

$$1. \quad a^{[l]} = \sigma(W^{[l]} a^{[l-1]} + b^{[l]}) \in \mathbb{R}^{n_l} \text{ for } l=2,3,\dots,L$$

$$a^{[1]} = x \in \mathbb{R}^{n_1}$$

$$\nabla_{a^{[L]}}(x) = \frac{\partial a^{[L]}}{\partial x}$$

$$\text{let } z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$$

$$\begin{aligned} \text{then } \frac{\partial a^{[L]}}{\partial x} &= \frac{\partial a^{[L]}}{\partial z^{[L]}} \cdot \frac{\partial z^{[L]}}{\partial a^{[L-1]}} \cdots \frac{\partial z^{[2]}}{\partial a^{[1]}} \cdot \frac{\partial a^{[1]}}{\partial x} \\ &= \text{diag}(\sigma'(z^{[L]})) \cdot W^{[L]} \cdots \text{diag}(\sigma'(z^{[2]})) \cdot W^{[2]} \cdot I \end{aligned}$$

Forward pass:

$$a^{[1]} = x$$

for $l=2$ to L :

$$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = \sigma(z^{[l]})$$

Backward pass:

$$J^{[1]} = I$$

for $l=2$ to L :

$$J^{[l]} = \text{diag}(\sigma'(z^{[l]})) \cdot W^{[l]} \cdot J^{[l-1]}$$

$$\text{return } \nabla_{a^{[L]}} = J^{[L]}$$