The University of British Columbia

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DATA 101

Lab 7 Solution

Date: November 16-20, 2020

Answer each question below, including the required lines of R code in each case.

1. If you toss a fair coin, the probability of getting heads is usually assumed to be 0.5, and if you toss such a coin repeatedly n times, the binomial distribution specifies the probability that you would get any number of heads, between 0 and n. The `dbinom()` function can be used to compute such probabilities. For example, if you toss a coin three times, the probability that you would see 2 heads is given by

```
dbinom(2, 3, 0.5)

## [1] 0.375
```

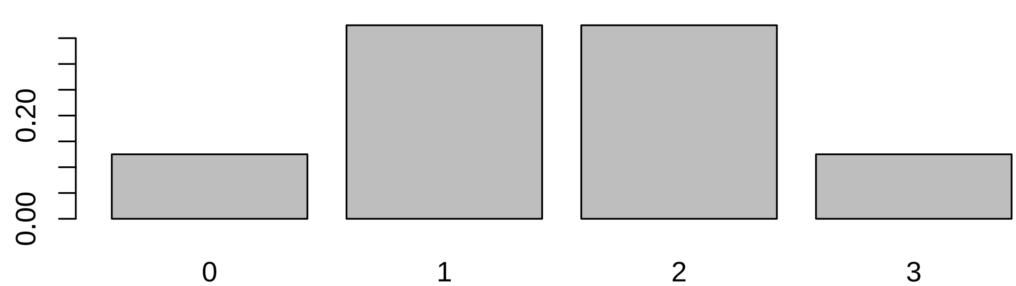
We can also simultaneously find the probability of getting 0, 1 or 3 heads by using the fact that the `dbinom` takes a vector parameter as its first argument:

```
dbinom(c(0, 1, 3), 3, 0.5)

## [1] 0.125 0.375 0.125
```

so we see that the probability of getting 0 heads is 1/8 and the probability of getting 3 heads is also 1/8, for example.

If we want to see the whole distribution all at once, we can plot the probabilities with the `barplot` function. To get decent tick labels on the horizontal axis, you can use the `names` function to name the elements of the vector of probabilities with the corresponding number of heads:

```
x <- dbinom(0:3, 3, 0.5) # x contains the probabilities
names(x) <- 0:3  # 0, 1, 2 or 3 heads
barplot(x)
```



(a) Suppose you toss a coin 5 times. Find the probability of getting 0 heads. Next, find the probabilities of getting 1 head, 2 heads, 3 heads, 4 heads, and then 5 heads.

```r
dbinom(0, 5, 0.5)

## [1] 0.03125

dbinom(1, 5, 0.5)

## [1] 0.15625

dbinom(2, 5, 0.5)

## [1] 0.3125

dbinom(3, 5, 0.5)

## [1] 0.3125

dbinom(4, 5, 0.5)

## [1] 0.15625

dbinom(5, 5, 0.5)

## [1] 0.03125
```

or

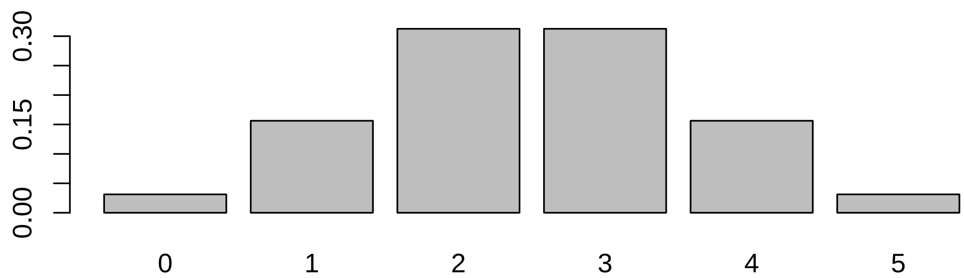
```r
for (i in 0:5) {
    print(dbinom(i, 5, 0.5))
}

## [1] 0.03125
## [1] 0.15625
## [1] 0.3125
## [1] 0.3125
## [1] 0.15625
## [1] 0.03125
```

or

```r
dbinom(0:5, 5, 0.5)

## [1] 0.03125 0.15625 0.31250 0.31250 0.15625 0.03125
```

(b) Construct a bar plot of the probability distribution of the numbers of heads obtained in 5 tosses of a fair coin.

```r
x <- dbinom(0:5, 5, 0.5)
names(x) <- 0:5
barplot(x)
```



2. If you toss a six-sided die, the probability of getting a one is 1/6. If the die is tossed n times, the binomial distribution can be used to specify the number of ones that are observed. Again, the `dbinom` function can be used to evaluate these probabilities. For example, if we toss the die 4 times, the probabilities of seeing 0 one, 1 one, ..., 4 ones is

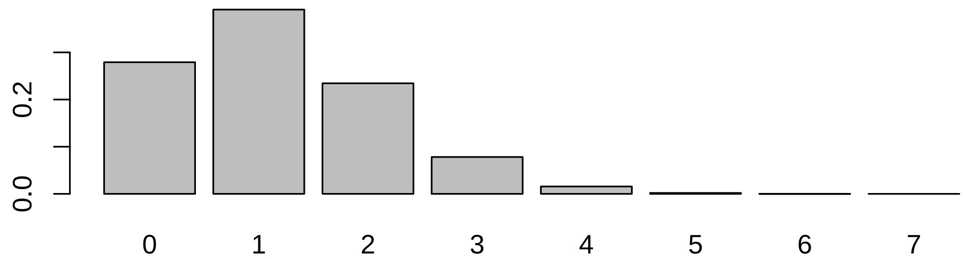
```
Px <- dbinom(0:4, 4, 1/6)
names(Px) <- 0:4
Px

##            0            1            2            3            4
## 0.4822530864 0.3858024691 0.1157407407 0.0154320988 0.0007716049
```

(a) Write a function called `barbinom` that takes `n` and `p` as arguments and displays a bar plot of the probability distribution for the number of successes obtained in `n` trials, where the probability of success at each trial is `p`. (e.g. `p` was 0.5 for the coin and 1/6 for the die.)

```
barbinom <- function(n, p) {
    Px <- dbinom(0:n, n, p)
    names(Px) <- 0:n
    barplot(Px)
}
```

(b) Use the function created in the previous question to display the probability distribution on the number of ones obtained in 7 tosses of a fair die.

```r
barbinom(7, 1/6)
```



3. Suppose you are to toss a six-sided die repeatedly until you first obtain a one. The number of tosses taken before you succeed has a geometric distribution which has probabilities that can be calculated using the `dgeom` function. To find the probability that you succeed immediately corresponds to this variable taking the value 0:

```r
dgeom(0, 1/6)

## [1] 0.1666667
```

This result should not be surprising, since this is the probability of getting a one on your next toss. If we want the probability that we will toss 3 other values before getting our first one, we use

```r
dgeom(3, 1/6)

## [1] 0.09645062
```

(a) Find the probability that you will get your first one on your 7th toss.

```r
dgeom(6, 1/6)
## [1] 0.05581633
```

(b) Noting that the probability of getting a one or a two can occur with probability 1/3, find the probability that you would have to toss the die 5 times until you see a one or two.

```r
dgeom(4, 1/3)
## [1] 0.06584362
```

(c) What is the probability that you could toss a die 10 times in a row without seeing a single one or two, and then see a one or two on the 11th toss?

```r
dgeom(10, 1/3)
```

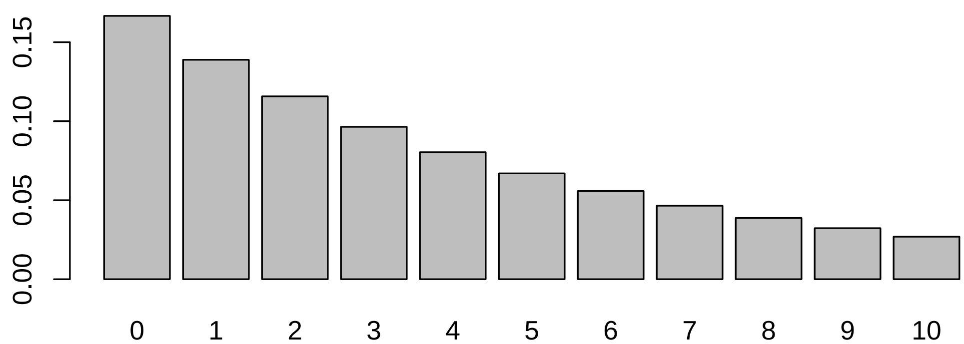
```
## [1] 0.00578051
```

4. (a) Write a function called `bargeom` which takes `p` as an argument and returns a bar plot of the first 11 values of the geometric probability distribution with parameter `p`. That is, the function needs to assign `dgeom(0:10, p)` to an object called `Px` which should be assigned names `0:10`. A bar plot of `Px` should then be drawn.

```r
bargeom <- function(p) {
    Px <- dgeom(0:10, p)
    names(Px) <- 0:10
    barplot(Px)
}
```

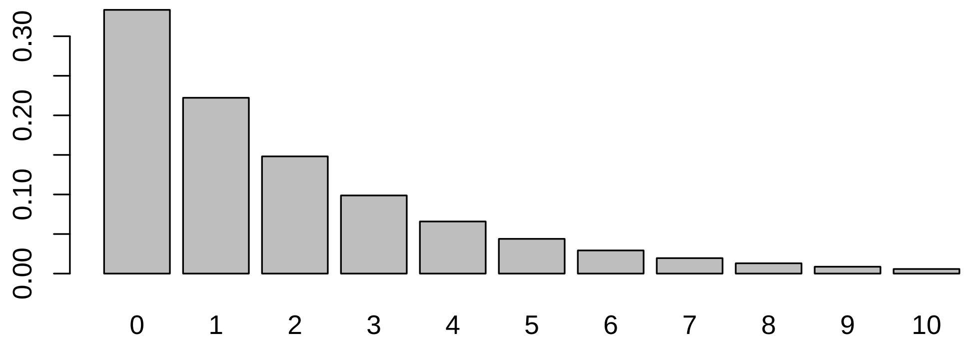
(b) Use the function just created to plot the probability distribution of the number of tosses of a six-sided die before obtaining the first one.

```r
bargeom(1/6)
```



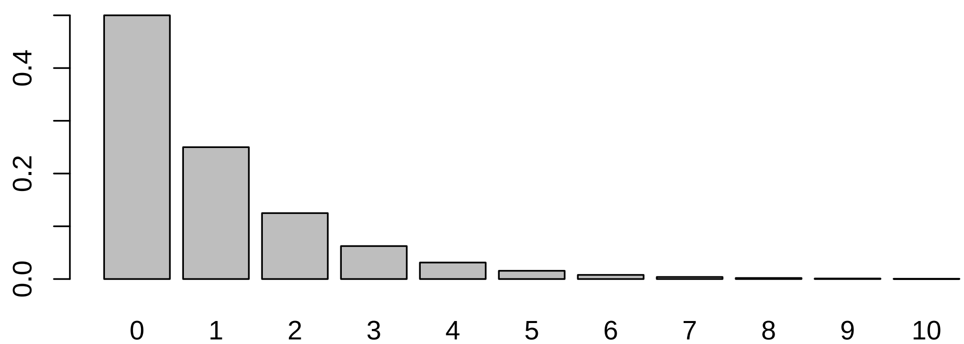
(c) Use the function just created to plot the probability distribution of the number of tosses of a six-sided die before obtaining the first one or two.

```r
bargeom(1/3)
```



(d) Use the function just created to plot the probability distribution of the number of tosses of a fair coin before obtaining the first head.

```
bargeom(1/2)
```



5. Suppose Ann tosses a die until she first sees a one, and Bob tosses a die until he first sees a one. Suppose Ann had a failures until her first success, and Bob had b failures until his first success. If x denotes the maximum of a and b, then x has a probability distribution given by

$$P(x) = 2p(1-p)^x + (1-p)^{2x+2} - (1-p)^{2x}, \quad x = 0, 1, 2, \ldots$$

where $p = 1/6$.

(a) Write a function called `dmaxgeom` which takes arguments `x` and `p` and returns the probability $P(x)$.

```
dmaxgeom <- function(x, p) {
    2*p*(1-p)^x + (1 - p)^(2*x + 2) - (1-p)^(2*x)
}
```

(b) Use your new function to evaluate the probability that at both Ann and Bob failed no more than 5 times before tossing their first ones.

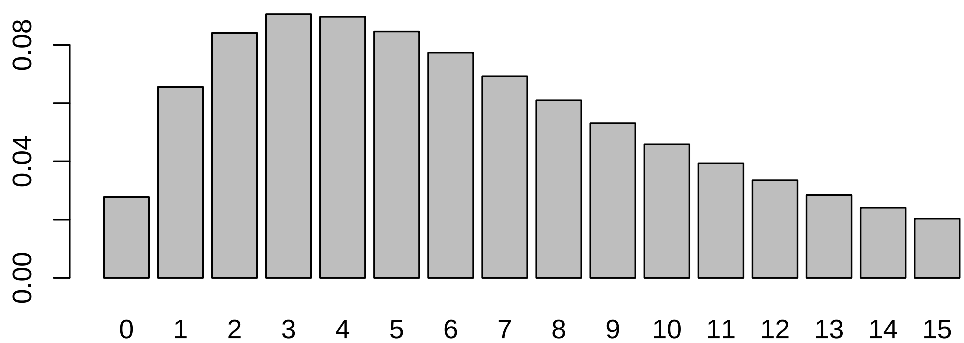
```
dmaxgeom(5, 1/6)

## [1] 0.08461026
```

6. (a) Write a function called `barmaxgeom` which takes `p` as an argument and returns a bar plot of the first 16 values of the probability distribution described in the previous question. That is, the function needs to assign `dmaxgeom(0:15, p)` to an object called `Px` which should be assigned names `0:15`. A bar plot of `Px` should then be drawn.

```
barmaxgeom <- function(p) {
    Px <- dmaxgeom(0:15, p)
    names(Px) <- 0:15
    barplot(Px)
}
```

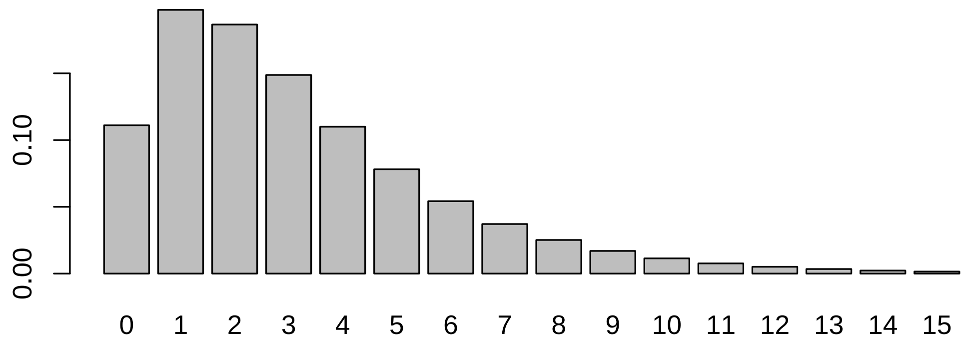
(b) Use the function just created to plot of the probability distribution when $p = 1/6$.

```
barmaxgeom(1/6)
```



(c) Use the function just created to plot the probability distribution when $p = 1/3$.

```
barmaxgeom(1/3)
```



(d) Use the function just created to plot the probability distribution when $p = 1/2$.

```
barmaxgeom(1/2)
```