Rigid motions

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August 2021

1 Rigid motion

A rigid motion in an IPS V is a transformation $f : V \to V$ **preserving the distance** between point st

$$||f(x) - f(y)|| = ||x - y||, \forall x, y \in V$$

Note that we do not assume the transformation to be linear.
Any unitary transformation is a rigid motion.

2 Properties of rigid motion

3 Thm 7.1

Let f be a rigid motion in a real IPS X, and let $T(x) : f(x) - f(0)$. Then T is an orthogonal transformation.
This is saying that any rigid motion in a real inner product space is a composition of an orthogonal transformation and a translation.

4 Lem 7.2

Let T be as defined in Thm7.1. Then for all $x, y \in X$:

1. $||Tx|| = ||x||$
2. $||Tx - Ty|| = ||x - y||$
3. $(T(x), T(y)) = (x, y)$