22nd Aug 2022

200050103

HW2

1) RegExps over $\Sigma = \{a,b\}$

1.1) Do not end with aa

$$(a+b)^*(ab+ba+bb) + a+b+\epsilon$$

1.2) Even number of b's

$$a^*(ba^*b)^*a^*$$

1.3) Do not contain the substring ba

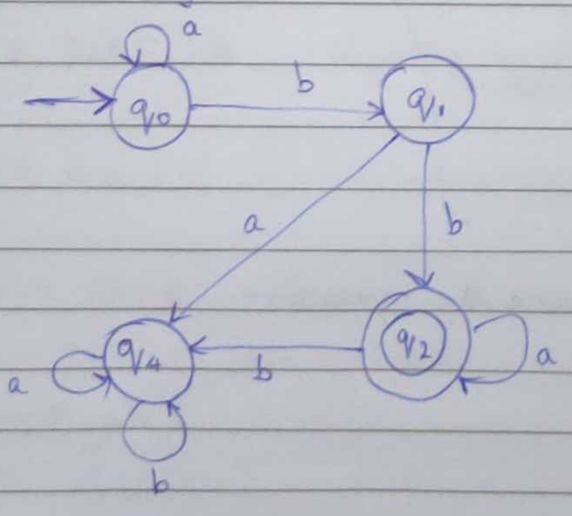
$$a^*b^*$$

1.4) Odd/Even-length block of a's is immediately followed by an odd/even-length block of b's

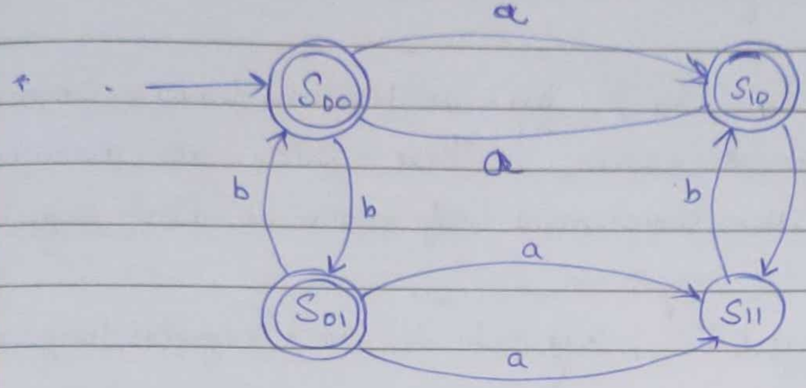
$$b^*((aa)^*(bb)^* + a(aa)^*b(bb)^*)^*$$

2) Minimal DFA

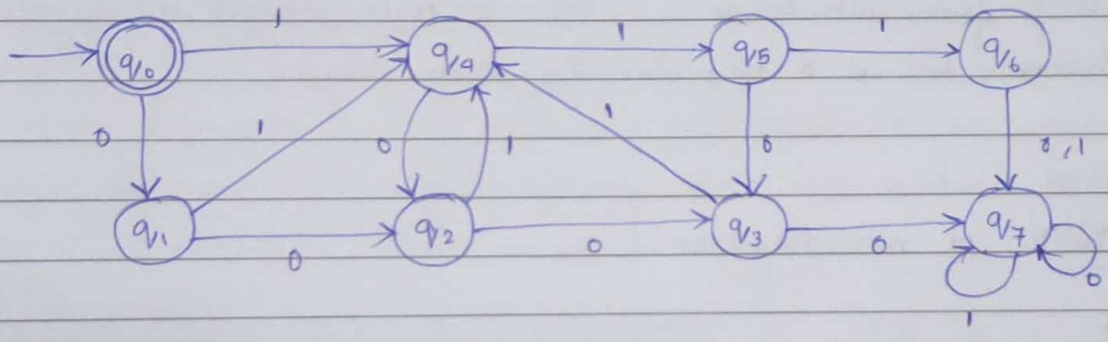
2.1) $L = \{a^* b b a^*\}$



2.2) $L = \{ w \in (a+b)^* \mid n_a(w) \equiv 0 \pmod 2$ OR $n_b(w) \equiv 0 \bmod 2 \}$



2.3) $L = \{ w \in (0+1)^* \mid$ every block of 4 consecutive symbols contains the substring 01$\}$



3) L_1, L_2 are regular languages over Σ, Show:

3.1) $L_1^c = \Sigma^* - L_1$ is a regular language

L_1 is regular, so DFA (D) can be constructed for it. In D, if we make $F' = Q - F$ (reject states as accept states and vice-versa) and call this new DFA: D', we can see that D' accepts every string D doesn't and rejects every string D does. $\therefore$ D' accepts L^c $\Rightarrow L^c$ is a regular language.

3.2) $L_1 \cap L_2$ is a regular lang.

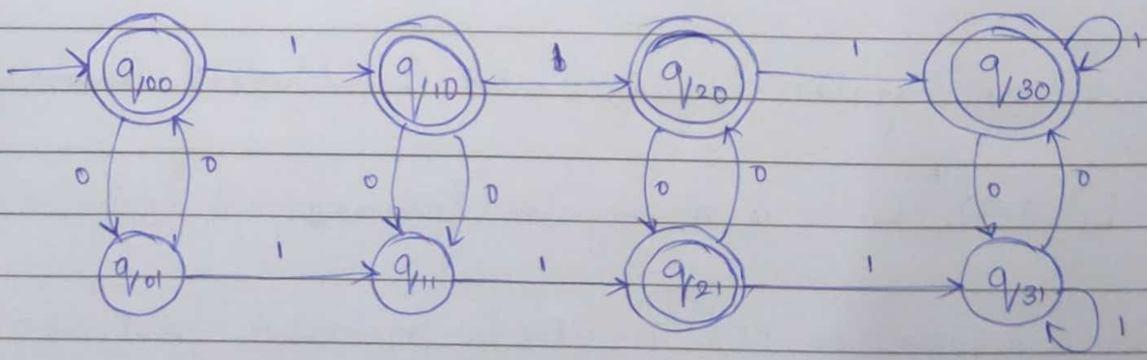
$L_1 \cap L_2$ is same as $(L_1^c \cup L_2^c)^c$ by De-Morgan's Law. from 3.1 we know L_1^c, L_2^c are regular. Their union is also regular by closure and then the complement of union is also regular

3.3) $L_3 = \{ x \in \Sigma^* \mid \exists y \in \Sigma^*, xy \in L_1 \}$ is a regular language.

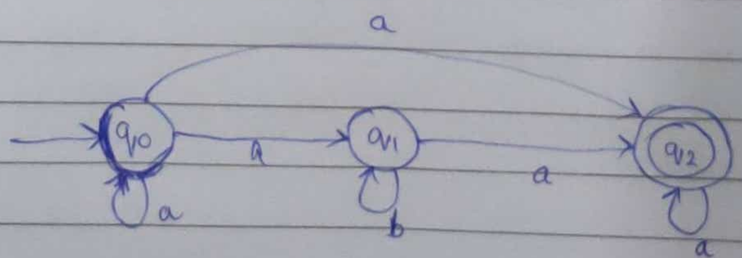
D is DFA constructed for reg. lang. L_1. Now in D, we make a new accepting state p. Now from each existing state q, draw an ϵ transition from q to p, if there is a path from q to any of the final states in D. This ϵ-NFA accepts all prefixes of L_1, hence L_3 is a regular language.

NFA or not regular

4.1) $\{ w \in \{0,1\}^* \mid w$ contains an even no. of 0s, or exactly 2 1s$\}$



4.2) $L(ab^*a^*) \cap L(a^*b^*a)$



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4.3) $L = \{www \mid w \in \{a,b\}^*\}$

Suppose L is accepted by DFA D with N states, consider w with more than N states, while accepting at least one state should be repeated ie. there is a loop before even w ends. Let's say $w = pqr$ then D accepts pqrpqrpqr and since there is a loop, it also accepts pqq...qrpqrpqr but this is not in L. Hence L is not regular.

4.4) $L = \{w \in \{a,b\}^* \mid w$ has more a's than b's$\}$

DFA of L : D with N states, consider w with N+1 b's followed by N+2 a's. this should be accepted by D. At least after N b's one of the state must be repeated ⇒ we can have indefinitely may b's followed by N+2 a's which doesn't ∈ to L but is accepted by D. Hence L is not regular.

4.5) didn't get idea

4.6) ~~$\{w \in \{a,b\}^* \mid n_a(w) < n_b(w)\}$~~ $L = \{a^i b^j a^k \mid k \neq i+j\}$

If L is a reg. lang then L^c should also be a reg. lang. Suppose L^c is accepted by DFA D with N states, consider w with N+1 a's followed by N b's followed by 2N+1 a's which is accepted by L. Therefore there must be a loop before first N+1 a's are exhausted which means indefinite no. of a's followed by N b's followed by 2N+1 a's which is not accepted by L^c. Hence L is not regular.

4.7) $L = \{a^n \mid n$ is not a perfect square$\}$

Consider L^c ie. n is a perfect square. Consider DFA D with N states and consider a^{N^2}, there must be a loop after traversing of N a's, max length of loop, l is N, so loop once again gives us a^{N^2+l} and this too is accepted by D but N^2+l is not a perfect square sin $1 <= l \leq N$. Hence L is not regular.

4.8) $L = \{a^p \mid p \text{ is a prime number}\}$

(not able to ~~p~~rove)