Q1 is autograded, answer is (b) and (c) – quite self explanatory.

Q2. i) Each player has 2 strategies: Player I: T and B, Player II: t and b

ii) To find the game matrix, we need to find the utilities of each player for every strategy profile

e.g., for (T, t), $u_I(T,t) = \frac{1}{4} \times 12 + \frac{3}{4} \times 8 = 9$

$u_{II}(T,t) = \frac{1}{4} \times 16 + \frac{3}{4} \times 4 = 7$

do similar exercise for other profiles.

The matrix is therefore

I \ II	t	b
T	9, 7	4, 7
B	13, 16	13, 16

iii) PSNEs are (B, t) and (B, b)

Q3. Observe that there is no PSNE in this game.

Also, L is weakly dominated by M for player 2.

Hence, in every MSNE of this game L will have zero probability (this problem can also be solved without using this extra result – the answer will be the same).

Hence the support of player I is $\{T, B\}$, and that of player II is $\{M, R\}$. The game essentially reduces to

		M q	R $1-q$
p	T	0, 2	2, 0
$1-p$	B	1, 0	−1, 3

using the MSNE characterization theorem:

$2p = 3(1-p) \Rightarrow \frac{p}{1-p} = 3/2$

$p = \frac{3}{5}$;

$2(1-q) = q - (1-q) \Rightarrow 3 = \frac{q}{1-q} \Rightarrow q = 3/4$

The answer is $\left(\left(\frac{3}{5}, \frac{2}{5}\right), \left(\frac{3}{4}, \frac{1}{4}\right)\right)$ on this reduced game. The answer in the original game is

$$\left(\left(\frac{3}{5}, \frac{2}{5}\right), \left(0, \frac{3}{4}, \frac{1}{4}\right)\right)$$

Q4. (i) Player II

(ii) Player II follows player I and puts 0 to the sibling of the node that player I has assigned 1. This strategy of player II ensures that every AND node gives a value of 0. The root node (OR) will gives 0 and player II wins. irrespective of what player I chooses.

(iii) Player II

iv) The argument is similar. Use induction from the leaf nodes and argue that every last 2 levels of any graph will have 0 at the OR level. The rest of the network will always lead to 0 at the root irrespective of how large k is. The strategy of player II remains the same – follow player I's move and set 0 to the sibling of I's chosen node.

[Aside: prove that if the depth was 2k+1, then player I would have a winning strategy. What is it?]

Q5. i) $\forall\, x, y, \quad x^T A y \geq \min_y x^T A y$ (by defn. of minima)

take max wrt x on both sides

$\forall y$: $\max_x x^T A y \geq \max_x \min_y x^T A y$ (RHS now is a constant, while LHS is a function of y)

Since this is true for all y, take minima wrt y

$$\min_y \max_x x^T A y \geq \max_x \min_y x^T A y.$$

ii) let $x^{*T} A y^* = v^*$

and by definition of MSNE

$$x^{*T} A y^* \geq x^T A y^* \quad \forall x \rightarrow \text{player 1} \quad \text{— (1)}$$

and $$x^{*T} A y^* \leq x^{*T} A y \quad \forall y \rightarrow \text{player 2} \quad \text{— (2)}$$

from (1), since this holds for all x, can take max on the RHS

$$\Rightarrow v^* \geq \max_x x^T A y^* \geq \min_y \max_x x^T A y \quad \text{— (3)}$$

this is a fixed choice

ineq. holds by defn. of minima.

from (2), using similar arguments,

$$\Rightarrow \max_x \min_y x^T A y \geq v^* \quad \text{- - - (4)}$$

From (i) we know min max $\geq$ max min, hence

$$v^* \geq \min_y \max_x x^T A y \geq \max_x \min_y x^T A y \geq v^*$$

same

They better all be equal.

Hence $x^{*T} A y^* = \min_y \max_x x^T A y = \max_x \min_y x^T A y$.