Assignment 1
P.T

1) $1000n = O(n^2+5n)$ but $n^2+5n \neq O(1000n)$

$f(n) = O(g(n))$ iff $\exists$ some +ve constant c such that $f(n) \leq c * g(n)$

$\forall\ n \geq n_0$

← another +ve constant (changes with c)

A) if $f(n) = 1000n$

then there exists c such that $f(n) \leq c(n^2+5n)\ \forall\ n \geq n_0$

eg. $c = 2$

$n_0 = 495$

ie; $1000n \leq 2 * (n^2+5n)\ \forall\ n \geq 495$

$\therefore$ ~~for~~ ~~1000n~~ $1000n = O(n^2+5n)$ is true.

B) if $f(n) = n^2+5n$

& $g(n) = 1000n$,

$n^2+5n \quad , \quad c(1000n)$

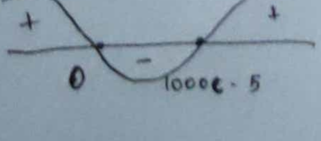
for any +ve constant c, there exists n_0 such that n^2+5n is greater than $c(1000n)$

$n^2+5n = c(1000n)$

$n\,(n+5 - 1000c) = 0$

$\therefore\ n = 0$ (or)

$n = 1000c - 5$



$\therefore\ f(n) \neq O(g(n))$

$\Rightarrow n^2+5n \neq O(1000n)$

Hence proved

2)
P.T $n|\sin n| = O(n)$

A) $-1 \leq \sin n \leq 1$

$0 \leq |\sin n| \leq 1$

$0 \leq n|\sin n| \leq n$ (n is a +ve number)

$n|\sin n| \leq n \quad \forall n \in R^+$

$\therefore n|\sin n| = O(n)$ ($c = 1$, $n_0 = 1$ in defn of $O()$)

PT $n \neq O(n|\sin n|)$

B) if there exists c, n_0 such that

$n \leq c * n|\sin n| \quad \forall n \geq n_0$ then

$n = O(n|\sin n|)$.

for above to be true,

$cn|\sin n| - n \geq 0 \quad \forall n \geq n_0$

$n(c|\sin n| - 1) \geq 0 \quad \forall n \geq 0$

case 1: $0 < c < 1$

$0 \leq |\sin n| \leq 1$

$0 < c|\sin n| < 1$

$-1 < c|\sin n| - 1 < 0$

we know $n > 0$

$\therefore$

$-n < n(c|\sin n| - 1) < 0$

$\Rightarrow$ the LHS is -ve $\therefore$ the statement is false

Case 2: $c_1 \geq 1$

$0 \leq \cancel{t} |\sin n| \leq 1$

$0 \leq c|\sin n| \leq c$

$-n \leq n(c|\sin n| - 1) \leq n(c-1)$

Assuming there exists a n_0, ~~...~~ then there is an $n \geq n_0$ such that

$n(c|\sin n| - 1) = -n$ which is $< n$

$\Rightarrow$ the ~~statement~~ proposition, is false

~~...~~ Proposition is false in case 1 & 2

$\Rightarrow \quad n \neq O(n|\sin n|)$