{q D_p \frac{\partial p}{\partial x}}_{\text{Diffusion}}.$$





$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{K_B T}{q}. \quad \text{— Einstein relation.}$$

$$\frac{K_B T}{q} = V_T \left(\begin{matrix}\text{Thermal}\\ \text{voltage}\end{matrix}\right). = 0.026V \text{ at } 300K.$$

Graded semi-conductors:-

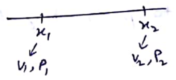
Net hole current $= 0$.

— " — electron " $= 0$.

$$p\mu_p qE - qD_p \frac{dp}{dx} = 0$$

$$E = \frac{D_p}{\mu_p p} \frac{dp}{dx} \quad \Rightarrow -\frac{dV}{dx} = \frac{V_T}{p}\frac{dp}{dx}$$

$\Rightarrow$ ~~$-\frac{\partial V}{\partial x} = D_p$~~ $\Rightarrow -dV = \frac{V_T}{p} dp$



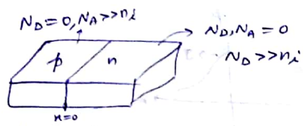
$$\Rightarrow \int_{x_1}^{x_2} -dV = \int_{x_1}^{x_2} \frac{V_T}{p} dp$$

$$\Rightarrow V_{21} = V_2 - V_1 = V_T \ln(p_1/p_2)$$

$$\Rightarrow \frac{p_1}{p_2} = e^{(V_2-V_1)/V_T}$$

$$\Rightarrow \begin{cases} p_1 = p_2 e^{V_{21}/V_T} \\ n_1 = n_2 e^{-V_{21}/V_T} \end{cases} \quad (V_{21} = V_2 - V_1).$$

$(n_1 p_1 = n_2 p_2)$.



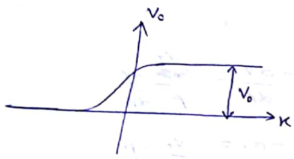
$p_p = N_A$ $\quad n_n = N_D$

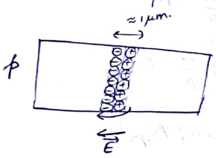
$n_p = \frac{n_i^2}{N_A}$ $\quad p_n = \frac{n_i^2}{N_D}$

$V_0 = V_T \ln\left(\frac{P_1}{P_2}\right) = V_T \ln\left(\frac{p_p}{p_n}\right)$

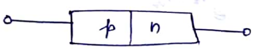
$\cancel{= V_T \ln\left(\frac{N_A}{(n_i^2/N_A)}\right)} = \cancel{2V_T \ln\left(\frac{N_A}{n_i}\right)}$

$= V_T \ln\left(\frac{N_A}{(n_i^2/N_D)}\right) = \boxed{V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)}$.



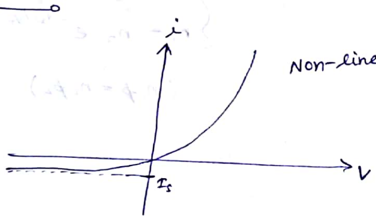


e^- diffuses from n to p creating holes(+ve) in n.



Diode

(passive)



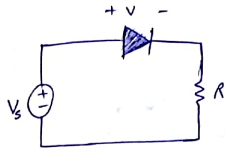
Non-linear

$i = I_s(e^{V/\eta V_T} - 1)$

$\eta = [1, 2]$.

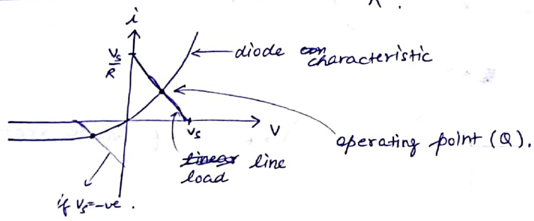
V_T: thermal voltage

$V_T = \frac{k_B T}{q} \approx 0.026$ ~~mV~~ V (300K).



$i = I_s(e^{V/\eta V_T} - 1)$

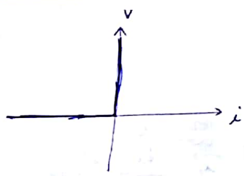
$V_s = V + iR \Rightarrow i = \frac{V_s - V}{R}$.



Forward bias (low resistance).

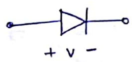
Reverse bias (high resistance).

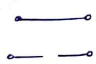
Ideal diode:-



$V<0 \Rightarrow i=0$.

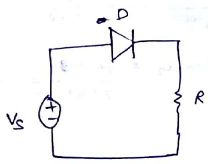
$V>0 \Rightarrow i=\infty$.



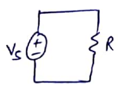


$V=0, i>0$

$i=0, V\leq 0$.



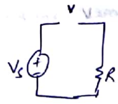
Assume D is ON.



$i = \frac{V_s}{R} > 0$

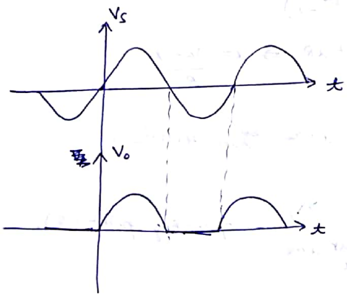
$\Rightarrow$ assumption OK.

Assume D is off

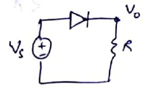


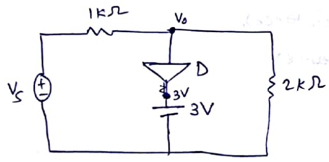
$V = V_s > 0$ (V should be ≤ 0).

$\Rightarrow$ assumption wrong.



(Half wave rectifier).





$V_s \rightarrow$ forward bias diode.

$3V \rightarrow$ reverse biase diode.

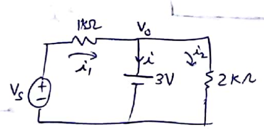
Assume D is ON $\Rightarrow$

i must be +ve

~~$\Rightarrow V_s > 4.5$~~.

$\Rightarrow V_s > 4.5V$.

$V_o = 3V$.



$i_1 = \frac{V_s - 3}{1000}$, $i_2 = \frac{3}{2000}$

$i = i_1 - i_2 = \frac{2V_s - 9}{2000} = \frac{V_s - 4.5}{1000}$.

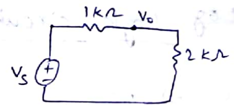
Assume D is OFF:-

$V_o < 3V$

$\Rightarrow \frac{2V_s}{3} < 3$

$\Rightarrow V_s < 4.5V$.

$V_o = \frac{2V_s}{3}$.

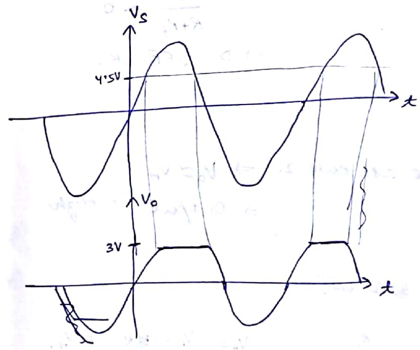


$V_o = \frac{2V_s}{3}$.

~~$V_o < 3V$~~

Voltage across $D = V_o - 3V$ should be ≤ 0

$$V_o = \begin{cases} \frac{2V_s}{3} & V_s < 4.5V \\ 3V & V_s > 4.5V \end{cases}$$

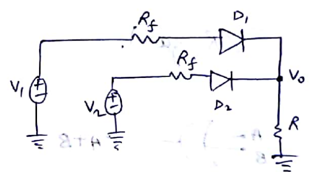


$\leftarrow$ Diode clipper circuit

Logic circuits:-

Low voltage = 0, High voltage = 1 } 2 levels. +ve logic

= 1, = 0 } -ve logic.

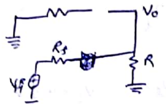


~~Log~~ Logic circuits

1) $V_1 = 0$, $V_2 = 0$ $\Rightarrow V_o = 0$

2) $V_1 = 0$, $V_2 = +V_H$ $\Rightarrow D_2 = ON$

Assume $D_1 = OFF$



$i_2 > 0 \Rightarrow D_2$ ON. OK.

$V_o = i_2 R = \frac{V_H R}{R+R_f} > 0$

$\Rightarrow D_1$ OFF OK.

If $R >>> R_f$

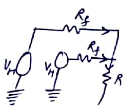
$\Rightarrow V_o \simeq V_H$.

$\Rightarrow$ Output is High

3) $V_1 = V_H$, $V_2 = 0 \Rightarrow$ Same as case 2. $\Rightarrow V_o \simeq V_H$

$\Rightarrow$ Output is High

4) $V_1 = V_H$, $V_2 = V_H \Rightarrow$ Assume both ON.



$V_o = \frac{V_H \cdot 2R}{2R+R_f} < V_H$.

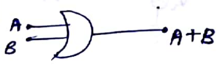
$\Rightarrow$ Both ON OK.

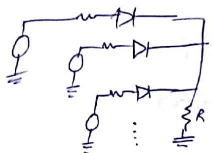
$\Rightarrow V_o \simeq V_H$ if $R >> R_f$.

Truth Table

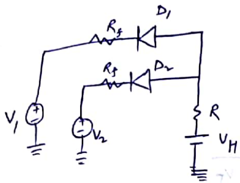
V_1	V_2	V_o
0	0	0
1	0	1
0	1	1
1	1	1

(OR gate).

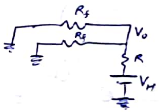




(OR gate).



1) $V_1 = 0$, $V_2 = 0 \xRightarrow{\text{Assume}}$ Both diodes ON.



$V_o = \frac{V_H R_f}{R_f + 2R}$.

$R >> R_f$

$\Rightarrow V_o \simeq 0$.

2) $V_1 = 0$, $V_2 = V_H \xRightarrow{\text{Assume}} D_1$ ON, D_2 OFF.

$\Rightarrow V_o = \frac{V_H R_f}{R_f + R}$ $R >> R_f$

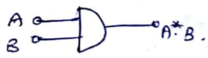
$\Rightarrow V_o \simeq 0$.

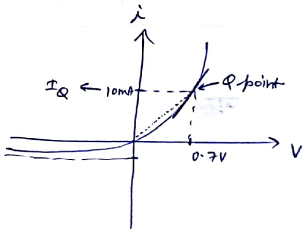
3) $V_1 = V_H$, $V_2 = 0 \Rightarrow$ same as case 2 $\Rightarrow V_o \simeq 0$. , D_1 OFF, D_2 ON.

4) $V_1 = V_H$, $V_2 = V_H \Rightarrow$ Both OFF $\Rightarrow V_o \simeq V_H$.

V_1	V_2	V_o
0	0	0
0	1	0
1	0	0
1	1	1

(And gate)





d.c. Resistance $R = \frac{0.7\,V}{10\,mA}$

$= 70\,\Omega$.

a.c. resistance

$V = 0.7V + V_{a.c}$

$i = 10\,mA + I_{a.c.}$

$g = \frac{i_{ac}}{V_{ac}}$ — ~~increment in i~~ (incremental conductance)

$r = \frac{1}{g} = \frac{V_{ac}}{i_{ac}}$ (incremental resistance)

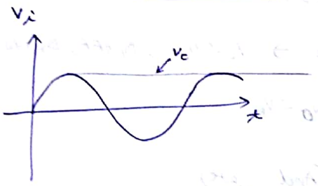
$g = \frac{di}{dV}$, $r = \frac{dV}{di}$.

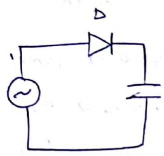
$i = I_s\left(e^{V/\eta V_T} - 1\right)$

$\frac{di}{dV} = \frac{I_s}{\eta V_T} e^{V/\eta V_T} = I_s e^{V/\eta V_T} \cdot \frac{1}{\eta V_T}$.

~~$r = \eta I$~~

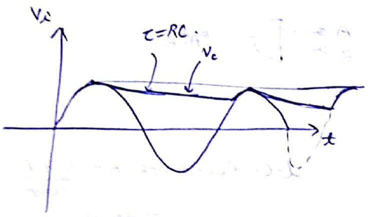
$$\boxed{r = \frac{\eta V_T}{I_Q}}$$

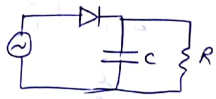


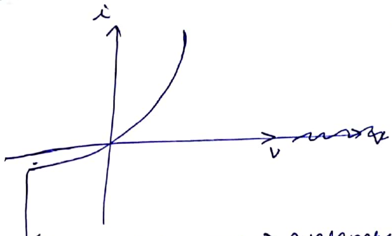


~~when~~

Peak Detector.

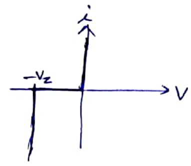


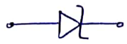




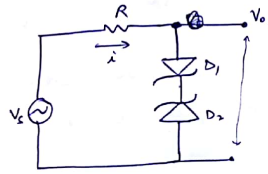
Breakdown. → 1) Avalanche
2) Tunneling

Zener diodes





(ideal Zener diode).



Assume $i > 0$ ⇒ D_1 Forward Biased
D_2 Reverse Biased

$V_{D_1} = 0$, $V_{D_2} = -12V$.

$V_s > 12V$.

$V_s = 24\sin\omega t$.

$V_Z = 12V$.

Assume $i < 0$ ⇒ D_1 Reverse, D_2 Forward

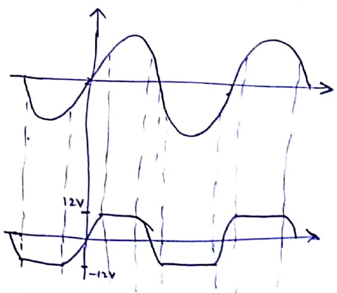
$V_{D_1} = -12V$, $V_{D_2} = 0V$

$V_s < -12V$.

Assume $i = 0$ ⇒ Assume $V_s = 7V$.

$V_R = 0$ $V_o = V_s$.

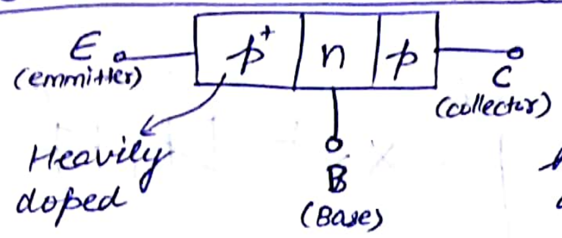
~~$V_r = 8V$~~.



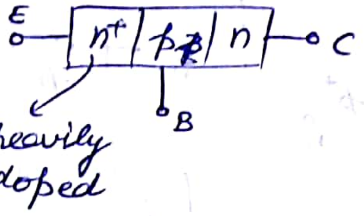
Voltage limiter.

Scanned by CamScanner

Bipolar Junction Transistors (BJT) :-

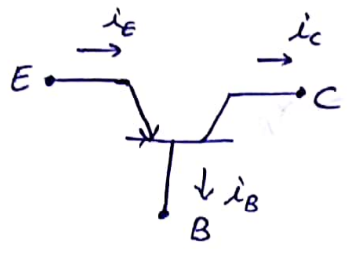


pnp BJT

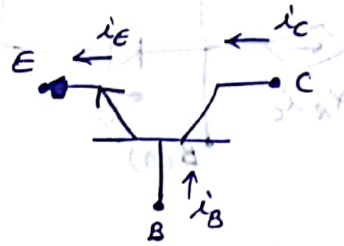


npn BJT

emitter doping > Base doping > Collector doping



pnp

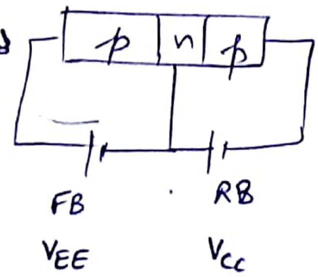


npn

$$i_E = i_B + i_c$$

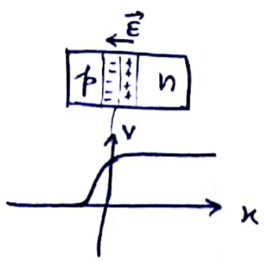
$$V_{EB} + V_{BC} + V_{CE} = 0.$$

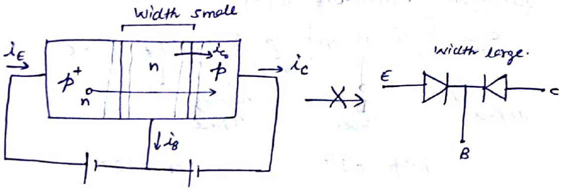
E–B junction	~~B–E~~ C–B junction	Mode
FB	FB	Saturation
FB	RB	Active
RB	FB	Reverse–Active (Inverted)
RB	RB	cut-off



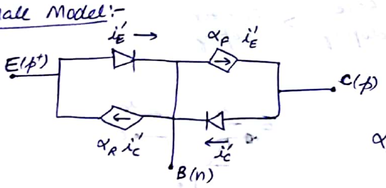
Active mode : amplifi~~er~~cation

sat., cut-off : logic





Ebers – Moll Model :-



$\alpha_F > \alpha_R$.

$$i_E' = I_{ES}\left(e^{V_{EB}/V_T} - 1\right)$$

$$i_C' = I_{CS}\left(e^{V_{CB}/V_T} - 1\right)$$

$$i_E = I_E' - \alpha_R i_C' = I_{ES}\left(e^{V_{EB}/V_T} - 1\right) - \alpha_R I_{CS}\left(e^{V_{CB}/V_T} - 1\right)$$

$$i_C = \alpha_F i_E' - i_C' = \alpha_F I_{ES}\left(e^{V_{EB}/V_T} - 1\right) - I_{CS}\left(e^{V_{CB}/V_T} - 1\right)$$

$$i_B = i_E - i_C$$

Assume active mode,

$$e^{V_{EB}/V_T} - 1 = \frac{i_E - \alpha_R I_{CS}}{I_{ES}}$$

$$i_C = \alpha_F I_{ES}\left(\frac{i_E - \alpha_R I_{CS}}{I_{ES}}\right) + I_{CS}$$

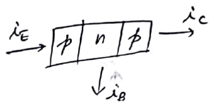
$$= \underset{\alpha}{\alpha_F}\, i_E + \underbrace{I_{CS}(1 - \alpha_F \alpha_R)}_{i_{C_0} \to 0}$$

$$i_C = \alpha i_E + i_{C_0} \Rightarrow \boxed{i_C = \alpha i_E}.$$

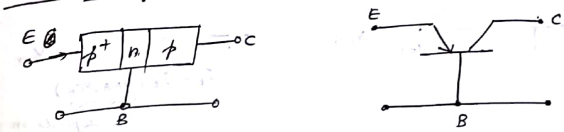
$$i_E = i_B + i_C$$

$$\Rightarrow i_B = i_E \underbrace{\left(1 - \frac{1}{\alpha}\right)}_{\beta}$$

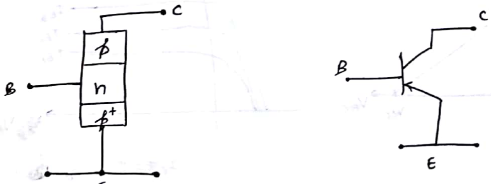
$$\Rightarrow i_C = \underbrace{\left(\frac{\alpha}{1-\alpha}\right)}_{\beta} i_B. \qquad \Rightarrow \boxed{i_C = \beta i_B}.$$



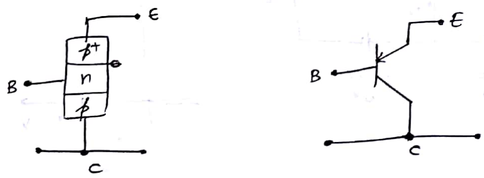
Circuit configuration :-



Common Base configuration

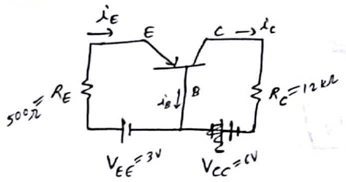


Common emmitter configuration



Common collector

Common Base :-



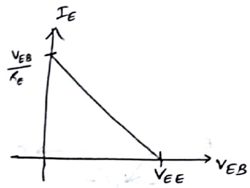
Input side

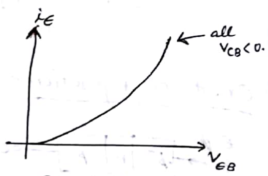
$V_{EE} = i_E R_E + V_{EB}$ (load line)

Output side

$V_{CC} = i_C R_C + V_{BC}$

Input side $\rightarrow$

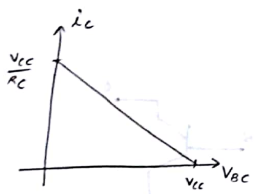


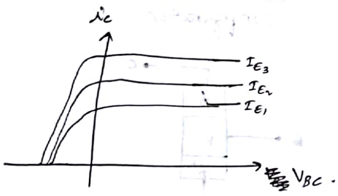


$I_E = I_E(\underbrace{V_{EB}, V_{CB}})$

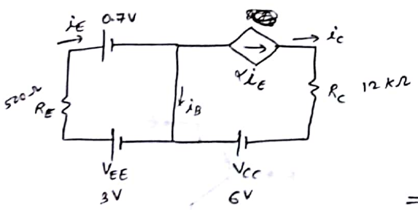
I_E depends on both.

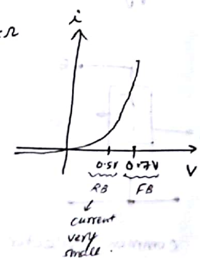
Output side $\rightarrow$





$I_C = I_C(V_{CB}, I_E)$.





$V_{EB} - V_{EE} + R_E i_E = 0$.

$0.7 - 3 + 500 i_E = 0 \Rightarrow i_E = 4.6$ mA.

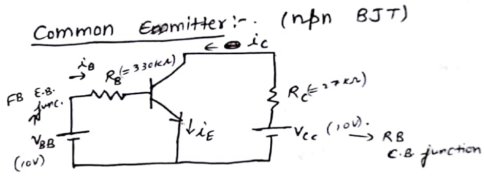
$V_{CB} - R_C i_c + 6 = 0$

$i_c = \alpha i_E \approx i_E$

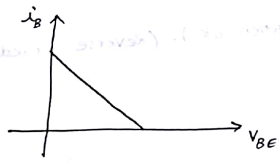
$\Rightarrow V_{CB} - 12000 \times 4.6 \times 10^{-3} + 6 = 0$

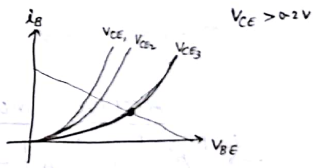
$\Rightarrow V_{CB} = -0.48V$. (Assumption OK). (Reverse biased).

Common Emitter :- (npn BJT)



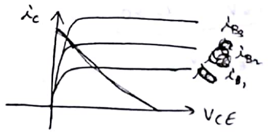
Input $\rightarrow$ $V_{BB} = i_B R_B + V_{BE}$



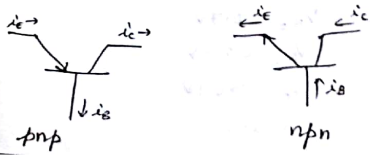


$i_B = i_B(V_{BE}, V_{CE})$

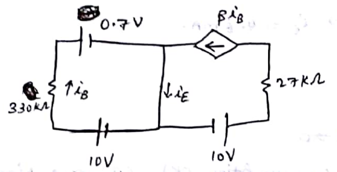
Output $\rightarrow$ $V_{CC} = i_C R_C + V_{CE}$.



$i_c = i_c(V_{CE}, i_B)$



Assume active mode :-



$$i_B = \frac{10-0.7}{330\times10^3} = 28.2\,\mu A.$$

$$i_c = \underset{100}{\beta}\, i_B = 2.82\ mA.$$

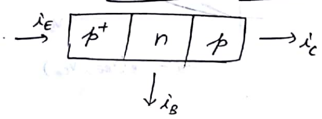
$$V_{CE} = (-2.7\times10^3)\, i_c + 10 = 2.39V.$$

$$\Rightarrow V_{BC} = V_{BE} - V_{CE}$$

$$= 0.7V - 2.39V$$

$\Rightarrow V_{BC} = -1.69\ V$ (Assumption OK). (Reverse biased).

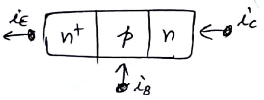
ACTIVE MODE



$$\boxed{V_{EB} = 0.7V,\ V_{CB} < 0.5V,\ i_B > 0}$$

$$V_{EC} = V_{EB} + V_{BC} = V_{EB} - V_{CB}$$

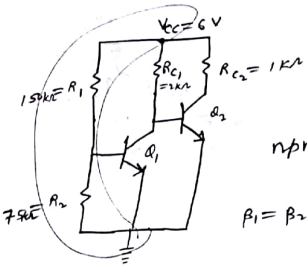
$$\Rightarrow \boxed{V_{EC} > 0.2V.}$$



$V_{BE} = 0.7V$

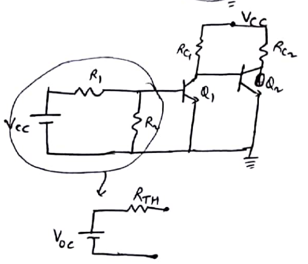
$\left\{ \begin{array}{l} V_{BC} < 0.5V \\ V_{CE} > 0.2V \end{array} \right.$

$i_B > 0.$

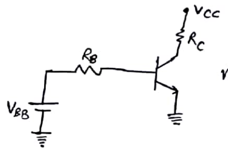


npn BJTs.

$\beta_1 = \beta_2 = 100.$



Assume both Active.



npn

Active mode

$V_{BE} = 0.7V.$

Cut-off mode

Both junctions R.B. $i_B = 0 = i_c$.

V_{BB} small

↓ increase V_{BB}

Active mode

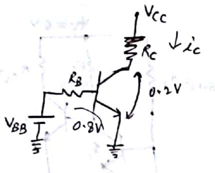
↓ increase V_{BB}

Saturation mode.

Saturation mode :-

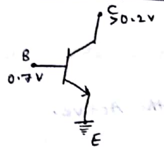
$V_{BE}\,(sat.) = 0.8V$.

$V_{CE}\,(sat.) = 0.2V$.

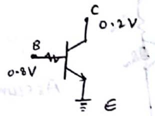


Switch.

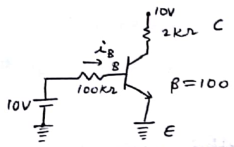
Check: $i_B > \frac{i_C}{\beta}$.



Active Mode



Sat. mode



Assume active mode, $i_B = \frac{10-0.7}{100k\Omega} = 93\,\mu A$

$i_C = \beta i_B = 9.3$ mA .

$V_{CE} = 10 - i_C R_C$

$= -8.6$ V .

V_{CE} must be $> 0.2V$ for active mode.

$\Rightarrow$ Incorrect assumption.

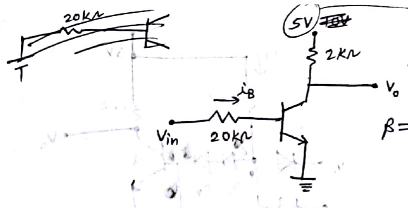
Assume saturation mode,

$i_B = \frac{10-0.8}{100k\Omega} = 92\,\mu A$ $\quad i_C = \frac{10-0.2}{2k\Omega} = 4.9$ mA .

~~$i_C = \beta i_B = 92$ mA~~.

$\beta i_B = 9.2$ mA $> i_C$. $\Rightarrow$ Correct .

$\boxed{\beta i_B > i_C}$



$\beta = 100$.

Low voltage level $= 0.2V$

High — " — $= 5V$.

1) V_{in} is small $V_{in} = 0.2V$.

Both reverse biased

$\Rightarrow Q = $ OFF cut-off mode

$\Rightarrow V_o = 5V$.

2) $V_{in} = 5V$.

Assume $Q =$ ON . (saturation).

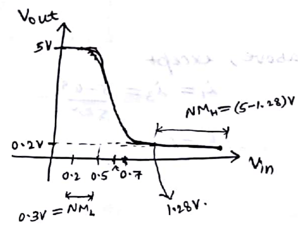
$i_B = \frac{5-0.8}{20k\Omega} = 0.21$ mA

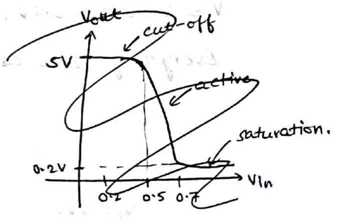
$i_C = \frac{5-0.2}{2k\Omega} = 2.4$ mA

$\beta i_B = 0.21 \times 100 = 21 > \underset{i_C}{2.4}$ mA $\Rightarrow \beta i_B > i_C \Rightarrow$ correct.

$V_o = 5 - 2k\Omega \times 2.4$ mA

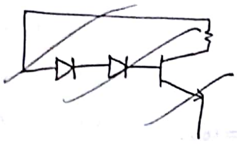
$\Rightarrow V_o = 0.2$ V.

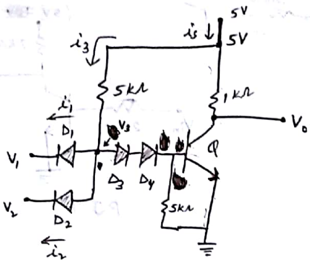




Need $i_B > \frac{i_C}{\beta}$ for sat. mode.

$\frac{V_{in}-0.8}{20k\Omega} > \frac{5-0.2}{2k\Omega \times 100} \Rightarrow \boxed{V_{in} > 1.28V}$.





1) $V_1 = V_2 = 0.2V$

Assume D_1, D_2 ON.

$\Rightarrow V_3 = \overset{V_1}{0.2} + 0.7 = 0.9V$

Assume D_3, D_4 OFF

$i_3 = \dfrac{5-0.9}{5k\Omega}$. $i_1 = i_2 = \dfrac{i_3}{2} = +ve$. ($D_1, D_2$ ON Correct).

$i_B = 0$, $V_{BE} = 0$.

Q = OFF

$V_0 = 5V$.

2) $V_1 = 0.2V$, $V_2 = 5V$.

Assume D_1 ON , D_2 OFF.

$V_3 = V_1 + 0.7 = 0.9V$.

Everything same as above, except

$i_1 = i_3 = \dfrac{5-0.9}{5k\Omega}$

$i_2 = 0$.

$V_0 = 5V$.

3) $V_1 = 5V$, $V_2 = 0.2V$ $\Rightarrow$ same as above

$i_1 = 0$

$i_2 = i_3$. $V_0 = 5V$.

4) $V_1 = 5V$, $V_2 = 5V$.

D_1, D_2 OFF. $i_1 = i_2 = 0$.

Assume $D_3, D_4 = $ ON.

Assume Q = ON (sat.)

$V_3 = 0.7 + 0.7 + 0.8 = 2.2V$.

$i_3 = \dfrac{5-2.2}{5k\Omega} = \dfrac{2.8}{5k\Omega} = 0.56$ mA. $i_B = i_3 - \dfrac{0.8}{5k\Omega} = 0.56 - 0.16$

$\Rightarrow \boxed{i_B = 0.4 \text{ mA}}$.

$\boxed{i_c = \dfrac{5-0.2}{1k\Omega} = 4.8 \text{ mA}}$.

If $\beta = 100 \Rightarrow \beta i_B = 40$ mA $> i_c$

$\Rightarrow$ correct assumption.

$\boxed{\dfrac{i_c}{i_B} = 12}$ if $\beta > 12 \Rightarrow$ correct assumption.

$V_0 = 0.2V$.

V_1	V_2	V_0	AND
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

NAND Gate

Power dissipation :-

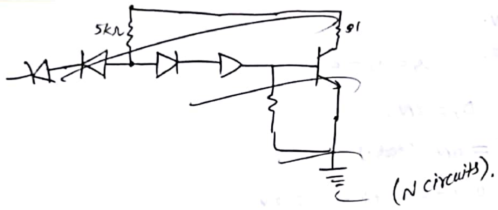
Case 4) $i_s = i_3 + i_c = 0.56 + 4.8 = 5.36$ mA.

Power $= Vi = 5 \times 5.36$

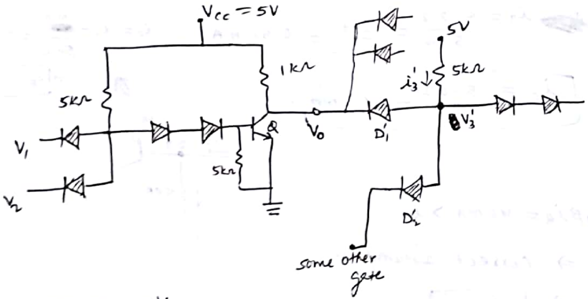
$= 26.8$ mW.

Cases 1,2,3) $i_s = i_3 + \cancel{i_c}^{0} = i_3 = 0.82$ mA.

Power $= Vi = 5 \times 0.82 = 4.1$ mW.



(N circuits).



If $V_1 = 5V$, $V_2 = 5V$

$V_0 = 0.2V$

Q in saturation.

$\Rightarrow D_1'$ is ON. $\Rightarrow V_3' = 0.2 + 0.7 = 0.9V$.

$$i_3' = \frac{5-0.9}{5k\Omega} = 0.82\ mA.$$

$i_c = \underbrace{i_R}_{\frac{5-0.2}{1k\Omega}} + N(0.82\ mA)$ → worst case if all current flows through D_1'.

$i_B = 0.4\ mA$ (calculated last time).

<u>Need</u>: $\beta i_B \geq i_c$

$\Rightarrow \beta(0.4) \geq (4.8 + N \times 0.82)$

Let $\beta = 40$

$\Rightarrow 16 \geq 4.8 + N \times 0.82 \Rightarrow N \leq \frac{11.2}{0.82} \Rightarrow N \leq 13.7$.

$\Rightarrow \boxed{N = 13}$. at max.

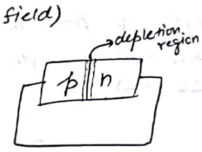
$\Rightarrow \boxed{\text{Fan out} = 13}$.

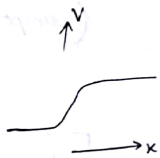
FET (Field Effect Transistors)

- 1) JFET – Junction FET
- 2) MOSFET – Metal Oxide Semiconductor FET.

Unipolar (majority carriers)

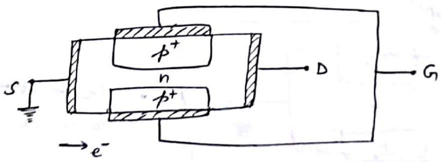
Voltage controlled
(electric field)



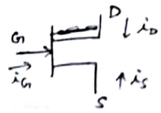


Depletion regions increases in reverse bias.

Can control the depletion region by Voltage.

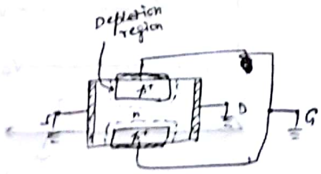


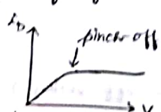
Source, Drain, Gate. (n-channel JFET)



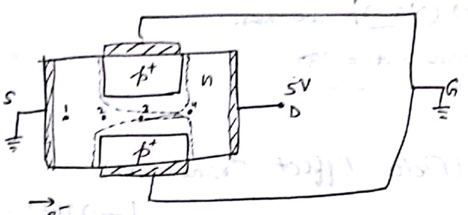
Apply $V_{DS} \geq 0$.

Everything grounded $\Rightarrow$



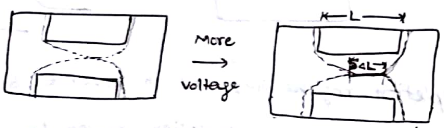


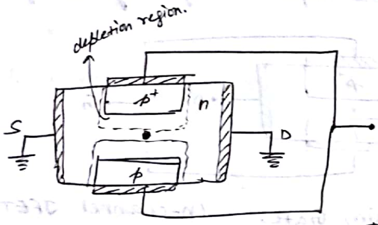
(long channel JFET)



pinch-off → when the depletion regions touch.

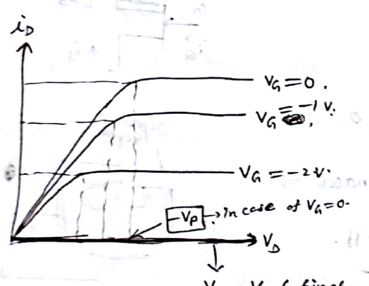
(Assumption: length of pinch-off is much less than total length ($\Delta L \ll L$)).





G ~~(-ve voltage)~~ ($V_{GS} \to -ve$ Votage).

If $V_{GS} \to +ve$ then forward biased diodes current from source may flow to gate through them



$V_G = V_P$ (pinch-off voltage).

$V_P = -ve$.

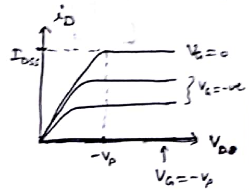
$\boxed{V_{D,sat.} = V_G - V_P}$.

$$\rightarrow i_D = I_{DSS}\left[2\left(1-\frac{V_{GS}}{V_P}\right)\frac{V_{DS}}{-V_P} - \left(\frac{V_{DS}}{V_P}\right)^2\right]$$

$i_D = i_{D,sat}$ (saturation/active).

If V_{DS} is small (compared to V_P),

$$R = \frac{V_{DS}}{i_D} = \frac{V_P^2}{2 I_{DSS}(V_{GS}-V_P)}$$



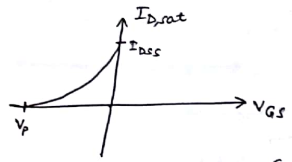
$I_{DSS} = I_{D,sat}(V_G = 0)$.

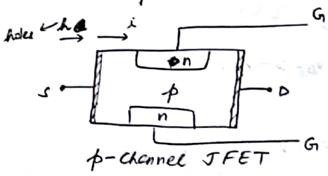
$$\frac{\partial i_D}{\partial V_D} = I_{DSS}\left[2\left(\frac{V_{GS}-V_P}{V_P^2}\right) - \frac{2V_{DS}}{V_P^2}\right]$$

$$\Rightarrow \boxed{V_{DS} = V_{GS} - V_P} = V_{D,sat}.$$

$$i_{D,sat} = I_{DSS}\left[2\left(\frac{V_{GS}-V_P}{V_P}\right)^2 - \left(\frac{V_{GS}-V_P}{V_P}\right)^2\right]$$

$$\Rightarrow i_{D,sat} = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2.$$

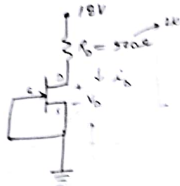




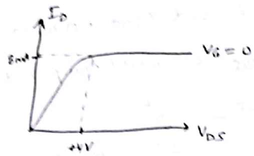
$V_{GS} = +ve$.
$V_{DS} = -ve$.

p-channel JFET

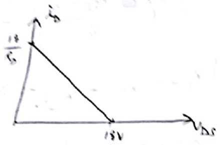
JFET- Depletion Type device



Given: $I_{DSS} = 8\,mA$.
$V_P = -4\,V$.



$18 = i_D R_D + V_{DS}$ (load line)



Assume JFET in active mode,

$i_{D,sat.} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 8\left(1 - \frac{0}{-4}\right)^2$

$\Rightarrow i_{D,sat} = 8\ mA.$

$18 = (8mA)\times(500\Omega) + V_{DS}$

$\Rightarrow \boxed{V_{DS} = 14\ V}$ (ok).

$V_{DS} > -V_P$.

If $R_D = 2k\Omega \Rightarrow V_{DS} = 2V$, $V_{DS} \ngtr -V_P$ ✗
wrong assumption.

Assume ohmic mode, — using main eqn.

$i_D = (8\times10^{-3})\left[2\left(1-\frac{0}{-4}\right)\frac{V_{DS}}{4} - \left(\frac{V_{DS}}{-4}\right)^2\right]$

$\Rightarrow \frac{18 - V_{DS}}{2000} = \frac{8}{1000}\left(\frac{V_{DS}}{2} - \frac{V_{DS}^2}{16}\right)$

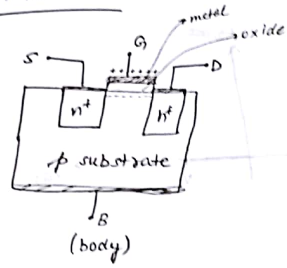
$\Rightarrow \cancel{72 - 4V_{DS} = 8V_{DS} - V_{DS}^2}$

$\Rightarrow \cancel{V_{DS}^2 - 12V_{DS} + 72 = 0}.$

$\Rightarrow V_{DS}^2 - 9V_{DS} + 18 = 0$

$\Rightarrow V_{DS} = 3, 6.$ ✓ ✗ $(V_{DS} < 4V)$.

MOSFET



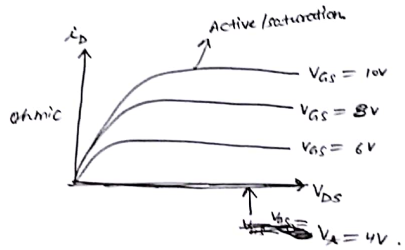
n-channel MOSFET (NMOS)

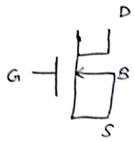
$V_G = 0$

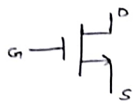
Apply $+ve$ V_G.

large $+V_G$, surface inversion.

$V_G > V_t$ — threshold voltage







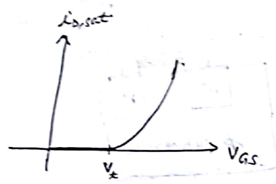
MOSFET → Enhancement Mode.

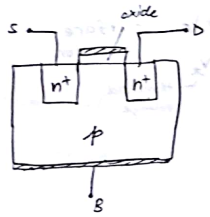
* $i_D = K\left[2(V_{GS} - V_t)V_{DS} - V_{DS}^2\right]$

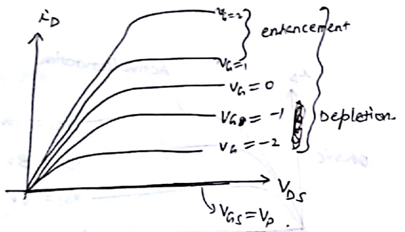
$$\frac{\partial i_D}{\partial V_{DS}} = K\left(2(V_{GS} - V_t) - 2V_{DS}\right)$$

$$\Rightarrow \boxed{V_{DS} = V_{GS} - V_t} = V_{D,sat}.$$

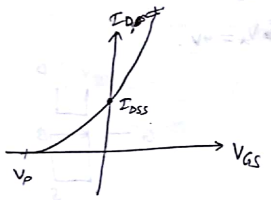
$i_{D,sat} = K(V_{GS} - V_t)^2.$

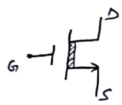


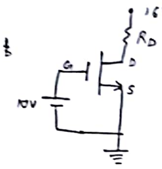




Depletion MOSFET



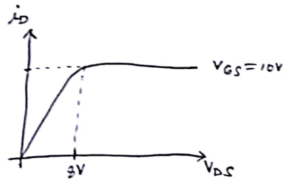




n-channel enhancement mode

$k = 0.25\ mA/V^2$

$V_T = 2V.$



$V_{D,sat} = V_{GS} - V_T = 10 - 2 = 8V.$

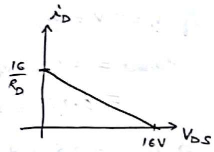
$i_{D,sat} = K(10-2)^2 = K(V_{GS} - V_T)^2.$

$= 0.25 \times 64\ mA$

$\Rightarrow i_{D,sat} = 16\ mA.$

Load line,

$16 = V_{DS} + i_D R_D$



Let $R_D = 250\Omega$.

Assume active (saturation) $\Rightarrow i_D = i_{D,sat} = 16\ mA.$

$16 = V_{DS} + 16mA \times 250V$

$\Rightarrow V_{DS} = 12V.$

$V_{DS} > 8V \Rightarrow$ Assumption OK

Let $R_D = 1K\Omega$.

Assume active (sat.) $\Rightarrow V_{DS} = 0V < 8V \Rightarrow$ WRONG Assumption.

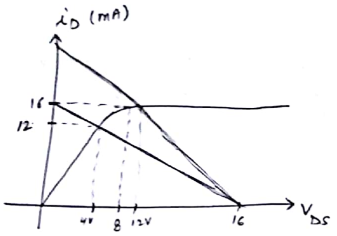
Assume ohmic mode $\Rightarrow$ $i_D = K[2\overbrace{(V_{GS} - V_T)}^{8} V_{DS} - V_{DS}^2]$ —(i)

$16 = V_{DS} + i_D R_D$ —(ii)

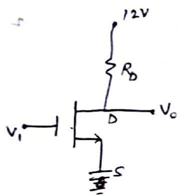
$\Rightarrow V_{DS} = 4V$ or $\cancel{16V}$ $\Rightarrow V_{DS} = 4V; i_D = 12m$

(ohmic mode)

Scanned by CamScanner



1) $R_D = 250\,\Omega$



$V_t = 2V$

$k = 0.25\ mA/V^2$

$R_D = 1k\Omega$

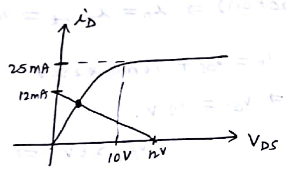
If $V_1 = 12V = V_{GS}$

$i_{D,sat} = k(V_{GS} - V_T)^2$

$= 0.25(12-2)^2\ mA$

$\Rightarrow i_{D,sat} = 25\ mA.$

$V_{D,sat} = V_{GS} - V_T = 12 - 2 = 10V.$



$12 = V_{DS} + i_D R_D$

$V_{DS} < 10V$

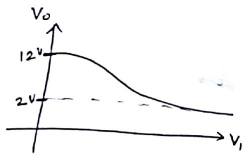
Assumption wrong $\Rightarrow$ assume ohmic & solve.

If $V_1 = 12V$ (high)

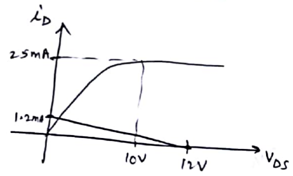
$V_o =$ low $\approx 2V$, $i_D = \frac{12-2}{1k\Omega} = 10mA.$

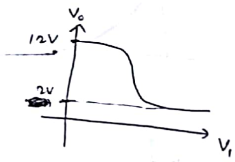
If $V_1 = 2V$ (low)

$\Rightarrow V_o = 12V.$



If $R_D = 10k\Omega$. $12 = V_{DS} + i_D R_D$





(better inverter)

Replace R_D by MOSFET.

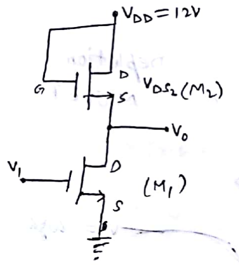
$\longrightarrow$ active or cut-off

Never ohmic mode

n-channel enhancement

$V_{D,sat} = V_{GS} - V_t = V_{DS} - V_t.$

$\Rightarrow$ Always active.



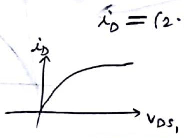
$V_t = 2V$

$k = 0.25 mA/V^2$

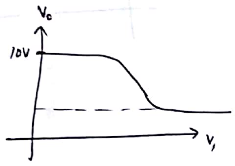
M_2: $i_D = i_{Dsat} = k_2(V_{GS} - V_t)^2$

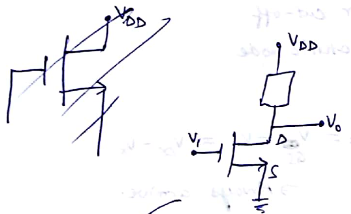
$= k_2(V_{DD} - V_{DS_1} - V_t)^2.$

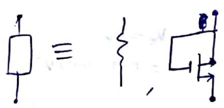
$i_D = (2.5\times10^{-4})(V_{DS_1} - 10)^2.$



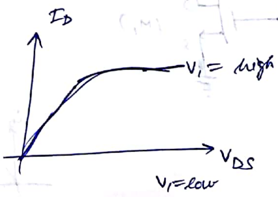
$\leftarrow \left(i_D = 0.25 (V_{DS_1} - 10)^2 \text{ mA} \right)$

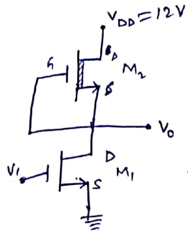






Depletion MOSFET





n-channel depletion MOSFET

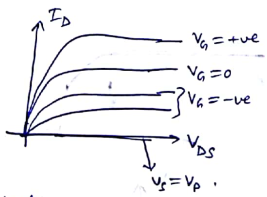
n-channel enhancement MOSFET

M_1 : $k = 0.25$ mA/V^2

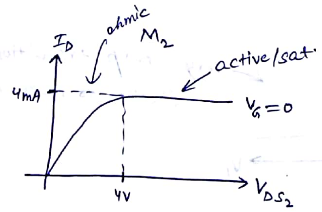
$V_t = 2V$

M_2 : $I_{DSS} = 4$mA

$V_p = -4V$.



Depletion mode



Assume M_2 in ohmic

$$I_D = I_{DSS} \left[2 \left(\frac{\cancel{V_{GS_2}}^{\,0} - V_p}{V_p^2} \right) V_{DS_2} - \left(\frac{V_{DS_2}}{V_p} \right)^2 \right]$$

$$= 4 \left[\frac{2 \times V_{DS_2}}{(4)^2} - \frac{(V_{DS_2})^2}{(4)^2} \right] \qquad V_p = -4V$$

$$V_{DS_2} = V_{DD} - V_{DS_1}.$$

$$i_D = -(2.5 \times 10^{-4}) \left(V_{DS_1}^2 - 16 V_{DS_1} + 48 \right).$$

Need : $V_{DS_2} = V_{DD} - V_{DS_1} \leq 4V$.

$\Rightarrow V_{DS_1} \geq 12 - 4 \Rightarrow V_{DS_1} \geq 8V$.

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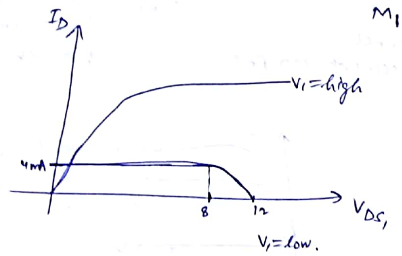
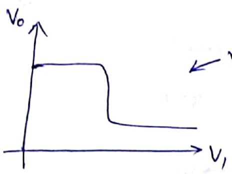
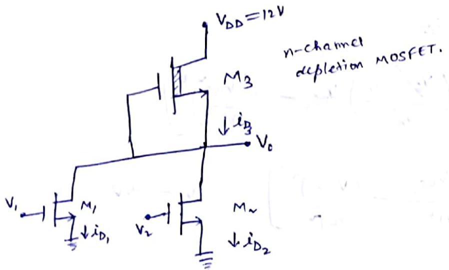
If $V_{DS_1} < 8V \Rightarrow M_2$: sat./active

$i_D = 4\,mA$.





← very sharp transition (better to make inverter).



1) $V_1 = 0$, $V_2 = 0$

$M_1 = M_2 = OFF$

$\Rightarrow i_{D_3} = 0$.

$\Rightarrow V_0 = V_{DD} = 12V$.

2) $V_1 = 0$, $V_2 = V_{DD}$

$M_1 = OFF$ ⇒ same as inverter

$V_0 = low$.

3) $V_1 = V_{DD}$, $V_2 = 0$

$M_2 = OFF$ ⇒ same as inverter

$V_0 = low$

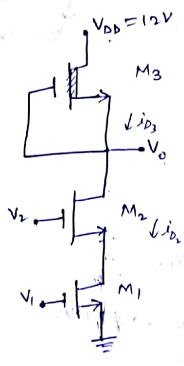
4) $V_1 = V_{DD}$, $V_2 = V_{DD}$

$M_1, M_2 = ON$

$V_0 = low$.

V_1	V_2	V_0
0	0	1
1	0	0
0	1	0
1	1	0

NOR



If $V_1 = 0$

$M_1 = OFF \Rightarrow i_D = 0$

$\Rightarrow V_0 = V_{DD}$.

1) $V_1 = 0$, $V_2 = 0 \Rightarrow V_0 = high$. $M_1 = OFF$. Find out M_2?

2) $V_1 = 0$, $V_2 = V_{DD} \Rightarrow V_0 = V_{DD}$. $M_1 = OFF$. State of M_2?

3) $V_1 = V_{DD}$, $V_2 = 0 \Rightarrow M_1 = ON$, $M_2 = OFF$

$\Rightarrow i_D = 0 \Rightarrow V_0 = V_{DD}$.

4) $V_2 = high$, $M_1 = ON$, $M_2 = ON \Rightarrow V_0 =$ small.

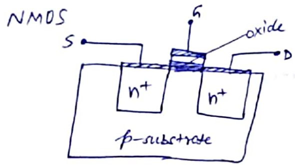
V_1	V_2	V_0	
0	0	1	$i_D = 0$
1	0	1	
0	1	1	
1	1	0	$\rightarrow i_D \neq 0 \Rightarrow$ Dissipation

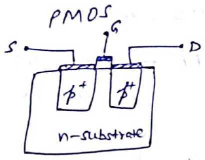
NAND.

CMOS

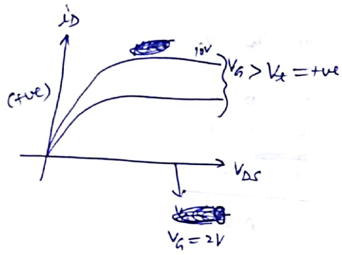
↓

complementary

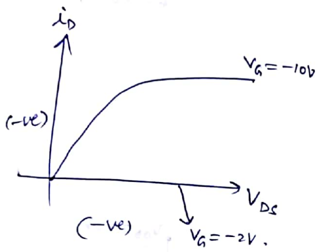


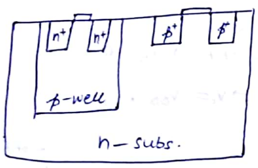


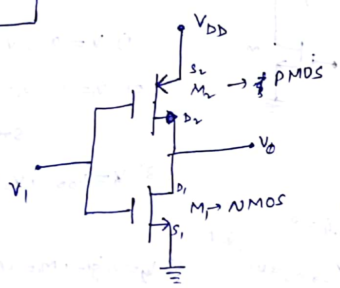
NMOS ⇓



⇓







Drains together.

If

1) $V_1 = 0 \Rightarrow M_1 = OFF \Rightarrow i_D = 0$

M_2 = ON $(V_{GS_2} = V_1 - V_{DD} = -V_{DD})$.

$\Rightarrow V_o$ = high. $(V_{DS} = 0$ if $i_D = 0)$

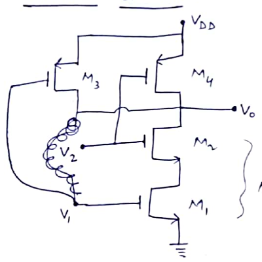
2) $V_1 = V_{DD} \Rightarrow M_1 = ON$

$M_2 = OFF$ $(V_{GS_2} = V_1 - V_{DD} = 0)$.

$\Rightarrow i_D = 0$.

$\Rightarrow V_{DS_1} = 0 \Rightarrow V_o = 0$.

CMOS NAND GATE :-



If $V_1 = 0 \Rightarrow M_1 = OFF \Rightarrow i_D = 0$.

M_3 = ON.

$V_o = V_{DD}$.

$\boxed{M_3 \Rightarrow V_{GS} = 0 - V_{DD} = -V_{DD} \equiv ON.}$

~~1) $V_1 = 0$, $V_2 = 0$~~

~~$V_o = V_{DD}$~~

1) $V_1 = 0, V_2 = 0 \Rightarrow V_o = V_{DD}$, $M_1 = OFF$, $M_2 = OFF$, $M_3 = ON$, $M_4 = ON$

2) $V_1 = 0, V_2 = V_{DD} \Rightarrow V_o = V_{DD}$, $M = OFF$, ~~$M_2 = ON$~~, $M_3 = ON$, ~~$M_4 = OFF$~~.

Assume M_2 ON.

$\Rightarrow V_{DS_2} = 0$

$\Rightarrow V_{DS_1} = V_o - V_{DS_2} = V_{DD}$

$\Rightarrow V_{GS_2} = V_2 - V_{DS_1} = 0$ (X).

Assume M_2 OFF. ...

3) $V_1 = V_{DD}, V_2 = 0 \Rightarrow M_1 = ON, M_3 = OFF$, $M_4 = ON$.

$M_2 = OFF$.

but $i_D = 0$

$V_{DS_4} = 0$

$\Rightarrow V_o = V_{DD}$.

4) $V_1 = V_{DD}$, $V_2 = V_{DD}$. $\Rightarrow$ $M_1 = ON$; $\underbrace{M_3, M_4 = OFF}$

$\Rightarrow i_D = 0 \leftarrow$ both off

M_2 : $V_{GS_2} = V_2 - V_{DS_1} = V_{DD} - V_{DS_1}$

M_1 ON but no current $\Rightarrow V_{DS_1} = 0$ $\Rightarrow V_{GS_2} = V_{DD}$

$\Rightarrow M_2 = ON$.

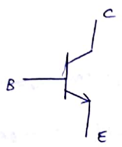
M_2 ON but no current $\Rightarrow V_{DS_2} = 0$ $\Rightarrow \boxed{V_0 = 0}$.

$\Rightarrow \cancel{V_0 = V_{DD}}$.

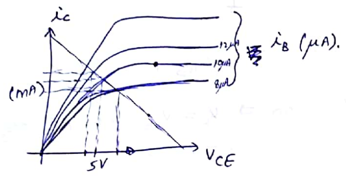
V_1	V_2	V_0
0	0	1
1	0	1
0	1	1
1	1	0

$\Rightarrow$ NAND Gate.

CMOS NOR GATE :- HW

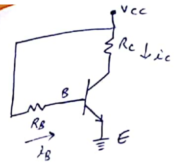


npn BJT
common emitter.



$\frac{I_C}{I_B} = \beta = 100$ $\Rightarrow$ $\frac{\Delta I_C}{\Delta I_B} = 100$.

(active) current amplification



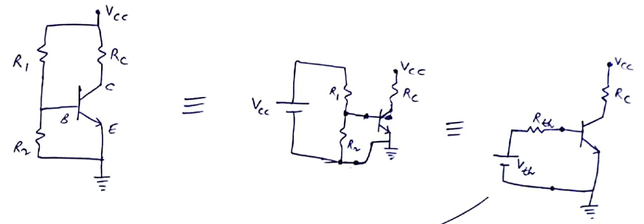
npn BJT

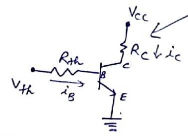
Assume active mode $\Rightarrow V_{BE} = 0.7V$.

$i_B = \frac{V_{CC} - 0.7}{R_B}$ $i_C = \beta i_B$

$V_{CE} = V_{CC} - i_C R_C$

Need $V_{CE} > 0.2V$.





$R_{th} = \frac{R_1 R_2}{R_1 + R_2}$

$V_{th} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$.

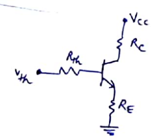
$V_{th} = V_{BE} + i_B R_{th} + R_E(i_B + i_C)$

Assume active $\Rightarrow V_{BE} = 0.7V$

$\cancel{i_B = \frac{V_{th} - 0.7}{R_{th}}}$.

$i_C = \beta i_B$

$V_{CE} = V_{CC} - i_C R_C$. Need $V_{CE} > 0.2V$.



Self biasing circuit.

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4) $V_1 = V_{DD}$, $V_2 = V_{DD}$. $\Rightarrow$ $M_1 = ON$; $M_3, M_4 = OFF$

$\Rightarrow i_D = 0$ ← turn off

M_2 : $V_{GS_2} = V_2 - V_{DS_1} = V_{DD} - V_{DS_1}$

M_1 ON but no current $\Rightarrow V_{DS_1} = 0$ $\Rightarrow V_{GS_2} = V_{DD}$

$\Rightarrow M_2 = ON$.

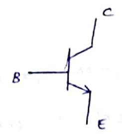
M_2 ON but no current $\Rightarrow V_{DS_2} = 0$ $\Rightarrow \boxed{V_0 = 0}$.

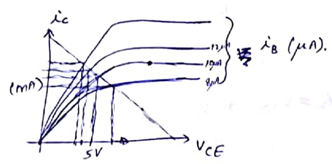
$\Rightarrow \cancel{V_0 = V_{DD}}$.

V_1	V_2	V_0
0	0	1
1	0	1
0	1	1
1	1	0

$\Rightarrow$ NAND Gate.

CMOS NOR GATE :- HW

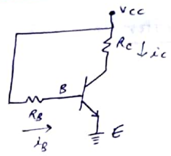




npn BJT
common emitter.

$\frac{I_C}{I_B} = \beta = 100$ $\Rightarrow$ $\frac{\Delta I_C}{\Delta I_B} = 100$.

(active) current amplification



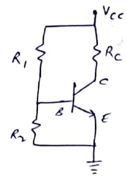
npn BJT

Assume active mode $\Rightarrow V_{BE} = 0.7V$.

$i_B = \frac{V_{CC} - 0.7}{R_B}$ $i_C = \beta i_B$

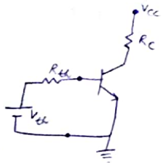
$V_{CE} = V_{CC} - i_C R_C$

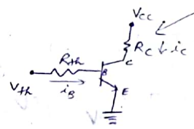
Need $V_{CE} > 0.2V$.



$\equiv$

$\equiv$

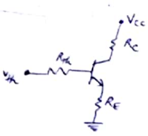




$R_{th} = \frac{R_1 R_2}{R_1 + R_2}$

$V_{th} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$.

$V_{th} = V_{BE} + i_B R_{th} + R_E(i_B + i_C)$



similar

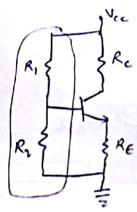
Assume active $\Rightarrow V_{BE} = 0.7V$

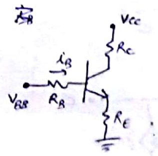
$\cancel{i_B = \frac{V_{th} - 0.7}{R_{th}}}$

$i_C = \beta i_B$

$V_{CE} = V_{CC} - i_C R_C$. Need $V_{CE} > 0.2V$.

Self biasing circuit.

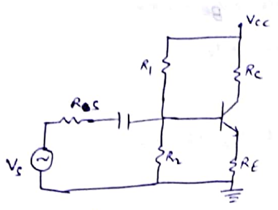




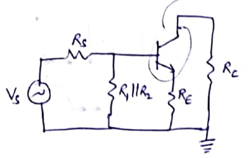
$i_B = \frac{V_{BB} - 0.7}{R_B + R_E(1+\beta)}$

$\boxed{\beta : h_{FE}}$.

Coupling capacitor



$Z = \frac{1}{j\omega C}$.



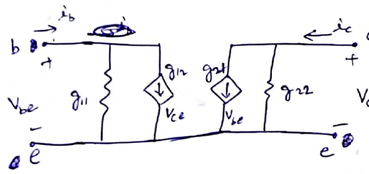
$I_B(V_{BE}, V_{CE})$, $I_C(V_{BE}, V_{CE})$

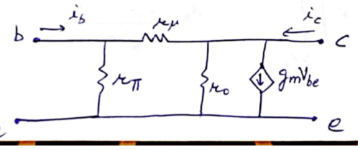
$$\underbrace{\delta I_B}_{i_B} = \underbrace{\left(\frac{\partial I_B}{\partial V_{BE}}\right)_{V_{CE}}}_{g_{11}} \underbrace{\delta V_{BE}}_{v_{be}} + \underbrace{\left(\frac{\partial I_B}{\partial V_{CE}}\right)_{V_{BE}}}_{g_{12}} \underbrace{\delta V_{CE}}_{V_{ce}}$$

$$\underbrace{\delta I_C}_{i_c} = \underbrace{\left(\frac{\partial I_C}{\partial V_{BE}}\right)_{V_{CE}}}_{g_{21}} \underbrace{\delta V_{BE}}_{v_{be}} + \underbrace{\left(\frac{\partial I_C}{\partial V_{CE}}\right)_{V_{BE}}}_{g_{22}} \underbrace{\delta V_{CE}}_{V_{ce}}.$$

$i_1 = g_{11} v_{be} + g_{12} v_{ce}$

$i_2 = g_{21} v_{be} + g_{22} v_{ce}$





$$i_b = \frac{v_{be}}{r_\pi} + \frac{v_{bc}}{r_\mu}$$

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o} + \frac{\overbrace{v_{cb}}^{-v_{bc}}}{r_\mu}.$$

Use:- $v_{bc} = v_{be} - v_{ce}$

$$i_b = \frac{v_{be}}{r_\pi} + \frac{v_{be}}{r_\mu} - \frac{v_{ce}}{r_\mu}$$

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o} + \frac{v_{ce}}{r_\mu} - \frac{v_{be}}{r_\mu}.$$

$g_{11} = \frac{1}{r_\pi} + \frac{1}{r_\mu}$, $g_{12} = -\frac{1}{r_\mu}$

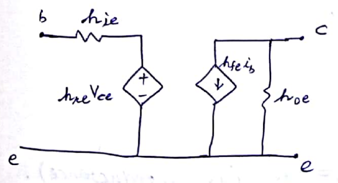
$g_{21} = g_m - \frac{1}{r_\mu}$, $g_{22} = \frac{1}{r_o} + \frac{1}{r_\mu}$.

Choose I_B, V_{CE}

$V_{BE}(I_B, V_{CE})$, $I_C(I_B, V_{CE})$

$$v_{be} = \underbrace{\left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}}}_{h_{ie}} i_b + \underbrace{\left(\frac{\partial V_{BE}}{\partial V_{CE}}\right)_{I_B}}_{h_{re}} v_{ce}$$

$$i_c = \cancel{\partial I_C} \; \underbrace{\left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}}}_{h_{fe}} i_b + \underbrace{\left(\frac{\partial I_C}{\partial V_{CE}}\right)_{I_B}}_{h_{oe}} v_{ce}$$



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$i_b = g_{11} V_{be} + g_{12} V_{ce}$

$i_c = g_{21} V_{be} + g_{22} V_{ce}$

Assume active region,

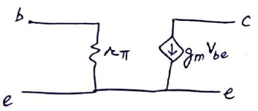
$g_{11} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)_{V_{CE}}$ $\qquad I_B = A e^{V_{BE}/V_T}$.

$= \frac{I_B}{V_T}$.

$g_{12} = \left(\frac{\partial I_B}{\partial V_{CE}}\right)_{V_{BE}} \simeq 0$.

$g_{21} = \left(\frac{\partial I_C}{\partial V_{BE}}\right)_{V_{CE}} = \frac{I_C}{V_T}$.

$g_{22} = \left(\frac{\partial I_C}{\partial V_{CE}}\right)_{V_{BE}} \simeq 0$.



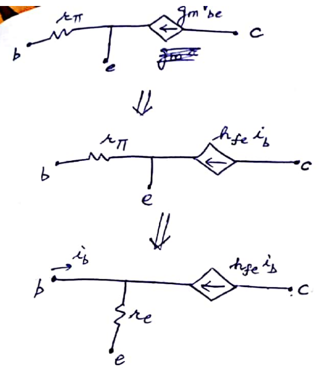
$r_\mu = \frac{-1}{g_{12}}$, $r_\pi = \frac{1}{g_{11}+g_{12}}$, $r_o = \frac{1}{g_{22}+g_{12}}$ ($g_{12} \to 0$, $g_{22} \to 0$)

$r_\mu = \frac{-1}{g_{12}}$, $r_\pi = \frac{1}{g_{11}}$, $r_o \approx \infty$. $\to$ open circuit.

$g_m = g_{21} - g_{12} = g_{21}$.

$r_\pi = \frac{V_T}{I_B}$, $g_m = \frac{I_C}{V_T}$

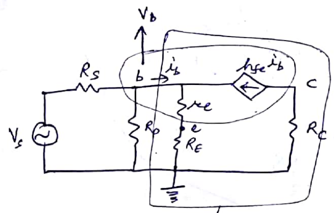
$r_\pi g_m = \frac{I_C}{I_B} = \frac{i_c}{i_b} = h_{fe}$ $\qquad g_m = \frac{h_{fe}}{r_\pi}$ = (transconductance).

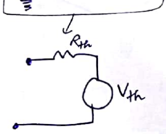


$i_b = \frac{V_{be}}{r_e} - h_{fe} i_b$

$\Rightarrow i_b(1+h_{fe}) = \frac{V_{be}}{r_e}$.

$\Rightarrow r_e = \frac{V_{be}}{i_b} \cdot \frac{1}{(h_{fe}+1)} = \frac{r_\pi}{(1+h_{fe})}$.

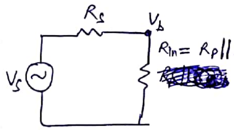




~~$R_{th} = r_e + R_E$~~ $\qquad R_{th} = (1+h_{fe})(R_E + r_e)$

~~V_{th}~~ $\qquad V_{th} = 0$.

$R_b = R_{th}$



$R_{in} = R_p || R_b$.

$R_{th} = \frac{V}{I}$.

$V = (I + h_{fe} I)(R_E + r_e)$

$\Rightarrow \frac{V}{I} = (1+h_{fe})(R_E + r_e)$.

$= R_{th}$.

$V_{oc} = 0 \Rightarrow$ No i_b & no voltage.

$\Rightarrow V_{th} = 0$.

ac Voltage gain

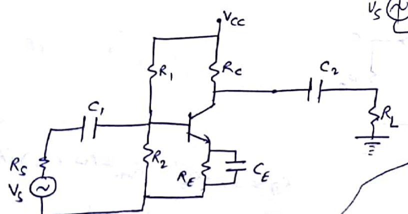
$$A_v = \frac{V_c}{V_b} = \frac{-R_c h_{fe} \cancel{i_b}}{R_b \cancel{i_b}}$$

$$\Rightarrow \boxed{A_v = \frac{-R_c}{r_e + R_E}}.$$

$$A_{vs} = \frac{V_c}{V_s}$$

$$\boxed{V_b = \frac{R_{in}}{R_{in}+R_s} \cdot V_s}$$

$$A_{vs} = \frac{V_c}{V_b} \cdot \frac{V_b}{V_s} = A_v \cdot \frac{R_{in}}{R_{in}+R_s}$$



$$R_b \approx (1+h_{fe})(r_e + R_E)$$

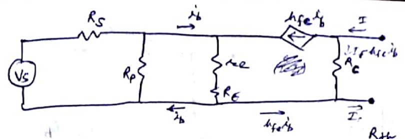
$$R_{in} = R_2 \| R_b$$

??

$$A_v = \frac{-R_T}{r_e} \quad R_T \to R_c \| R_p$$

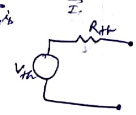
$$\frac{-R_c}{r_e + R_E}$$

$$A_{vs} = \frac{A_v R_{in}}{R_{in}+R_s}$$



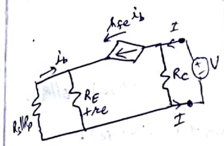
$V_{TH} = ?$

$R_{th} = ?$



$$(r_e + R_E)(i_b + h_{fe} i_b) = (R_s \| R_p)\, i_b.$$

$$i_b = \frac{R_c I}{h_{fe} R_c + (r_e + R_E)(1+h_{fe})}.$$



$i_b = 0$

$$\boxed{R_{TH} = R_c}.$$

$$R_c\left(- \frac{h_{fe} R_c}{(\quad)} \right)$$

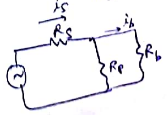
$$R_c\left(1 - \frac{R_c h_{fe}}{h_{fe} R_c + (r_e + R_E)(1+h_{fe})} \right)$$

$$= \frac{R_c \cdot (r_e + R_E)(1+h_{fe})}{h_{fe} R_c + (r_e + R_E)(1+h_{fe})}.$$

ac current gain

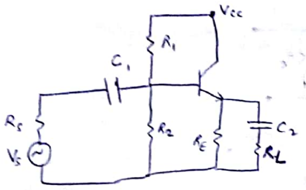
$$A_i = \frac{i_L}{i_b} = \frac{-\left[h_{fe} \cancel{i_b} \cdot \left(\frac{R_c}{R_c + R_L}\right)\right]}{\cancel{i_b}}$$

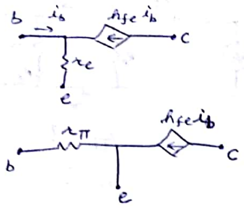
$$\Rightarrow A_i = \frac{-h_{fe} R_c}{R_c + R_L}.$$

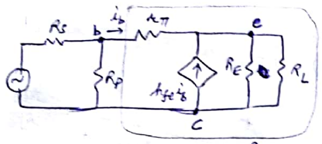


$$i_b = \frac{R_p i_s}{R_p + R_b}$$

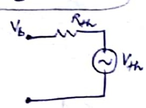
$$\Rightarrow A_{is} = \frac{i_L}{i_s} = \frac{i_L}{i_b} \cdot \frac{i_b}{i_s} =$$







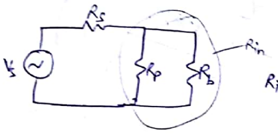
$R_T = R_E || R_L$.



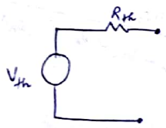
$V_{th} = 0$.

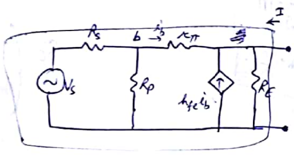
$R_{th} = r_\pi + (1+h_{fe})(R_E||R_L)$.

$R_{th} = r_\pi + R_T(1+h_{fe})$.



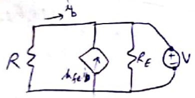
$R_{in} = R_P || R_b$.

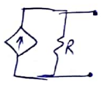




R_E

$R = (R_S || R_P + r_\pi) || R_E$





$i_b = -V/R$

$i_{R_E} = V/R_E$.

$\Rightarrow i_b + h_{fe} i_b - \frac{V}{R_E} + I = 0$

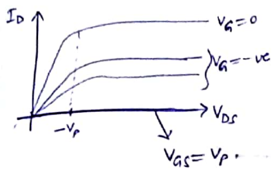
$\Rightarrow (1+h_{fe}) i_b = V/R_E - I$.

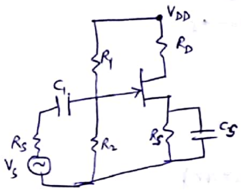
$i_b(r_\pi + R_P||R_S) = R_E(1+h_{fe}) i_b + R_E I$.

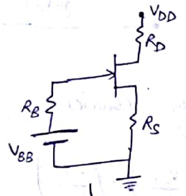
$\Rightarrow \boxed{V/I = R_E || \frac{R}{(1+h_{fe})}}$.

Buffer = $\begin{cases} R_o \text{ small} \\ R_{in} \text{ large} \\ A_{vs} \approx 1 \end{cases}$.

n-channel JFET



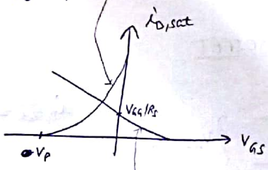




$i_D = I_{DSS}\left[2\left(\frac{V_{GS} - V_P}{V_P^2}\right)V_{DS} - \left(\frac{V_{DS}}{V_P}\right)^2\right]$ — ohmic region.

$i_{D,sat} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$ — sat.

$V_{D,sat} = V_{GS} - V_P$



Load line: $V_{GG} = i_D R_S + V_{GS}$.

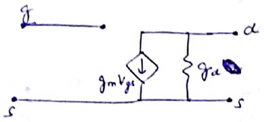
Need small signal model:-

$I_D\ (V_{GS}, V_{DS})$

$I_G\ (V_{GS}, V_{DS})$

$$i_d = \underbrace{\left(\frac{\partial I_D}{\partial V_{DS}}\right)_{V_{GS}}}_{g_d} V_{ds} + \underbrace{\left(\frac{\partial I_D}{\partial V_{GS}}\right)_{V_{DS}}}_{g_m} V_{gs}$$

$i_g \approx 0$



$$g_m = I_{DSS} \cdot \frac{2V_{DS}}{V_P^2} \quad \text{(ohmic)}$$

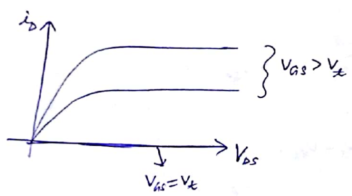
$$= 2 I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right) \cdot \left(\frac{-1}{V_P}\right) \quad \text{(active)}$$

$$\left(1 - \frac{V_{GS}}{V_P}\right) \to \left(\frac{I_D}{I_{DSS}}\right)^{1/2}$$

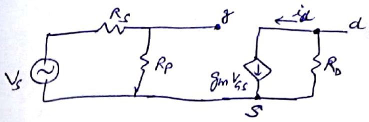
$$g_m = \frac{-2}{V_P} \sqrt{I_D \cdot I_{DSS}}$$

(enhancement)

n-channel MOSFET



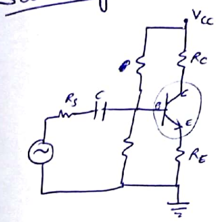
$i_{D,sat} = K(V_{GS} - V_t)^2$.

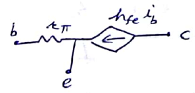


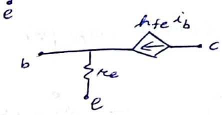
$A_V = \frac{V_d}{V_s} = -R_d \cdot g$

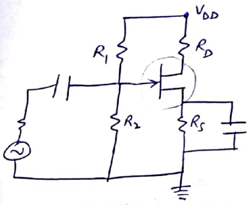
$V_{DS} = -R_D i_d = -R_D\, g_m V_g$.

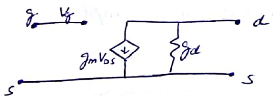
Summary:-

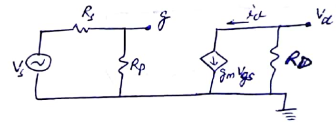




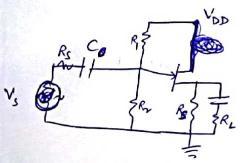








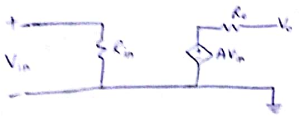
$V_D = -g_m V_{gs} R_D$.



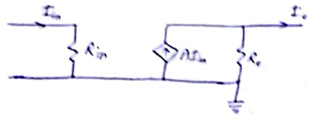
Common-drain amplifier

buffer

$V_o \approx V_i$, $R_{out} \approx 0$, $R_{in} \to \infty$.



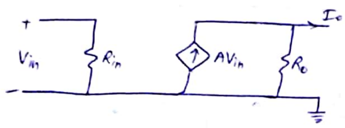
Voltage amplifier



Current amplifier

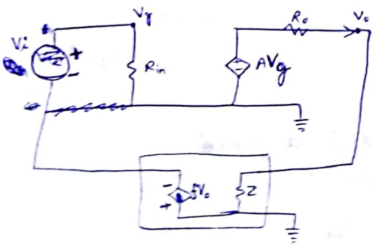
A: units of resistance.

trans-resistance ampl.



A: units of conductance.

trans-conductance amp.



Assume $Z >> R_o$ (no loading)

series – shunt feedback

$$\left.\begin{aligned} V_o &= AV_g \\ V_g &= V_i - fV_o \end{aligned}\right\} \Rightarrow V_g = \frac{V_o}{A}$$

$$\Rightarrow \frac{V_o}{A} = V_i - fV_o$$

$$\Rightarrow V_o = V_i \underbrace{\left(\frac{A}{1+Af}\right)}_{\text{overall gain}}.$$

$$\boxed{A_F = \frac{A}{1+Af}}.$$

If $Af = +ve$, $|\text{overall gain}| \downarrow$. (−ve feedback)

If $Af = -ve$, $|\text{overall gain}| \uparrow$. (+ve feedback)

If $Af = -1$, $|A_F| \to \infty$.

If $Af = -2$, $A_F = \frac{A}{-1} = -A$ ✗

for $|A_F| < 1$, $A_F = A(1 - Af + (Af)^2 - \dots)$

Gain $= A$

feedback $= -f$.

Add $-fV_o$ to V_i

extra input $= -fAV_i$

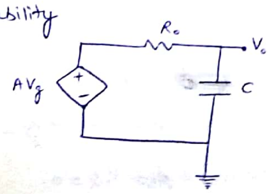
Net output $= AV_i - AfAV_i$.

extra input $= fAfAV_i$

⋮

Net output $= AV_i\,(1 - Af + (Af)^2 - \dots)$

Stability



$$C\frac{dV_o}{dt} = i$$

$$AV_g = iR_o + V_o = C\frac{dV_o}{dt}R_o + V_o$$

Put $V_g = V_i - fV_o$

$$CR_o \frac{dV_o}{dt} + (1+Af)\,V_o = AV_i$$

Put $V_i = 0$. Is $V_o = 0$ stable ??

Try $V_o = V_o(t=0).\,e^{-t/\tau}$

$\Rightarrow \frac{CR_o}{-\tau} + (1+A_f) = 0$

$\Rightarrow \tau = \frac{CR_o}{1+A_f}$. If $Af \le -1$, $\tau < 0$

V_o diverges.

$\Rightarrow$ Not stable.

$$A_F = \frac{A}{1+Af}$$

Af = loop gain

If $Af = -1$, ~~oscillating~~ oscillations start

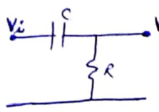
Barkhausen criteria

__loading__

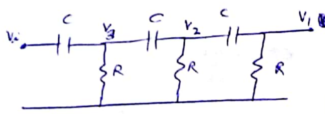
$$V_o = AV_g \frac{Z}{R_o+Z} = \underbrace{\left(\frac{AZ}{R_o+Z}\right)}_{A'} V_g$$

$$\left.\begin{aligned} V_o &= A'V_g \\ V_g &= V_i - fV_o \end{aligned}\right\} \quad A_F = \frac{A'}{1+A'f}.$$

$A'f = -1 \rightarrow$ Barkhausen criteria.



$$V_o = \frac{V_i}{R + \frac{1}{j\omega C}} \cdot R$$



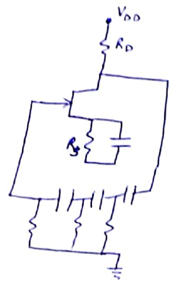
KCL at each node

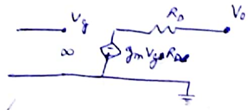
$$\begin{cases} j\omega C(V_3 - V_i) + j\omega C(V_3 - V_2) + \frac{V_3}{R} = 0 \\ j\omega C(V_2 - V_3) + j\omega C(V_2 - V_1) + \frac{V_2}{R} = 0 \\ j\omega C(V_1 - V_2) + V_1/R = 0. \end{cases}$$

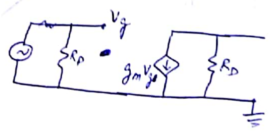
Need

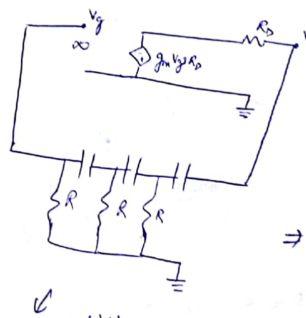
$\frac{V_1}{V_i} = \ldots$

At $\omega = \omega_0 = \frac{1}{RC\sqrt{6}}$, $\frac{V_1}{V_i} = \frac{-1}{29}$.









Phase shift oscillator

$R >> R_D$ (no loading)

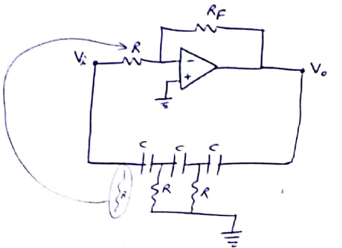
$\Rightarrow A = -g_m R_D$

$f = \frac{1}{29}$ at $\omega = \omega_0 = \frac{1}{RC\sqrt{6}}$.

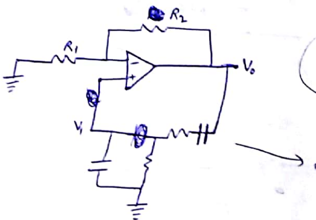
Need $Af \le -1$.

$\frac{-g_m R_D}{29} \le -1 \Rightarrow g_m R_D \ge 29$

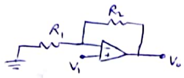
$\Rightarrow \boxed{g_m \ge \frac{29}{R_D}}$ — oscillations at $\omega = \omega_0$.



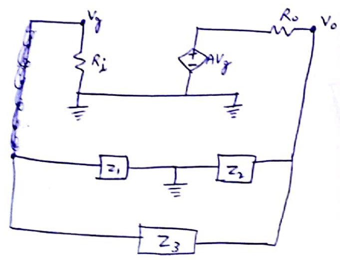
$A = -\frac{R_F}{R}$. (gain)



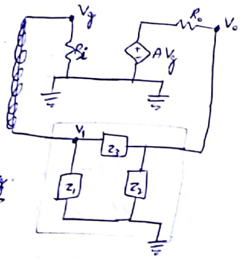
$A = 1 + \frac{R_2}{R_1}$.



→ HW.



$\equiv$



$$\frac{V_1}{Z_1} + \frac{V_1 - V_o}{Z_3} = 0 \quad (R_i \to \infty).$$

$$\Rightarrow V_1 = \left(\frac{Z_1}{Z_1+Z_3}\right) V_o \rightarrow -f.$$

Put

$$\frac{V_o}{Z_2} + \frac{V_o - V_1}{Z_3} + \frac{V_o - AV_g}{R_o} = 0$$

~~Put~~

$V_o = A'V_g$

$$A' = \frac{AZ_2(Z_1+Z_3)}{R_o(Z_1+Z_2+Z_3) + Z_2(Z_1+Z_3)}.$$

Loop gain $= A'f = \frac{-AZ_1Z_2}{R_o(Z_1+Z_2+Z_3) + Z_2(Z_1+Z_3)}$.

Assume $Z = jX$

$X = \omega L$ (Ind.)

$X = \frac{-1}{\omega C}$ (cap.)

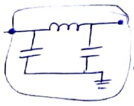
$$= \frac{AX_1X_2}{R_o(X_1+X_2+X_3) - X_2(X_1+X_3)}$$

Put $X_1 + X_2 + X_3 = 0$

loop gain $= \frac{-AX_1}{X_1+X_3} = \frac{AX_1}{X_2}$.

Choose $X_1 =$ cap.
$X_2 =$ cap.
$X_3 =$ ind.

Resonant circuit osc. → (L–C circuit)



Colpitts oscillator.

$$\frac{-1}{\omega C_1} - \frac{1}{\omega C_2} + \omega L_3 = 0 \Rightarrow \omega = \frac{1}{\sqrt{L_3 \frac{C_1C_2}{C_1+C_2}}}.$$

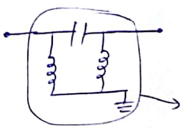
$A = -g_m R_D$

$R_D g_m \frac{C_2}{C_1} \geq 1$.

$X_1 =$ ind.
$X_2 =$ ind.
$X_3 =$ cap.

$$\omega L_1 + \omega L_2 - \frac{1}{\omega C_3} = 0 \Rightarrow \omega = \frac{1}{\sqrt{C_3(L_1+L_2)}}$$

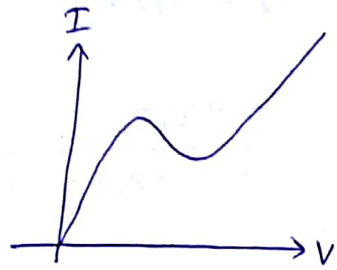
$R_D g_m \frac{L_1}{L_2} \geq 1$.



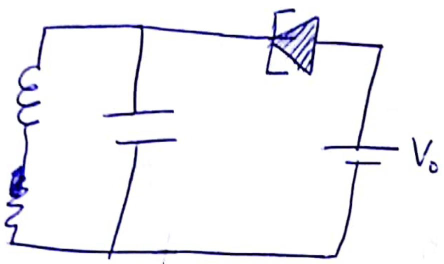
Hartley oscillator.

~~Colpitts oscillator.~~

RESONANT TUNNELING DIODE (RTD)



differential resistance $= \frac{dV}{dI} = -ve.$



$$\left(\int fg\,dx\right)^2 \leq \left(\int f^2 dx\right)\left(\int g^2 dx\right)$$

$$\boxed{\mathrm{Var}(\hat{\theta};\theta) \geq \frac{1}{I(\theta)}}$$

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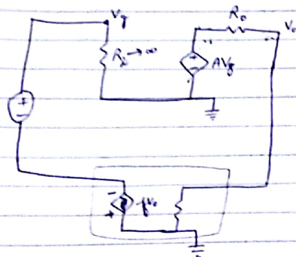
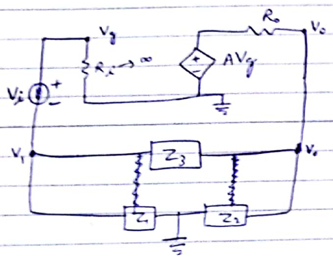


$$V_o = AV_g$$

$$V_g = V_i - fV_o$$

$$\Rightarrow V_o = \frac{A}{1+Af} V_i$$



$$\frac{V_1}{Z_1} + \frac{V_1 - V_o}{Z_3} = 0$$

$$\Rightarrow V_1 = V_o \underbrace{\left(\frac{Z_1}{Z_1+Z_3}\right)}_{=-f}.$$

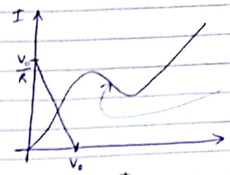
$$\frac{V_o}{Z_2} + \frac{V_o - V_1}{Z_3} + \frac{V_o - AV_g}{R_o} = 0$$

$$\Rightarrow V_o = A'V_g.$$

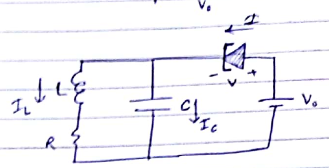
$$V_g = V_i + V_1$$
$$= V_i - fV_o.$$

Resonant Tunneling Diode (RTD)

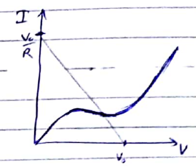
Vander Pol oscillator



$$\frac{dV}{dI} = -ve$$



$$V_o = V + IR$$



$$I = I_c + I_L$$

Put $I_c = C\frac{d}{dt}(V_o - V).$

$$= -C\frac{dV}{dt}.$$

$$V_L + i_L R = V_o - V.$$

$I = f(V)$ ← some function.

$$\Rightarrow L\frac{di_L}{dt} + i_L R = V_o - V.$$

$$\Rightarrow L\frac{d}{dt}(I - \cancel{C}I_c) + i_L R = V_o - V \Rightarrow L$$

$$\Rightarrow L\left(\frac{df}{dt}\right) + LC\frac{d^2V}{dt^2} + \left(f + C\frac{dV}{dt}\right)R = V_o - V$$

$$\frac{df}{dt} \to \frac{df}{dV}\cdot\frac{dV}{dt}$$

$$\Rightarrow LC\frac{d^2V}{dt^2} + \left[L\frac{df}{dV} + RC\right]\frac{dV}{dt} + Rf + V = V_o.$$

If $V = \text{const.} \Rightarrow V = V_0$.

Put $V = V_1 + v(t)$

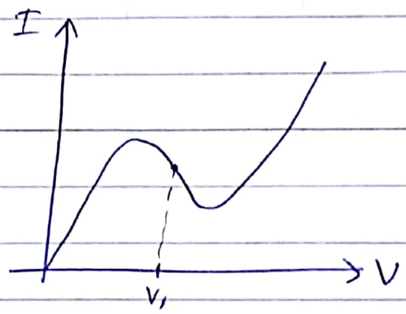
$$LC\frac{d^2v}{dt^2} + \left[L\left(\frac{df}{dV}\right) + RC\right]\frac{dv}{dt} + Rf + v = 0.$$

$$\Rightarrow \frac{d^2v}{dt^2} + \left[\frac{1}{C}\left(\frac{df}{dV}\right)_{V_1} + \frac{R}{L}\right]\frac{dv}{dt} + \cancel{Rf} + \frac{v}{LC} = 0$$

Series RLC circuit

Use $v = e^{st} \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0.$

$$\Rightarrow s = \left(-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}\right)\Big/2.$$



$$\ddot{x} + \mu(x^2-1)\dot{x} + \omega^2 x = 0$$

if $|x|<1$, damping $= -ve$
$|x|>1$, damping $= +ve$

Magnetic circuits

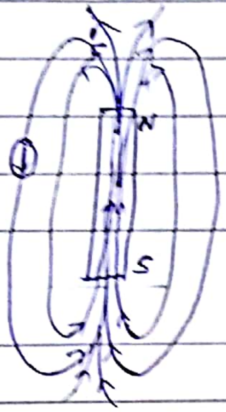
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\,\hat{r}$$

$\}$ mag. field

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \rho(\vec{r}')\,\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\cdot d^3r'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int J(\vec{r}')\times\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,d^3r' \Rightarrow H = \frac{1}{4\pi}\int\_\_\_\_ .$$

Permeability of vacuum.



flux lines.

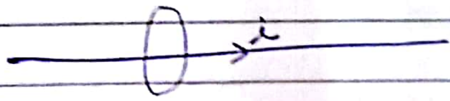
$B = \frac{\Phi}{A}$. Φ : $\frac{\text{Webers}}{m^2} = \text{tesla}$

$$\vec{F} = q(\vec{v}\times\vec{B}) + q\vec{E}.$$

B = Magnetic flux density.
(magnetic induction) $B = \mu H$.

H = Magnetic field

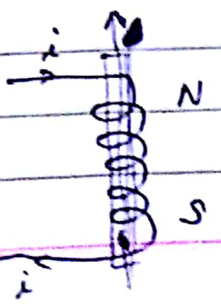
* $\nabla\times\vec{H} = \vec{J} \Rightarrow \vec{H}\cdot d\vec{l} = i$



$H \cdot 2\pi r = i$

$$\Rightarrow H = \frac{i}{2\pi r}.$$

$$\Rightarrow B = \frac{\mu i}{2\pi r}.$$



$$B = \frac{\mu N i}{L}.$$

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flux $\phi = BA$

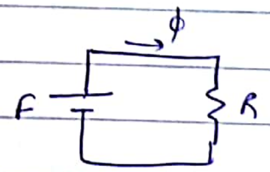
$\phi = \frac{\mu N A}{l} i$.

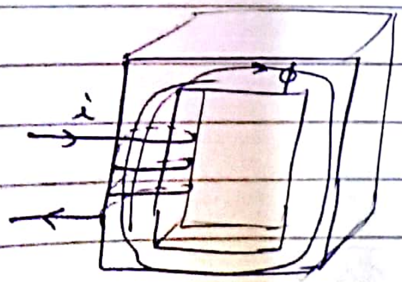
Magnetomotive force $\mathcal{F} = Ni$.

$\phi = \frac{\mathcal{F}}{R}$, $R = \frac{l}{\mu A}$.

R: Reluctance.

Electrical	Magnetic
Current i	Flux ϕ
Voltage V	~~reluctance R~~ mmf $\mathcal{F}$
Resist. R	reluctance R
Ohm's law $V = iR$	$\mathcal{F} = \phi R$
$\left(R = \frac{\rho l}{A}\right)$.	$\left(R = \frac{l}{\mu A}\right)$.



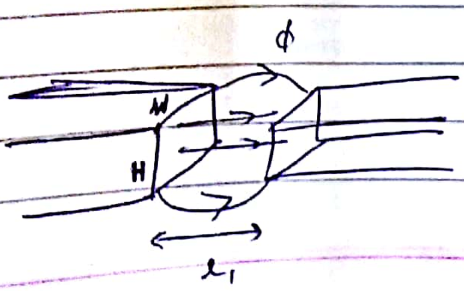


$R_c = \frac{l}{\mu A}$ (core)

$\mathcal{F} = Ni$

$\left.\right\} \phi = \frac{\mathcal{F}}{R_c}$

$B = \frac{\phi}{A}$.



Fringing flux.

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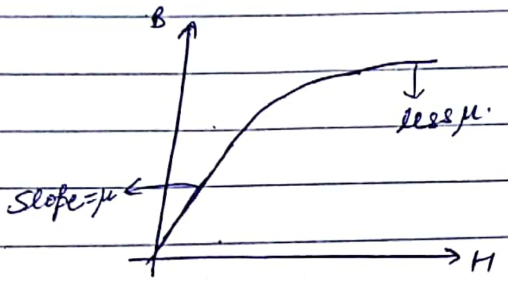
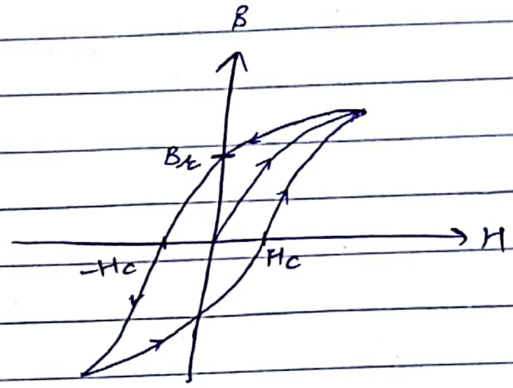
Effective area $= (W + l_1)(h + l_1)$

$$\boxed{\mu = \mu_0}$$

$R = R_c + R_{gap}$ $\qquad$ $R_{gap} >> R$.

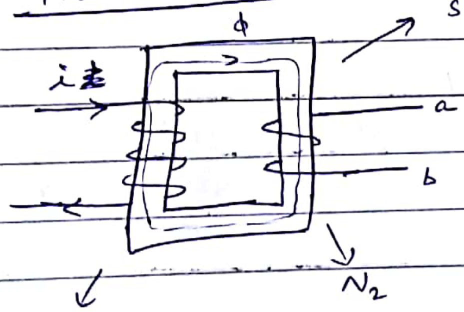
$\mu \approx 1000\mu_0$, $B = \mu H$.





Hysteresis,
↓
(loss).

- TRANSFORMER :-



secondary coil

N_1
↓
Primary coil

Magnetically coupled.

$\phi = \frac{N_1 i}{R}$

inductance:

$V(t) = \frac{d\phi}{dt}$

↓

$V = L\frac{di}{dt}$.

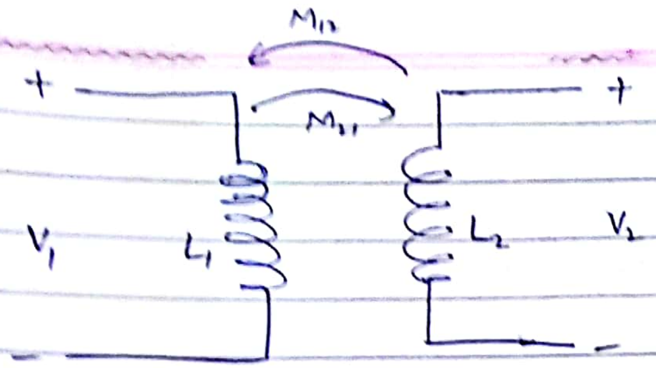
L → self inductance
↓
Henry (H).

$V_{ab} = N_2 \frac{d\phi}{dt}$.

$= \frac{N_2 N_1}{R} \frac{di}{dt}$

$\Rightarrow V_{ab} = M\frac{di}{dt}$

M: mutual inductance.



$M_{12} = M_{21} = M.$

$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}.$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}.$$

$$\text{Energy stored } (W) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$V = L\frac{di}{dt}$$

$$\text{Energy} = \int vi\,dt$$

$$= \int Li\frac{di}{dt}.dt$$

$$= \frac{1}{2}Li^2$$

$$\text{Energy} = \int (V_1 i_1 + V_2 i_2)\,dt$$

$$= \int\left(L_1 i_1\frac{di_1}{dt} + L_2 i_2\frac{di_2}{dt} + M i_1\frac{di_2}{dt} + M i_2\frac{di_1}{dt}\right)dt$$

$$= \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2.$$

$M \leq \sqrt{L_1 L_2}$. Put $i_2 = x i_1$.

$W \geq 0 \;\forall x \in R.$

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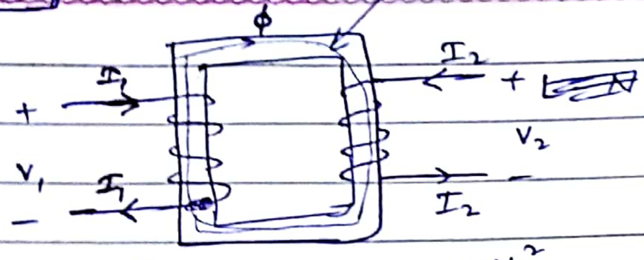
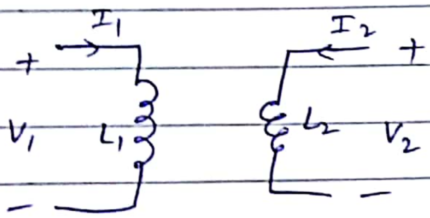


$M \leq \sqrt{L_1 L_2}$

$L_1 = \frac{N_1^2}{R}$. $L_2 = \frac{N_2^2}{R}$

If $M = \sqrt{L_1 L_2} = \frac{N_1 N_2}{R}$. $\Rightarrow k = 1$.

Coupling coefficient (K) $= \frac{M}{\sqrt{L_1 L_2}}$.



$\bar{V_1} = j\omega L_1 \bar{I_1} + j\omega M \bar{I_2}$

$\Rightarrow \bar{I_1} = \frac{V_1 - j\omega M \bar{I_2}}{j\omega L_1}$

~~$\bar{I_2} = \bar{V_1} - j\omega$~~

$\bar{V_2} = j\omega L_2 \bar{I_2} + j\omega M \bar{I_1}$

$\bar{V_2} = \frac{M \bar{V_1}}{L_1} - \frac{j\omega M^2 \bar{I_2}}{L_1} + j\omega L_2 \bar{I_2}$

Assume $k = 1 \Rightarrow M = \sqrt{L_1 L_2}$.

$\bar{V_2} = \sqrt{\frac{L_2}{L_1}}\, V_1 = N \bar{V_1}$.

$L_2 = \frac{N_2^2}{R}$, $L_1 = \frac{N_1^2}{R}$ $\Rightarrow N = \frac{N_2}{N_1}$.

$N > 1 \Rightarrow$ step up transformer
$N < 1 \Rightarrow$ step down —"—
$N = 1 \Rightarrow$ Isolation.

$\frac{\bar{V}_1}{j\omega L_1} - \frac{M\bar{I}_2}{L_1}$ $\sqrt{L_1L_2}$

$= -N\bar{I}_2.$

~~$= -NM$~~

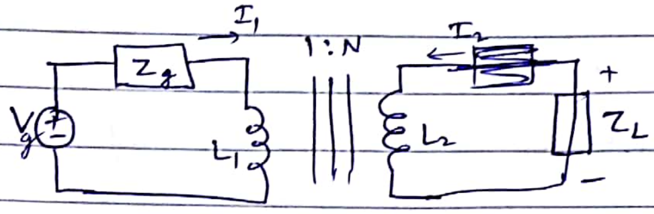
$L_1 \to \infty$

$L_2 \to \infty$

$\underbrace{N = \text{constt.}}_{\text{Ideal transformer}}$ $K=1$

$\bar{I}_1 = -N\bar{I}_2$ or $\bar{I}_2 = \frac{-\bar{I}_1}{N}$

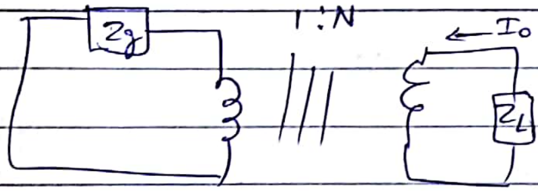
$\bar{V}_2 = N\bar{V}_1.$



$V_{oc} = ?$

$V_g = Z_g I_1 + V_1 = V_1$

$\Rightarrow V_{oc} = V_2 = NV_1 = NV_g.$



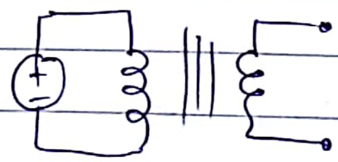
$V_1 = -Z_g I_1$

$= -Z_g(-NI_o)$

$= NZ_g I_o$

$V_o = NV_1 = N(NZ_gI_o)$

$Z_o = \frac{V_o}{I_o} = N^2 Z_g.$



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