Week 1 Assignment

3/16/2026

1. $h(x_1, x_2) = \sigma(b + w_1 x_1 + w_2 x_2)$ $\quad \sigma(z) = \frac{1}{1+e^{-z}}$

$(x_1, x_2, y) = (1, 2, 3)$ $\quad \theta^0 = (b, w_1, w_2) = (4, 5, 6)$

$$\theta^{n+1} = \theta^n - \alpha \nabla_\theta \text{Loss}$$

$$\theta^1 = \theta^0 - \alpha \cdot \nabla_\theta \left[(y - h(x_1, x_2))^2 \right]$$

$$\frac{\partial \text{Loss}}{\partial b} = 2(y-h) \cdot (-1) \cdot \sigma'(b + w_1 x_1 + w_2 x_2)$$

$$\frac{\partial \text{Loss}}{\partial w_1} = 2(y-h) \cdot (-1) \cdot \sigma'(b + w_1 x_1 + w_2 x_2) \cdot x_1$$

$$\frac{\partial \text{Loss}}{\partial w_2} = 2(y-h) \cdot (-1) \cdot \sigma'(b + w_1 x_1 + w_2 x_2) \cdot x_2$$

$$\sigma'(z) = \sigma(z) \cdot (1 - \sigma(z))$$

$$b^1 = b^0 - \alpha \frac{\partial \text{Loss}}{\partial b} = 4 - (-2\alpha (y-h) \sigma'(z))$$

$$= 4 + 2\alpha (3 - \sigma(21)) \sigma(21)(1 - \sigma(21))$$

$$w_1^1 = w_1^0 + 2\alpha (y-h) \sigma'(z) x_1$$

$$= 5 + 2\alpha (3 - \sigma(21)) \sigma(21)(1 - \sigma(21)) \cdot 1$$

$$w_2^1 = w_2^0 + 2\alpha (y-h) \sigma'(z) x_2$$

$$= 6 + 2\alpha (3 - \sigma(21)) \sigma(21)(1 - \sigma(21)) \cdot 2$$

$$\theta^1 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 2\alpha (3 - \sigma(21)) \sigma(21)(1 - \sigma(21)) \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

2. (a) $\sigma(x) = \frac{1}{1+e^{-x}}$

$$\sigma'(x) = -(1+e^{-x})^{-2}(-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2} = \sigma(x)(1-\sigma(x))$$

$$\sigma''(x) = \sigma'(x)(1-\sigma(x)) + \sigma(x)(-\sigma'(x))$$
$$= \sigma'(x) - 2\sigma(x)\sigma'(x)$$
$$= \sigma'(x)(1-2\sigma(x))$$
$$= \sigma(x)(1-\sigma(x))(1-2\sigma(x))$$

$$\sigma'''(x) = \sigma''(x)(1-2\sigma(x)) + \sigma'(x)(-2\sigma'(x))$$
$$= \sigma''(x)(1-2\sigma(x)) - 2(\sigma'(x))^2$$
$$= \sigma(x)(1-\sigma(x))(1-2\sigma(x))(1-2\sigma(x))$$
$$- 2(\sigma(x)(1-\sigma(x)))^2$$
$$= \sigma(x)(1-\sigma(x))\left[(1-2\sigma(x))^2 - 2\sigma(x)(1-\sigma(x))\right]$$

$$= \sigma(x)(1-\sigma(x))(1-6\sigma(x)+6\sigma(x)^2)$$

(b) $\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

$$\tanh\left(\frac{x}{2}\right) = \frac{e^{\frac{x}{2}} - e^{-\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}$$

$$\frac{1}{2}\left(1+\tanh\frac{x}{2}\right) = \frac{1}{2}\left(1 + \frac{e^{\frac{x}{2}} - e^{-\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}\right)$$

$$= \frac{1}{2}\,\frac{2e^{\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}} = \frac{e^{\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}$$

$$= \frac{1}{1+e^{-x}}$$

$$\sigma(x)$$