TD1
EX1

a) $L = (L - R) \cup R \longrightarrow$ false

$L = \underline{\{a, c\}}$, $R = \{a, b\}$, $L - R = \{c\}$, $(L-R) \cup R = \underline{\{a,b,c\}}$

b) $L = (L \cup R) - R \longrightarrow$ false.

$L = \underline{\{a, c\}}$, $R = \{a, b\}$, $L \cup R = \{a, b, c\}$, $(L \cup R) - R = \underline{\{c\}}$

c) true, symmetric difference.

d) $L(R \cup S) = LR \cup LS \longrightarrow$ true

proof $\subseteq$) Let $w \in L(R \cup S)$

$\Rightarrow \exists\, \underline{u \in L}, \underline{v \in R \cup S}$ sth $w = uv$

$\Rightarrow \underline{v \in R}$ or $v \in S$

$\Rightarrow \left\{ \begin{array}{l} w = uv \in LR \\ \text{or } w = uv \in LS \end{array} \right| \Rightarrow w \in LR \cup LS.$

proof $\supseteq$) Let $w \in LR \cup LS$

$\Rightarrow w \in LR$ ①

or $w \in LS$ ②

① $\Rightarrow \exists\, u \in L, \underline{v \in R}$ sth $w = uv$

$\Rightarrow \exists\, u \in L, v \in R \cup S$ sth $w = uv$

$\Rightarrow \underline{w \in L(R \cup S)}$.

② $\Rightarrow \exists\, u' \in L, v' \in S$ sth $w = u'v'$

$\Rightarrow \exists\, u' \in L, v' \in (R \cup S)$ sth $w = u'v'$

$\Rightarrow \underline{w \in L.(R \cup S)}$

① & ② $\Rightarrow w \in L(R \cup S)$.

e) $L(R \cap S) = LR \cap LS \longrightarrow$ false (true $\subseteq$)

proof $\subseteq$)

Let $w \in L(R \cap S)$

$\Rightarrow \exists u \in L, v \in R \cap S$ sth $w = uv$

$v \in R \cap S \Rightarrow v \in R$ and $v \in S$

$\Rightarrow w = uv \in LR$ and $w \in LS$

$\Rightarrow w \in LR \cap LS$

proof trial $\supseteq$)

Let $w \in LR \cap LS$

$\Rightarrow w \in LR$ and $w \in LS$ X

Counter-example.

$L = \{a, ab\}$, $R = \{b\}$, $S = \{bb\}$

$\rightarrow L(R \cap S) = L.\emptyset = \emptyset$

$\rightarrow LR \cap LS = \{ab, abb\} \cap \{abb, abbb\} = \{abb\}$

f) $(L \cap R)^* = L^* \cap R^* \longrightarrow$ false

counter-example.

$L = \{ab\}$, $R = \{a, b\}$: $(L \cap R)^* = \emptyset^* = \{\varepsilon\}$

$L^* \cap R^* = L^* = \{\varepsilon, ab, abab, \dots\}$

g) $L(R^* \cap S^*) = LR^* \cap LS^* \longrightarrow$ false.

CE: $L = \{\varepsilon, a\}$, $R = \{ab\}$, $S = \{b\}$

$R^* = \{\varepsilon, ab, abab, \dots\}$, $S^* = \{\varepsilon, b, bb, b^3, \dots\}$; $L(R^* \cap S^*) = \{\varepsilon, a\}$

$LR^* = \{\underline{\varepsilon}, \underline{ab}, abab, \cdots, a, aab, aabab, \cdots\}$; $LS^* = \{\underline{\varepsilon}, b, bb, \cdots, a, \underline{ab}, abb, \cdots\}$

$LR^* \cap LS^* \ni ab.$

EX2 a) $\{a,b\}^* = \{a\}^* (\{b\}\{a\}^*)^*$ no proof

ex: $\underline{aaa}\ \underline{b\,a\,a}\ \underline{b}\ \underline{b}\ \underline{a}\ \underline{b}$

b) $L = L' \cup \{\varepsilon\}$, $R = R' \cup \{\varepsilon\}$

$L\Sigma^* R = (L' \cup \{\varepsilon\})\Sigma^*(R' \cup \{\varepsilon\})$

$= L'\Sigma^* R' \cup L'\Sigma^* \cup \Sigma^* R' \cup \underbrace{\Sigma^*} = \Sigma^*$

EX3 a) $\{w : \exists u \in \Sigma.\Sigma ,\ w = u u^R u\}$

$u = abb$, $u^R = bba$.

$\Sigma\Sigma = \{a,b\}.\{a,b\} = \{aa, ab, ba, bb\}$

abbaab in , aaaaaa in

a , ba , ... not in

b) $\{w : ww = www\} = \{\varepsilon\}$ $(2|w| = 3|w|)$

c) $\{w : \exists u,v \in \Sigma^*, uvw = wuv\} = \Sigma^*$

d) $\{w : \exists u \in \Sigma^*, www = uu\}$

$u = aaa$, $w = aa$ in , $w = bb$ in

$w = ab$ not in $\underline{ab\,a}|\underline{b\,ab}$

EX4 $\underline{\mathbb{N}, +}$ $\forall p, q \in \mathbb{N}$, $p+q \in \mathbb{N}$

$\underline{\mathbb{N}, -}$ is not closed.

$\mathbb{Z}$ fermeture de $\mathbb{N}$ % $-$

L ? • : $\boxed{L^*} \supseteq L$.