# Tutorial Sheet 6

Polynomial Interpolation - Week 1

1. Let $x_0, x_1, \cdots, x_n$ be distinct nodes. If $p(x)$ is a polynomial of degree less than or equal to $n$, then show that

$$p(x) = \sum_{i=0}^{n} p(x_i) l_i(x),$$

where $l_i(x)$ is the $i^{\text{th}}$ Lagrange polynomial.

2. Given distinct nodes $x_0$, $x_1$, $\ldots$, $x_n$, for $n \geq 1$, show that

$$\sum_{k=0}^{n} l_k(x) = 1,$$

where $l_k(x)$ denotes the $k^{\text{th}}$ Lagrange polynomial.

3. Given distinct nodes $x_0$, $x_1$, $\ldots$, $x_n$, for $n \geq 1$, let

$$L(x) = \prod_{i=0}^{n} (x - x_i), \ \ x \in [a, b].$$

For each $k = 0, 1, \ldots, n$, show that

$$l_k(x) = \frac{L(x)}{(x - x_k) L'(x)}, \ \ x \in [a, b],$$

where $l_k(x)$ is the $k^{\text{th}}$ Lagrange polynomial for the given nodes.

4. Using Lagrange form of interpolating polynomial for the function $g(x) = 3x^2 + x + 1$, express the rational function

$$f(x) = \frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$$

as a sum of partial fractions.

5. Find the Lagrange form of interpolating polynomial $p_2(x)$ that interpolates the function $f(x) = e^{-x^2}$ at the nodes $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$. Further, find the value of $p_2(-0.75)$ (use 6-digit rounding arithmetic). Compare the value with the true value $f(-0.75)$ (use 6-digit rounding arithmetic). Find the percentage error in this calculation.

6. Construct the divided difference table for the data

| $x$ | $-3$ | $-1$ | $0$ | $3$ | $5$ |
|---|---|---|---|---|---|
| $f(x)$ | $-30$ | $-22$ | $-12$ | $330$ | $3458$ |

   Find the Newton form of interpolating polynomial for the above data and use the interpolating polynomial to get an approximate value of $f(2.5)$.

7. Let $x_0, x_1, \cdots, x_n$ be distinct nodes, and $f$ be a given function on $\mathbb{R}$. Define $w(x) = \prod_{i=0}^{n}(x - x_i)$. Prove that

$$f[x_0, x_1, \cdots, x_n] = \sum_{i=0}^{n} \frac{f(x_i)}{w'(x_i)}.$$