CS6001: Game Theory and Algorithmic Mechanism Design

Quiz 1 – Semester 1, 2022-23, Computer Science and Engineering,
Indian Institute of Technology Bombay
Total Points: 20, Time: 40 minutes, ATTEMPT ALL QUESTIONS

1. Consider the following ways an instrutor informs the students of a course about a make-up class.

   (a) Writes the information about the make-up class on the course webpage.

   (b) Posts about it on some *learning management system (LMS)* (e.g., Piazza) where all students are enrolled and chooses the setting so that everyone can see the post. Students can see each other on the LMS. (Assume that students check their LMS accounts daily)

   (c) Announces the information in the class. If a student is absent, (s)he gets the information from others by the end of the day.

   (d) Sends individual emails (not a mass email where To or CC has all the email addresses) to every student.

   Assume that no other communication happens (among the students or between the students and the instructor) regarding the information on the make-up class apart from the ones described above. In which of these cases the information is a **common knowledge** on the next day?

   (i) All of them.

   (ii) (a) and (b).

   (iii) (b) and (c).

   (iv) (c) and (d).

   2 points.

2. Describe the following game (Figure 1) in normal form (let Player I choose the row, Player II choose the column).

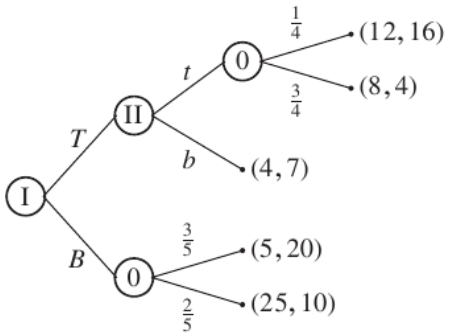

Figure 1: Game tree of problem 2.

   (i) How many strategies do players I and II have respectively?

   (ii) Draw the game matrix along with the utilities.

   (iii) Find all the PSNEs, if any exist.

   1 + 2 + 1 = 4 points.

3. Compute all the MSNEs (which include PSNEs if they exist) for the following game (Figure 2), where Player I is the row player and Player II is the column player. **4 points.**

	L	*M*	*R*
T	1, 1	0, 2	2, 0
B	0, 0	1, 0	−1, 3

Figure 2: Game matrix of problem 3.

4. **OR-AND** is a two-player game played on a full binary tree with a root, of depth n (e.g., see Figure 3 when $n = 2$). Players I and II in turn choose a leaf of the tree that has not previously been selected, and assigns it a logic value 1 and 0 respectively. After all the leaves have been assigned a value, a value for the entire tree is calculated as shown in the figure. The first step involves calculating the value of the vertices at one level above the level of the leaves: the value of each such vertex is calculated using the logical **AND** function, operating on the values assigned to its children. Next, a value is calculated for each vertex one level up, with that value calculated using the logical **OR** function, operating on the values previously calculated for their respective children. The values of all the vertices of the tree are alternately calculated in this manner recursively, with the value of each vertex calculated using either the **AND** or **OR** functions, operating on values calculated for their respective children. Player I wins (i.e., Player II loses) if the value of the root vertex is 1, and loses (i.e., Player II wins) if the value of the root vertex is 0.

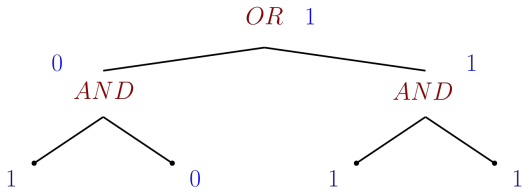

Figure 3: Game tree of problem 4.

   (i) Which player has a winning strategy in a game played on a tree of depth two?

   (ii) Explain your answer to the previous question.

   (iii) Which player has a winning strategy in a game played on a tree of depth $2k$, where k is any positive integer?

   (iv) Explain your answer to the previous question.

1 + 1 + 1 + 1 = 4 points.

5. Consider a two-player zero-sum game. The payoff of Player I is given by the matrix A. The mixed strategies of players I and II are given by the vectors x and y respectively ($x \in [0,1]^{|S_I|}, x^\top \mathbf{1} = 1$, $y \in [0,1]^{|S_{II}|}, y^\top \mathbf{1} = 1$). Hence, the utility of player I for a strategy profile (x, y) is given by

$$x^\top A y.$$

   (i) Prove that

$$\min_y \max_x \; x^\top A y \geqslant \max_x \min_y \; x^\top A y.$$

   (ii) If (x^*, y^*) is an MSNE of this game, then prove that

$$\min_y \max_x \; x^\top A y = \max_x \min_y \; x^\top A y = {x^*}^\top A y^*.$$

2 + 4 = 6 points.

Good Luck!