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by Mr Adnan

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Research Topic

Determining all the edge-to-edge tilings of the double branched cover of the sphere by regular spherical polygons & filling gaps of the archimedean tilings and johnson solids tilings

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Introduction

Tiling theory in mathematics has a rich history, particularly in classifying spherical tilings with regular polygons. The tiling of the double branched cover of the sphere is an important part of tiling theory. The double branched cover of the sphere is obtained by taking the unit sphere and cutting it along a longitude from the North pole to the South pole and widening it out into a hemisphere. Then take a second copy of this, and glue them together along these cut open borders. It looks like the result is just another copy of the sphere, but we think of the angle of each cut open sphere at the north and south pole to remain 360 degrees even once it is widened, so once we glue together, we obtain a "sphere" that has angle 720 degrees at the north and south pole and an equator of length 4π instead of the usual 2π. But otherwise, locally at each point, it has the usual spherical geometry. For over 50 years, we have known all the edge-to-edge tilings of the sphere by regular spherical polygons. For the double branched cover, although significant progress has been made in the study of determining all the edge to edge tilings, notable gaps still have remained.In particular, for vertices of the type that appear in Archimedean tilings, it has not yet been verified that the only possible tilings are those obtained by unwinding an Archimedean tiling of the sphere to the double branched cover of the sphere. A similar gap exists for vertex types related to the Johnson solids.

This research study therefore aims to:

- Fill up the remaining gaps for determining the tilings of the double branched cover of the sphere when considering the cases of archimedean tilings and johnson fields and to provide a comprehensive and rigorous account of all potential tilings.
- Check whether other types of tilings of the double branched cover of the sphere work when considering different vertex types like singular or half singular vertex, varying orders of regular polygons, and other conditions.
- Prove mathematically whether Archimedean tilings of the double branched cover arise from unwinding Archimedean tilings of the sphere and analyze tilings involving vertex types corresponding to Johnson solids and other non-Archimedean structures.

Importance of the Research

- It will extend the classification of spherical polygonal tilings to double branched covers of the spheres. New tilings will be introduced that were previously unexplored.

- It will contribute to the broader mathematical fields of geometry, topology, and group theory, with applications in crystallography, polyhedral geometry, and symmetry studies.

- After filling all the gaps, it will provide a foundation for future explorations of tilings of n-fold branched covers. And this will establish a new area of mathematics that demonstrates how combinatorics, algebra, and spherical geometry can be combined to solve fascinating problems.

Literature Review

The classification of convex polyhedra with regular polygonal faces has long been a central problem in geometry. In 1966, the **Johnson solids**, 92 strictly convex polyhedra with regular polygonal faces that are neither Platonic, Archimedean, prisms, nor antiprisms were first classified. [1] It helped extend the understanding of which combinations of regular polygons can form convex polyhedra (and by extension, spherical tilings). In 1967, [2] the classification of all convex polyhedra composed of regular polygons was completed. This work established that such polyhedra consist of the 5 Platonic solids, the 13 Archimedean solids, the 92 Johnson solids, and the infinite families of prisms and antiprisms. This classification has a direct correspondence with tilings of the 2-sphere: every convex polyhedron circumscribable about a sphere induces a tiling of the sphere by projecting its faces outward, while conversely, every edge-to-edge tiling of the 2-sphere by regular polygons can be realized as a convex polyhedron in 3-space with regular polygonal faces. Through this correspondence the complete list of edge-to-edge tilings of the 2-sphere by regular polygons was identified: the 5 Platonic solids, the 13 Archimedean solids, 25 of the 92 Johnson solids that are circumscribable, and the infinite categories of prisms and antiprisms [3].

Recent research has extended this perspective to the double branched cover of the sphere. In a 2024 study, all non edge-to-edge tilings of the double branched cover of the sphere involving regular spherical polygons with three or more sides were determined. These findings revealed five continuous families of kaleidoscope tilings, fifteen continuous families of 2-hemisphere tilings, four lunar tilings, five sporadic tilings, five composed tilings, and one magic triangle tiling [4]. Then it was [5] proved that all edge-to-edge tilings of the double branched cover of the 2-sphere by regular polygons are either lifts of 2-sphere tilings or belong to a finite, explicit set of new families. These include prism and antiprism tilings, sunflower tilings, ice cream cone tilings, decomposable bigon tilings (of several types), decomposable polygon tilings (pentagonal, octagonal, or decagonal), chrysanthemum tilings, and the so-called radioactive tiling. This work demonstrates that while many tilings of the branched cover arise as natural lifts, the geometry of the surface also admits novel configurations that have no spherical analogue. But still it has some gaps in the archimedean tilings and johnson fields. This research will fill up these gaps.

Research Question

- What are the complete families of edge-to-edge tilings of the double branched cover of the sphere by regular polygons?

- When we do tiling in the branched cover of the sphere, we need to consider the side lengths of the regular polygons that fit. In the context of tilings on the double branched cover of the sphere, how should one appropriately account for the side lengths and angles of regular polygons that fit into such tilings? While previous results have relied on the spherical laws of cosines and sines, along with basic spherical geometric properties, is the direct application of these standard spherical relations sufficient when analyzing polygons of the special cases like archimedean tilings or johnson fields on the branched cover, or do we need additional geometric constructions or lemmas to find these special tilings?

- If we are to obtain a new tiling on the double branched cover of the sphere, what specific aspects must we take into account? Is it primarily due to variations in the polygon orders of the tilings, or vertex configurations or for the constraints imposed by local symmetries?

- For a specific special tiling that fails to work on the double branched cover of the sphere, what are the main underlying reasons for this failure?

- Can the classification methods for the double branched cover extend to n-fold branched covers of the sphere?

Research Methodology

This study adopts a mixed-method approach, combining analytical, computational, and comparative techniques to systematically investigate tilings on the double branched cover of the sphere.

1. Analytical Phase (Qualitative & Theoretical Component)

- **Formalization of Constraints:** Develop precise combinatorial and geometric frameworks governing vertex configurations, polygonal side lengths, and angle relations for tilings on the double branched cover.

- **Archimedean Analysis:** Examine whether previously known Archimedean tilings appear solely via the "unwinding" mechanism on branched covers, identifying potential gaps.
- **Vertex Classification:** Systematically classify Johnson-type, non-standard, and singular vertex arrangements, analyzing their geometric feasibility under spherical constraints and branch point singularities.

2. Computational Phase (Quantitative & Experimental Component)

- **Algorithmic Enumeration:** Implement algorithms to enumerate all feasible tilings under prescribed vertex and polygonal constraints, ensuring exhaustive coverage of potential configurations.
- **Computational Modeling and Visualization:** Use computational models to construct candidate tilings, visualize geometric arrangements, and verify consistency with spherical laws (e.g., spherical cosine and sine laws) and combinatorial constraints.

3. Comparative Phase (Integrative Component)

- **Comparison with Known Tilings:** Quantitatively and qualitatively contrast tilings of the double branched cover with classical tilings of the sphere to identify novel features or deviations.
- **Generalization to n-Fold Covers:** Explore extensions to n-fold branched covers, investigating whether observed combinatorial and geometric patterns persist under higher-order coverings.

Project Interventions and Evaluations

- **Classification Completion:** Achieve a complete classification of edge-to-edge tilings on the double branched cover, accounting for Archimedean, Johnson-type, and special vertices.
- **Digital Archive Development:** Construct a comprehensive digital repository of tilings, including precise diagrams, formal proofs, computational models, and visualizations for reproducibility and accessibility.
- **Peer-Reviewed Dissemination, :** Collaborate on formal publication of results in mathematical journals

- **Verification and Validation:** Ensure correctness and completeness of findings via rigorous combinatorial, geometric, and spherical verification, supported by mathematical proofs and computational cross-validation.

Timeline (12 Months)

Over the course of 12 months, the project will begin with a review of relevant literature, formalization of tiling definitions on branched covers, and reconstruction of known results to ensure consistency during months 1 to 3. In months 4 to 6, the focus will shift to analyzing gaps in Archimedean tilings and classifying unwinding behavior and special cases. Months 7 to 9 will involve investigating Johnson-type and non-standard vertex configurations and extending the classification of tilings. Finally, during months 10 to 12, the research will explore generalizations to n-fold branched covers, finalize mathematical proofs, and prepare both the digital archive of tilings and the research publication.

Ethical Considerations

This research does not involve human participants. Ethical considerations focus on academic integrity, collaboration acknowledgment, and ensuring accessibility of results (e.g., open-access digital archive).

Conclusion

The research expects to provide a complete classification of all the edge-to-edge tilings of the double branched cover of the sphere. Ultimately, this research could open a new avenue in mathematics by demonstrating how tiling theory, spherical geometry, and combinatorial analysis can intersect to solve previously unexplored problems. It may also provide a framework for generalizing results to n-fold branched covers, offering insights into more complex geometric structures and potentially inspiring further work in mathematical tilings and polyhedral geometry.

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[4] Colin Adams, Cameron Edgar, Peter Hollander, and Liza Jacoby, "The Non-Edge-to-Edge Tilings of the Sphere by Regular Polygons," *Discrete & Computational Geometry*, vol. 72, pp. 1029–1085, 2024.

[5] Colin Adams and Olivia Straw, "All Edge-to-Edge Tilings of the Double Branched Cover of the Sphere by Regular Polygons," *arXiv:2009.11802*, pp. 1-87, 2020.

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