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COVER PAGE

Research Topic:

Optimal Control Model for a Country's Transition to Renewable Energy

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RESEARCH PROPOSAL

Research Aim:

With pollution levels rising and natural reserves of coal, oil, etc. decreasing, organizations such as the United Nations (UN) has deemed the global transition to renewable energy necessary **[1]**, especially due to the argument that renewable energy sources, such as wind and solar, emit little to no greenhouse gases, are readily available and in most cases cheaper than coal, oil or gas. As such, this research aims to analyze various strategies for a non-renewable energy reliant country to transition towards renewable energy using mathematical modelling and optimal control theory. The research formulates a simplified optimal control model and, through the consideration of three strategies such as constant investment, delayed investment and front-loaded investment, it will determine the most effective strategy in a simplified scenario for both profitability and a successful transition to renewable energy.

Research Questions:

1. How can a simplified mathematical model based on optimal control theory represent a country's transition to renewable energy?
2. How do different investment strategies affect the profitability of this transition?
3. In what ways can such a simplified model provide insight for planning in the real world?

Introduction:

The transition to renewable energy, although widely agreed upon as imperative, is a complex issue due to the extensive financial investments and long-term strategic planning required **[2]**. Additionally, a country's energy grid is a dynamic system where changes may have unprecedented socio-economic ramifications **[3]**. To emphasize the urgency, note that "Renewables will become the largest source of global electricity generation by 2026, surpassing coal." (latest forecasts from International Energy Agency, IEA) **[2]**. As such, this research proposes a mathematical model based on optimal control theory to provide a robust framework to find the most efficient strategy for this transition. Although a country's energy grid is highly complex, this research demonstrates not only that the simplified model can provide valuable insight into the fundamental trade-offs involved, but also the utility of mathematical models in solving real-world issues. The central hypothesis tested with this model is that a front-loaded investment strategy with a minimum baseline for investment will prove to be the most successful and effective strategy for a transition to renewable energy. Transparency is achieved by incorporating key pragmatic constraints (such as diminishing returns to capacity and a minimum baseline investment) to the model. Such constraints are often omitted in a purely theoretical model. Additionally, it avoids following overly complex models, such as those involving integro-differential equations, to improve transparency.

Literature Review:

Existing Research: Although mathematical models on energy and climate modelling already exist, such as ones used by institutions like the Intergovernmental Panel on Climate Change (IPCC), they are usually highly complex, and operate on a massive scale **[4]**. It is therefore difficult for policymakers to understand the reasoning behind their conclusions and to adopt it. The lack of transparency has been identified as a significant barrier, sometimes referred to as a "black box" problem, and can result in public and political distrust **[5]**. This research aims to improve transparency in such mathematical models via simplification, thus resulting in pragmatic and effective policy-making **[6]**.

The Role of Optimal Control Theory: Optimal control theory is a branch of control theory that deals with finding a control for a dynamical system over a period of time such that an objective function is optimized **[7]**. It has been successfully applied in the past across various fields, such as chemical engineering **[8]** and robotics **[9]**. According to optimal control theory, the solution requires three elements: a dynamic system, a control variable and an objective function **[7]**. In our case, the dynamic system is the country's energy grid, the control variable is the rate of investment in renewable energy, and the objective function is the total profit from the transition.

Research Gap: While complex, data-intensive models exist, they lack transparency; they are frequently described as "black boxes" where only inputs and outputs can be observed, but not the processes. This opaqueness can lead to distrust in policymakers, who may thus be hesitant to act on conclusions provided by such models **[10]**. Due to the importance of transparency in policy-relevant research, a simplified mathematical model is required to provide a clear and comprehensible way to analyze potential choices and outcomes for a country's energy transition. Existing models often also tend to not include parameters for various constraints or the necessity for a minimum baseline investment for the purpose of reducing computational complexity, and to avoid the necessity of numerical solutions **[11]**. On the other hand, extremely accurate models are often too complicated for policymakers, or anyone not specialized in mathematics, to fully comprehend.

Research Methodologies:

The proposed simplified control model will be designed to simulate the aforementioned energy transition over a defined time horizon, T. It seeks to determine the optimal investment strategy $I_{opt}(t)$, which maximizes net total profit while adhering to physical and economic limitations. It will also consider the total profit on individual year as a performance metric to make the inner processes of the model more transparent. The model itself will consist of a dynamic system, a control variable, and an objective function. The following terms and parameters will be used:

State Variables:

1. $R(t)$: Active renewable capacity in Gigawatts (GW) at time t
2. $A(t)$: Cumulative total renewable capacity at time t

Control Variable: $I(t)$: Investment rate of selected investment strategy in renewable energy in $ billions per year

Key parameters:

1. T: Total time being considered (such as 30 years)
2. K: Maximum theoretical renewable capacity

3. γ: Investment efficiency
4. β: Depreciation rate
5. δ: Minimum renewable capacity, used to model resilience
6. I_{max}, I_{min}: Maximum and minimum investment

The Dynamic System: The dynamic system is defined by the two aforementioned state variables. These variables **[7]** change over time according to the following forces:

1. $\gamma\sqrt{I(t)}$: This term represents productive investment. It is the rate at which new capacity is created, where γ is the efficiency and $\sqrt{I(t)}$ models diminishing returns to investment.
2. $\left(1-\frac{A(t)}{K}\right)$: This term represents market saturation. Here, $A(t)$ represents the total accumulated capacity and K is the theoretical maximum value of renewable capacity.
3. $\beta\left(\frac{1}{R(t)+\delta}\right)R(t)$: This term stands for risk-adjusted depreciation, which is the loss of operational renewable capacity. It is scaled by a factor of $\left(\frac{1}{R(t)+\delta}\right)$, meaning depreciation rate is highest when capacity $R(t)$ is small. This models the high fragility of a new project.

These terms together make up the dynamic system, which relies on the following two coupled ODEs (Ordinary Differential Equations):

$$\frac{dR}{dt} = \gamma\sqrt{I(t)}\left(1-\frac{A(t)}{K}\right) - \beta\left(\frac{1}{R(t)+\delta}\right)R(t)$$

$$\frac{dA}{dt} = \gamma\sqrt{I(t)}\left(1-\frac{A(t)}{K}\right)$$

Here, the capacity growth term, $\gamma\sqrt{I(t)}\left(1-\frac{A(t)}{K}\right)$ is present in both equations, used to show that, while investment creates new operational capacity R, it also permanently contributes to market saturation A. However, only the operational capacity function $R(t)$, is subject to risk-adjusted depreciation $\beta\left(\frac{1}{R(t)+\delta}\right)R(t)$.

The Objective Function and Constraints: The overall success of this transition is judged by comparing the cumulative net profit, J, over the time horizon T for the three strategies. This profit is the total value generated from the renewable capacity minus the total costs of investment. It is represented by the following integral:

$$J = \int_0^T [R(t) - I(t)]dt$$

Additionally, to improve transparency of the model, it will consider the instantaneous profit $P(t) = R(t) - I(t)$, which will be considered at a yearly basis to show financial implications of each strategy.

Investment Strategies: This research will consider three investment strategies, namely, front-loaded, constant, and delayed investment strategies. This will be done by assigning different functions as $I(t)$. The constraint, $I_{min} \leq I(t) \leq I_{max}$ will be applied for all three strategies **[8]**.

Constant: $I(t) = \frac{B}{T}$, where B is the total budget (determined from I_{max} and I_{min}).

Front-Loaded: $I(t) = I_{min} + (I_{max} - I_{min})\left(1 - tanh\left(\frac{t-t_c}{\tau}\right)\right)$

Here, $tanh\left(\frac{t-t_c}{\tau}\right)$ is used to ensure a smooth, continuous curve. The constant t_c determines the time when there is a major decrease in investment, and τ determines the smoothness of that decrease. For this model, $t_c = 15\ years$, and $\tau = 5.0$ will be used.

Delayed: $I(t) = I_{min} + (I_{max} - I_{min})\left(1 + tanh\left(\frac{t-t_c}{\tau}\right)\right)$

Here, the equation is similar, but with $\left(1 + tanh\left(\frac{t-t_c}{\tau}\right)\right)$ instead of $\left(1 - tanh\left(\frac{t-t_c}{\tau}\right)\right)$ to signify sudden increase instead of decrease. Similar to the last strategy, the constant t_c determines the time when there is a major decrease in investment, and τ determines the smoothness of that decrease. For this model, $t_c = 15\ years$, and $\tau = 5.0$ will be used.

To ensure fairness in comparison, the integral $\int_0^T I(t)\,dt$, should be equal to B, for all three strategies. The Budget will be determined from I_{max} and I_{min} , both of which will be fixed for all three strategies.

Data Acquisition: Data for the parameters and constants (including R_0 and A_0) mentioned above will be obtained from reputable research agencies such as World Bank or International Energy Agency (IEA), and it will be ensured that data is from an appropriate context (in this case, for a country not reliant on renewable energy).

Solutions and Research Practicalities:

The coupled ODEs will be solved via numerical methods. Specifically, it will use the Runge-Kutta 4th Order (RK4) method via its generalized, adaptive-step implementations found in the scipy.integrate.odeint function of the Python SciPy library. Using the time trajectories obtained from this, the objective function, J, will be determined via Numerical Quadrature, specifically the Trapezoidal Rule. The calculated scalar value J will be used as the main metric for determining effectiveness of the strategy. To validate robustness of the model and identify critical policy thresholds, a comprehensive sensitivity analysis will be performed on the seven key parameters (both one-way sensitivity and two-way sensitivity), and results will be displayed as tornado plots and heatmaps. Parameter thresholds where sensitivity changes will also be identified.

Potential Limitations of the Proposed Model:

Due to the emphasis on transparency and simplicity, the model makes numerous assumption. For example, it does not account for variable price structures or elasticity of demand. Additionally, the model is strictly deterministic (stochasticity is ignored), meaning variables such as political uncertainty and technological breakthroughs are ignored. It also assumes that the key parameters are constant, while they actually vary in reality **[12]**.

Conclusion:

The research proposes an optimal model system based on mathematical simulation and Optimal Control Theory, that can be adopted globally for an efficient transition to renewable energy leading to a sustainable future. The proposed model aims to strike a balance between accuracy and transparency, while still considering many complex variables such as irreversible market saturation and risk-adjusted depreciation. It is expected to demonstrate the importance of timing and quick action quantitatively, by showing that strategies focused on early investment, such as front-loaded investment, tend to have a higher Cumulative Net Profit J. Ultimately, it aims to find the globally optimal investment strategy $I^*(t)$ by using this model as a foundation, thereby offering the definitive, most cost-effective path to a successful energy transition.

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