New Mansoura University

Tutorial 4

Problem 1

Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

a. Find $P(X \le 2)$.

b. Find $P(X > 1)$.

c. Find μ_X.

d. Find σ_X^2.

a. $p(x \le 2) = P(0) + P(1) + P(2) = 0.4 + 0.3 + 0.15 = 0.85$

b. $p(x > 1) = P(2) + P(3) + P(4)$ = 0.15+0.1+0.05 = 0.3

c. $\mu_x = 0 * 0.4 + 1 * 0.3 + 2 * 0.15 + 3 * 0.1 + 4 * 0.05 = 1.1$

d. $\sigma_x^2 = \mu_{x^2} - (\mu_x)^2$

$\mu_{x^2} = 0^2 * 0.4 + 1^2 * 0.3 + 2^2 * 0.15 + 3^2 * 0.1 + 4^2 * 0.05 = 2.6$

$$\sigma_x^2 = 2.6 - (1.1)^2 = 1.39$$

Problem 2

Let X represent the number of tires with low air pressure on a randomly chosen car.

a. Which of the three functions below is a possible probability mass function of X? Explain.

	x				
	0	**1**	**2**	**3**	**4**
$p_1(x)$	0.2	0.2	0.3	0.1	0.1
$p_2(x)$	0.1	0.3	0.3	0.2	0.2
$p_3(x)$	0.1	0.2	0.4	0.2	0.1

$$\sum_{i=1}^{5} p_1(x_i) = 0.2 + 0.2 + 0.3 + 0.1 + 0.1 \neq 1$$

$$\sum_{i=1}^{5} p_2(x_i) = 0.1 + 0.3 + 0.3 + 0.2 + 0.2 \neq 1$$

$$\sum_{i=1}^{5} p_3(x_i) = 0.1 + 0.2 + 0.4 + 0.2 + 0.1 = 1$$

Problem 3

A survey of cars on a certain stretch of highway during morning commute hours showed that 70% had only one occupant, 15% had 2, 10% had 3, 3% had 4, and 2% had 5. Let X represent the number of occupants in a randomly chosen car.

a. Find the probability mass function of X.

b. Find $P(X \leq 2)$.

c. Find $P(X > 3)$.

d. Find μ_X.

e. Find σ_X.

(a)

x	1	2	3	4	5
$p(x)$	0.70	0.15	0.10	0.03	0.02

(b) 0.85

(c) 0.05

(d) 1.52

(e) 0.9325

Problem 4

Ex 2:- A pair of fair dice is thrown. Let x be the random variable which denotes maximum of two numbers. Find

1- Probability mass , graph it.

2- Cumulative distribution , graph it.

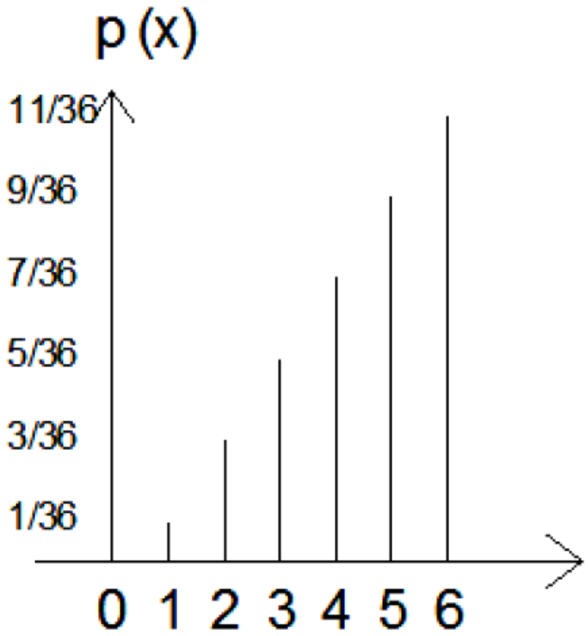
3- Mean , variance , standard deviation

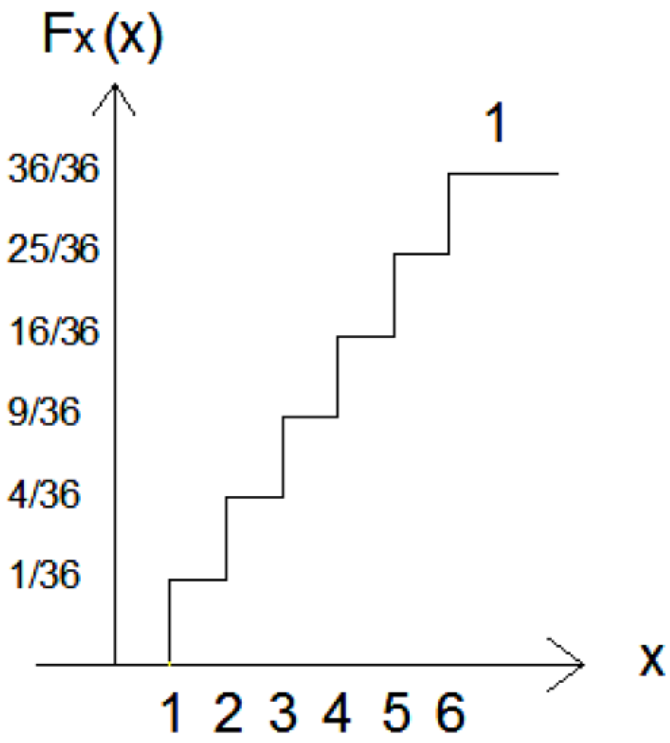
4- P ($1 \le x < 4$)

x	1	2	3	4	5	6
P(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
$F_x(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	$\frac{36}{36}$

1	2	3	4	5	6
2	2	3	4	5	6
3	3	3	4	5	6
4	4	4	4	5	6
5	5	5	5	5	6
6	6	6	6	6	6

Problem 4


p (x)
11/36
9/36
7/36
5/36
3/36
1/36
0 1 2 3 4 5 6


Fx (x)
1
36/36
25/36
16/36
9/36
4/36
1/36
x
1 2 3 4 5 6

Problem 4

- Mean $\equiv$ E (x) = $\Sigma\, x_i\, p_i$

  $= 1*\frac{1}{36} + 2*\frac{3}{36} + 3*\frac{5}{36} + 4*\frac{7}{36} + 5*\frac{9}{36} + 6*\frac{11}{36}$

  = 4.4722

- $E(x^2) = \Sigma\, x_i^2\, p(x_i)$

  $= 1^2*\frac{1}{36} + 2^2*\frac{3}{36} + 3^2*\frac{5}{36} + 4^2*\frac{7}{36} + 5^2*\frac{9}{36} + 6^2*\frac{11}{36}$

  = 21.9722

- $V(x) = E(x^2) - (E_{(x)})^2 = 21.9722 - (4.4722)^2$

  = 1.9716

- $\sigma_{(x)} = \sqrt{V(x)} = \sqrt{1.9716} = 1.4$

- $P(1 \leq x < 4) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} = \frac{9}{36}$

Problem 5

After manufacture, computer disks are tested for errors. Let X be the number of errors detected on a randomly chosen disk. The following table presents values of the cumulative distribution function $F(x)$ of X.

x	$F(x)$
0	0.41
1	0.72
2	0.83
3	0.95
4	1.00

a. What is the probability that two or fewer errors are detected?

b. What is the probability that more than three errors are detected?

c. What is the probability that exactly one error is detected?

d. What is the probability that no errors are detected?

Problem 5

(a) $P(X \le 2) = F(2) = 0.83$

(b) $P(X > 3) = 1 - P(X \le 3) = 1 - F(3) = 1 - 0.95 = 0.05$

(c) $P(X = 1) = P(X \le 1) - P(X \le 0) = F(1) - F(0) = 0.72 - 0.41 = 0.31$

(d) $P(X = 0) = P(X \le 0) = F(0) = 0.41$

Problem 6

A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$P(X = x)$	a	a	a	b	b	0.3

(a) Given that $E(X) = 4.2$ find the value of a and the value of b.

(b) Show that $E(X^2) = 20.4$

Problem 7

$$\sum_{i=1}^{6} p(x_i) = 1 \;\; \ldots\ldots a + a + a + b + b + 0.3 = 1$$

$$E(x) = \sum_{i=1}^{6} x\, p(x_i) = \;\; a + 2a + 3a + 4b + 5b + b + 0.3 * 6 = 4.2$$

$$6a + 9\,b = 2.4 \qquad\qquad \& \quad 3a + 2b = 0.7$$

$$A) \qquad\qquad a = 0.1 \qquad\qquad \& \quad b = 0.2$$

$$B) \quad E(x^2) = \sum_{i=1}^{6} x^2\, p(x_i)$$

$$= \quad 0.1 + 2^2 * 0.1 + 3^2 * 0.1 + 4^2 * 0.2 + 5^2 * 0.2 + 6^2 * 0.3 = 20.4$$

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Thank You