Orthogonality

Zexi Sun

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1 Orthogonality

Vector X Vector:
Two vectors u and v are orthogonal if $(u, v) = 0$, and we say $u \perp v$.
Vector X Subspace:
A vector v is orthogonal to a subspace E if v is orthogonal to all vectors w in E.
Subspace X Subspace:
Subspaces E and F are orthogonal if all vectors in E are orthogonal to all vectors in F
Vector systems :
A system of vectors $v_1, ..., v_n$ is orthogonal if any two vectors are orthogonal to each other (i.e, if $(v_j, v_k) = 0$ for $j \neq k$). .

2 Pythagorean identity

If $u \perp v$ we have $||u + v||^2 = ||u||^2 + ||v||^2$

3 Lem 2.3

Let E be spanned by vectors $v_1, ..., v_r$.
Then $v \perp E$ iff $v \perp v_k$, for all $k = 1, ..., r$.

4 Lem 2.5: Generalzed Pythagorean Identity

Let $v_1, ..., v_r$ be an orthogonal system. Then $|| \sum_{k=1}^{n} \alpha_k v_k||^2 = \sum_{k=1}^{n} |\alpha_k|^2 ||v_k||^2$. This formula looks particularly simple for orthonormal systems, where $||v_k|| = 1$.

5 Cor 2.6

Any orthogonal system $v_1, ..., v_n$ of non-zero vectors is linearly independent.

6 Orthogonal system

An orthogonal (orthonormal) system which is also a basis is called an **orthogonal (orthonormal) basis**.

If $dimV = n$ then any orthogonal system of n non-zero vectors is an orthogonal basis.

To find coordinates of a vector in an orthogonal basis one does not need to solve a linear system, the coordinates are determined by the formula $\alpha_k = \frac{(x,v_k)}{||v_k||^2}$.

Namely, if $v_1, ..., v_n$ is an orthonormal basis, any vector v can be represented as $v = \sum_{k=1}^{n}(v, v_k)v_k$, which is called the **abstract orthogonal Fourier decomposition**.