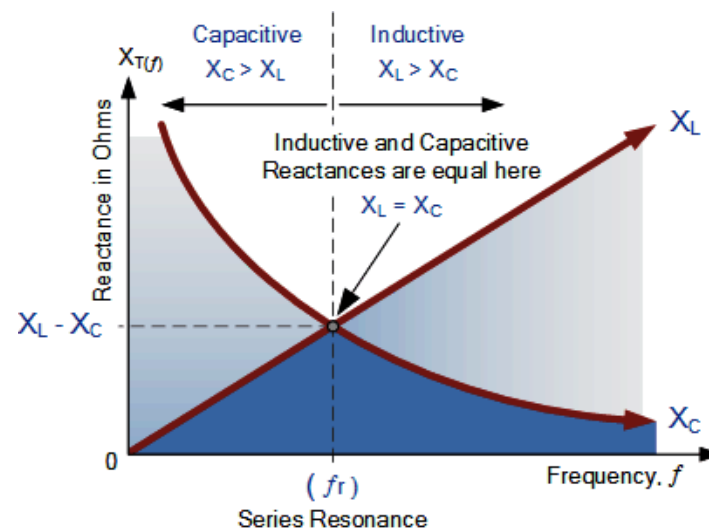


# Alternating Current (AC) Circuits II

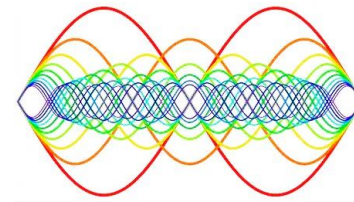
## Resonance

### CSE 113



# OUTLINES

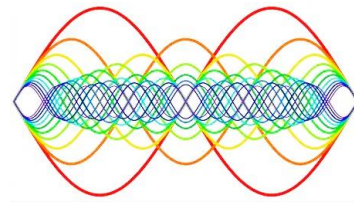
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- ❑ **AC Power**
- ❑ **Power Triangle**
- ❑ **AC Power Factor**
- ❑ **Power Factor Correction**
- ❑ **Resonance.**
  - **Series Resonant Circuit**
  - **Paralell Resonant Circuit**

# AC Power

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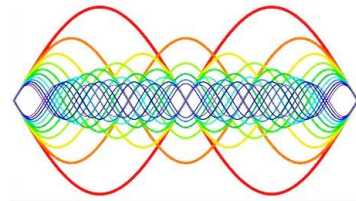
- AC Impedance (**Z**) is a **complex** quantity made up of real **resistance** (**R**) and imaginary **reactance** (**X**):

$$\vec{Z} = R + jX \quad (\Omega)$$

- AC Apparent Power (**S**) is a **complex** quantity made up of real **active** (**P**) power and imaginary **reactive** (**Q**) power:

$$\vec{S} = P + jQ \quad (VA)$$

# AC Power



- ❑ The Active power ( $P$ ) is the power that is dissipated in the resistance of the load.
- ❑ It uses the same formula used for DC ( $V$  &  $I$  are the *magnitudes*, not the phasors):

$$P = I^2 R = \frac{V^2}{R} \quad [\text{watts, W}]$$

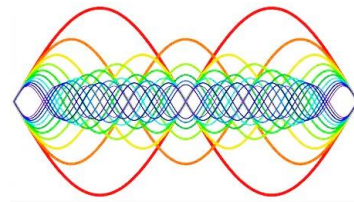
**WARNING! #1 mistake with AC power calculations!**

The **Voltage** in the above equation is the Voltage drop across the resistor, not across the entire circuit!

**CAUTION!**

REAL value of resistance ( **$R$** ) is used in REAL power calculations, not IMPEDANCE ( **$Z$** )!

# AC Imaginary (Reactive) Power (Q)



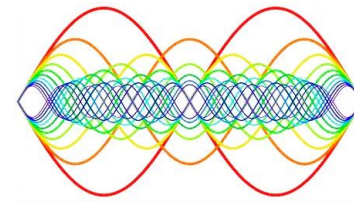
- ❑ The reactive power (Q) is the power that is exchanged between reactive components (inductors and capacitors).
- ❑ The formulas look similar to those used by the active power, but use reactance instead of resistances.

$$Q = I^2 X = \frac{V^2}{X^*} \quad [\text{VAR}]$$

**WARNING! #1 mistake with AC power calculations!**

The **Voltage** in the above equation is the Voltage drop across the reactance, not across the entire circuit!

- ❑ **Units: Volts-Amps-Reactive (VAR)**
- ❑ Q is negative for a capacitor by convention and positive for inductor.
- ❑ Just like X is negative for a capacitor! (-jX<sub>C</sub>)



# AC Apparent Power (S)

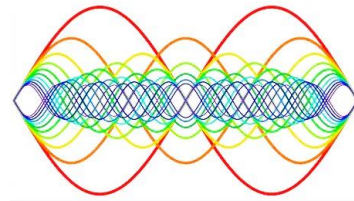
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- ❑ The Apparent Power (S) is the power that is “appears” to flow to the load.
- ❑ The magnitude of apparent power can be calculated using similar formulas to those for active or reactive power:

$$S = VI^* = I^2 Z = \frac{V^2}{Z^*} \quad [\text{VA}]$$

- ❑ Units: Volts-Amps (VA)
- ❑ V & I are the magnitudes, not the phasors.

# AC Power Relationships



Notice the  
relationship  
between **Z**  
and **S**:

$$\vec{\mathbf{Z}} = R + jX \quad (\Omega)$$

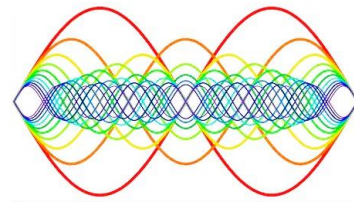
Apparent power calculated with Z

Real power calculated with R

Reactive power calculated with X

$$\vec{\mathbf{S}} = P + jQ \quad (VA)$$

# Power Triangle

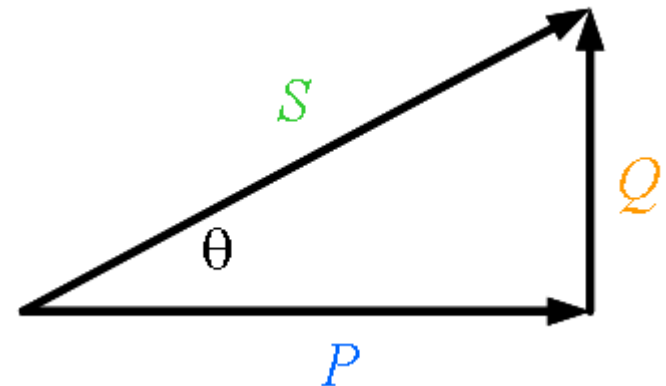
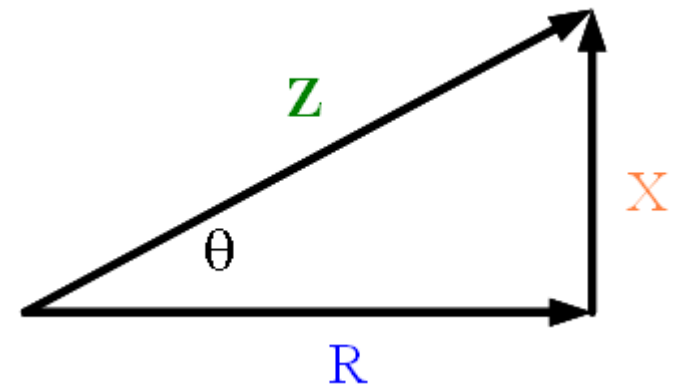


- The **power triangle** graphically shows the relationship between real (**P**), reactive (**Q**) and apparent power (**S**).

$$S = \sqrt{P^2 + Q^2}$$

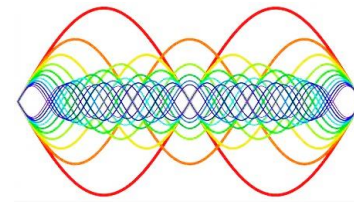
$$\vec{S} = P + jQ_L$$

$$\vec{S} = S \angle \theta$$

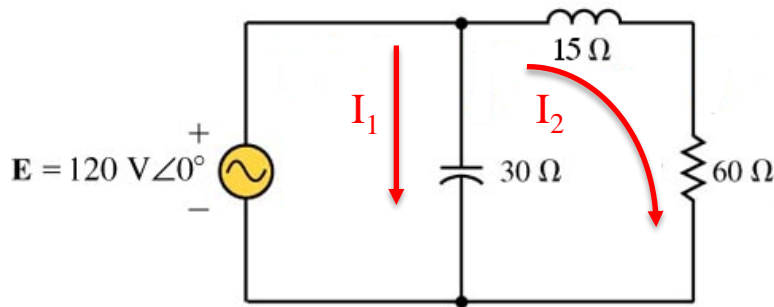




# Example Problem 1



Determine the real ( $P$ ) and reactive ( $Q$ ) power of each component.  
Determine the apparent ( $S$ ) power delivered by the source.



$$P = I^2 R = \frac{V^2}{R}$$

$$P_{R60} = I_2^2 R = (1.94A)^2 * (60\Omega) = \boxed{225.8W}$$

$$Q = I^2 X^* = \frac{V^2}{X^*}$$

$$Q_{L15} = I_2^2 X_{L15} = (1.94A)^2 * (15\Omega) = \boxed{56.5VAR}$$

$$Q_{C30} = I_1^2 X_{C30} = (4A)^2 * (-30\Omega) = \boxed{-480VAR}$$

$$Q_T = Q_{L15} + Q_{C30} = (56.5 - 480)VAR = \boxed{-423.5VAR}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(225.8)^2 + (-423.5)^2} = \boxed{480VA}$$

$$\vec{S} = (\vec{V})(\vec{I}^*) = (120V \angle 0)(4A \angle -62) = 480VA \angle -62^\circ \Rightarrow 225.35 - j423.81$$

$$Z_T = Z_{C30} / (Z_{L15} + Z_{R60}) = \left( \frac{1}{30\Omega \angle -90^\circ} + \frac{1}{(15\Omega \angle 90^\circ) + (60\Omega \angle 0^\circ)} \right)^{-1}$$

$$Z_T = 30\Omega \angle -62^\circ$$

$$I_S = \frac{E_S}{Z_T} = \frac{120V \angle 0^\circ}{30\Omega \angle -62^\circ} = 4A \angle 62^\circ$$

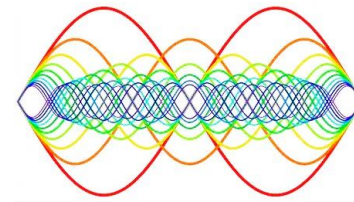
$$I_1 = I_S \frac{Z_T}{Z_{C30}} = (4A \angle 62^\circ) \frac{30\Omega \angle -62^\circ}{30\Omega \angle -90^\circ} = 4A \angle 90^\circ$$

$$I_2 = I_S \frac{Z_T}{Z_{L15} + Z_{R60}} = (4A \angle 62^\circ) \left( \frac{30\Omega \angle -62^\circ}{60\Omega + j15\Omega} \right) = 1.94A \angle -14^\circ$$

$\theta$  is from  $Z_T$

Real Power      Reactive Power

# Real and Reactive Power

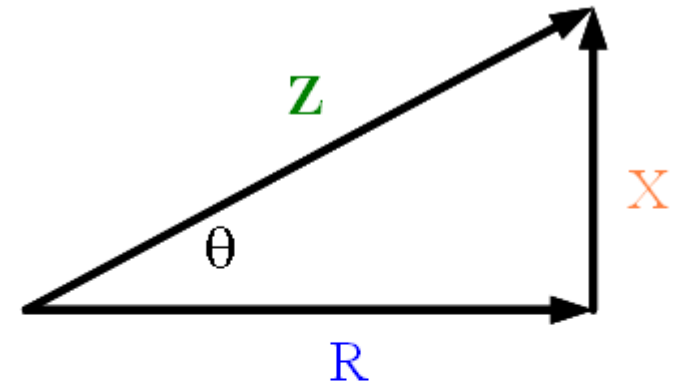


- The **power triangle** also shows that we can find real (**P**) and reactive (**Q**) power.

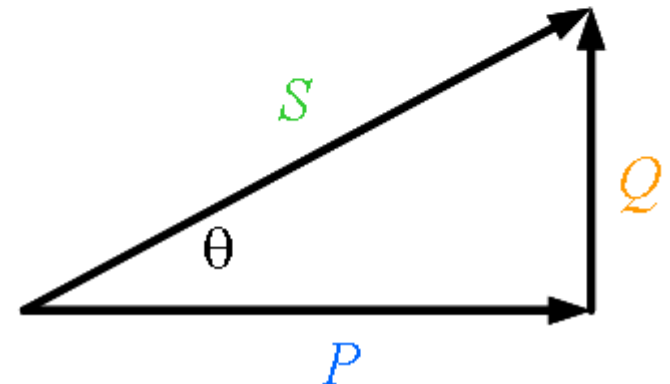
$$S = IV \quad (\text{VA})$$

$$P = S \cos \theta \quad (\text{W})$$

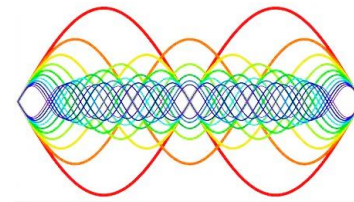
$$Q = S \sin \theta \quad (\text{VAR})$$



NOTE: The impedance angle and the “power factor angle” are the same value!



## Example Problem 2 [TUTORIAL]

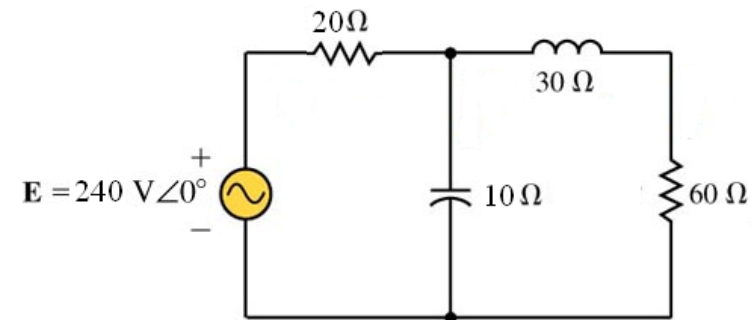


- Determine the apparent (S) power, total real (P) and reactive (Q) power using the following equations:

$$S = VI \quad (\text{VA})$$

$$P = S \cos \theta \quad (\text{W})$$

$$Q = S \sin \theta \quad (\text{VAR})$$



$$Z_T = Z_{R20} + [Z_{C10} // (Z_{L30} + Z_{R60})] = 20\Omega\angle 0^\circ + \left( \frac{1}{10\Omega\angle -90^\circ} + \frac{1}{(30\Omega\angle 90^\circ) + (60\Omega\angle 0^\circ)} \right)^{-1}$$

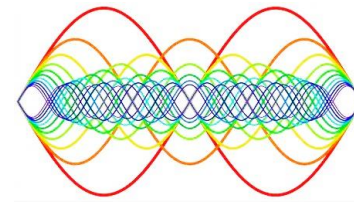
$$Z_T = 23.9\Omega\angle -26^\circ$$

$$I_s = \frac{E_s}{Z_T} = \frac{240V\angle 0^\circ}{23.9\Omega\angle -26^\circ} = 10A\angle 26^\circ$$

$$S_T = |VI^*| = |(240V\angle 0^\circ) * (10A\angle -26^\circ)| = 2.4kVA$$

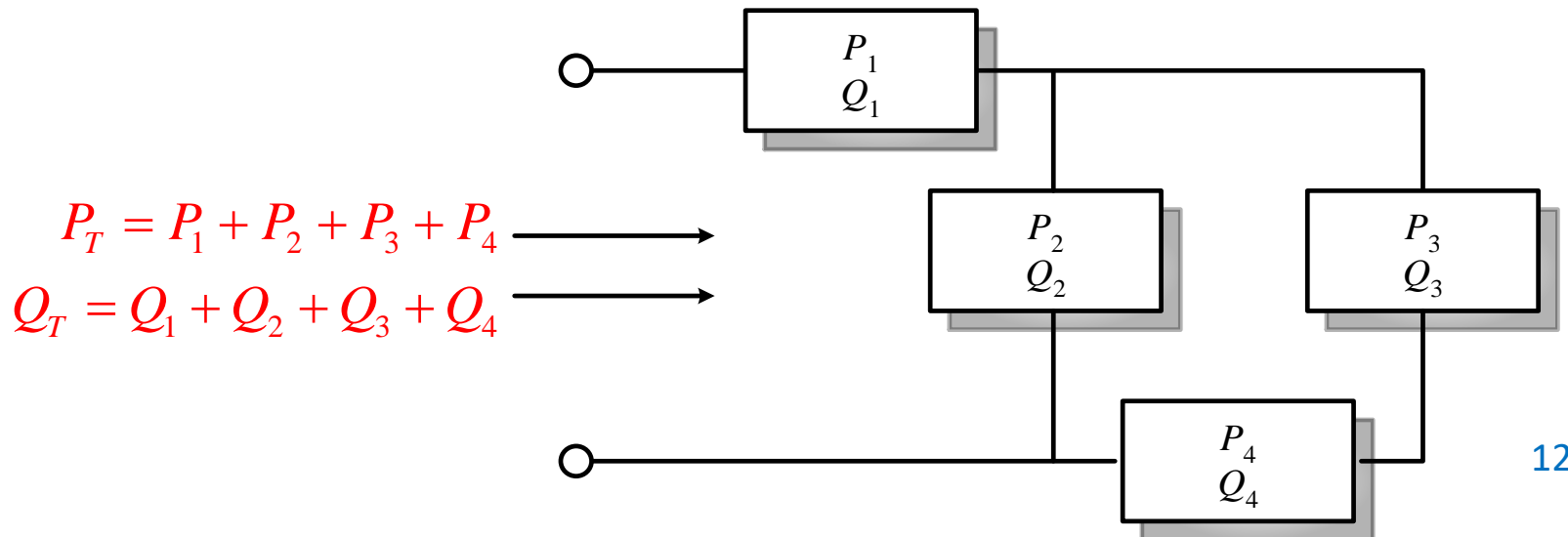
$$P_T = S_T \cos \theta = 2.4kVA * \cos(-26^\circ) = 2157W$$

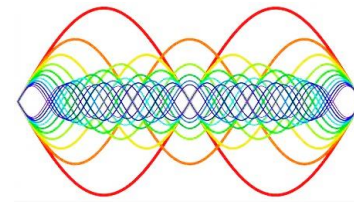
$$Q_T = S_T \sin \theta = 2.4kVA * \sin(-26^\circ) = -1052VAR$$



# Total Power in AC Circuits

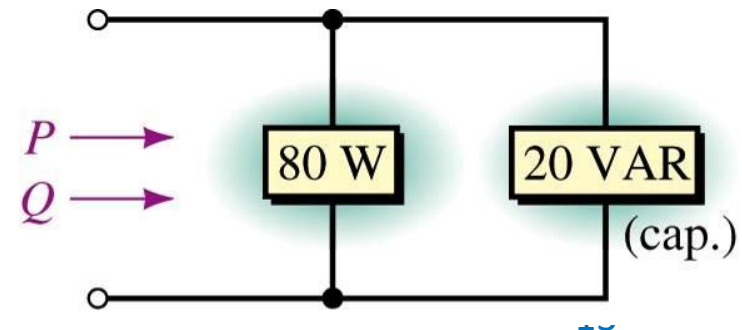
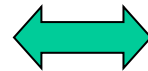
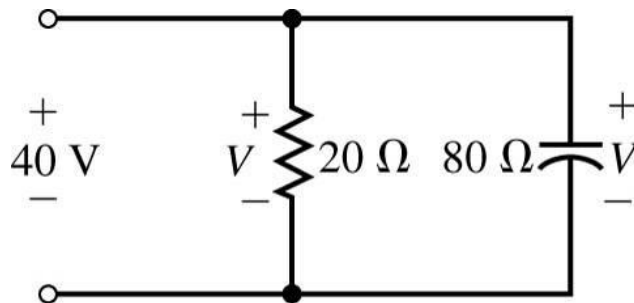
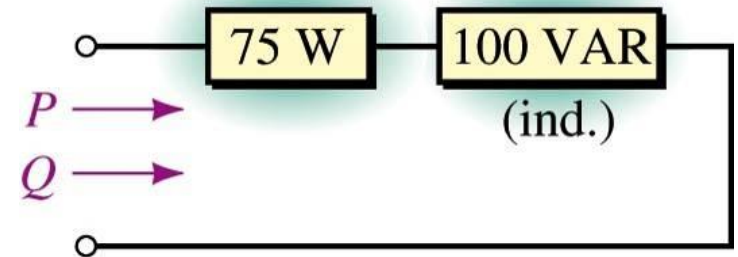
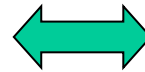
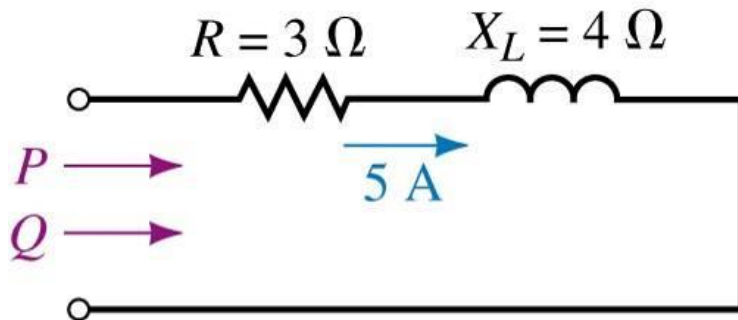
- The total power real ( $P_T$ ) and reactive power ( $Q_T$ ) is simply the sum of the real and reactive power for each individual circuit elements.
- *How elements are connected **does not** matter for computation of total power.*



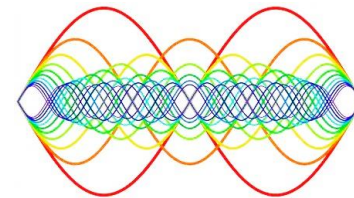


# Total Power in AC Circuits

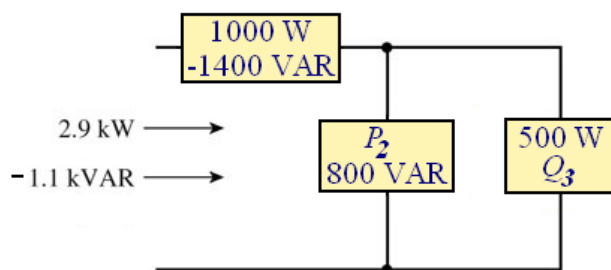
- Sometimes it is useful to redraw the circuit to symbolically express the real and reactive power loads.*



# Example Problem 3



- Determine the unknown real ( $P_2$ ) and reactive powers ( $Q_3$ ) in the circuit below.
- Determine total apparent power.
- Draw the power triangle.
- Is the unknown element in Load #3 an inductor or capacitor?



$$a) \quad P_T = P_1 + P_2 + \dots + P_N$$

Rearrange to find  $P_2$ :

$$P_2 = 2900W - 1000W - 500W = 1400W$$

$$a) \quad Q_T = Q_1 + Q_2 + \dots + Q_N$$

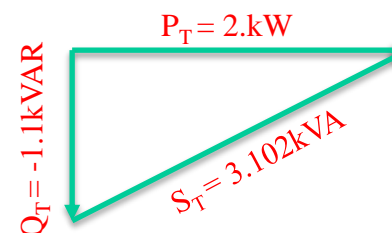
Like wise, for  $Q_3$ ; rearrange to find  $Q_3$ :

$$Q_3 = -1100VAR - (-1400VAR) - (800VAR) = -500VAR$$

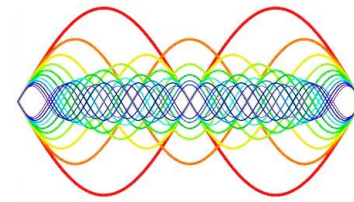
$$b) \quad S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(2900)^2 + (-1100)^2} = 3102VA$$

d) Because  $Q_3$  is negative, it is capacitive

c) Power Triangle



# Example Problem 4 [TUTORIAL]



- Determine the value of  $R$ ,  $P_T$  and  $Q_T$
- Draw the power triangle and determine  $S$ .

$$S_T = V * I = 600V * 30A = 18kVA$$

$$Q_T = 2750VAR - 17880VAR + 12960VAR = -2170VAR$$

$$P_T = \sqrt{S_T^2 - Q_T^2} = \sqrt{18kVA^2 - (-2170VAR)^2} = 17.87kW$$

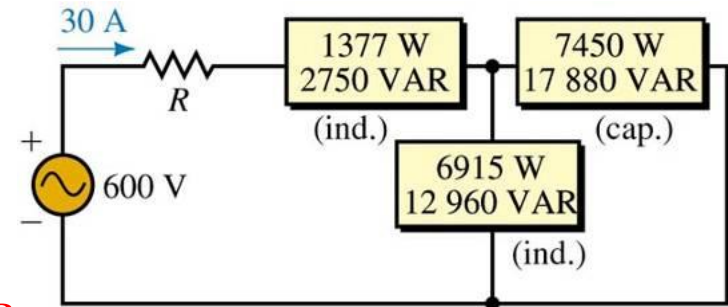
$$P_T = P_R + P_{L1} + P_{L2} + P_C$$

$$\Rightarrow P_R = P_T - P_{L1} - P_{L2} - P_C$$

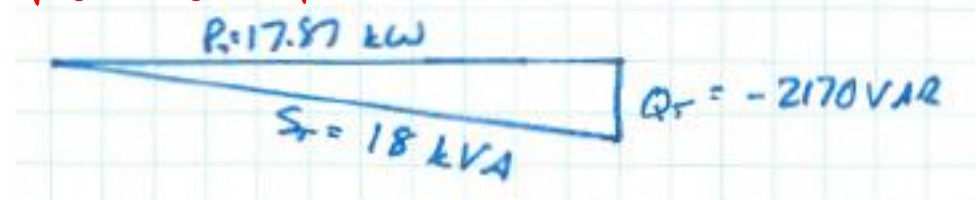
$$P_R = 17.87kW - 1377W - 7450W - 6915W = 2127W$$

$$P_R = I_T^2 * R$$

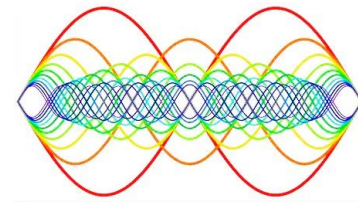
$$\Rightarrow R = \frac{P_R}{I_T^2} = \frac{2127W}{30A^2} = 2.36\Omega$$



$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{17.87kVA^2 + (-2170VAR)^2} = 18kVA$$



# Use of Complex Numbers in Power Calculations

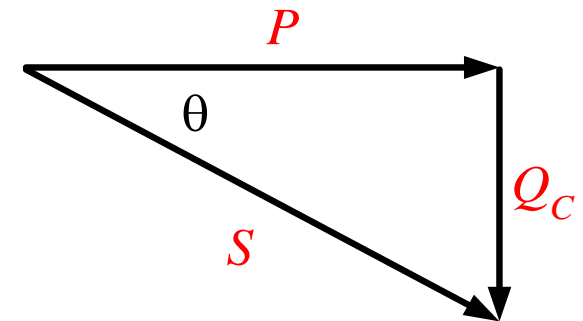


- ❑ AC power can be calculated using complex equations.
- ❑ Apparent Power can be represented as a complex number.
- ❑ The resultant can be used to determine real and reactive power by changing it to rectangular form.

$\mathbf{I}^*$  is complex conjugate of  $\mathbf{I}$

$$\bar{\mathbf{S}} = \bar{\mathbf{V}}\bar{\mathbf{I}}^* = P + jQ$$

$$\bar{\mathbf{S}} = \frac{|\bar{\mathbf{V}}|^2}{\bar{\mathbf{Z}}^*} = |\bar{\mathbf{I}}|^2 \bar{\mathbf{Z}}$$

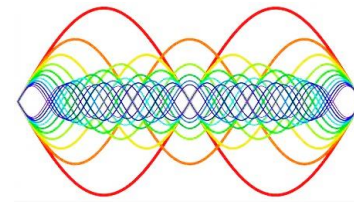


NOTE!

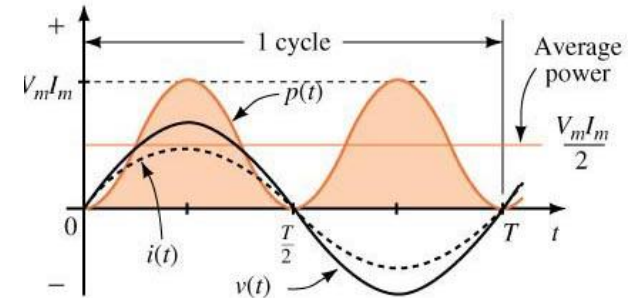
The complex conjugate of Current is used to make the power angle the same as the impedance angle!



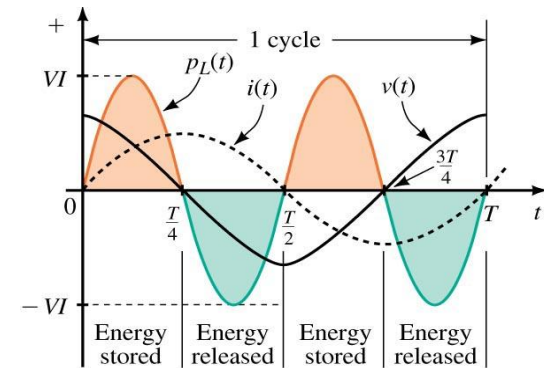
# Summary



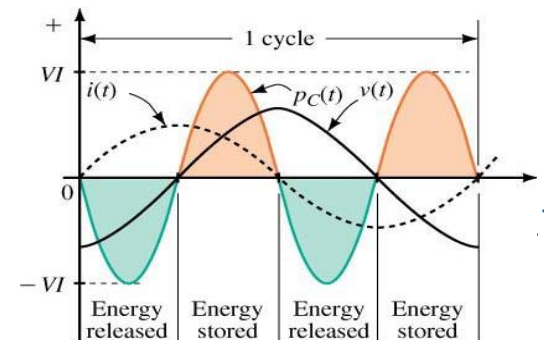
## AC Power to a Resistive Load

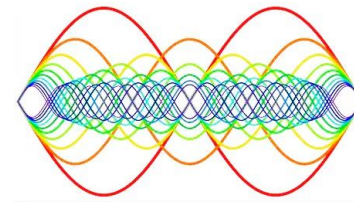


## AC Power to a Inductive Load



## AC Power to a Capacitive Load





# AC Power Factor

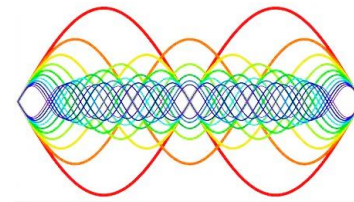
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- ❑ Power factor ( $F_P$ ) tells us what portion (or ratio) of the apparent power ( $S$ ) is actually real power ( $P$ ).
- ❑ Power factor is a ratio given by:

$$F_P = \cos \theta = \frac{P}{S}$$

- ❑ Power factor is expressed as a number between 0 to 1.0 (or as a percent from 0% to 100%).
- ❑ The closer to 1.0 the power factor gets, the more resistive.
- ❑ The closer to 0.0 the power factor gets, the more reactive.

# Power Factor



- From the power triangle it can be seen that:

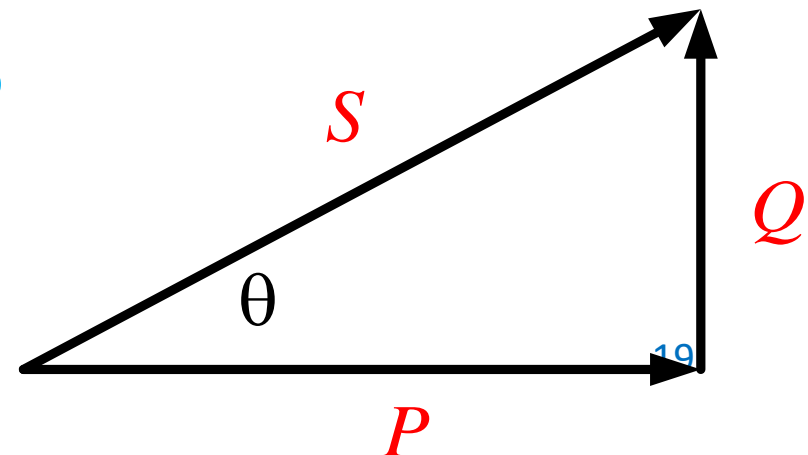
$$F_p = P / S = \cos \theta$$

- Power factor angle** is thus given:

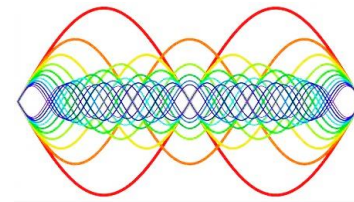
$$\theta = \cos^{-1}(P / S)$$

- For a **pure** resistance:  $\theta = 0^\circ$
- For a **pure** inductance:  $\theta = 90^\circ$
- For a **pure** capacitance:  $\theta = -90^\circ$

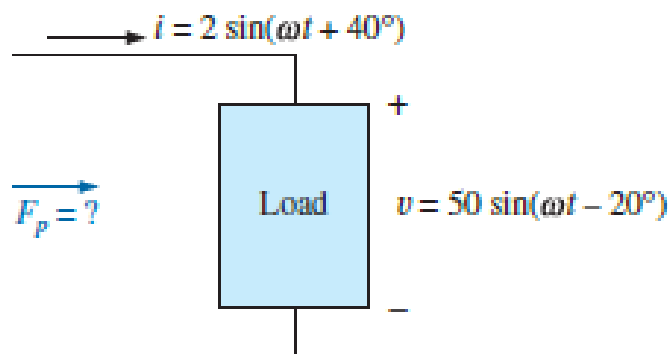
NOTE:  $\theta$  is the phase angle of  $\mathbf{Z}_T$ , not the current or voltage.



# Power Factor Leading or Lagging



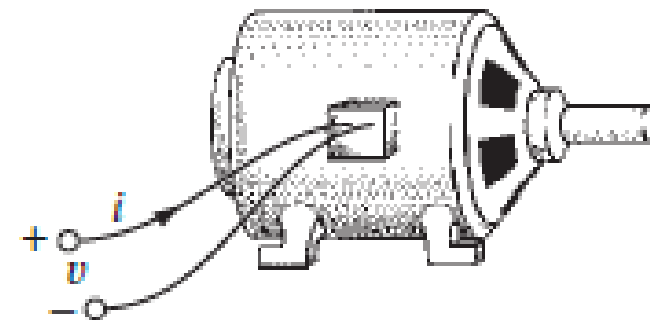
- The term leading and lagging are defined in *reference to the current* through the load.
  - If the *current leads* the voltage across the load then the load has a *leading power factor*.
  - If the *current lags* the voltage across the load then the load has a *lagging power factor*.



$$F_p = \cos \theta$$

$$F_p = \cos((-20^\circ) - 40^\circ)$$

$$F_p = \cos 60^\circ = 0.5 \text{ leading}$$

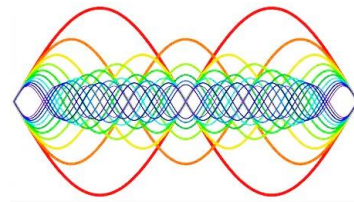


$$F_p = \cos \theta$$

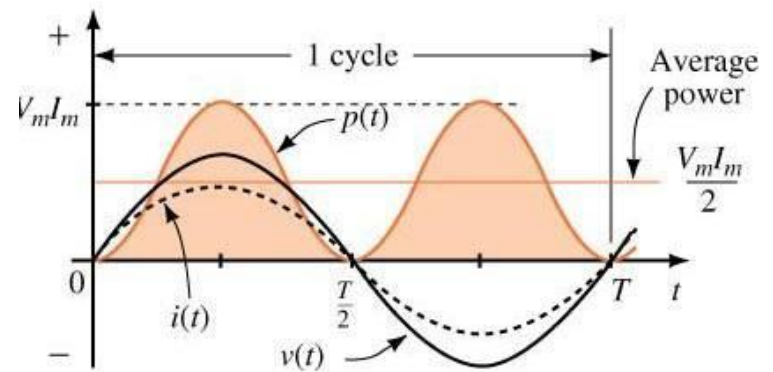
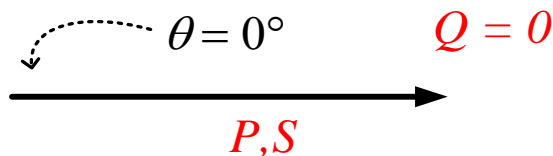
$$F_p = \cos(80^\circ - 30^\circ)$$

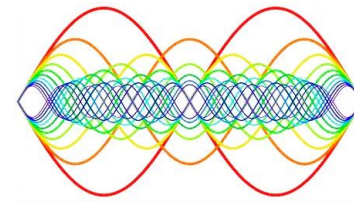
$$F_p = \cos 50^\circ = 0.64 \text{ lagging}$$

# Unity Power Factor ( $F_p = 1$ )



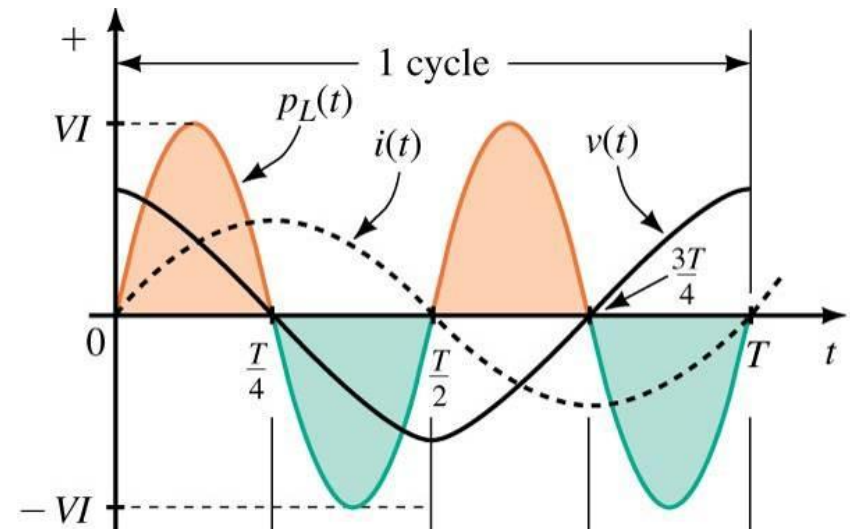
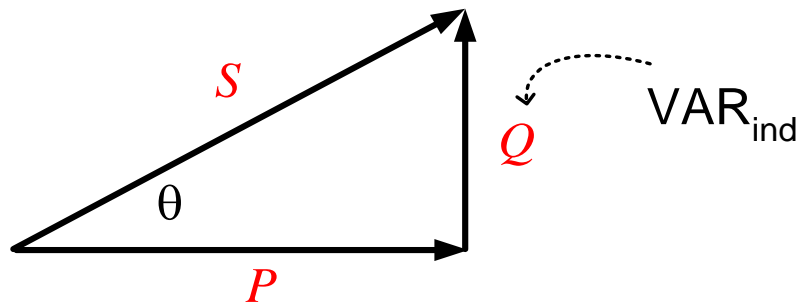
- Unity Power Factor implies that *all* of a load's apparent power is real power ( $S = P$ ).
- If  $F_p = 1$ , then  $\theta = 0^\circ$ .
- It could also be said that the load looks purely *resistive*.
- Load current and voltage are *in phase*.



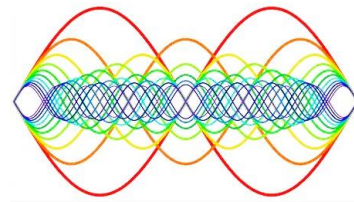


# Lagging Power Factor ( $\theta > 0^\circ$ )

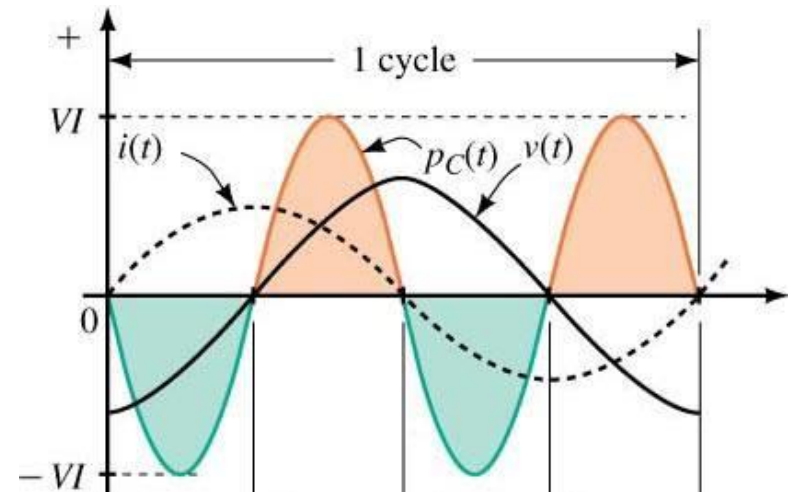
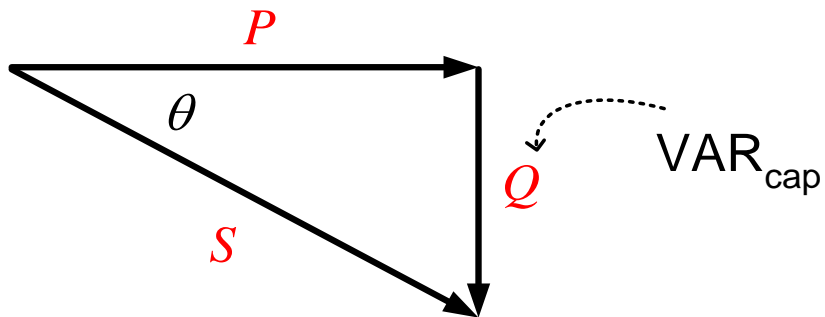
- ❑ The load current lags load voltage: **ELI**
- ❑ Implies that the load looks **inductive**.

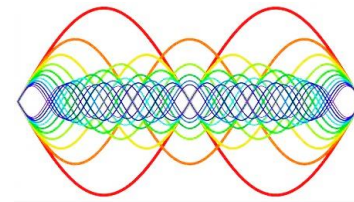


# Leading Power Factor ( $\theta < 0^\circ$ )



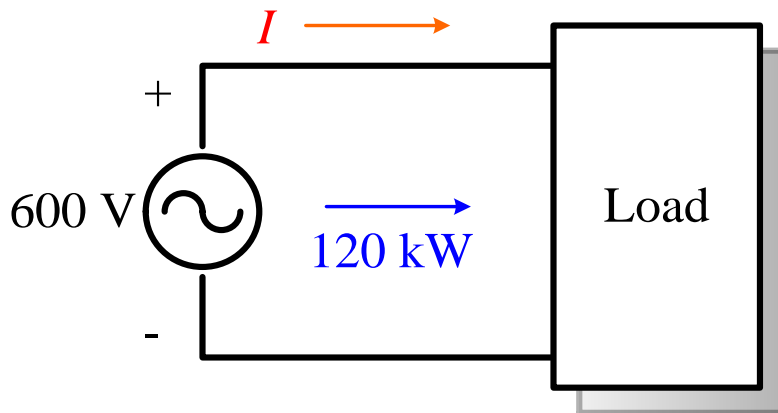
- The load current leads load voltage: **ICE**
- Implies that the load looks **capacitive**.





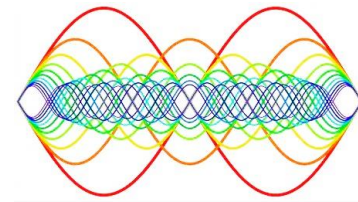
# Why is Power Factor Important?

- ❑ Consider the following example: A generator is rated at 600 V and supplies one of two possible loads.
  - ❑ Load1:  $P=120\text{kW}$ ,  $F_p=1$
  - ❑ Load 2:  $P = 120 \text{ kW}$ ,  $F_p = 0.6$
- ❑ Determining how much current ( $I$ ) is required is one such reason...

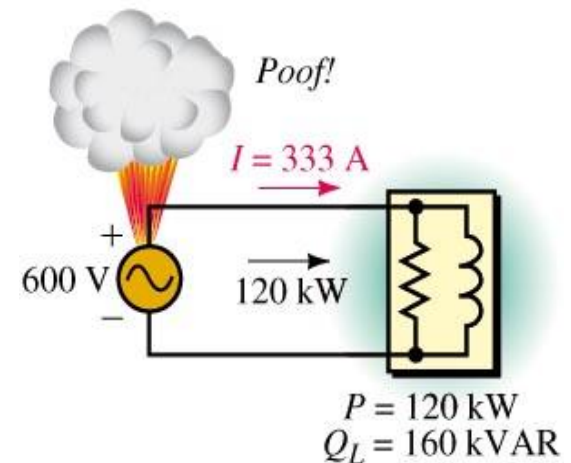
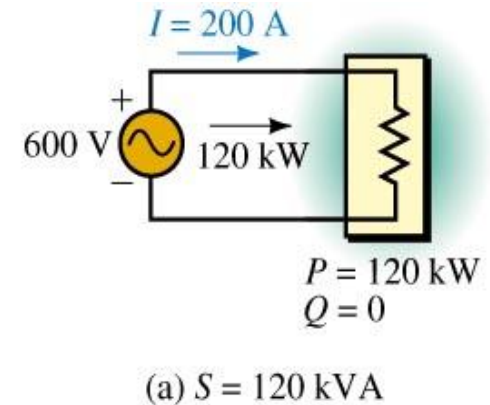




# Why is Power Factor Important?

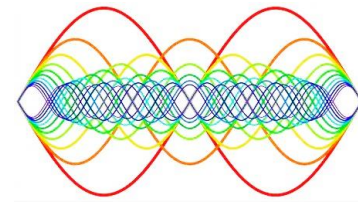


- For the load with  $F_p = 1$ , the generator had to supply 200 A
- For the load with  $F_p = 0.6$ , the generator had to supply 133 more amperes in order to do the same work ( $P$ )!
- Larger current means larger equipment (wires, transformers, generators) which cost more.
- Larger current also means larger transmission losses (think  $I^2R$ ).



# Why is Power Factor Important?

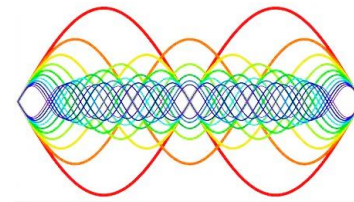
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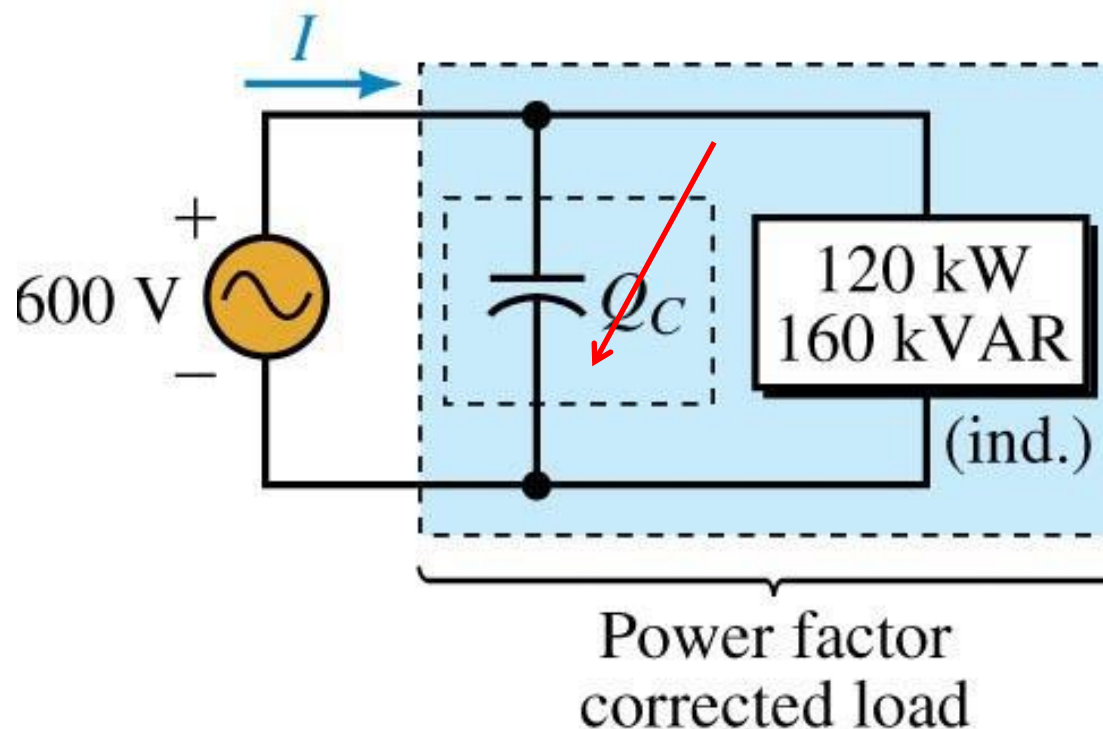
- ❑ Because of the wide variation in possible current requirements due to power factor, most large electrical equipment is rated using **apparent power ( $S$ )** in volt-amperes (VA) instead of **real power ( $P$ )** in watts (W).
- ❑ Is it possible to change the power factor of the load?

The answer is yes...through power factor correction...

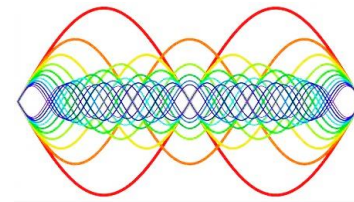
# Power-Factor Correction



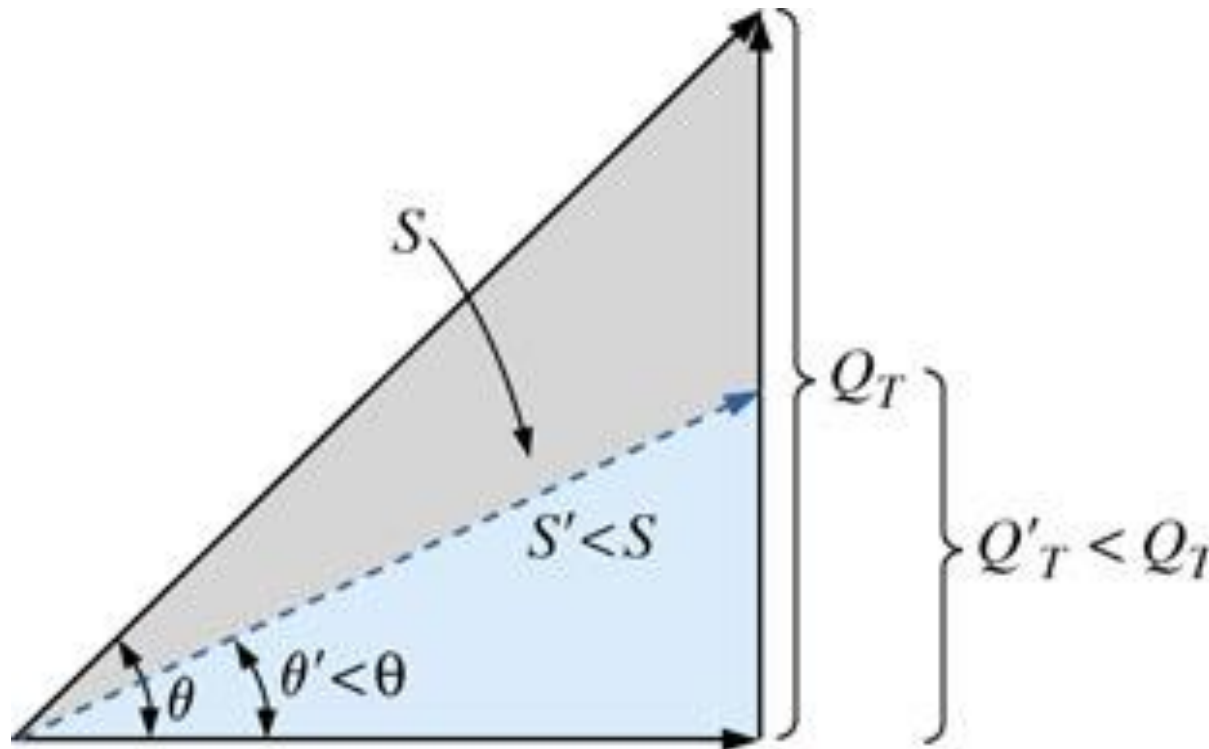
- In practice, almost all loads (commercial, industrial and residential) look **inductive** (due to motors, fluorescent lamp ballasts, etc.).
- Hence, almost all power factor correction consists of adding **capacitance**.



# Power-Factor Correction



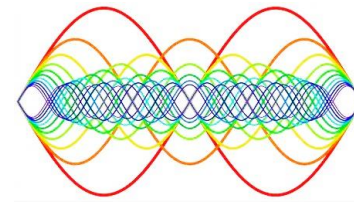
- How it changes the power triangle:



*Demonstrating the impact of power-factor correction on the power triangle of a network.*

# Power-Factor Correction Solution Steps

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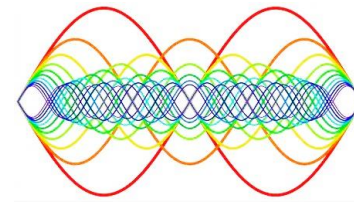


1. Calculate the reactive power ( $Q$ ) of the load.
2. Insert a component in parallel of the load that will cancel out that reactive power.

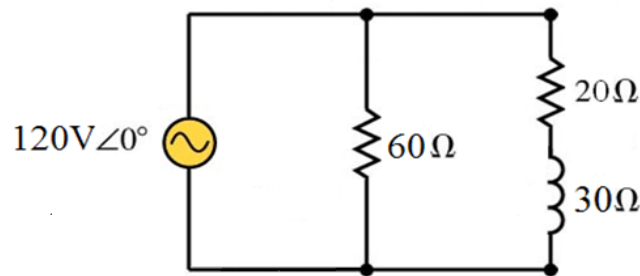
e.g. If the load has  $Q_{LD} = 512 \text{ VAR}$ , insert a capacitor with  $Q_C = -512 \text{ VAR}$ .

3. Calculate the reactance ( $X$ ) that will give this value of  $Q$  Normally the  $Q = V^2/X$  formula will work.
4. Calculate the component value ( $F$  or  $H$ ) required to provide that reactance.

# Example Problem 5



- Determine the value of the capacitance (in F) required to bring the power factor up to unity (freq of 60 Hz).
- Determine load current before and after correction.



b) Because the  $Q_T = 332 \text{ VAR}$ , we can insert a capacitor with  $Q_C = -332 \text{ VAR}$

a)

$$Z_T = \frac{1}{\frac{1}{60\Omega} * \frac{1}{20\Omega + j30\Omega}} = 25.3\Omega \angle 35.8^\circ$$

$$I_T = \frac{E_s}{Z_T} = \frac{120V \angle 0^\circ}{25.3\Omega \angle 35.8^\circ} = 4.74A \angle -35.8^\circ$$

Sign Change

$$S_T = E_s * (I_T^*) = (120 \angle 0^\circ) * (4.74A \angle 35.8^\circ)$$

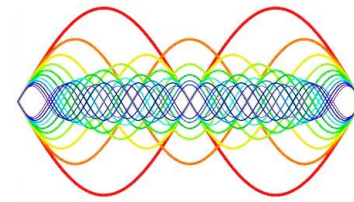
$$S_T = 568.7VA \angle 35.8^\circ = 462W + j332VAR$$

$\uparrow$   
 $P_T$

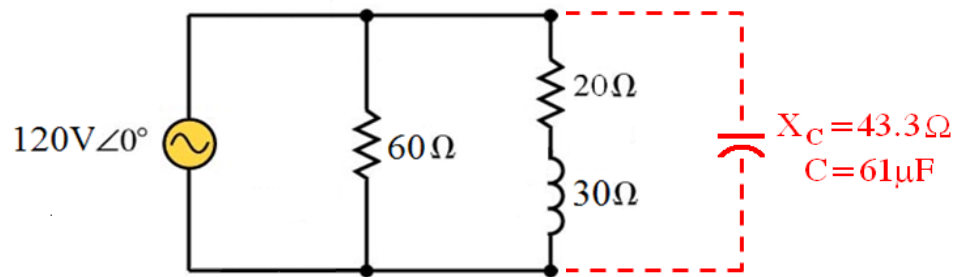
$\uparrow$   
 $Q_T$

$$F_P = \frac{P}{S} = \frac{462W}{568.7VA} = 0.81 \quad \text{or} \quad 81\%$$

# Example Problem 5 cont...



- Determine the value of the capacitance (in F) required to bring the power factor up to unity (freq of 60 Hz).
- Determine load current before and after correction.



Notice that  $X_C \neq X_L$ !  
Notice that  $X_C \neq X_{LD}$ !

**c) Adding a unity cap changes  $Z_T$ :**

$$Z_{T_{\text{Unity}}} = \frac{1}{\frac{1}{60\Omega} * \frac{1}{20\Omega + j30\Omega} + \frac{1}{-j43.4\Omega}} = 31.2\Omega \angle 0^\circ$$

**NEW Current:**

$$I_T = \frac{E_S}{Z_T} = \frac{120V \angle 0^\circ}{31.2\Omega \angle 0^\circ} = \boxed{3.85A \angle 0^\circ}$$

**b) Because the  $Q_T = 332 \text{ VAR}$ , we can insert a capacitor with  $Q_C = -332 \text{ VAR}$**

$$Q_{T_{\text{Unity}}} = -332 \text{ VAR} \Rightarrow \frac{V^2}{X_C}$$

$$X_C = \frac{V^2}{|Q_{T_{\text{Unity}}}|} = \frac{(120V)^2}{332} = 43.3\Omega$$

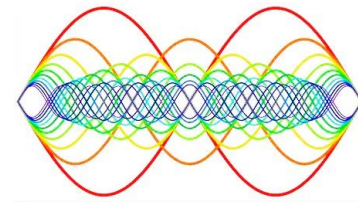
$$X_{C_{\text{Unity}}} = \frac{1}{2\pi f C_{\text{Unity}}} \Rightarrow$$

$$C_{\text{Unity}} = \frac{1}{2\pi f X_{C_{\text{Unity}}}} = \frac{1}{2\pi (60\text{Hz})(43.3\Omega)} = \boxed{61.1\mu F}$$

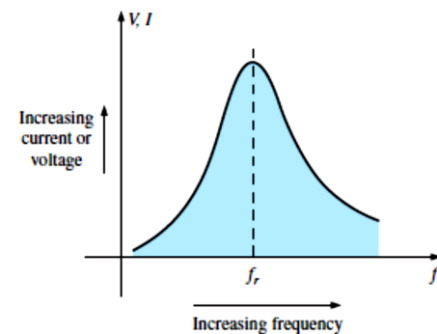
**OLD Current:**

$$I_T = \frac{E_S}{Z_T} = \frac{120V \angle 0^\circ}{25.3\Omega \angle 35.8^\circ} = 4.74A \angle -35.8^\circ$$

# Resonance



- ❑ **Resonant** (or **tuned** ) **circuits**, are fundamental to the operation of a wide variety of electrical and electronic systems in use today.
- ❑ The resonant circuit is a combination of **R**, **L**, and **C** elements having a frequency response characteristic similar to the one below:
- ❑ The resonant electrical circuit must have both inductance and capacitance.
- ❑ In addition, resistance will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve.
- ❑ When resonance occurs due to the application of the proper frequency (  $f_r$  ), the energy absorbed by one reactive element is the same as that released by another reactive element within the system.



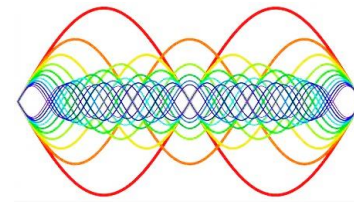
- ❑ **Remember:**

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \Rightarrow Z_C = -jX_C$$

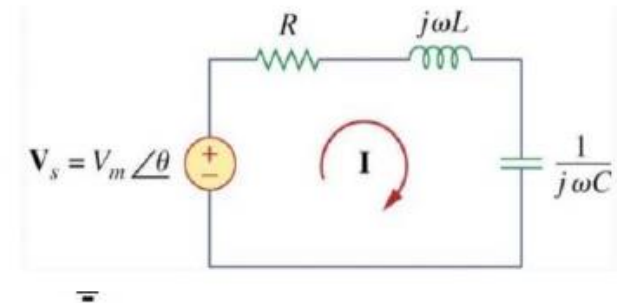
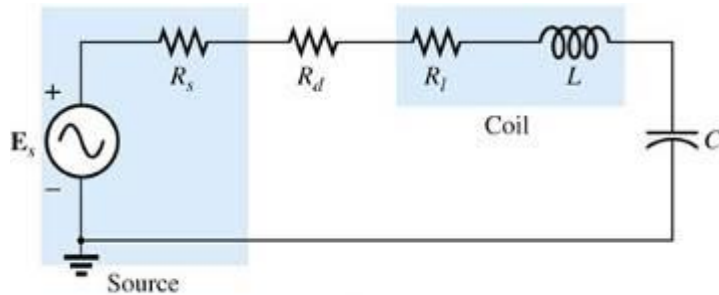
$$X_L = \omega L = 2\pi fL \Rightarrow Z_L = jX_L$$



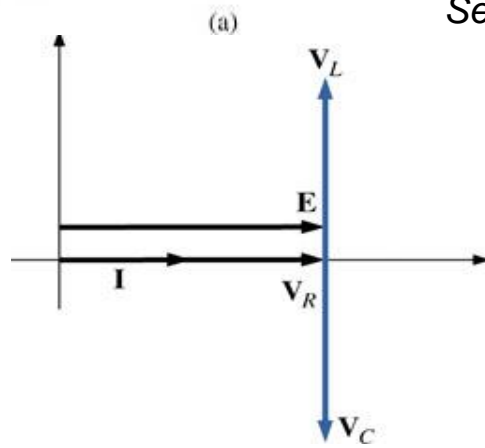
# Series Resonant Circuit



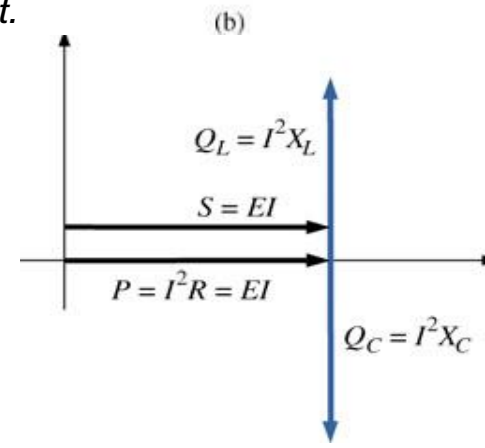
- The basic format of the series resonant circuit is a series R-L-C combination in series with an applied voltage source.



Series resonant circuit.

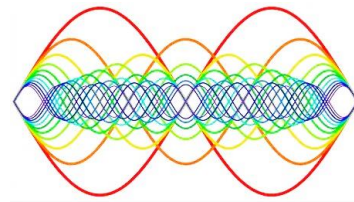


Phasor diagram for the series resonant circuit at resonance.



Power triangle for the series resonant circuit at resonance.

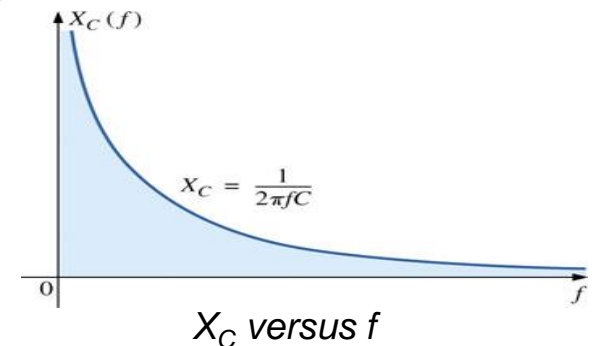
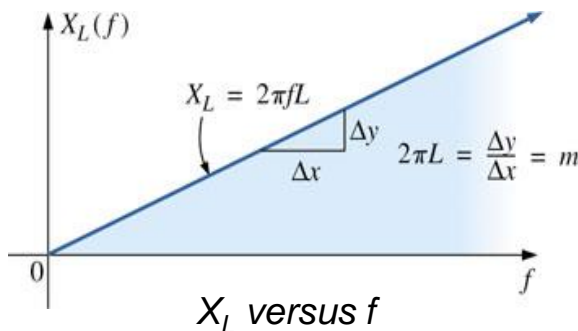
# $Z_T$ Versus Frequency



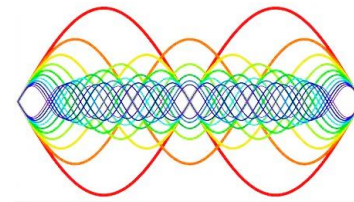
- Knowing that  $X_C$  and  $X_L$  are dependent upon frequency it can be stated:
  - Capacitor Impedance decreases as frequency increases.
  - Inductor Impedance increases as frequency increases.
- This implies that the total impedance of the series  $R$ - $L$ - $C$  circuit below, at any frequency, is determined by:

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z_T = R + jX_L - jX_C$$



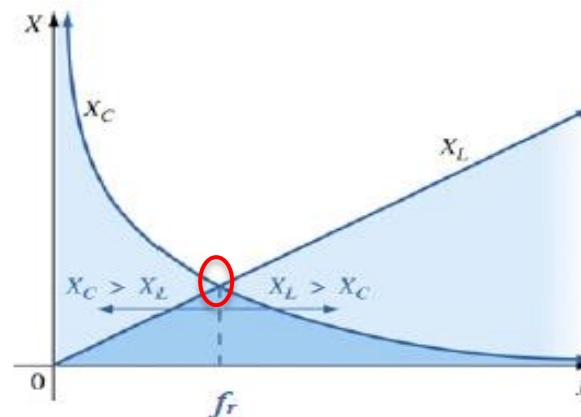
# $Z_T$ Versus Frequency cont.



- The total-impedance-versus-frequency curve for the series resonant circuit below can be found by applying the impedance-versus-frequency curve for each element of the equation previously shown, written in the following form:

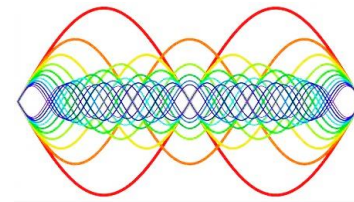
$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$

- When  $X_L = X_C$  the resonant frequency ( $f_r$ ) can be found.



Frequency response of  $X_L$  and  $X_C$  of a series R-L-C circuit on the same set of axes

# The Resonant Frequency ( $f_r$ )



- To find  $f_r$ , set the impedances equal and solve:

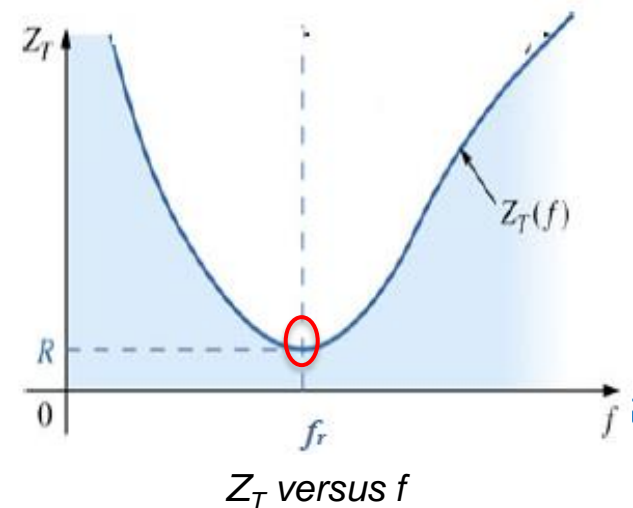
$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow \omega^2 = \frac{1}{LC}$$

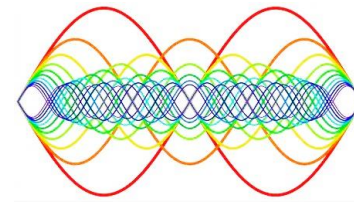
$$\omega = \frac{1}{\sqrt{LC}}, \text{ since } \omega = 2\pi f$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- This is the key equation for resonance. Total impedance at this point is shown to the right:

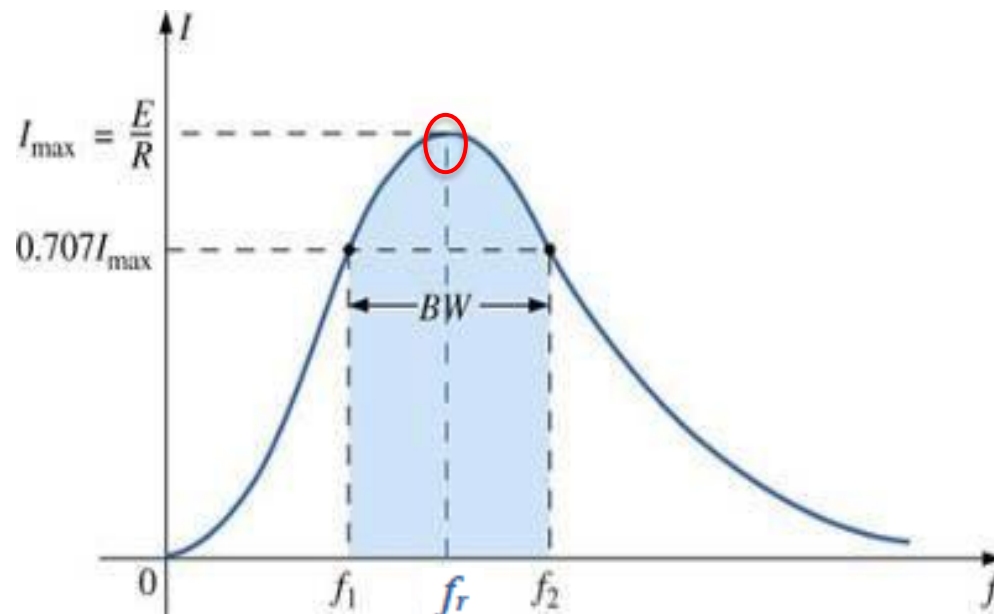


# Current Versus Frequency



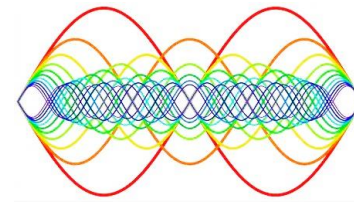
- If impedance is minimum at  $f_r$  current will be at a maximum:
- If we now plot the magnitude of the current versus frequency for a *fixed* applied voltage  $E$ , we obtain the curve showing that current is maximum at  $f_r$ :

$$I = \frac{E}{Z_T}$$



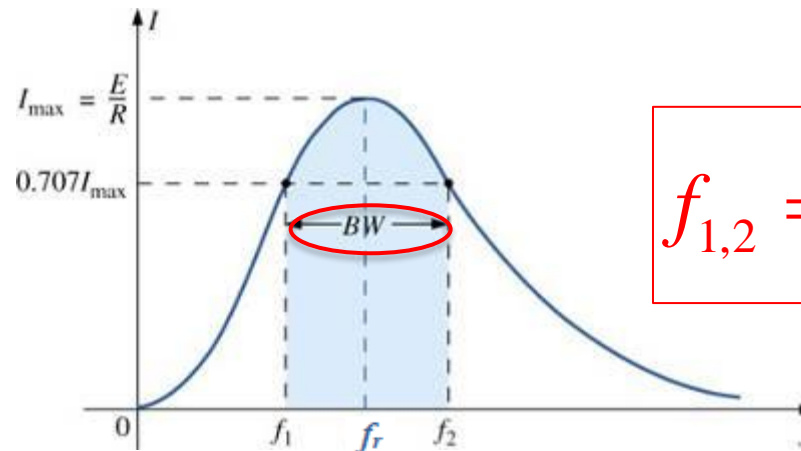
$I$  versus  $f$  for the series resonant circuit

# Bandwidth (BW)

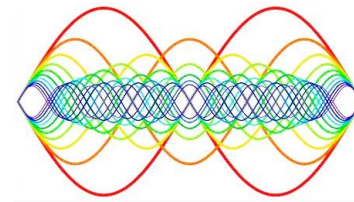


- ❑ Band frequencies are those that define the points on the resonance curve that are 0.707 (  $\frac{1}{\sqrt{2}} = 0.707$  ) of the peak current or voltage.
- ❑ Bandwidth (BW) is the range of frequencies between the band, or  $\frac{1}{2}$  power frequencies. Defined by:

$$BW = f_2 - f_1$$



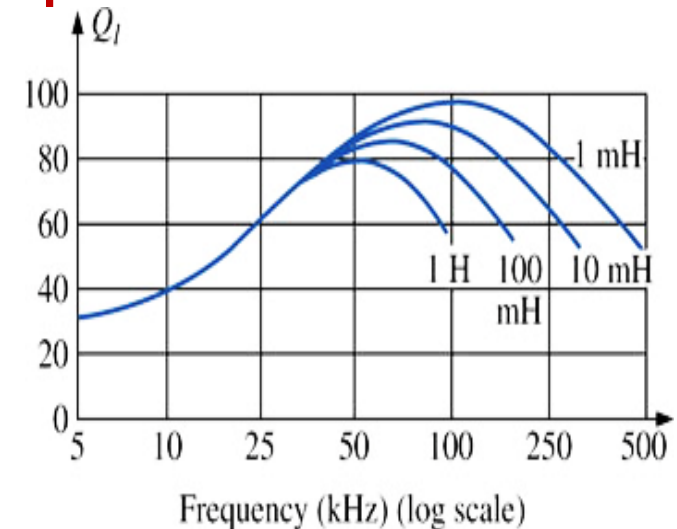
$$f_{1,2} = f_r \pm \frac{BW}{2}$$



# The Quality Factor (Q)

- The **quality factor (Q)** of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance.
- Q can be found several ways:

$$Q = \frac{f_r}{BW} = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$$



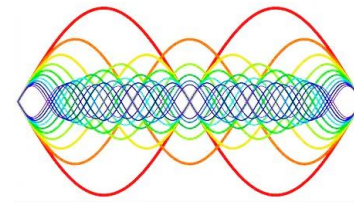
$Q_l$  versus frequency for a series of inductors

- This also gives an alternate way to find BW:

$$BW = \frac{f_r}{Q}$$

# OUTLINES

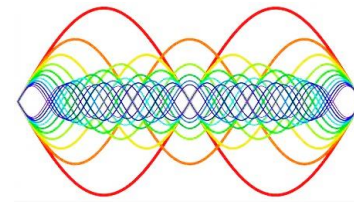
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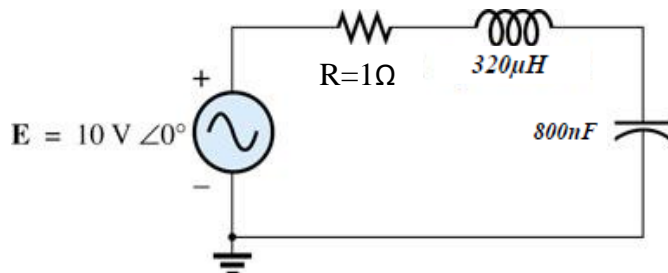
- ❑ **AC Power and Power Triangle**
- ❑ **AC Power Factor and Power Factor Correction**
- ❑ **Resonance.**
  - **Series Resonant Circuit**
  - **Parallel Resonant Circuit**



# Example Problem 6



Determine  $f_r$ ,  $Q$ ,  $BW$  and the current ( $I$ ) at resonance.  
Plot the current vs. frequency and label  $f_r$ ,  $f_1$ ,  $f_2$  and  $BW$ .



We know  $X_L$  so we can find  $Q$ :

$$Q = \frac{X_L}{R} = \frac{20\Omega}{1\Omega} = 20$$

We know  $Q$  so we can find  $BW$ :

$$BW = \frac{f_r}{Q} = \frac{10\text{kHz}}{20} = 500\text{Hz}$$

Now find  $I_{\max}$ :

$$I = \frac{E}{Z_T} = \frac{10\text{V}}{1\Omega} = 10\text{A}$$

Remember, since  $X_L = X_C$  at resonance, they cancel out for  $Z_T$  and only  $R$  is left.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(320\mu\text{H})(800\text{nF})}} = 10\text{kHz}$$

$$X_L = \omega_r L = 2\pi f_r L = 2\pi(10\text{kHz})(320\mu\text{H}) = 20\Omega$$

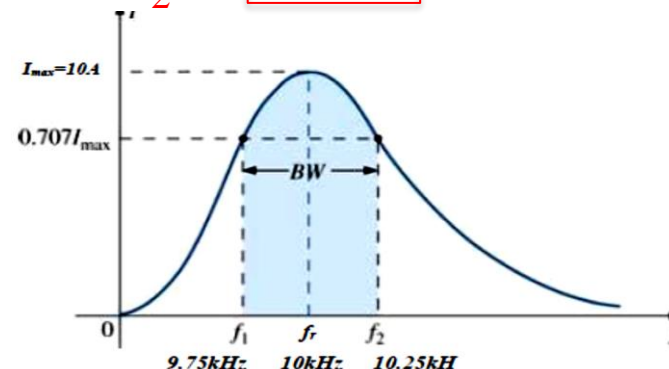
Because we are at  $f_r$  we know that  $X_L = X_C$ , but just to show it:

$$X_C = \frac{1}{\omega_r C} = \frac{1}{2\pi f_r C} = \frac{1}{2\pi(10\text{kHz})(800\text{nH})} = 20\Omega$$

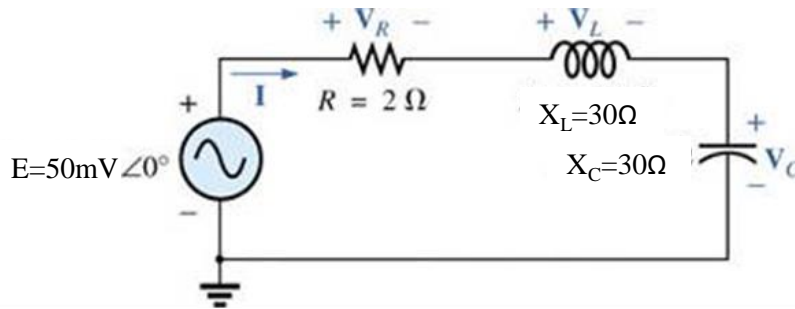
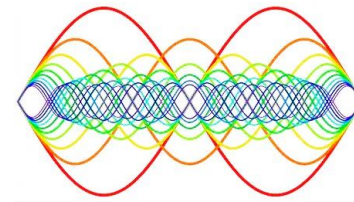
Now let's find  $f_1$  and  $f_2$  and then plot:

$$f_{1,2} = f_r \pm \frac{BW}{2} \Rightarrow f_2 = 10\text{kHz} + \frac{500\text{Hz}}{2} = 10.25\text{kHz}$$

$$\Rightarrow f_1 = 10\text{kHz} - \frac{500\text{Hz}}{2} = 9.75\text{kHz}$$



# Example Problem 7 [TUTORIAL]



- Find  $I$ ,  $V_R$ ,  $V_C$ ,  $V_L$  at resonance.
- Determine  $Q$  for the circuit.
- If the  $f_r$  is 5 kHz, what is the  $BW$ ?
- With  $f_r = 5\text{kHz}$  what are the values of  $L$  and  $C$ ?
- What is the power dissipated in the circuit at the *half-power* frequency?

$$a) Z_T = R + jX_L - jX_C = 2\Omega + j30\Omega - j30\Omega = 2\Omega$$

$$I = \frac{E}{Z_T} = \frac{50\text{mV} \angle 0^\circ}{2\Omega \angle 0^\circ} = \boxed{25\text{mA} \angle 0^\circ}$$

$$V_R = I * R = (25\text{mA} \angle 0^\circ) * (2\Omega \angle 0^\circ) = \boxed{50\text{mV} \angle 0^\circ}$$

$$V_L = I * X_L = (25\text{mA} \angle 0^\circ) * (30\Omega \angle 90^\circ) = \boxed{750\text{mV} \angle 90^\circ}$$

$$V_C = I * X_C = (25\text{mA} \angle 0^\circ) * (30\Omega \angle -90^\circ) = \boxed{750\text{mV} \angle -90^\circ}$$

This shows that at resonance  $V_C = V_L$

$$b) Q = \frac{X_L}{R} = \frac{30\Omega}{2\Omega} = \boxed{15}$$

$$c) BW = \frac{f_r}{Q} = \frac{5\text{kHz}}{15} = \boxed{333.3\text{Hz}}$$

$$d) X_L = 2\pi f_r L \Rightarrow$$

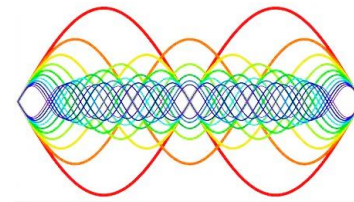
$$L = \frac{X_L}{2\pi f_r} = \frac{30\Omega}{2\pi(5\text{kHz})} = \boxed{955\mu\text{H}}$$

$$X_C = \frac{1}{2\pi f_r C} \Rightarrow$$

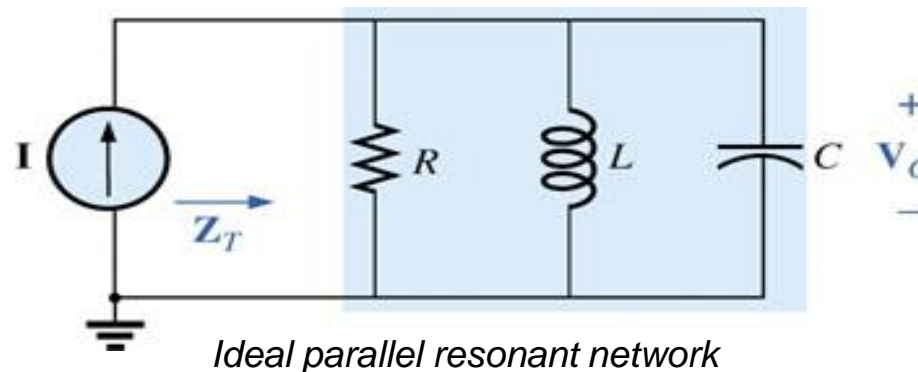
$$C = \frac{1}{2\pi f_r X_C} = \frac{1}{2\pi(5\text{kHz})(30\Omega)} = \boxed{1.06\mu\text{F}}$$

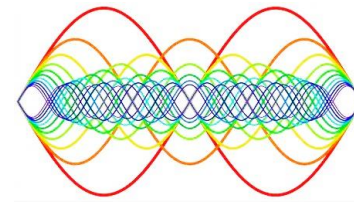
$$e) P = I_{0.707}^2 * R = (25\text{mA} * 0.707) * (2\Omega) = \boxed{35.4\text{mW}}$$

# Parallel Resonant Circuit



- ❑ The basic format of the parallel resonant circuit is a parallel R-L-C combination with an applied current source.
- ❑ The parallel resonant circuit has the basic configuration shown below:





# Parallel Resonant Circuit

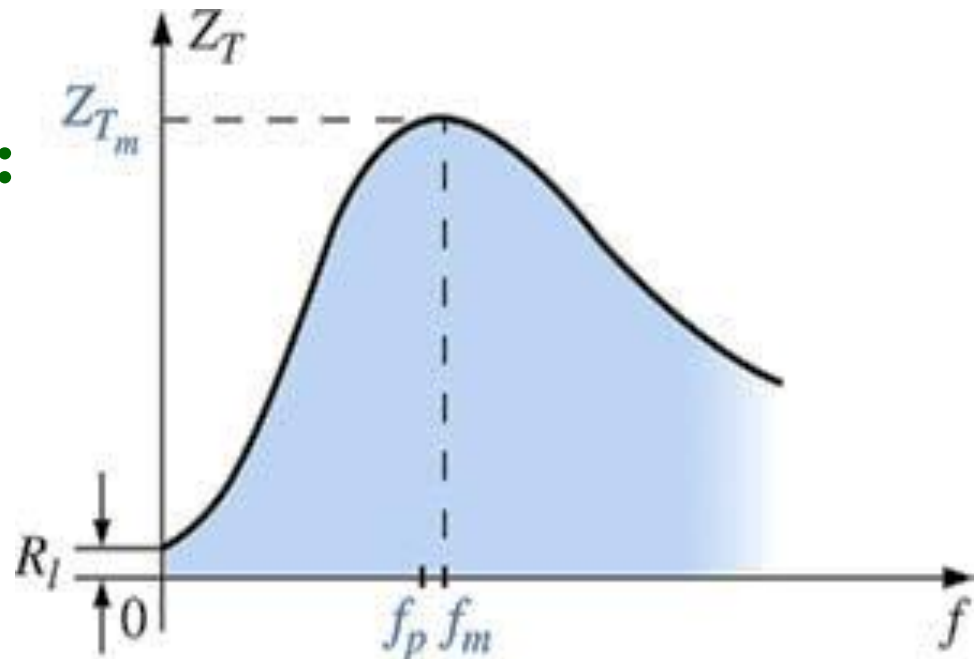
- **Unity Power Factor,  $f_p$ :**

$$f_p = f_r \sqrt{1 - \frac{R_l^2 C}{L}}$$

- **Maximum Impedance,  $f_m$ :**

$$f_m = f_r \sqrt{1 - \frac{1}{4} \left( \frac{R_l^2 C}{L} \right)}$$

$$f_r > f_p > f_m$$



$Z_T$  versus frequency for the parallel resonant circuit