

# Automata Homework-3 (CFG)

Q1)

a)  $S \rightarrow aSc | T$   
 $T \rightarrow \epsilon | bTc$

b)  $S \rightarrow \epsilon | aB | bA$

$A \rightarrow aC | bAA$

$B \rightarrow aD | bC$

$C \rightarrow aS | bCA$

$D \rightarrow aDB | bS$

b)  $S \rightarrow AC$

$A \rightarrow \epsilon | aAb$

$C \rightarrow \epsilon | bCc$

c)  $S \rightarrow BC$

$B \rightarrow AE$

$A \rightarrow a | aA$

$E \rightarrow \epsilon | aEb$

$C \rightarrow FD$

$F \rightarrow \epsilon | cFd$

$D \rightarrow d | dD$

d)  $L$

$S \rightarrow \epsilon | a^3sb^2$

e)  $S \rightarrow Ab$

$A \rightarrow a | aAb | aAa | bAa | bAb$

f)  $S \rightarrow aSd | T | V$

$S \rightarrow T$  is chosen if  $m \geq q$

$S \rightarrow V$  is chosen if  $m \leq q$

$T \rightarrow W | aTc$

$V \rightarrow W | bUd$

$W \rightarrow \epsilon | bWc$

g)  $S' \rightarrow aB' | bA' | \epsilon$

$A' \rightarrow aS' | bA'A'$

$B' \rightarrow bS' | aB'B'$

$S \rightarrow A' | SS$

$S', A', B'$  have respectively  $n_a = n_b, n_b + 1, n_b$  and  $S$  can be represented as a sequence of  $A$ 's

Q2)  $S \rightarrow SS | P | \epsilon$   
 $P \rightarrow 0$   
 $\uparrow$   
 Chomsky Normal Form

Q3) Greibach normal form

$S \rightarrow 0 | 0SA | 1A$   
 $A \rightarrow 1 | 1AS | 0S$

Q4)  $L = \{ ww \mid w \in (a+b)^* \}$

Consider strings of the form  $a^n b^n a^n b^n$ , there are type  $ww$  and PDA for  $L$  (if exists) should accept these languages. Now by pumping lemma for CFG,  $a^n b^n a^n b^n = uvwxz$  where  $|vwx| \leq n$ . Now either  $uvw$  is in  $a^n$ ,  $a^n b^n$ ,  $b^n$ ,  $b^n a^n$ , in each of these cases we can see that  $uv^iwx^iz$  is not part of the language. Hence there is no PDA for  $L$  which proves  $L$  is not CFG.