

Q3.

Truth Table:

| Input |   |   |   | 2's complement |   |   |   |   |
|-------|---|---|---|----------------|---|---|---|---|
| A     | B | C | D |                | W | X | Y | Z |
| 0     | 0 | 0 | 0 |                | 0 | 0 | 0 | 0 |
| 0     | 0 | 0 | 1 |                | 0 | 0 | 0 | 1 |
| 0     | 0 | 1 | 0 |                | 0 | 0 | 1 | 0 |
| 0     | 0 | 1 | 1 |                | 0 | 0 | 1 | 1 |
| 0     | 1 | 0 | 0 |                | 0 | 1 | 0 | 0 |
| 0     | 1 | 0 | 1 |                | 0 | 1 | 0 | 1 |
| 0     | 1 | 1 | 0 |                | 0 | 1 | 1 | 0 |
| 0     | 1 | 1 | 1 |                | 0 | 1 | 1 | 1 |
| 1     | 0 | 0 | 0 |                | 1 | 0 | 0 | 0 |
| 1     | 0 | 0 | 1 |                | 0 | 1 | 1 | 1 |
| 1     | 0 | 1 | 0 |                | 0 | 1 | 1 | 0 |
| 1     | 0 | 1 | 1 |                | 0 | 1 | 0 | 1 |
| 1     | 1 | 0 | 0 |                | 0 | 1 | 0 | 0 |
| 1     | 1 | 0 | 1 |                | 0 | 0 | 1 | 1 |
| 1     | 1 | 1 | 0 |                | 0 | 0 | 1 | 0 |
| 1     | 1 | 1 | 1 |                | 0 | 0 | 0 | 1 |

Using K-Maps to simplify for W,X,Y,Z:

For W: The only minterm that has a value of 1 is  $m_8$  which would result in  $A * \bar{B} * \bar{C} * \bar{D}$  so that is the equation for W

For the rest: The k-map for 4 variables is as follows:

|   |   |          |          |          |          |
|---|---|----------|----------|----------|----------|
|   |   |          |          | C        |          |
|   |   | $m_0$    | $m_1$    | $m_3$    | $m_2$    |
|   |   | $m_4$    | $m_5$    | $m_7$    | $m_6$    |
| A | B | $m_{12}$ | $m_{13}$ | $m_{15}$ | $m_{14}$ |
|   |   | $m_8$    | $m_9$    | $m_{11}$ | $m_{10}$ |
|   |   |          |          | D        |          |

For X the K-Map is as follows:

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |

The maximum grouping has been labelled on the figure above. This grouping would result in the equation of X to be:

$$(\bar{A} * B) + (C * A * \bar{B}) + (D * A * \bar{B}) + (B * \bar{C} * \bar{D})$$

For Y the K-Map is as follows:

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 |

The maximum grouping has been labelled on the figure above. This grouping would result in the equation of Y to be:

$$(A * C * \bar{D}) + (\bar{A} * C) + (C * \bar{D})$$

For Z the K-Map is as follows:

|   |   |   |   |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 |

The maximum grouping has been labelled on the figure above. This grouping would result in the equation of Z to be:  $D$