C H A P T E R **8**

Asset Valuation: Bonds

Chapter Objectives

By the end of this chapter you should be able to:

1. Discuss the theory of valuation
2. Show how bonds are valued
3. Compute yields to maturity on bonds

In this chapter we will investigate how to value financial assets. We will learn that the theory behind asset valuation is the same for all financial assets. Although the theory is not difficult, the calculation of accurate market prices can be challenging. There is room for a great deal of error in the process of valuing many assets, which is why some security prices can be very volatile. An understanding of asset valuation is important for many financial decisions.

In this chapter we draw together many of the skills learned earlier in this text. In Chapter 7 we learned about risk and return. In Chapter 6 we studied the time value of money. In Chapters 2 through 5 we learned how the markets function and who the players are. In this chapter, we put all this information to work to show how financial asset values are established.

This chapter appears in the Investments section of this text. It could as easily have appeared in the Markets section, because that is where asset values are established. It also could have appeared in the section of the book dealing with corporate assets, because virtually all corporate decisions are made with an eye toward maximizing the firm's security prices. Assets are not valued in isolation. It is the interaction of market traders with each other that establishes prices. Asset valuation appears here because the pricing of investments is fundamental to the investment decision.

USE PRESENT VALUE TO PRICE *ANY* BUSINESS ASSET

The value of all financial assets are found the same way. ***The current value is the present value of all future cash flows.*** Recall the meaning of present value from Chapter 6. If you have the present value of a future cash flow, you can exactly reproduce that future cash flow by investing the present value amount at the discount rate. For example, the present value of $100 that will be received in 1 year is $90.91 if the discount rate is 10%. An investor is completely indifferent between having the $90.91 today and having the $100 in 1 year because the $90.91 can be invested at 10% to provide $100.00 in the future ($90.91 X 1.10 = $100). This represents the essence of value. The current price must be such that the seller is indifferent between continuing to receive the cash flow stream provided by the asset and receiving the offer price.

Valuing Investments

Suppose you are interested in investing in a Yeats painting called *Tinker's Encampment: The Blood of Abel.* If you think you can hold the painting for 2 years and then resell it for $1,200,000, how much would you pay for the painting today? The only cash flow from this investment is the final sales price. No periodic payments will be received. To compute the current price of the painting, you will first need to determine an appropriate discount rate to use for computing the present value of the cash flows. Investments in art are high risk and therefore require a high discount rate. Let us assume you require a 16% return to compensate you for the risk of the investment.

To find the current price of the painting, discount the future cash flow back to the present at 16%. This provides a present value of $891,795.48 [$\$1,200,000/(1.16^2)$]. In fact, this painting actually sold recently for $883,123. The buyer must have expected a slightly lower future price or required a slightly higher return than assumed by our example.

A commercial building is valued using the same technique. An appraisal of commercial property is composed of several methods of valuation. Paramount among these is the income approach, which involves computing the present value of the building's cash flows and using this to estimate the property's value. Other methods used by appraisers include the cost of construction and the sales prices of similar properties. The property appraiser reconciles the *income* approach with the other valuation methods to arrive at a final price. Suppose that you wanted to find the value of a four-unit student apartment building. Assume that the revenues from rents total $2,000 per month and total expenses average $1,000 per month to pay for insurance, utilities, and damage (such as holes in walls and collapsed balconies). To find the value of the building we also have to estimate the useful life of the building. If we assume that it can be rented for 30 years before needing major renovation, then the current value is the present value of the net cash flows ($2,000 - $1,000 = $l,000/month) for 30 years. If a discount rate of 12% is appropriate, the value of the building is $\$1,000 \times PVIFA_{360,1\%} = \$97,218.33$ (note that the interest rate is 1% because of monthly compounding).

Why did the Yeats painting not sell for the present value of its expected cash flows and why might the apartment building sell for an amount other than $97,218.33? It is because competition for valuable investments in the market causes prices to adjust so that they represent the best estimate of value by market participants. Not everyone agrees about what the future cash flows are going to be. Assets will sell to the investor who either expects the largest cash flows or sees the least risk in the investment. We will return to this concept later in the chapter.

Let us summarize how to find the value of any business asset.

1. Identify the cash flows that result from owning the asset.
2. Determine what discount rate is required to compensate the investor for holding the asset.
3. Find the present value of the cash flows estimated in step 1 using the discount rate determined in step 2.

The rest of this chapter focuses on how one important asset is valued: bonds. In the next chapter we will study stock valuation. Later in the text we look at how to value businesses.

FINDING THE PRICE OF BONDS

Chapter 2 introduced bonds. Recall that a bond usually pays interest semiannually in an amount equal to the coupon interest rate times the face amount (or par value) of the bond. When the bond matures, the holder also receives a lump sum payment equal to the face amount. Most corporate bonds have a face amount of $1,000. Basic bond terminology is reviewed in Table 8.1.

The issuing corporation usually sets the coupon rate close to the rate available on similar outstanding bonds at the time the bond is offered for sale. Unless the bond has

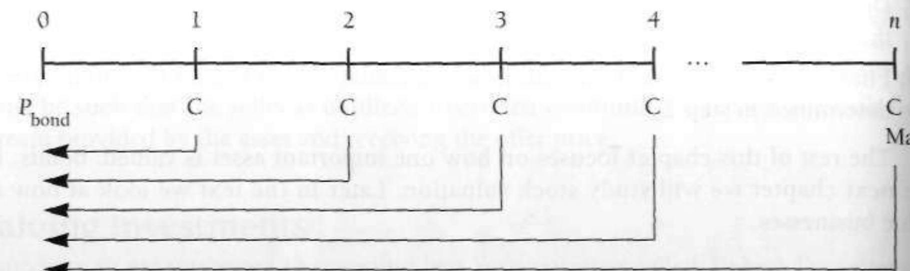
TABLE 8.1 Bond Terminology

Coupon interest rate	The stated annual interest rate on the bond. It is usually fixed for the life of the bond.
Current yield	The coupon interest payment divided by the current market price of the bond.
Face amount	The maturity value of the bond. The holder of the bond will receive the face amount from the issuer when the bond matures. Face amount is synonymous with par value.
Indenture	The contract that accompanies a bond and specifies the terms of the loan agreement. It includes management restrictions, called covenants.
Market rate	The interest rate currently in effect in the market for securities of like risk and maturity. The market rate is used to value bonds.
Maturity	The number of years or periods until the bond matures and the holder is paid the face amount.
Par value	The same as face amount.
Yield to maturity	The yield an investor will earn if the bond is purchased at the current market price and held until maturity.

an adjustable rate, the coupon interest payment remains unchanged throughout the life of the bond.

The first step in finding the value of the bond is to identify the cash flows the holder of the bond will receive. The value of the bond is the present value of these cash flows. The cash flows consist of the interest payments and the final lump sum repayment.

In the second step these cash flows are discounted back to the present using an interest rate that represents the yield available on other bonds of like risk and maturity. We can now draw a time line to show the cash flows:



The equation for the value of a bond is provided by Equation 8.1:

$$P_{bond} = \frac{C_1}{(1 + k_d)^1} + \frac{C_2}{(1 + k_d)^2} + \cdots + \frac{C_n}{(1 + k_d)^n} + \frac{Mat}{(1 + k_d)^n} \tag{8.1}$$

In summation form, this is

$$P_{bond} = \sum_{n=1}^{N} \frac{C_n}{(1 + k_d)^n} + \frac{Mat}{(1 + k_d)^n}$$

where

P_{bond} = current market value of the bond
C = coupon interest payment, equal to the coupon rate times the par value of the bond
Mat = maturity value of the bond (usually $1,000 for corporate bonds)
n = number of periods until the bond matures
k_d = the market interest rate (i.e., the interest rate that the market has set for bonds of similar risk and maturity)

Equation 8.1 becomes tedious to use if there are many cash flows. Because all of the coupon payments are the same, the equation for an annuity can be used. Using present value interest factor notation, Equation 8.1 is rewritten as Equation 8.2:

$$P_{bond} = C(PVIFA_{kdn}) + Mat(PVIF_{kdn}) \tag{8.2}$$

A financial calculator can also be used to easily find bond values. Many financial calculators have bond value functions that allow input of the exact maturity date along with the current date to give very precise answers. We will demonstrate how to use the more typical calculators to solve bond valuation problems in the next example.

EXAMPLE 8.1 Bond Valuation: Annual Interest Payments

Suppose you found a bond in your great, great aunt's attic. It has 2 years before it matures (it has been in that shoebox for 28 years), a 10% coupon rate, and a $1,000 par value. If interest is paid annually and bonds with similar risk currently have an interest rate of 9%, what is the current value of this bond?

Solution: Numerical Equation

The coupon interest payment is $100 (0.10 × $1,000 = $100). Putting the numbers from the example into Equation 8.1 yields

$$P_{\text{bond}} = \frac{\$100}{(1 + 0.09)^1} + \frac{\$100}{(1 + 0.09)^2} + \frac{\$1{,}000}{(1 + 0.09)^2}$$

$$P_{\text{bond}} = \$91.74 + \$84.17 + \$841.68 = \$1{,}017.59$$

Solution: Financial Factors

Using Equation 8.2 results in the following:

$$P_{\text{bond}} = \$100(\text{PVIFA}_{2,9\%}) + \$1{,}000(\text{PVIF}_{2,9\%})$$

$$P_{\text{bond}} = \$100(1.7591) + \$1{,}000(0.8417)$$

$$P_{\text{bond}} = \$175.91 + \$841.70 = \$1{,}017.61$$

Solution: Financial Calculator

There are five keys on the financial calculator that deal with the time value of money. Bond valuation problems use all of them:

2	9		100	1000
N	I	PV	PMT	FV
		= −1017.59		

If you do not get the right answer, make sure that your calculator is set to one payment per period. Note that the answer is *negative* $1,017.59. The negative number indicates a cash outflow. If you invest $1,017.59 (an outflow), you will receive two payments of $100 and one payment of $1,000 (inflows). It is important to keep track of the direction of your cash flows and to be consistent when you input the signs.

Your great, great aunt did not exactly make you rich. Maybe she should have bought General Electric stock instead.

Self-Test Review Question*

Suppose your great, great aunt had deposited the $1,000 in an account earning 10%. How much would be in this account and why is the value so different from the value of a bond?

*The future value of a $1,000 deposit earning 10% for 28 years is $14,420.99 [$1,000(1.10^{28}) = $14,420.99]. A bond pays interest, but the interest does not get added back to the balance to earn additional interest. In other words, bond interest is not compounded.

Study Tip

By far the most common mistake made by students attempting to value bonds is to use the coupon interest rate in the denominator instead of the market interest rate. The coupon rate is used only to compute the coupon payment.

Pricing of Bonds with Semiannual Compounding

Let us look at a more realistic example. Most bonds pay interest semiannually. To adjust the cash flows for semiannual payments, divide the coupon payment by 2 because only half of the annual payment is paid each 6 months. Similarly, to find the interest effective during one-half of the year, the market interest rate must be divided by 2. The final adjustment is to double the number of periods because there will be two periods per year. With semiannual compounding, Equation 8.2 becomes Equation 8.3:[1]

$$P_{\text{semiannual bond}} = \sum_{n=1}^{2n} \frac{C/2}{\left(1 + \frac{k_d}{2}\right)^n} + \frac{\text{Mat}}{\left(1 + \frac{k_d}{2}\right)^{2n}} \tag{8.3}$$

EXAMPLE 8.2 Bond Valuation: Semiannual Payment

Let us compute the price of Safeway bonds recently listed in the Wall Street Journal. The bonds have a 10.95% coupon rate and a $1,000 par value (maturity value), and mature in 2021 ($n = 20$). Assume semiannual compounding and that the market rate of interest is 12%.

Solution

Begin by identifying the cash flows. Compute the coupon interest payment by multiplying 0.1095 times $1,000 to get $109.50. Because the coupon payment is made each 6 months, it will be half of $109.50, or $54.75. The final cash flow consists of repayment of the $1,000 face amount of the bond. This does not change because of semiannual payments.

Next, we need to know what market rate of interest is appropriate to use for computing the present value of the bond. We are told that bonds being issued today with similar risk have coupon rates of 12%. Divide this amount by 2 to get the interest rate over 6 months. This gives an interest rate of 6%.

Finally, find the present value of the cash flows. Note that with semiannual compounding the number of periods must be doubled. This means that we discount the bond payments for 40 periods.

Solution: Financial Factors

It is not reasonable to compute the present value of 40 separate cash flows using the numeric equation method. Below we find the value of the bond using financial factors, which take advantage of the fact that the constant interest payments are an annuity:

$$P_{\text{bond}} = \$54.75(\text{PVIFA}_{40,6\%}) + \$1{,}000(\text{PVIF}_{40,6\%})$$
$$P_{\text{bond}} = \$54.75(15.0463) + \$1{,}000(0.0972)$$
$$P_{\text{bond}} = \$823.78 + \$97.20 = \$920.98$$

[1] There is a theoretical argument for discounting the final cash flow using the full-year interest rate with the original number of periods. Derivative securities are sold in which the principal and interest cash flows are separated and sold to different investors. The fact that one investor is receiving semiannual interest payments should not affect the value of the principal-only cash flow. However, virtually every text, calculator, and spreadsheet computes bond values by discounting the final cash flow using the same interest rate and number of periods as are used to compute the present value of the interest payments. To be consistent, we will use that method in this text.

Solution: Financial Calculator

40	6		54.75	1000
N	I	PV	PMT	FV
		= −921.01		

Note the small difference in value between the financial factors and the financial calculator approaches. The calculator is the most accurate. The market price of this bond should be $921.01. The value of the bond computed with annual compounding is $921.57.[2]

Notice that the market price is below the $1,000 par value of the bond. When the bond sells for less than the par value, it is selling at a **discount.** When the market price exceeds the par value, as in Example 8.1, the bond is selling at a **premium.** Notice that in the previous paragraph the price of the semiannual bond is less than the price of the annual bond. This is always true for *discount* bonds. The price of semiannual premium bonds is always above that of similar annual bonds.

EXTENSION 8.1

Computing Semiannual Bond Prices Using the Effective Annual Rate

In the last section we computed the price of semiannual bonds using a discount rate found by simply dividing the annual rate by 2. While this initially seems intuitive and easy to do, it results in bond prices that may not make sense. Consider the following example.

Suppose that you have an annual 10% coupon bond with 5 years to maturity and a $1,000 par value. What is the current market price if market rates are currently 12%? Solving this with a calculator where N = 5, I = 12%, PMT = $100, and FV = $1,000, we get a price of $927.90.

Now let us find the price of the bond if the interest payments are made semiannually. If we solve this as shown in the last section, we divide the interest payment and market interest rate by 2 and double the number of periods. Again, solving using the calculator method with N = 10, I = 6%, PMT = $50, and FV = $1,000, we get a price of $926.40.

When we compare the price of the bond with annual payments with the one with semiannual payments, we find that by using this method the price of the semiannual bond is less than that of the annual bond. Clearly, investors would rather have the semiannual bond and should therefore be willing to pay more for it. What went wrong?

[2]The method for computing semiannual bond prices shown here is a simplification that gives only an approximate answer. See Extension 8.1 for a more complex but accurate approach.

The problem is that by discounting the cash flows of the semiannual bond at an interest rate computed by simply dividing the annual rate in half, we have ignored the effect of compounding discussed in Chapter 6. To correctly adjust for semiannual compounding, we must compute the periodic interest rate by using the effective rate of return. We do this by manipulating Equation 6.3,

$$\text{Effective Rate} = \left(1 + \frac{i}{m}\right)^{m} - 1 \tag{6.3}$$

We know that the effective rate is 12%, since the effective rate for the annual and semiannual bonds must be the same. Let us call the periodic rate i' and rearrange Equation 6.3 to solve for i':

$$\textit{Effective Rate} = (1 + i')^{m} - 1 \tag{8.4}$$

$$i' = (1 + \textit{Effective Rate})^{1/m} - 1$$

Using Equation 8.4 solve for the periodic rate,

$$i' = (1 + 12)^{1/2} - 1$$

$$i' = .0583 \text{ or } 5.83\%$$

Now if we use i' as the periodic rate and compute the price of the semiannual bond as above (N = 10, I = 5.83, PMT = \$50, and FV = \$1,000), we find the semiannual bond worth \$938.41. This is above the value of the bond that pays interest annually (\$938.41 versus \$927.90).

EXAMPLE 8.3 Bond Valuation of Semiannual Bonds Using Effective Annual Rate Calculation

Compute the price of a semiannual bond with a 10% coupon rate, 10 years to maturity, and a \$1,000 par, assuming a 7.5% market rate. Use the effective rate method.

Solution

Begin by computing the period rate using Equation 8.4:

$$i' = (1 + \textit{Effective Rate})^{1/m} - 1$$

$$i' = (1 + .075)^{1/2} - 1$$

$$i' = .0368 = 3.68\%$$

Now solving using the calculator approach, with N = 20, I = 3.68, PMT = \$50, and FV = \$1,000, the value is \$1,184.58. The price of the same bond paying interest annually would be \$1,171.60.

Study Tip

The relationship between interest rates and value can be used to check your answers to bond value problems. If the market interest rate is above the coupon rate, your bond value should be less than \$1,000. If the market interest rate is below the coupon rate, the bond value should be more than \$1,000.

Interest Rate Risk

What determines whether a bond will sell for a premium or a discount? Suppose that you are asked to invest in an old bond that has a coupon rate of 10% and \$1,000 par. You would not be willing to pay $ 1,000 for this bond if new bonds with similar risk were available yielding 12%. The seller of the old bond would have to lower the price on the 10% bond to make it an attractive investment. In fact, the seller would have to lower the price until the yield earned by a buyer of the old bond exactly equaled the yield on similar new bonds. This means that as interest rates in the market rise, the value of bonds with fixed interest rates falls. Similarly, as interest rates available in the market on new bonds fall, the value of old bonds with fixed interest rates rises.

Many investors think that bonds are a very low risk investment because the cash flows are fairly certain. It is true that high-grade bonds seldom default, but bond investors face price fluctuations due to market interest rate movements in the economy. In Chapter 7 we defined risk as fluctuations in the cash flows. As interest rates rise and fall, the value of bonds changes in the opposite direction. This does not cause a loss to investors who do not sell their bonds, but many investors do not hold their bonds until maturity. If they attempt to sell their bonds after interest rates have risen, they will receive less than they paid. The possibility of suffering a loss because of interest rate changes is called **interest rate risk.**

Review Table 8.2. This table shows the market price of 10% coupon bonds with 1, 10, and 20 years to maturity at three different market interest rates. Notice that when the market rate is the same as the coupon rate the bond is valued at par. In other words, the value of the bond does not change unless the market interest rate changes, regardless of how many years there are to maturity. The other important feature to note in Table 8.2 is that the change in the value of the bonds is much greater for bonds with longer maturities. A 1% drop in interest rates results in a $9.17 increase in price if the bond has 1 year to maturity. That same 1% drop in interest rates results in a $91.28 increase in price if there are 20 years to maturity. We can conclude from this that ***interest rate risk increases with increasing maturity.***

Recall the discussion of the yield curve from Chapter 4. We found that bonds with longer maturities had higher interest rates and discussed several theories to explain the shape of the yield curve. Interest rate risk is one reason for the upward slope. Longer-term bonds are more risky because interest rate risk is greater the longer the term to maturity. Investors demand higher returns to compensate for the increased risk (see Box 8.1).

TABLE 8.2 Price of $1,000 Par, 10% Coupon Bond with Different Maturities and Market Interest Rates

	Market Rate		
Term	**9%**	**10%**	**11%**
1	$1,009.17	$1,000.00	$990.99
10	1,064.18	1,000.00	941.11
20	1,091.28	1,000.00	920.37

Box 8.1 Investors Can Select Desired Interest Rate Risk

Many investors in bonds want and expect low risk. When interest rates rise and fall the values of their bonds change, possibly for the worse. Some investment companies attempt both to educate investors about the perils of interest rate risk and to offer investment alternatives that match their investors' risk preferences.

Vanguard Group, for example, offers eight separate high-grade bond mutual funds. In its prospectus, Vanguard separates the funds by the average maturity of the bonds they hold. Three funds invest in bonds with average maturities of 1 to 3 years, which Vanguard rates as having low interest rate risk. Three different funds hold bonds with average maturities of 5 to 10 years, which Vanguard rates as having medium interest rate risk. Two funds hold long-term bonds with maturities of 15 to 30 years, which Vanguard rates as having high interest rate risk.

The Vanguard prospectus demonstrates the effect on interest rates by computing the percentage change in bond value resulting from a 1% increase and decrease in interest rates.

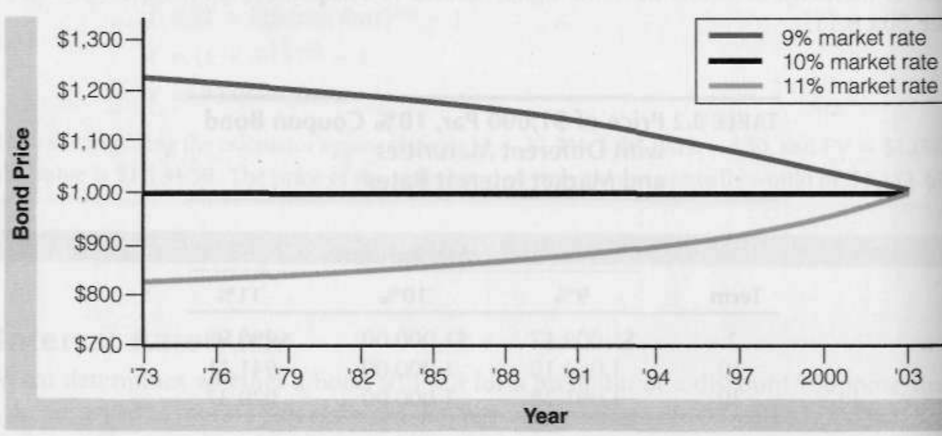
Bond Price Changes over Time

In Table 8.2 we see that as the time to maturity falls, the closer the values of both the 9% and 11% bonds are to par ($1,000). In other words, as the maturity date approaches, the value of the bonds approaches the maturity value of the bond, regardless of whether it initially sold at a premium or a discount. To see why this occurs, consider what you would pay for a bond the day before it matures. You would pay almost $1,000, regardless of the coupon rate or the market rate, because you would know that the very next day you would be paid the face amount of $1,000.

The relationship between the time to maturity and the price of a discount bond (one with a coupon rate below market rates), a premium bond (one with a coupon rate above market rates), and a par value bond is graphed in Figure 8.1. All mature in the year 2003.

When you buy a bond at a discount, you will receive the benefit of price appreciation as the maturity date approaches. This is in addition to the interest payment you will

FIGURE 8.1
Price for Various Years of a 10% Coupon Bond at 9% and 11% Market Rates



receive from the issuing firm. If you buy a premium bond, you will suffer a price decline as the maturity date approaches.

The 10% coupon bond shown in Figure 8.1 initially sold for $1,225.16 when market interest rates were 8%. The market price falls the closer to maturity in 2003. The 10% coupon bond initially sold for $838.90 when market interest rates were 12%. Its price rises the closer to maturity in 2003. The 10% coupon bond has a constant value of $1,000 when market rates are also 10%. The value of the bonds in all three situations is $1,000 at maturity.

Computing the Yield to Maturity

Does the coupon rate offered on a bond accurately reflect the yield an investor will receive from a bond? No, the coupon rate is only part of the return an investor earns. Remember that the issuer of a bond pays the holder the face amount of the bond when it matures. In Example 8.2, an investor pays $921.01 (calculator method) for a bond with a coupon rate of 10.95%. The investor receives semiannual interest payments equal to 10.95% of $1,000 *plus a capital gain from an increase in the price of the bond from $921.01 at purchase to $1,000 at maturity.* The total return consists of the interest income plus the capital gain:

Total return = Interest payment + Change in price of the bond

The total return an investor earns on a bond from both interest payments and a change in the price of the bond is called the **yield to maturity** (YTM). The YTM is computed by solving for the discount rate that sets the present value of the interest and principal payments equal to the current market price. In Example 8.2 the YTM is 12%. Equation 8.5 shows the role of YTM in the bond valuation equation:

$$P_0 = \frac{C}{(1 + YTM)^1} + \frac{C}{(1 + YTM)^2} + \cdots + \frac{C}{(1 + YTM)^n} + \frac{Mat}{(1 + YTM)^n} \quad (8.5)$$

If interest rates in the market are above the coupon rate, then the price of the bond must be below par so that investors will earn an extra return from the increase in price to make up for a low coupon rate. If interest rates in the market are below the coupon rate, then the price of the bond must be above par so that investors will incur a loss on the principal. In either case the price of the bond adjusts so that the YTM on the bond is exactly equal to the return available on similar bonds selling in the market. Box 8.2 discusses an unusual situation in which the YTM was negative.

Calculation of the YTM is most easily done on a financial calculator. Simply input the current market price as the PV, the face amount as the FV, the number of periods to maturity as N, and the coupon payments as PMT, and compute I.[3]

Study Tip

It is easy to confuse the market rate of interest and the YTM. In fact, they are the same thing. If you are given the current market price of the security, you can solve for the YTM. If you are given the current market interest rate, you can solve for the market price of the bond.

[3] On most calculators you must accurately input the signs of the cash flows. If you input all cash flows with a positive sign, you will get an error message. Input the current market price as a negative value and the interest payments and par value as positive values.

Box 8.2 Are Negative T-Bill Rates Possible?

We normally assume that interest rates must be positive. Negative rates mean that in the future investors get back less when their securities mature than was originally invested. Negative rates would seem impossible then—why would anyone invest when they could simply hold cash?

Despite this argument, in November 1998, interest rates on Japanese 6-month Treasury bills became negative, yielding an interest rate of-0.004%. This was an extremely unusual event; no other country has seen negative interest rates during the past 50 years.

Why did the Treasury rates go negative? We cannot look to the weakness of the Japanese economy nor to their negative inflation rate. These factors can explain low rates but not negative ones. The actual reason is because the convenience of holding large sums of money in bonds rather than in cash made T-bills more desirable than cash. Obviously, this convenience factor cannot justify taking the rates much below zero, but it did give rise to a seemingly impossible situation.

EXAMPLE 8.4 Yield to Maturity

What is the YTM of a $1,000 par bond with a 10% coupon, 2 years to maturity, and a current market price of $966.20?

Solution

The solution is found by solving the following equation for YTM:

$$\$966.20 = \frac{\$100}{(1 + \text{YTM})^1} + \frac{\$100}{(1 + \text{YTM})^2} + \frac{\$1{,}000}{(1 + \text{YTM})^2}$$

There is no easy algebraic method to solve for YTM in this equation. There are three non-algebraic approaches.

Financial calculator: In this example PV = –$966.20, PMT = $100, FV = $1,000, and N = 2. I computes as 12%.

Plug and chug: Pick an interest rate and plug it in for YTM in Equation 8.5. Then calculate whether the right-hand side equals the current market price. If it does not, pick another interest rate and try again. You can use what you know about the price of bonds to help pick interest rates. If the price is below par, then we know that the yield to maturity is above the coupon rate. You might begin by trying 11%. If you plug 11% into the above equation and solve for the price, you get $982.87. Because this is still above the actual market price, try 12%.

Approximation equation: An estimation equation has been developed that gives reasonable approximations of the yield to maturity. Equation 8.6 can be used when calculators are not available:[+]

$$\text{YTM} = \frac{C + \dfrac{\text{Par} - \text{Market}}{n}}{\dfrac{\text{Par} + 2\,\text{Market}}{3}} \tag{8.6}$$

[+]Equation 8.6 differs from the YTM estimation equation that is widely reported. Ricardo Rodriguez demonstrates in the *Journal of Financial Education*, Fall 1988, that the above provides consistently better results.

Study Tip

A common mistake made when applying Equation 8.6 is to use the coupon interest *rate* in the numerator rather than the coupon interest *payment* where

C = coupon interest payment, equal to the coupon rate times the par value of the bond
Par = face amount of the bond, usually $1,000
Market = current market price of the bond
n = number of periods until maturity

Equation 8.6 is u... 8.4:

$$\text{YTM} = \frac{\$100 + \dfrac{\$1{,}000 - \$966.20}{2}}{\dfrac{\$1{,}000 + 2(\$966.20)}{3}} = 11.96\%$$

This is a reasonable approximation of the actual YTM of 12%.

Comparing Yield to Maturity to Current Yield

An investor earns the YTM in effect at the time the bond is purchased only if the bond is held until it matures. As interest rates rise and fall over time, the YTM changes. This does not matter to the investor as long as the bond is not sold. Because many investors do not know how long they will hold a bond, another measure of a bond's return is often quoted. The **current yield** is the coupon interest payment divided by the current market price of the bond and is computed using Equation 8.7:

$$\text{Current yield} = \frac{C}{\text{Market}} \tag{8.7}$$

where

C = coupon interest payment, equal to the coupon rate times the par value of the bond
Market = current market price of the bond

The current yield is widely reported. Be aware that it only approximates the actual yield to maturity. It tells you what you will earn in interest payments, but it ignores the earnings from appreciation or depreciation in the price of the bond.

EXAMPLE 8.5 Current Yield

What is the current yield for the Safeway bond described in Example 8.2? The coupon interest rate is 10.95% and we computed the market price to be $921.01.

Solution

The coupon interest payment is found by multiplying the coupon interest rate times the $1,000 par value (0.1095 x $1,000 = $109.50). Substituting the factors into Equation 8.7, you obtain

$$\text{Current yield} = \frac{\$109.50}{\$921.01} = 11.89\%$$

The current yield is well above the coupon rate because investors pay less than $1,000 for the investment.

We can now put current yield together with the YTM and the **capital gain** (the percentage change in the price of the bond):

$$\text{Yield to maturity} = \text{Current yield} + \text{Capital gain} \quad (8.8)$$

In Example 8.5 the yield to maturity is 12%, since the market rate of interest is given as 12%. The current yield is 11.89%. If we solve for the capital gain, we get the following:

$$12\% = 11.89\% + \text{Capital yield}$$
$$\text{Capital gain} = 12\% - 11.89\% = 0.11\%$$

We can verify this figure by computing the price appreciation of the bond. The bond price was originally computed with 20 years to maturity. The bond price with 19 years to maturity is $922.06. The price has increased by $1.05. The percentage increase is $1.05/$921.01 = 0.0011 = 0.11%, as computed above.[3]

Self-Test Review Questions*

1. What is the current market price of a $1,000 par bond with a 6% coupon paid semiannually and 5 years to maturity if current market rates on similar bonds are 8%?
2. What is the current yield on this bond?
3. If the price goes to $950, what is the yield to maturity?

INVESTING IN BONDS

Bonds are one of the most popular long-term alternatives to investing in stocks. Bonds are lower risk than stocks because they have a higher priority of payment. This means that when the firm is having difficulty meeting its obligations, bondholders get paid before stockholders. Additionally, should the firm have to liquidate, bondholders must be paid before stockholders.

Even healthy firms with sufficient cash flow to pay both bondholders and stockholders often have very volatile stock prices. This volatility scares many investors out of

*1. $P_b = \$30(PVIFA_{4\%,10\,yr}) + \$1{,}000(PVIF_{4\%,10\,yr}) = \$243.33 + \$675.60 = \$918.93.$
2. $CY = \$60/\$918.93 = 0.06529 = 6.53\%.$
3. Use the estimation equation:

$$YTM = \frac{\$30 + \dfrac{\$1{,}000 - \$950}{10}}{\dfrac{\$1{,}000 + 2(\$950)}{3}} = 0.0362 = 3.62\%$$

Because the bond is semiannual, the interest rate computed by the estimation equation must be multiplied by 2 to get the annual interest rate: $3.62 \times 2 = 7.24\%$

[5]The bond price is found by computing the present value with $n = 38$, PMT = $54.75, $i = 6\%$, and FV = $1,000 because the bond has semiannual payments.

Careers in Finance — Credit Manager

Many firms sell their products on credit, giving their customers varying amounts of time to pay, The credit manager evaluates the customers and decides whether to extend them credit. Once credit has been given, the credit manager is responsible for making sure that payments are made as agreed and initiating collection efforts when problems arise. In some firms the amount of credit extended can be very large and evaluating customer creditworthiness is very similar to being a bank loan officer. The credit manager must balance the firm's desire to sell its products against the risk that the customer could fail to pay. Credit managers should earn between $30,000 and $65,000 depending on the firm's size.

the stock market. Bonds are the most popular alternative. They offer security and dependable cash payments, making them ideal for retired investors and those who want to live off their investment.

One important lesson in this chapter is that even though bonds may be less risky than stock, they are still subject to interest rate risk. Recall that this means that changing interest rates can result in changing bond prices. If an investor were to buy long-term bonds when interest rates were low, a loss may be suffered if rates rise. For this reason prudent investors match the maturity of their bond portfolios with their return and risk requirements. Box 8.3 discusses the importance of the yield curve when selecting bonds.

Box 8.3 Watch That Yield Curve

In Chapter 4 we introduced the yield curve and noted that it is usually upward sloping, meaning that long-term bonds provide higher returns than short-term bonds. We also learned that the yield curve changes its shape based on a number of factors. Investors in bonds should carefully examine the yield curve before buying bonds.

At times there is a large premium for investing in long-term bonds. At other times, such as when the yield curve is flat, there are very little additional earnings for taking the risk of buying a long-term security. For example, in December 1997, the yield curve was flat and investors earned only about 0.75% for investing in 30-year bonds rather than 1-year bonds. Yet with interest rates very low, those long-term bonds are subject to large losses in principal value if interest rates in the market were to increase. By contrast, in 1992, 30-year bonds paid 3.78% more than 1-year bonds, yet with the higher interest rates then offered, the chance of a principal loss was lower.

Bond investors should consider the level of interest rates in the market, the shape of the yield curve, and the likelihood that rates will rise in the future when deciding which bonds to purchase.

By December 2000, the yield curve was inverted. Short-term interest rates were high due to rate hikes by the Federal Reserve, but long-term rates were low due to expectations that rates would be dropped in the future to spur the economy. Investors at this time were actually penalized, rather than rewarded, for buying long-term bonds with their accompanying high interest rate risk.

CHAPTER SUMMARY

The values of all business assets are computed the same way: by computing the present value of the cash flows that will go to the holder of the asset. For example, a commercial building is valued by computing the present value of the net cash flows the owner will receive. We compute the value of bonds by finding the present value of the cash flows, which consist of periodic interest payments and a final principal payment.

The value of bonds fluctuates with current market prices. If a bond has an interest payment based on a 5% coupon rate, no investor will buy it if new bonds are available for the same price with interest payments based on 8% coupon interest. To sell the bond the holder will have to discount the price until the yield to the holder equals 8%. The amount of the discount is greater the longer the term to maturity.

The return a bond holder will earn if the bond is held until it matures is its yield to maturity. Because investors often do not plan to hold a bond this long, many prefer to look at the current yield, which ignores the final principal payment. The yield to maturity is the sum of the current yield and the percentage change in the price of the bond.

Bond values can be computed accurately because all of the inputs are known with a high degree of confidence. This is not the case for stock valuation. We explore this topic in Chapter 9.

KEY WORDS

capital gain 216	discount 209	premium 209
current yield 215	interest rate risk 211	yield to maturity 213

DISCUSSION QUESTIONS

1. How is any investment asset valued?
2. Explain why an investor is willing to pay a price equal to the present value of the cash flows for an investment.
3. What cash flows will an investor receive from the purchase of a bond?
4. If you buy a bond, are you borrowing money from a corporation or lending money to the corporation? Explain your answer.
5. Under what conditions will an investor actually receive a return on a bond equal to the yield to maturity?
6. What is interest rate risk?
7. Is interest rate risk more or less for long-term bonds than for short-term bonds?
8. What two points are made by Table 8.2?
9. As a bond approaches maturity, what happens to its market price?
10. What portion of the yield to maturity is measured by the current yield? What other return does the investor earn?

PROBLEMS

1. A firm is considering buying another company. The acquisition will increase revenues by 510,000 per year for 5 years. If the appropriate discount rate is 15%, what is the value of this acquisition? (Hint: Find the present value of the cash flows.)
2. What is the value of a building that will generate net revenues (after all costs and maintenance) of $15,000 per year for 20 years if the appropriate discount rate is 10%?
3. A bond pays $80 per year in interest (8% coupon). The bond has 5 years before it matures, at which time it will pay $1,000. Assuming a discount rate of 10% and annual payments, what should be the price of the bond?
4. A bond has a 10% coupon rate of interest (paid annually). If it matures in 4 years and market interest rates for similar bonds are 7%, what is the price of the bond?
5. A zero coupon bond has a par value of $1,000 and matures in 20 years. Investors require a 10% annual return on these bonds. For what price should the bond sell? (Note: Zero coupon bonds pay no interest. The entire return is from appreciation.)
6. Consider the two bonds described here:

	Bond A	*Bond B*
Maturity	15 yr	20 yr
Coupon rate (paid semiannually)	10%	6%
Par value	**$1,000**	**$1,000**

a. If both bonds had a required return of 8%, what would the bonds' prices be?
b. Describe what it means when a bond sells at a discount, at a premium, and at its face amount (par value). Are these two bonds selling at a discount, a premium, or par?
c If the required return on the two bonds rose to 10%, what would the bonds' prices be?

7. A bond sells for $1,525, matures in 10 years, has a par value of $1,000, and has a coupon rate of 10%, paid semiannually. What is the bond's yield to maturity?
8. A 10-year bond pays interest of $35 semiannually, has a par value of $ 1,000, and is selling for $737. What are its coupon rate and its yield to maturity?
9. A bond has 5 years to maturity, a coupon rate of 10%, and a par value of $1,000, and the market rate for similar bonds is 12% (yield to maturity = 12%). Assume annual compounding.
   a. What is the current price of the bond?
   b. What is the current yield?
   c What is the capital gain?
   d. What will be the price of the bond when it has 4 years to maturity?
   e. What is the percentage increase in price during the first year? Is it the same as you found in part c?
10. A bond has 10 years to maturity, a coupon rate of 8%, and a par value of $1,000, and the market rate for similar bonds is 6% (yield to maturity = 6%).
    a. What is the current price of the bond? Assume annual compounding.
    b. What is the current yield?
    c What is the capital gain?
    d. What will be the price of the bond when it has 9 years to maturity?
    e. What is the percentage decrease in price during the first year?
11. A bond pays interest semiannually, has a par value of $1,000, a coupon rate of 7%, and 6 years until it matures. Assuming a market rate of 6%, what is the value of the bond computed using the effective annual rate? (Extension 8.1)
12. A bond pays interest semiannually, has a par value of $1,000, a coupon rate of 5%, and 10 years until it matures. Assuming a market rate of 8%, what is the value of the bond computed using the effective annual rate? (Extension 8.1)

SELF-TEST PROBLEMS

1. A building has an expected life of 30 years. Its net cash flows are projected to be $120,500 per year. If a discount rate of 17% is appropriate, what is the value of the building?
2. If in problem 1 the discount rate were to increase to 23% because of a perception of increased risk in the real estate market, what would the value of the building become?
3. A bond's par value is $1,000. The bond has 5 years until maturity and its coupon rate is 7%. What is the value of the bond if the market rate is 10%, assuming annual compounding?
4. Given the bond in problem 3, what would the current value be if the market rate were 5%?
5. Again use looking at the bond in problem 3, what would the value be if it had 30 years to maturity instead of 5 years, assuming all other features are as given in problem 3?
6. What is the price of a bond with features as given in problem 3 except that it matures in 1 year rather than in 5 years?
7. Review your solutions to problems 5 and 6. Which bond is subject to the greatest interest rate risk?
8. What would be the value of the bond discussed in problem 3 if interest payments made were semiannually rather than annually?
9. What would be the value of the bond discussed in problem 4 if interest payments were made semiannually rather than annually?
10. What will be the price of the bond discussed in problem 3 when there are 4 years before it matures?

11. What is the percentage change in price of the bond discussed in problem 10 between when there are 5 years before it matures and when there are 4 years before it matures?
12. What is the current yield of the bond discussed in problem 3?
13. Is the sum of the current yield and the change in price found in problem 11 the same as the market rate?
14. A bond has a current market price of $1,125. It has an annual coupon rate of 6%, its par value is $1,000, and it matures in 10 years. What is its yield to maturity?
15. A bond has a current market price of $825. It has an annual coupon rate of 6%, its par value is $1,000, and it matures in 10 years. What is its yield to maturity?
16. What would be the yield to maturity for the bond discussed in problem 14 if the payments were made semiannually rather than annually? If you use a calculator, remember to multiply your answer by 2.
17. What would be the yield to maturity for the bond discussed in problem 15 if the payments were made semiannually rather than annually?

*Some of these problems require the use of a financial calculator since some of the interest rates are not in the tables.

WEB EXPLORATION

1. Investment companies attempt to explain to investors the nature of the risk they incur when buying shares in its mutual funds. For example, Vanguard attempts to carefully explain interest rate risk and to offer alternative funds with different interest rate risk. Go to majestic.vanguard.com/FP/DA.
   a. Select the bond fund you would recommend to an investor who has very low tolerance for risk and a short investment horizon. Justify your answer.
   b. Select the bond fund you would recommend to an investor who has very high tolerance for risk and a long investment horizon. Justify your answer.
2. *Saving bond redemption.* In this chapter we have discussed bonds as if there were only one type: long-term interest-paying corporate bonds. In fact there are also discount bonds. A discount bond is sold at a low price, and the whole return comes in the form of price appreciation. You can easily compute the current price of a discount bond using the financial calculator at app.ny.frb.org/sbr/.

   To compute the redemption values for savings bonds, fill in the information at the site and click on the Compute Values button. A *maximum* of 5 years of data will be displayed for each computation.

MINI CASE

Your favorite uncle learned that you have completed a course in finance. Uncle Bob has been investing in stocks for years through his company pension plan. He has also bought some stock outside the company plan. Bob does not think he is saving enough for his retirement and wants your help in choosing where he should put his extra funds. He wants to get together with you several times, and during the first session he wants to talk about bonds.

a. Explain how any financial asset is valued. Why is this method reasonable?
b. What is a bond? What are the primary features of a bond?
c What cash flows will your uncle receive from investing in a bond?
d. Your Uncle Bob has become very excited about bonds because he sees that if he buys bonds issued by a solid company, there is no risk. Explain the concept of interest rate risk. Discuss when interest rate risk is greatest and lowest.
e. Your uncle tells you that he checked online and has found that he can buy a $1,000 face value bond for only $950. He thinks this sounds like a bargain and wants your opinion. What additional information would you need to accurately advise him about this bond? Why is the bond probably selling below par? What are the two sources of return from a bond?

f. Uncle Bob has a list of bonds he has found for sale. He asks you to compute what the return on each of the following bonds will be:
   i. $1,000 par, 8.5% coupon, 10 years to maturity, current price $907.83
   ii. $1,000 par, 7.75% coupon, 15 years to maturity, current price $828.86
   iii. $1000 par, 12.5% coupon, 5 years to maturity, current price $1,094.77

g. What do the results you found in part f suggest about the relative risk among the three bonds?

h. Your uncle looks at what you computed for part f and wants to know how the current market prices were computed. Demonstrate how bond prices are found by computing the price of the following bonds:
   i. $1,000 par, 8.5% coupon, 10 years to maturity, current YTM 8.5%
   ii. $1,000 par, 8.5% coupon, lOyears to maturity, current YTM 6.8%
   iii. $1,000 par, 8.5% coupon, 10 years to maturity, current YTM 9.25%

i. Explain to your uncle why the current yields to maturity reported in part h could be different.

j. Your uncle's last question is whether you think he should include some bonds in his investment portfolio. How would you respond?