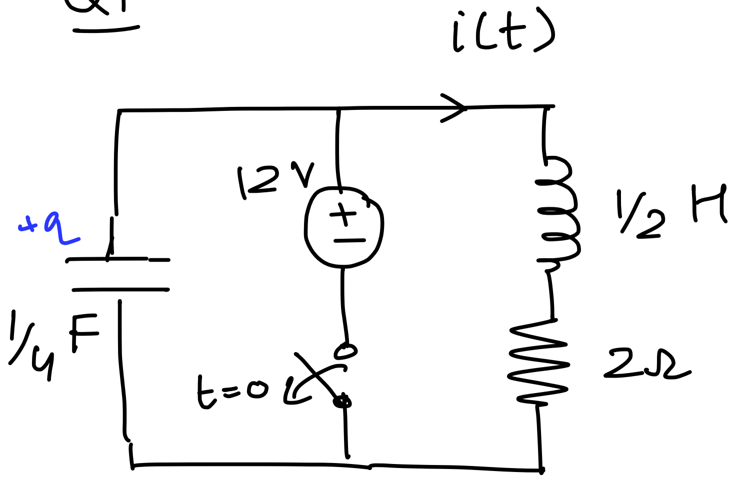
Endsem 2018

Q1



$i(0) = 6\,A$

$V_c(0) = 12\,V$

$t \geq 0$

$V_c = \frac{1}{2}\frac{di}{dt} + 2i$

$i = -\frac{dq}{dt}$

$$4q = -\frac{d^2q}{2\,dt^2} - 2\frac{dq}{dt}$$

$$\frac{d^2q}{dt^2} + 4\frac{dq}{dt} + 8q = 0$$

$$\frac{-4 \pm \sqrt{-16}}{2} = -2 \pm 2i$$

$$q = e^{-2t}(A\cos 2t + B\sin 2t)$$

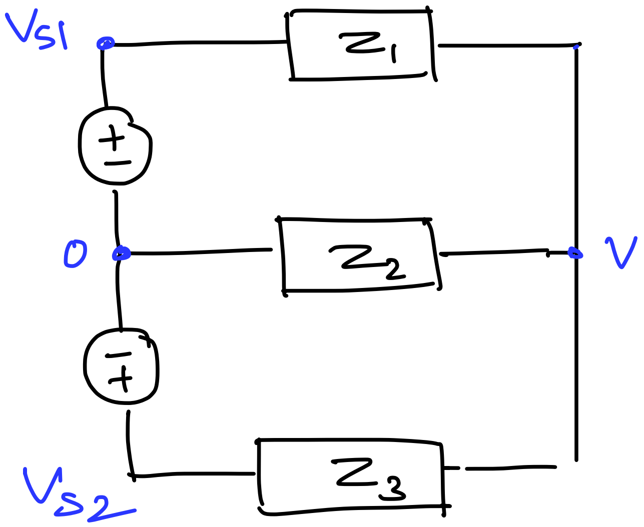
$$q(0) = C\,V_c(0) = 3 = A$$

$$\frac{dq}{dt}(0) = 2B - 2A = -i(0) = -6 \Rightarrow B = 0$$

$$i = -e^{-2t}((2B-2A)\cos(2t) - (2A+2B)\sin(2t))$$

$$= 6(\sin(2t) + \cos(2t))e^{-2t}\ A$$

Q2.



$Z_1 = Z_2 = Z_3 = (10-10j)\,\Omega$

$V_{s1} = 2\cos(\omega t)$

$V_{s2} = 2\sin(\omega t)$

$= 2\cos(\omega t - 90)$

$= 2\angle{-90}\ \cos(\omega t)$

$= -2i\ \cos(\omega t)$

$$\frac{V - V_{s1}}{Z_1} + \frac{V - V_{s2}}{Z_3} + \frac{V}{Z_2} = 0$$

$$\Rightarrow V = \frac{V_{s1} + V_{s2}}{3}$$

$$= \left(\frac{2-2i}{3}\right) \cos(\omega t)$$

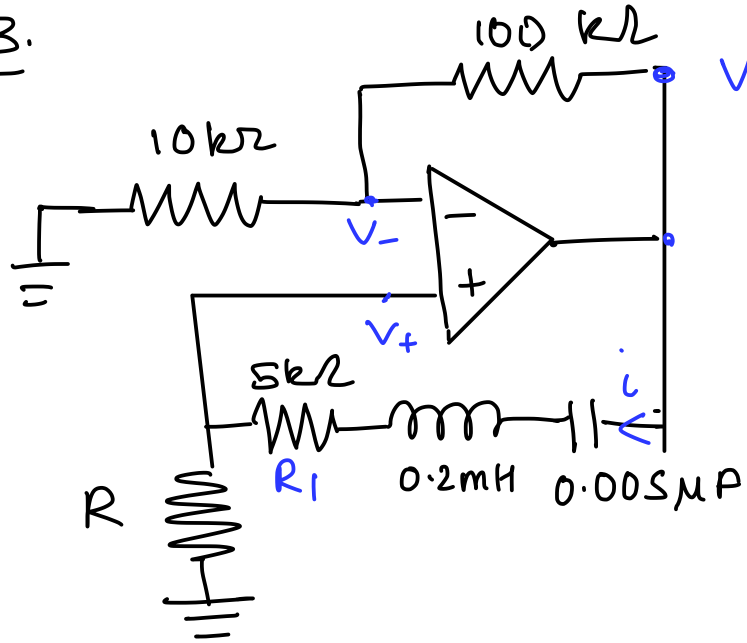
$$P_1 = \frac{1}{2}\left(\frac{V - V_{s1}}{Z_1}\right)^2 |Re(Z_1)|$$

$$= \frac{1}{2} \times \frac{(16+4)}{9 \times (200)} \times 10 = \frac{1}{18}\ W$$

$$P_2 = \frac{1}{2}\left(\frac{V}{Z_2}\right)^2 |Re(Z_2)| = \frac{1}{45}\ W$$

$$P_3 = \frac{1}{18}\ W$$

Q3.



Assuming virtual short.

$V_- = V_+$

$$\frac{V_0 - V_-}{100} = \frac{V_- - 0}{10}$$

$$\Rightarrow V_0 = 11 V_-$$

$$\frac{V_+}{R} = \frac{V_0 - V_+}{|Z_{eq}|} \Rightarrow 10R = |Z_{eq}|$$

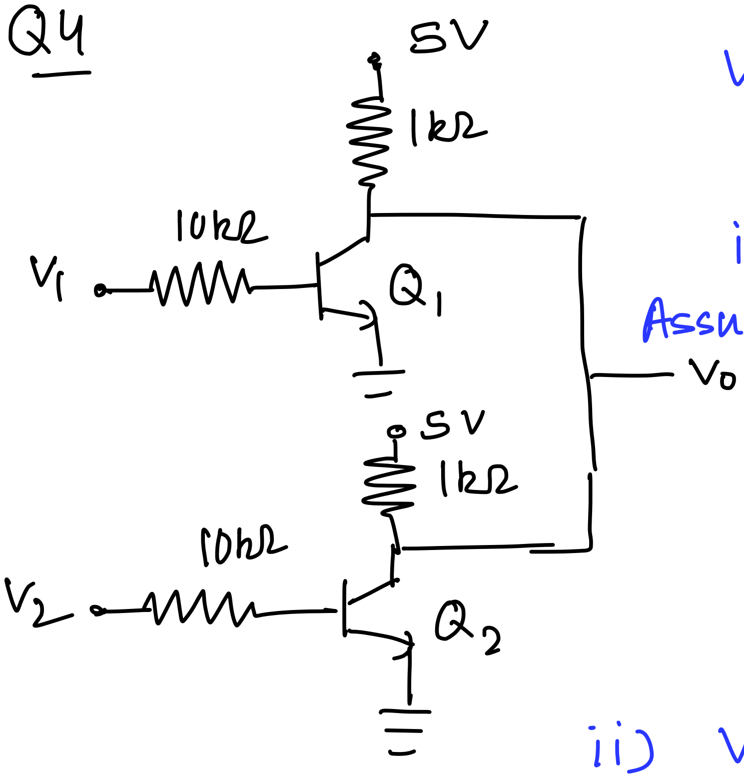
$$(10R)^2 = 5^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \times 10^{-6}$$

$$\Rightarrow 10R \geq 5 \quad \Rightarrow R \geq 0.5\ k\Omega$$

For $R = 500\,\Omega$,

$$\omega = \frac{1}{\sqrt{LC}} = 10^6 \text{ rad/s}$$

Q4



$V_{low} = 0.2V$ $V_{High} = 5V$

$\beta = 50.$

i) $V_1 = V_2 = V_{low}$

Assume $Q_1, Q_2 \rightarrow$ cutoff.

$V_{C1} = V_{C2} = \boxed{V_o = 5V}$

$V_{BE1} = V_{BE2} = 0.2 < 0.5$

Assumption correct

ii) $V_1 = 5V$, $V_2 = 0.2V$

Assume $Q_2 \rightarrow$ cutoff, $Q_1 \rightarrow$ sat.

$V_{B1} = 0.8V$ $i_{B1} = \frac{4.2}{10k\Omega} = 0.42\,mA$

$V_{C1} = \boxed{V_o = 0.2V}$

$i_{C1} = \frac{(5 - 0.2) \times 2}{1k\Omega} = 9.6\ mA$

$\frac{i_{C1}}{i_{B1}} < \beta$ and $V_{BE} = 0.2 < 0.5$

Assumption correct

iii) $V_1 = 0.2V$ $V_1 = 5V$

same as ii)

$\boxed{V_o = 0.2V}$

iv) $V_1 = V_2 = 5V$

Assume $Q_1, Q_2 \rightarrow$ sat.

$V_{B1} = V_{B2} = 0.8\,V \qquad i_{B1} = i_{B2} = 0.42\,mA$

$V_{C1} = V_{C2} = \boxed{V_0 = 0.2\,V}$

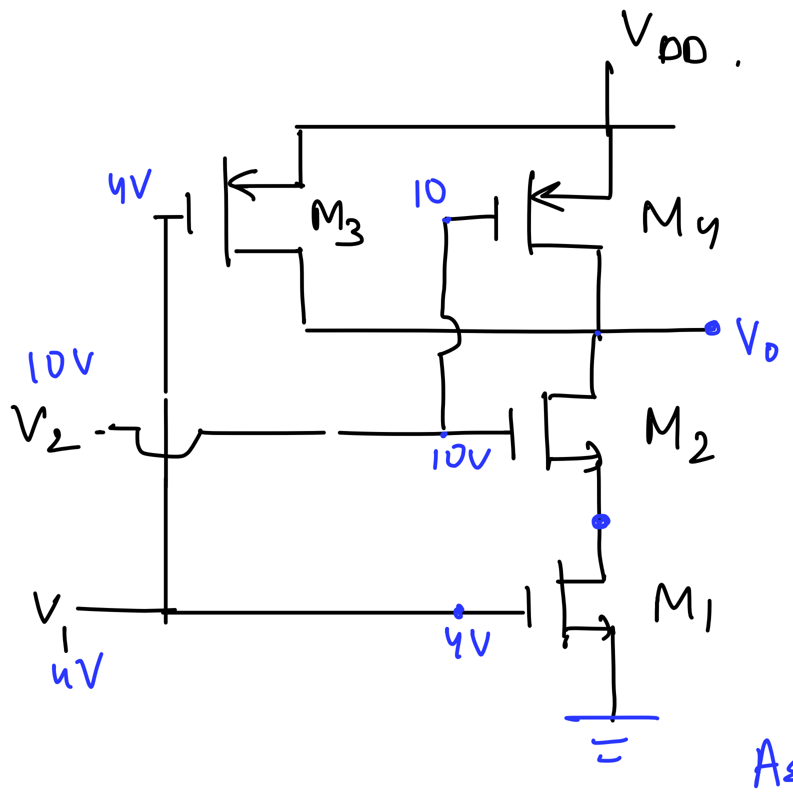
$i_{C1} = i_{C2} = \dfrac{(5-0.2)}{1k\Omega} = 4.8\,mA$

$\dfrac{i_{C1}}{i_{B1}} = \dfrac{i_{C2}}{i_{B2}} < \beta$ and $V_{BE1} = V_{BE2} = 0.2 < 0.5$

Assumption correct.

NOR GATE

Q5.



$V_1 = 4V \quad V_2 = 10V$

$V_{DD} = 10V$

$K = 0.25 \ mA/V^2$

$V_T(M_1, M_2) = 1V$

$V_T(M_3, M_4) = -1V$

$M_4 \rightarrow$ OFF

$V_{GS4} = 0 > V_{T4}$

$V_{GS3} = -6V < V_{T3}$

Assume M_1, M_2 active and M_3 in ohmic.

$i_3 = i_2 = i_1$

$i_1 = K(V_{GS1} - V_T)^2 = K(4-1)^2 = 9K$

$i_2 = K(V_{GS2} - V_T)^2 = K(9 - V_{DS1})^2$

$i_1 = i_2 \Rightarrow \quad V_{DS1} = 6V$

$i_3 = K(2(V_{GS3} - V_T)V_{DS3} - V_{DS3}^2)$

$= K(2(-5)V_{DS3} - V_{DS3}^2)$

$-10V_{DS3} - V_{DS3}^2 = 9$

$V_{DS3}^2 + 10V_{DS3} + 9 = 0$

$\Rightarrow V_{DS3} = -1$ or ~~-9~~ as for active region $V_{DS} > V_{GS} - V_T = -5$

$V_o = 10 - 1 = 9V$

$$V_{DS3} = -1\,V \geq V_{GS3} - V_T = -5\,V$$

$\Rightarrow M_3$ is in ohmic region

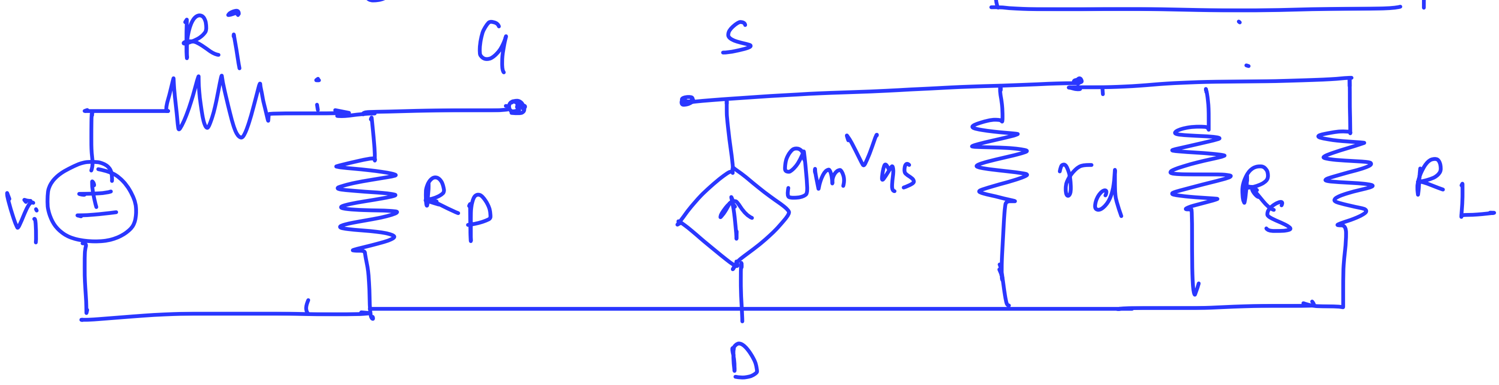
$$V_{DS2} = V_o - V_{DS1} = 3\,V \geq V_{GS2} - V_T = 3\,V$$

$$V_{DS1} = 6\,V \geq V_{GS1} - V_T = 3\,V$$

Q6

small signal model

$$\boxed{R_p = R_1 \parallel R_2}$$

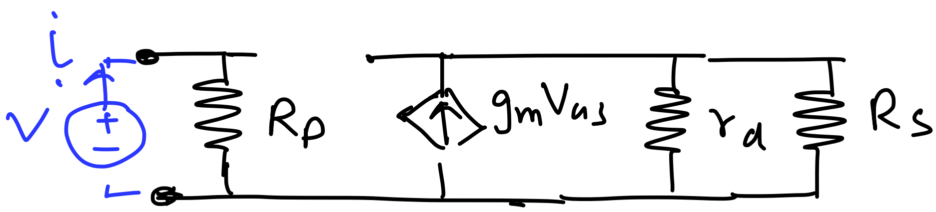


$$V_L = g_m V_{gs} \left(r_d \parallel R_s \parallel R_L \right)$$

$$V_{gs} = \frac{V_i R_p}{R_i + R_p} - V_L$$

$$\frac{V_L}{V_i} = g_m \frac{R_p}{R_p + R_i} \frac{\left(r_d \parallel R_s \parallel R_L \right)}{\left(1 + g_m r_d \parallel R_s \parallel R_L \right)}$$

Calculating Rin

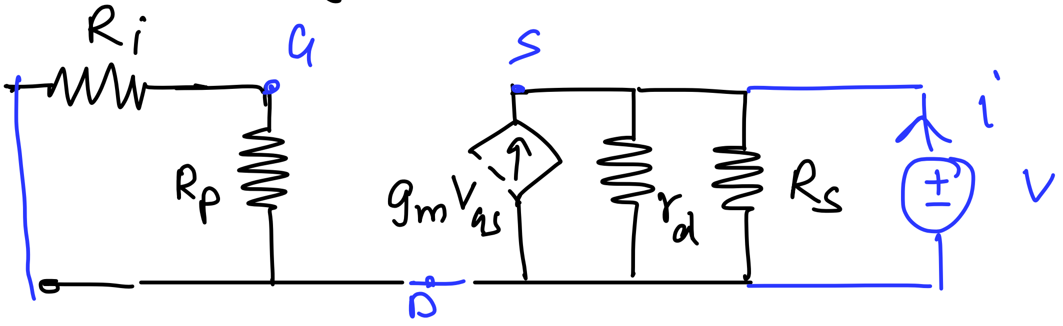


$$V = R_p i \Rightarrow \underline{R_{th}} = \frac{V}{i} = R_p$$

$$R_{in} = R_p = R_1 \parallel R_2$$

Calculating R_{out}

set V_i to 0



$$V_{gs} = -V$$

$$i = \frac{V}{r_d} + \frac{V}{R_s} + g_m V$$

$$\frac{V}{i} = r_d \parallel R_s \parallel \frac{1}{g_m} = R_{out}$$