1	1	2	3	Q
2	2	4	Q	Q
3	4	6	Q	Q
5	5	8	Q	Q
7	8	Q	Q	Q

7. Here is an interesting problem from Prof. Abhiram Ranade. A piecewise linear function on [0,1] may be represented by a sequence of special x and y values such as the table below:

x	0	0.3	0.5	1
f(x)	1.3	1.2	4.1	0.3

Thus, a good representation is f.NumberOfSegments, f.x (array of x values), f.y (array of y values). Now given two functions f and g, let h be the minimum of f and g. Clearly, it is also a piecewise linear function. Compute the representation of h.

Week 9 (30th Sep. - 5th October)

- **Graphs**
  Graphs - the basic definitions. Directed and undirected graphs. Paths and cycles. Subgraphs. Connectedness, connected components. Trees and forests. Number of edges and connectivity. Spanning trees. Nice (Eulerian) cycles. Degree of a node. The graph ADT. Various methods. Lecture - 24 Graphs

- **Data Structures for Graphs**
  The Edge-Vertex adjacency list. The adjacency matrix and the traditional adjacency list. Various extensions of the adjacency list. The Breadth First Search. Examples and the queue implementation. The O(E+V) time analysis. Predecessors and the BFS tree.
  Lecture - 25 Data Structures for Graphs lec25.ppt.pdf

- **Two Applications of Breadth First Search**
  Connected components and shortest paths. Bipartite graphs, odd cycles and detection using BFS.Diameter of a graph.
  Lecture - 26 Two Applications of Breadth First Search

Tutorial 8

1. The graph is an extremely useful modelling tool. Here is how a Covid tracing tool might work. Let V be the set of all persons. We say (p,q) is an edge (i) in E1 if their names appear on the same webpage, and (ii) in E2 if they have been together in a common location for more than 20 minutes. What significance does the connected components in these graphs and what does the BFS do? Does the second graph have epidemiological significance? If so, what? If not, how would you improve the graph structure to get a sharper epidemiological meaning?
2. Let us take a plane paper and draw circles and lines to divide the plane into various pieces. There is an edge (p,q) between two pieces if they share a common boundary of intersection (which is more than a point). Is this graph bipartite? Under what conditions is it bipartite?
3. There are three containers A, B and C, with capacities 5,3 and 2 liters respectively. We begin with A having 5 liters of milk and B and C being empty. There are no other measuring instruments. A buyer wants 4 liters of milk. Can you dispense this? Model this as a graph problem with the vertex set V as the set of configurations c=(c1,c2,c3) and an edge from c to d if d is reachable from c. Begin with (5,0,0). Is this graph directed or undirected? Is it adequate to model the question: How to dispense 4 liters?
4. Suppose that there are M workers in a call center for a travel service which gives travel directions within a city. It provides services for N cities - C1,...,CN. Not all workers are familiar with all cities. The number of requests from each city per hour are R1,...,RN. A worker can handle K calls per hour. How would you model this problem? Assume that R1,...,RN and K are small numbers.
5. There is a set of bureaucrats B={b1,...bm}. Subgroups of them keep meeting and making decisions of n attributes, e.g., Parking is to be allowed (Y/N), Garba can take place (Y/N) etc, . Let us call these as boolean variable P1,P2,...,Pn. Each meeting M has a time-stamp and a decision to change these boolean values. The new assignment is carried with the bureaucrats who participated. Model this as a graph. What questions can be answered using this model? For example, given a meeting in which two bureaucrats bi and bj have opposite decisions on an attribute, can they decide who of the two has the latest information?
6. Study the BFS code from Prof. Naveen's slides. Argue that at any time during the running of the algorithm the d-values of vertices in the queue can only take 1 or 2 consecutive values.
7. List the properties of the white, grey and black vertices. In line 10, while processing u, can we encounter a vertex v which is gray? Or black? In such cases what are the values possible for d[v]?

```
BFS(G,s)
01 for each vertex u ∈ V[G]-{s}
02    color[u] ← white
03    d[u] ← ∞
```

```
    π[u] ← NIL
color[s] ← gray
d[s] ← 0
π[u] ← NIL
Q ← {s}
while Q ≠ ∅ do
    u ← head[Q]
    for each v ∈ Adj[u] do
        if color[v] = white then
            color[v] ← gray
            d[v] ← d[u] + 1
            π[v] ← u
            Enqueue(Q,v)
    Dequeue(Q)
    color[u] ← black
```

8.

Week 10

- Depth First Search
- Applications of DFS
- DFS in Directed Graphs
- Applications of DFS in Directed Graphs
- Minimum Spanning Trees
- The Union
- Prims Algorithm for Minimum Spanning Trees
- Single Source Shortest Paths
- Correctness of Dijkstra's Algorithm
- Single Source Shortest Paths