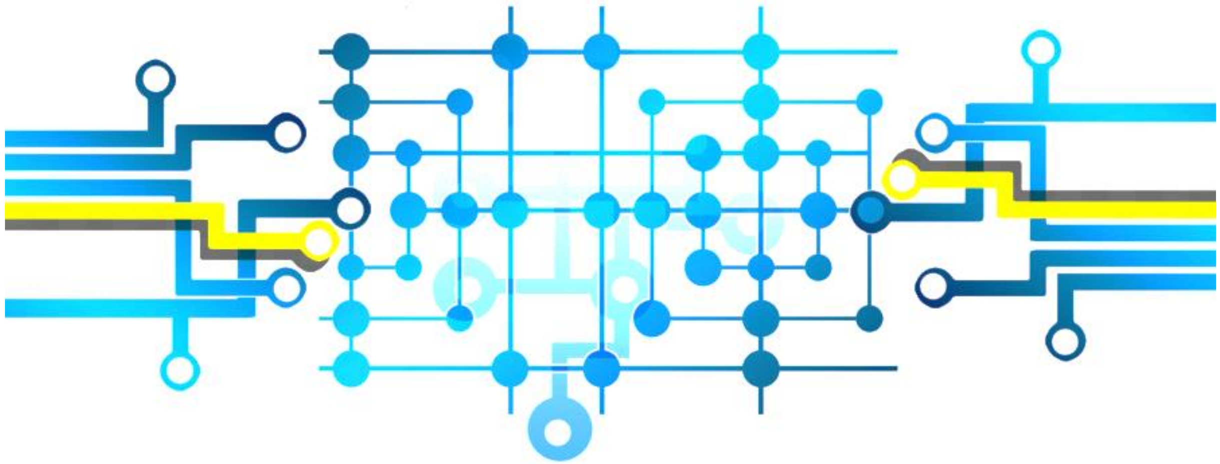
Circuit Analysis and Circuit Theorems

CSE 113



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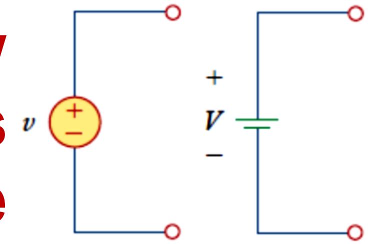
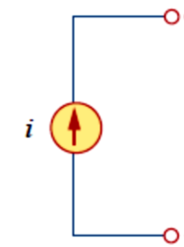
OUTLINES

- Independent Vs. Dependent Sources
- Kirchhoff's Laws
- Nodal Analysis
- Supernode
- Mesh Analysis
- Super-Mesh
- Circuit Theorems
  - Circuit linearity
  - Superposition
  - Source transformation
  - Thevenin’s theorem
  - Norton theorem

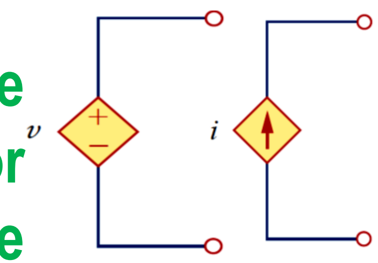
Independent Vs. Dependent Sources



- Sources in which the voltage is completely independent of the current, or the current is completely independent of the voltage; these are termed independent sources.
  - An independent voltage source is characterized by the terminal voltage which is completely independent of the current through it.

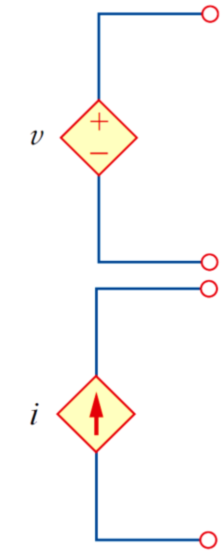


  - For an independent current source, the current through the element is completely independent of the voltage across it.

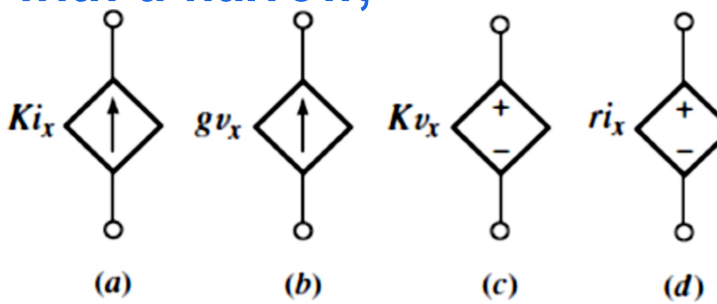


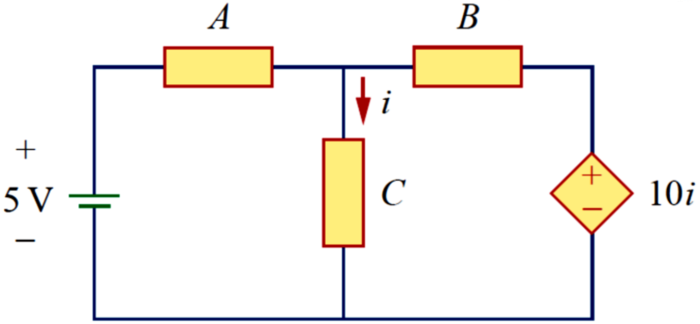
- Particlular kinds of sources for which either the source voltage or current depends upon current or voltage elsewhere in the circuit; such sources are referred to as dependent sources.

Dependent Sources



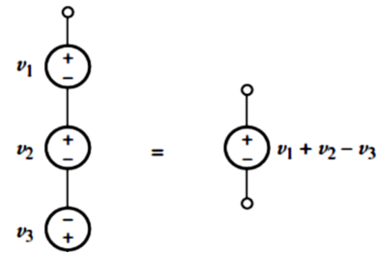
- Dependent sources are useful in modelling elements such as transistors, operational amplifiers, and integrated circuits.
- Dependent sources are usually designated by diamond-shaped symbols.
- Voltage source comes with polarities (+ −) in its symbol, while a current source comes with a narrow, irrespective of what it depends on.
- The four different types of dependent sources:
  - (a) current-controlled current source;
  - (b) voltage-controlled current source;
  - (c) voltage-controlled voltage source;
  - (d) current controlled voltage source.



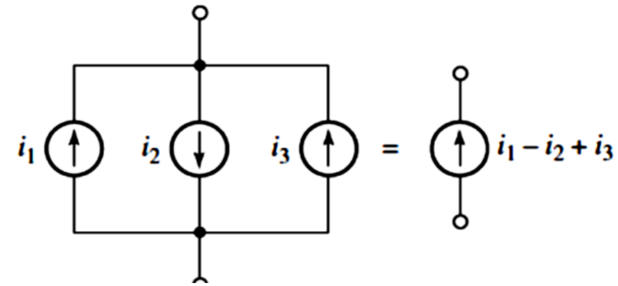


Series and parallel connection of sources

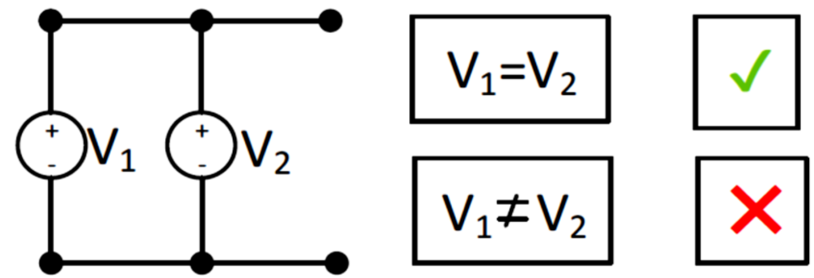
- **Series-connected voltage sources can be replaced by a single source.**



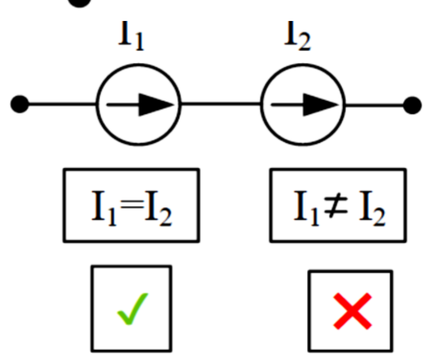
- **Parallel current sources can be replaced by a single source.**



- **Voltage Sources in Parallel**



- **Current Sources in Series**



Kirchhoff's Laws

- Ohm's law is not sufficient for circuit analysis.
- Kirchhoff's laws complete the needed tools.

There are two laws:

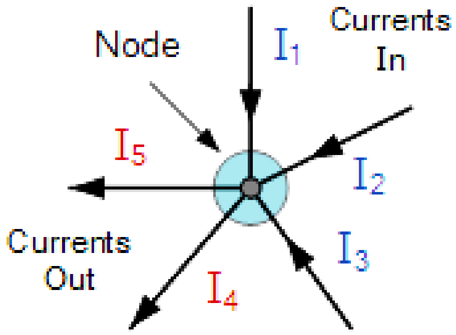
  - Current law.
  - Voltage law.

Kirchhoff's current law (KCL)

- Kirchhoff's current law is based on conservation of charge.
- It states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- It can be expressed as:

$$\sum_{n=1}^{N} i_n = 0$$

Currents Entering the Node
Equals
Currents Leaving the Node

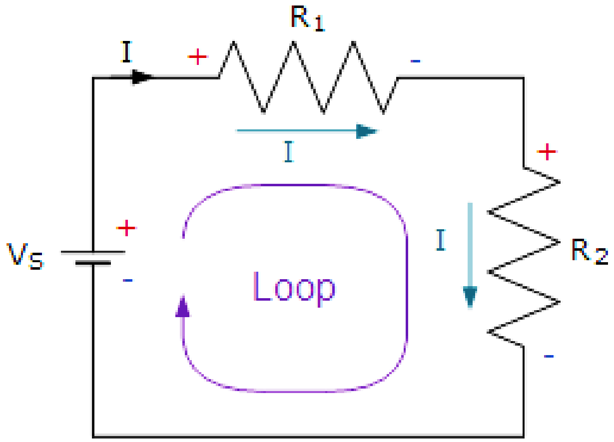


$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$

Kirchhoff's voltage law (KVL)

- Kirchhoff's voltage law is based on conservation of energy.
- It states that the algebraic sum of currents around a closed path (or loop) is zero.
- It can be expressed as

$$\sum_{m=1}^{M} v_m = 0$$



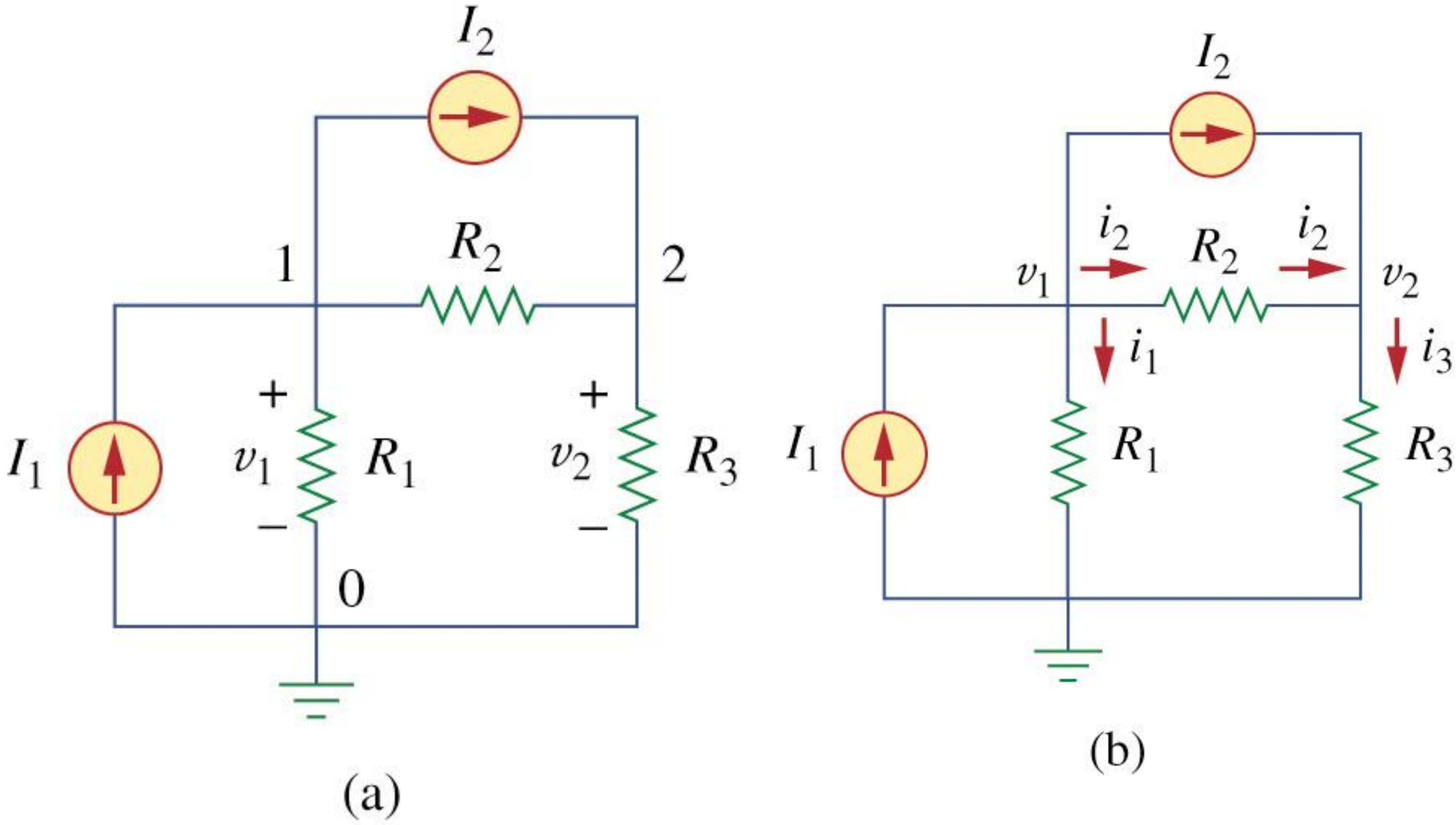
Nodal Analysis

Steps to Determine Node Voltages:

- Select a node as the reference node. Assign voltage v_1, v_2, …v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
- Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
- Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 2

- Typical circuit for nodal analysis



Example 2

$$I_1 = I_2 + i_1 + i_2$$

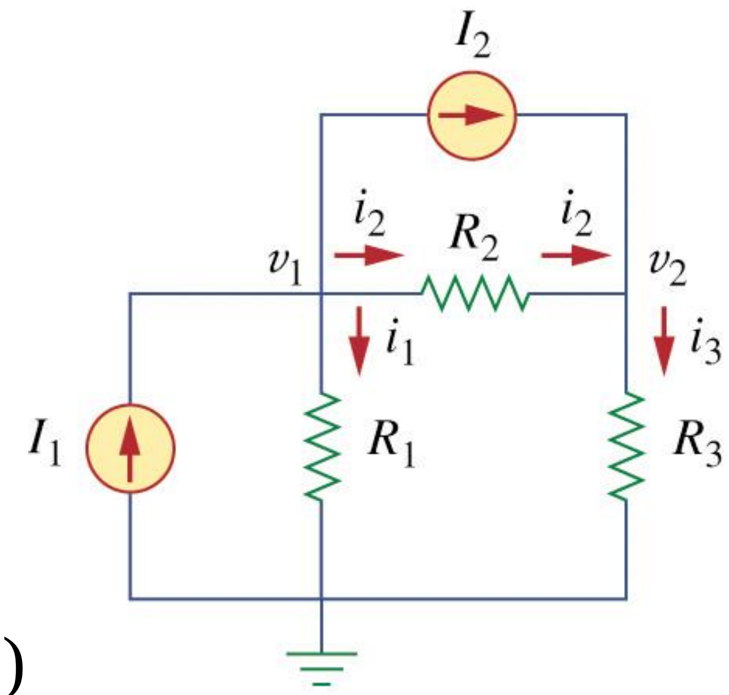
$$I_2 + i_2 = i_3$$

$$\boxed{i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}}$$

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2 (v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$



(b)

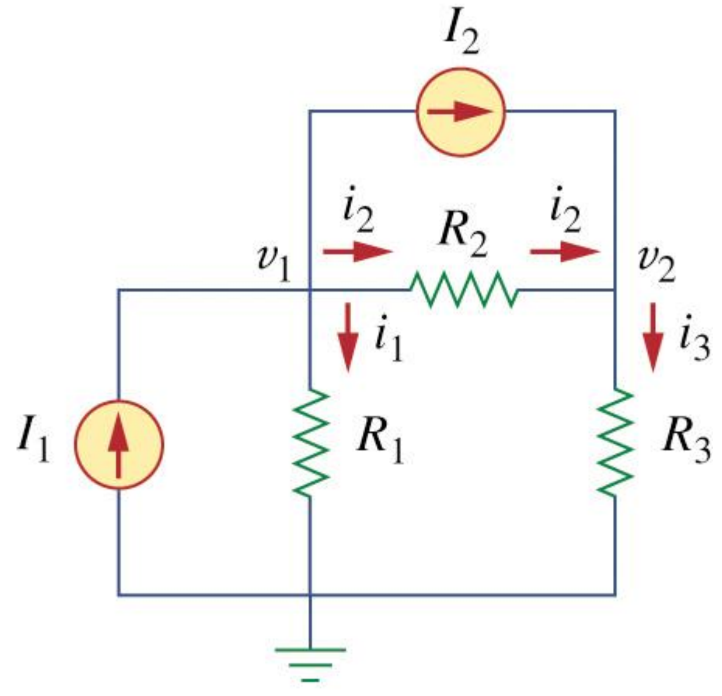
Example 2

$$\Rightarrow I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

$$\Rightarrow I_1 - I_2 = G_1 v_1 + G_2 (v_1 - v_2)$$

$$I_2 = -G_2 (v_1 - v_2) + G_3 v_2$$



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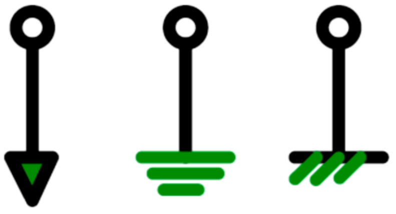
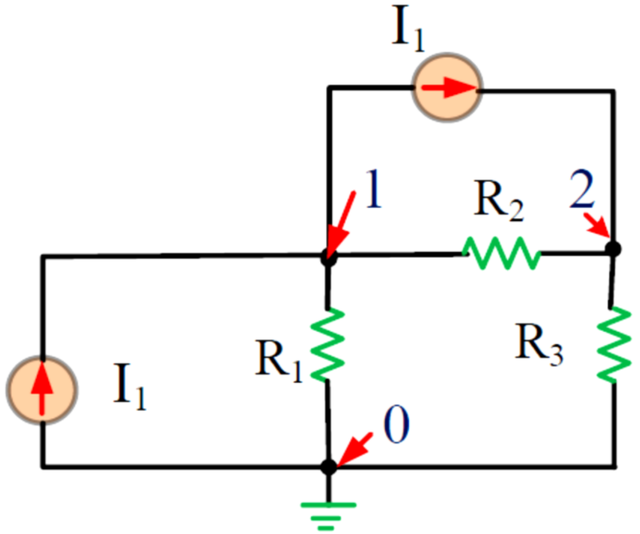
Nodal Analysis with Voltage Sources

- **Case 1:** The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.



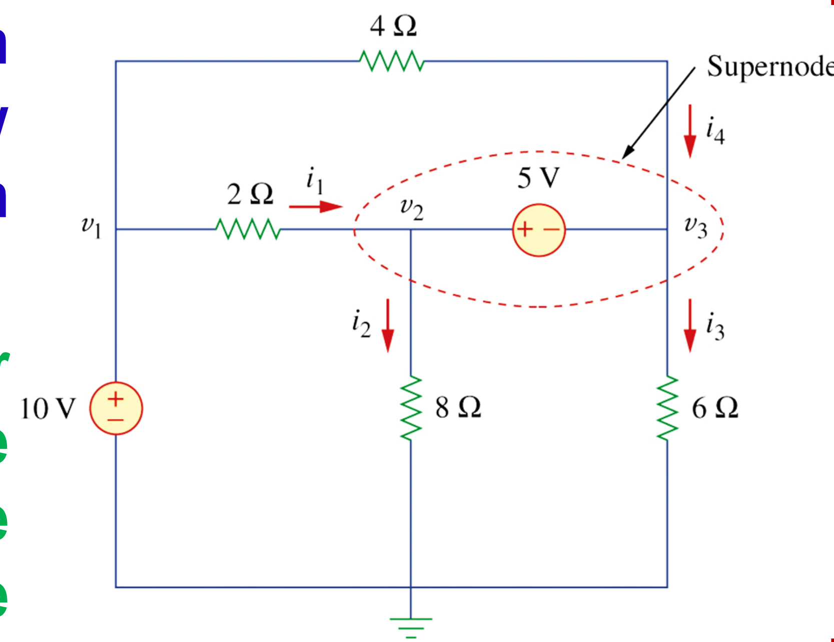
- **Case 2:** The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

- Node 0 is Reference node
- Nodes 1, 2 are Non-Reference node



Supernode

- **A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.**
- **The required two equations for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the relationship of node voltages due to the voltage source.**



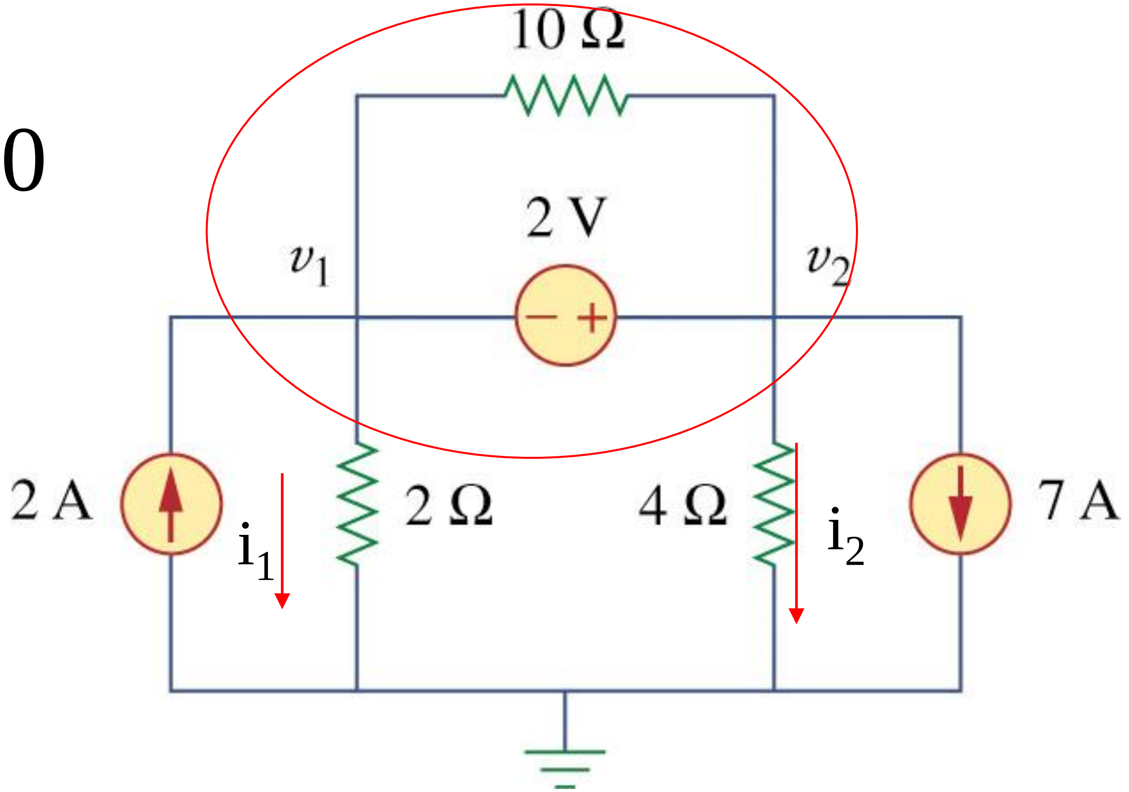
Example 1

- For the circuit shown in Fig, find the node voltages.

$$2 - 7 - i_1 - i_2 = 0$$

$$2 - 7 - \frac{v_1}{2} - \frac{v_2}{4} = 0$$

$$v_1 - v_2 = -2$$

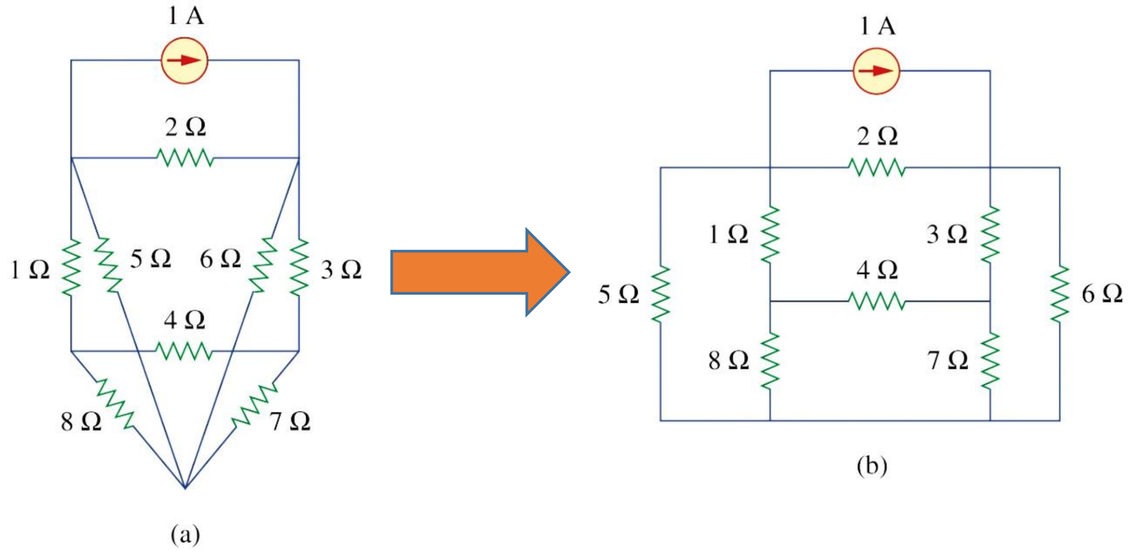


Mesh Analysis

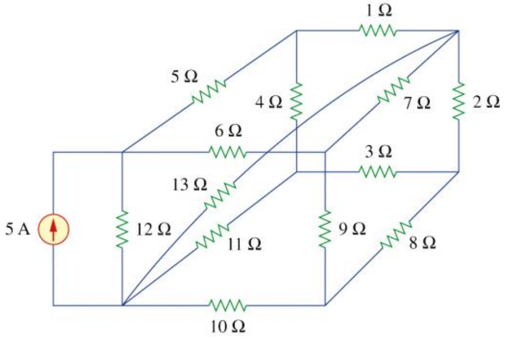
- Mesh analysis helps us to solve complex electrical networks
- Loop: It is a closed path with no node passed more than once.
- Mesh: A mesh is a loop that does not contain any other loop within it.
- Mesh analysis is only applicable to a circuit that is planar.
- Planar Circuit: A planar circuit is one that can be drawn in a plane with no branches crossing one another.
- Non-Planar Circuit: A nonplanar circuit is one that can be drawn in a plane with branches crossing one another.

Mesh Analysis

- A Planar circuit with crossing branches,
- The same circuit redrawn with no crossing branches.

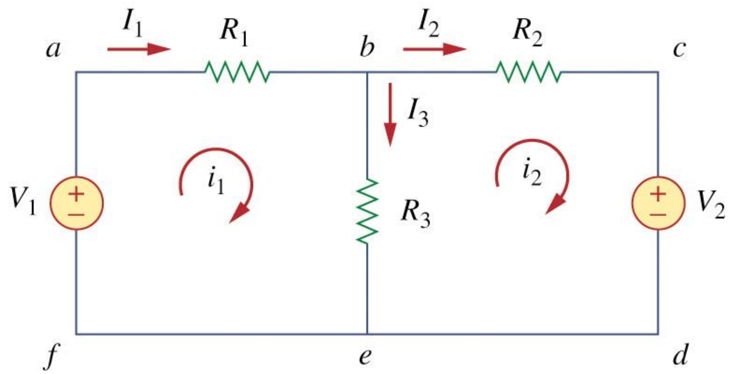


- A nonplanar circuit.



Mesh Analysis

- **Steps to Determine Mesh Currents:**
  - Assign mesh currents $i_1, i_2, .., i_n$ to the n meshes.
  - Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
  - Solve the resulting n simultaneous equations to get the mesh currents.



- **A circuit with two meshes.**
  - Apply KVL to each mesh. For mesh 1,

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

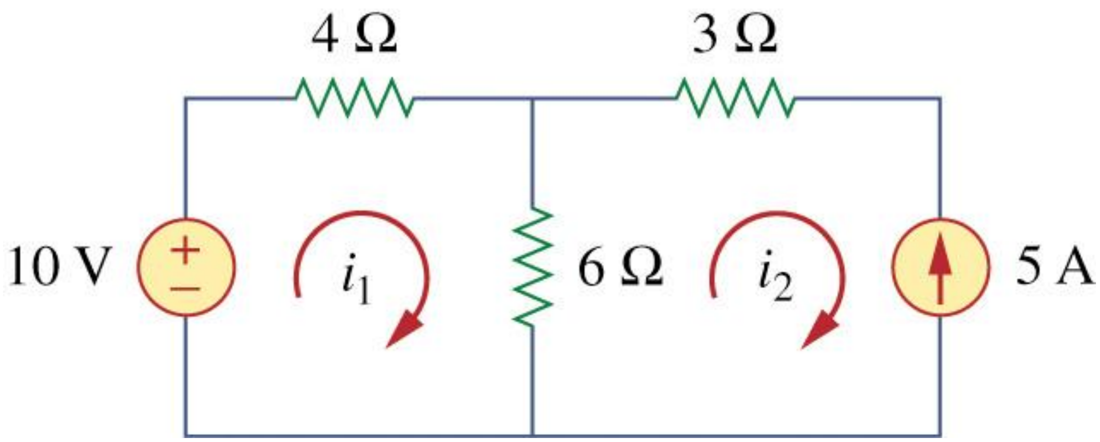
  - For mesh 2,

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

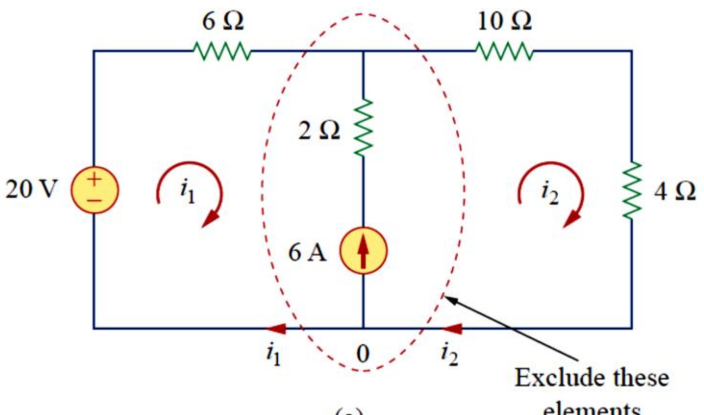
$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

Mesh Analysis with Current Sources

- Two cases arises in these circuits,
  - Current source exist in arm of a mesh



  - Current source exist in between meshes.



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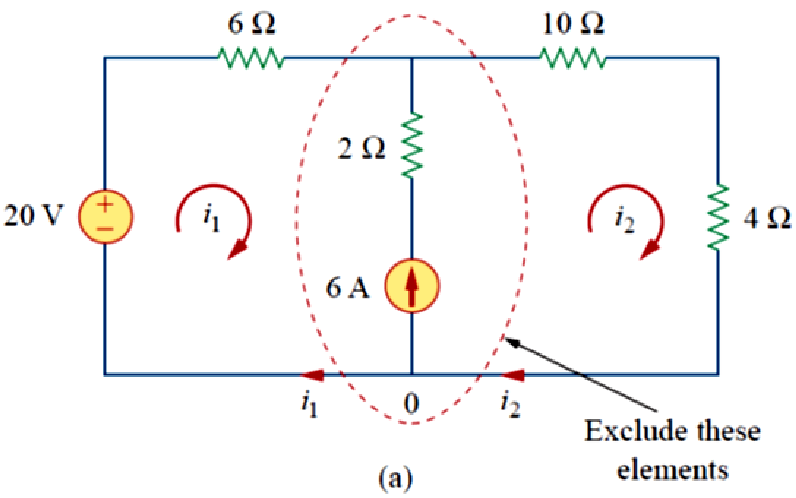
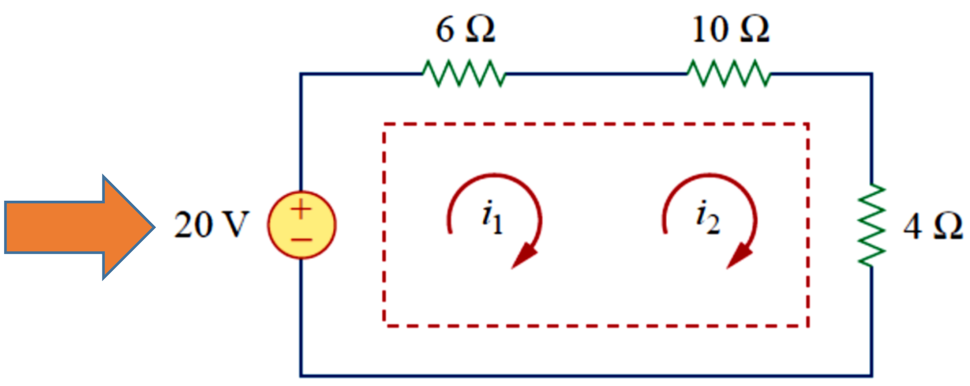
Super Mesh Concept

- a super-mesh results when two meshes have a (dependent, independent) current source in common.
- We create a super-mesh by excluding the current source and any elements connected in series with it while taking KVL.





Super Mesh Concept

- **Applying KVL to the super-mesh.**

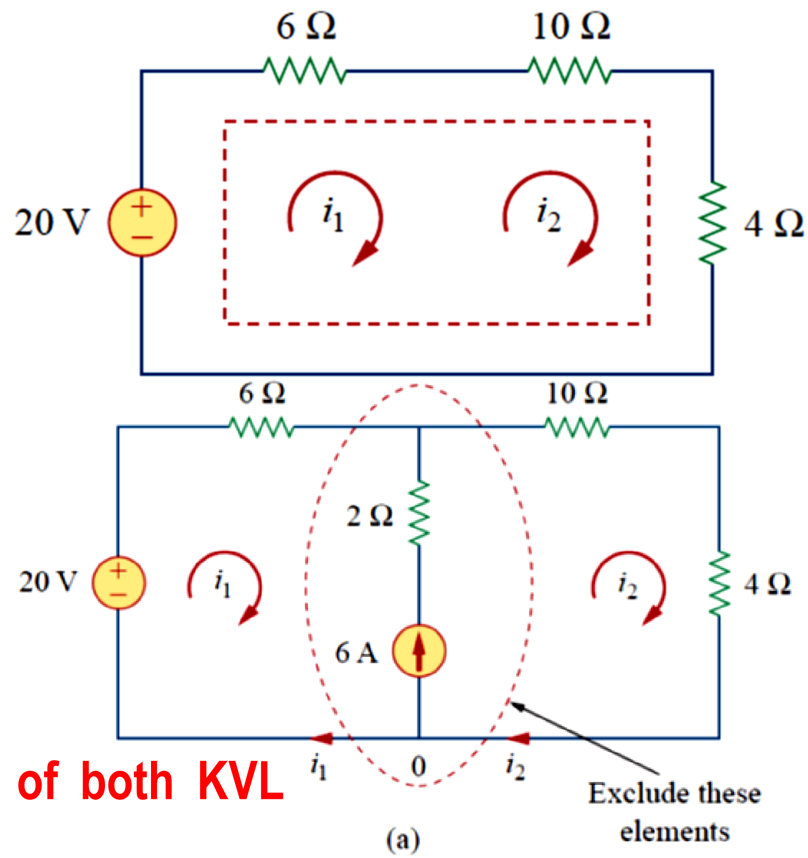
$$6i_1 + 10i_2 + 4i_2 = 20$$

$$6i_1 + 14i_2 = 20$$

- **applying KCL at bottom node,**

$$i_1 - i_2 = -6$$

- **A super-mesh requires the application of both KVL and KCL.**
- **Similarly, a super-mesh formed from three meshes needs three equations: one is from the super-mesh and the other two equations are obtained from the two current sources.**



NODAL ANALYSIS VS MESH ANALYSIS

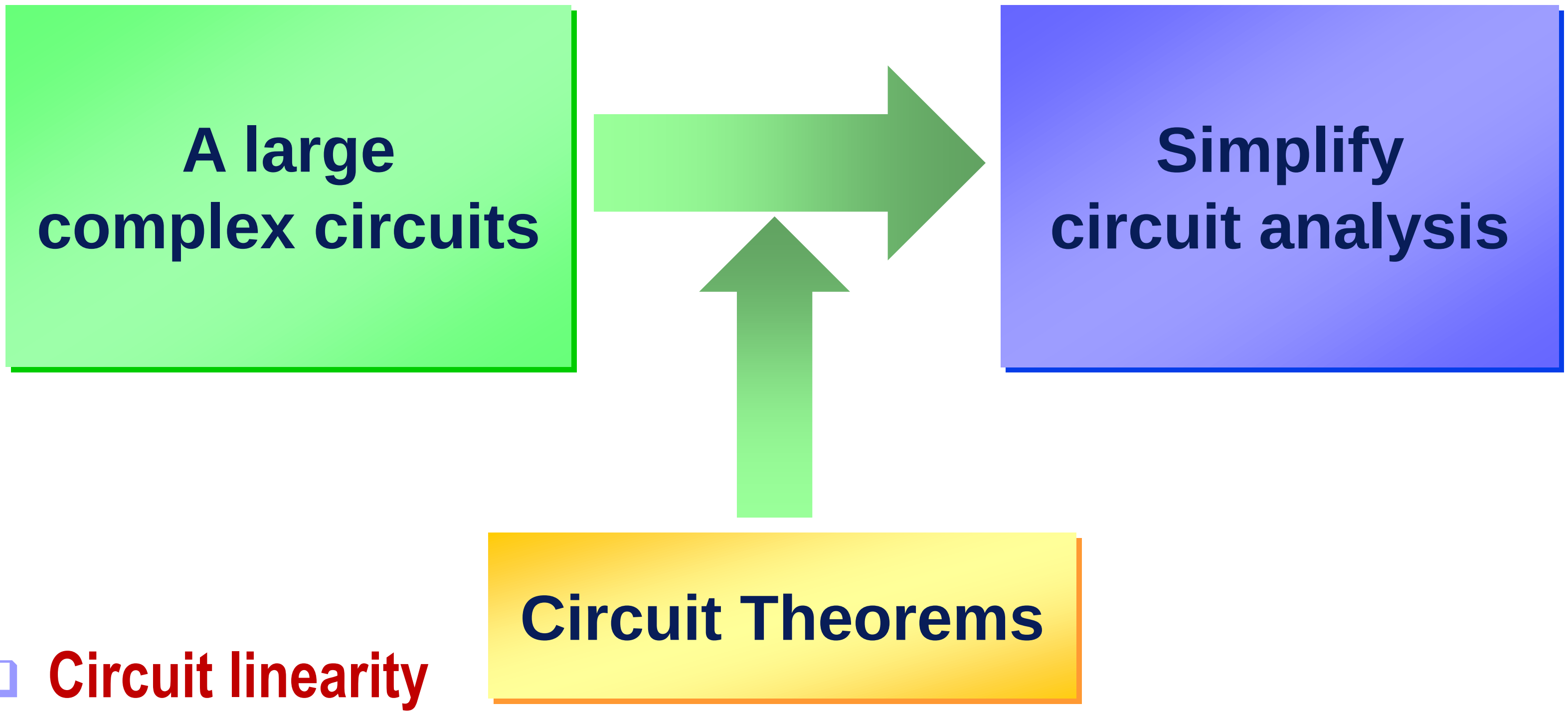
Nodal Analysis

- The number of voltage variables, and hence simultaneous equations to solve, equals the number of nodes minus one.
- Every voltage source connected to the reference node reduces the number of unknowns by one.
- Nodal analysis is thus best for circuits with voltage sources.

Mesh analysis

- The number of current variables, and hence simultaneous equations to solve, equals the number of meshes.
- Every current source in a mesh reduces the number of unknowns by one.
- Mesh analysis is thus best for circuits with current sources.

Circuit Theorems



- Circuit linearity
- Superposition
- Source transformation
- Thevenin’s theorem
- Norton theorem

Linearity Property

The property is a combination of both the homogeneity (scaling) property and the additivity property.

- **The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.**

$$V = IR \rightarrow KV = KIR$$

- **A linear circuit is one whose output is linearly related (or directly proportional) to its input.**

Linearity Property

- The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

$$v_1 = i_1 R \quad \textbf{and} \quad v_2 = i_2 R$$

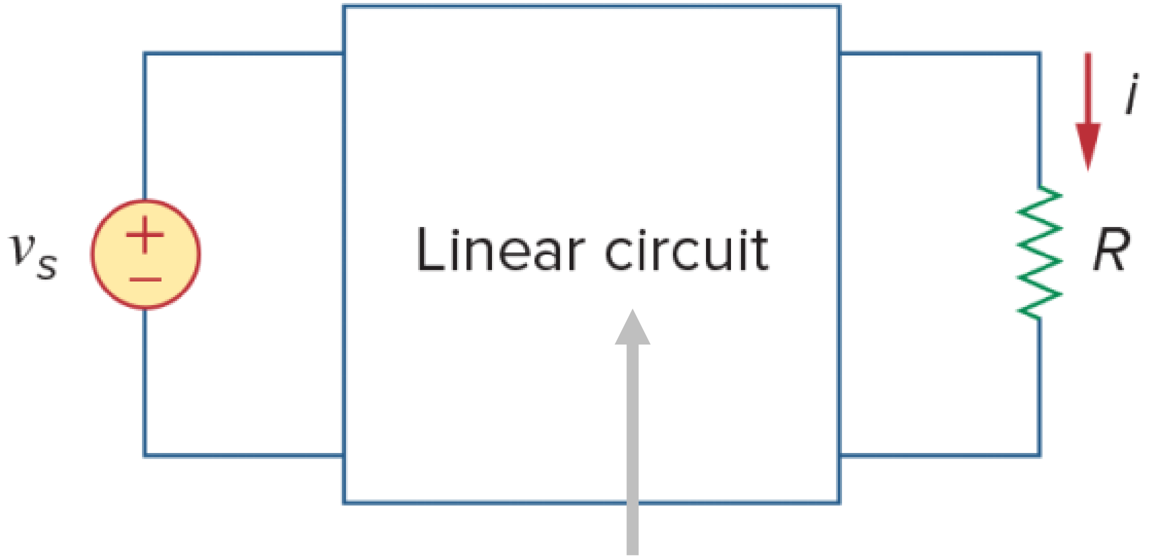
then applying $(i_1 + i_2)$ gives

$$v = \boxed{(i_1 + i_2)R} = i_1 R + i_2 R = v_1 + v_2$$

- We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

Linearity Property

- A linear circuit is one whose output is linearly related (or directly proportional) to its input



the input

the output

The circuit is terminated by a load R

has no independent sources inside it

- Example

$$v_s = 10\text{V} \rightarrow i = 2\text{A}$$

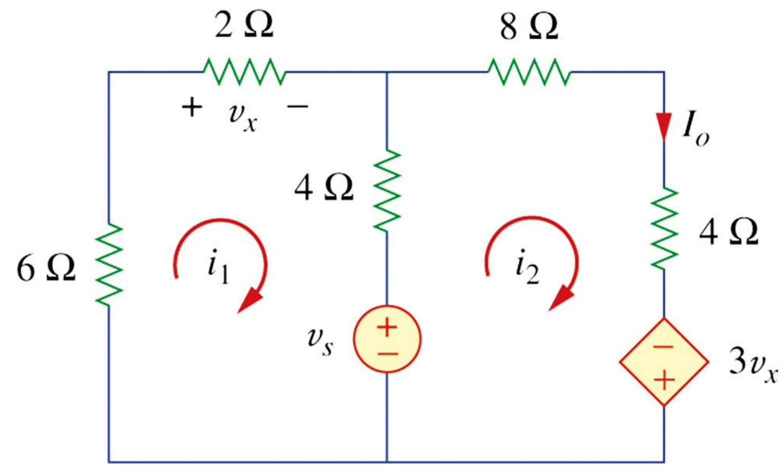
$$v_s = 1\text{V} \rightarrow i = 0.2\text{A}$$

$$v_s = 5\text{mV} \leftarrow i = 1\text{mA}$$

$$p = i^2 R = \frac{v^2}{R} \quad : nonlinear$$

Example 3

- For the circuit in fig, find I_0 when v_s=12V and v_s=24V.
- KVL



$$12i_1 - 4i_2 + v_s = 0 \qquad (1.1)$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad (1.2)$$

$$But\, v_x = 2i_1$$

$$\therefore -10i_1 + 16i_2 - v_s = 0 \qquad (1.3)$$

From eqs(1.1) and (1.3) we get

$$2i_1 + 12i_2 = 0 \rightarrow i_1 = -6i_2$$

Example 3

Eq(1.1), we get

$$-76i_2 = v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$

When

$$v_s = 12\text{V}$$

$$I_0 = i_2 = \frac{12}{76}\text{A}$$

When

$$v_s = 24\text{V}$$

$$I_0 = i_2 = \frac{24}{76}\text{A}$$

- **Showing that when the source value is doubled, I_0 doubles.**

Superposition

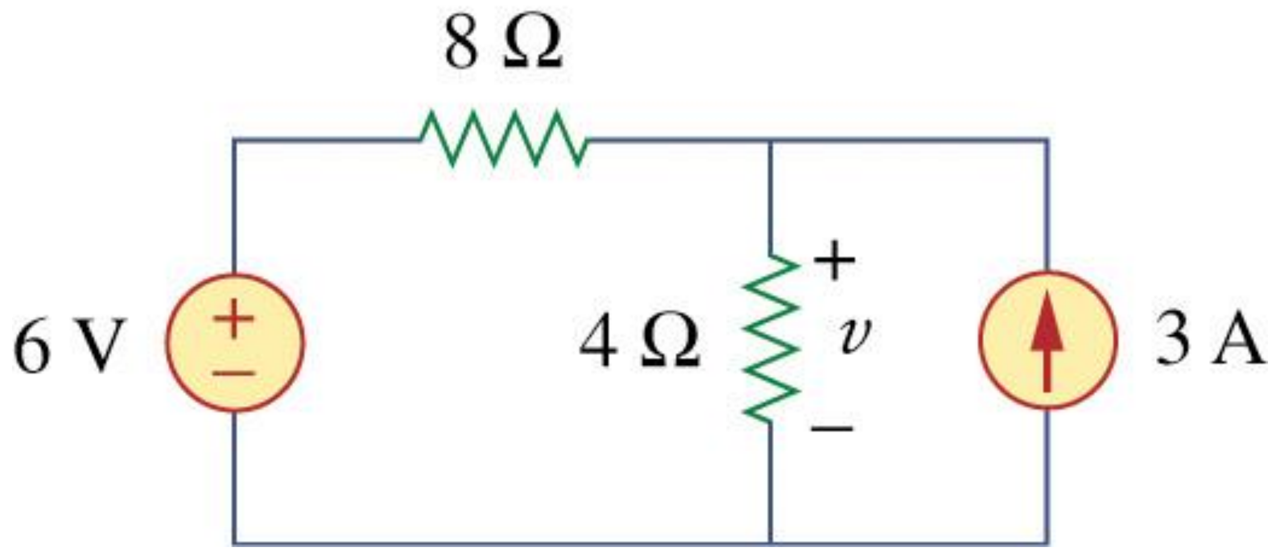
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Turn off, killed, inactive source:
  - independent voltage source: 0 V (short circuit)
  - independent current source: 0 A (open circuit)
- Dependent sources are left intact because they are controlled by circuit variables.

Superposition

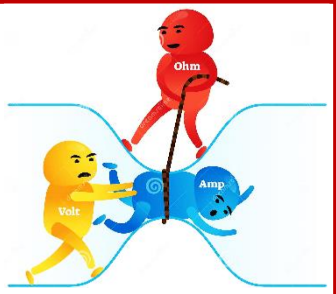
- **Steps to apply superposition principle:**
  1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
  2. Repeat step 1 for each of the other independent sources.
  3. Find the total contribution by adding algebraically all the contributions due to the independent sources.
- **How to turn off independent sources**
  - Turn off voltages sources = short voltage sources; make it equal to zero voltage
  - Turn off current sources = open current sources; make it equal to zero current

Example 4

Use the superposition theorem to find **v** in the circuit in Fig.



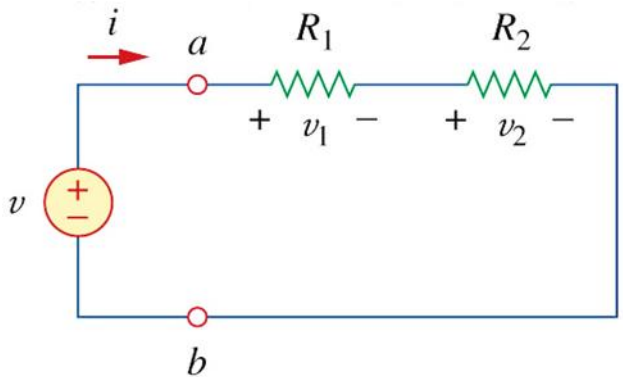
Voltage Division



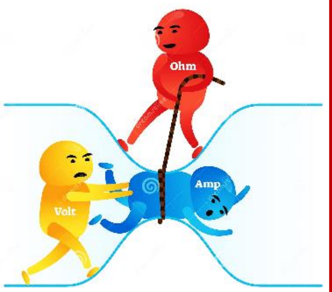
- The voltage drop across any one resistor can be known.
- The current through all the resistors is the same, so using Ohm's law:

$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

- This is the principle of voltage division.



Current Division

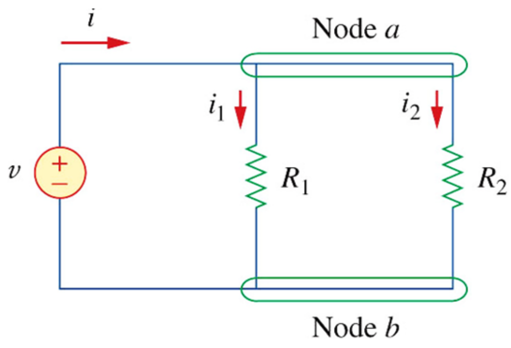


- Given the current entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors.

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

- This can be used in combination with Ohm's law to get the current through each resistor:

$$i_1 = \frac{iR_2}{R_1 + R_2} \qquad i_2 = \frac{iR_1}{R_1 + R_2}$$



Example 4

- Since there are two sources,
- let $v = v_1 + v_2$
- set the current source to zero

  Applying KVL to the loop

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5\ \text{A}$$

$$v_1 = 4i_1 = 2\ \text{V}$$

- set the voltage source to zero

  Current division, to get

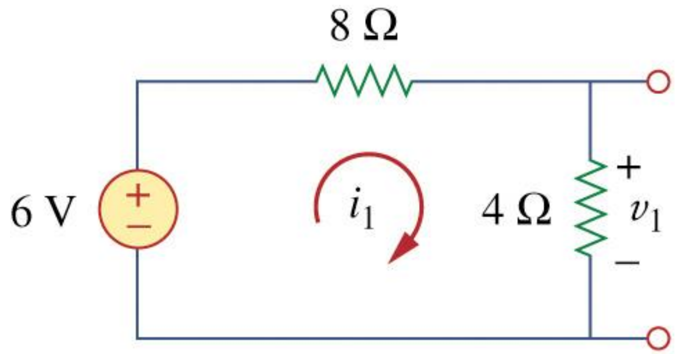
$$i_3 = \frac{8}{4+8}(3) = 2\text{A}$$

- Hence

$$v_2 = 4i_3 = 8\text{V}$$

- And we find

$$v = v_1 + v_2 = 2 + 8 = 10V$$



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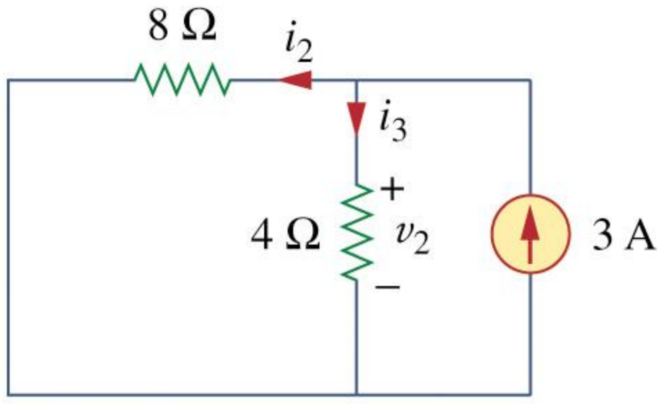

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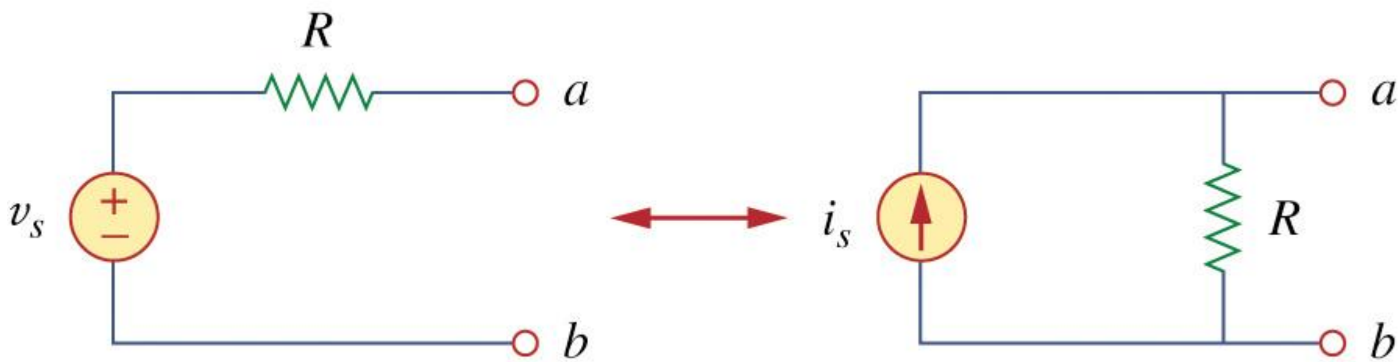
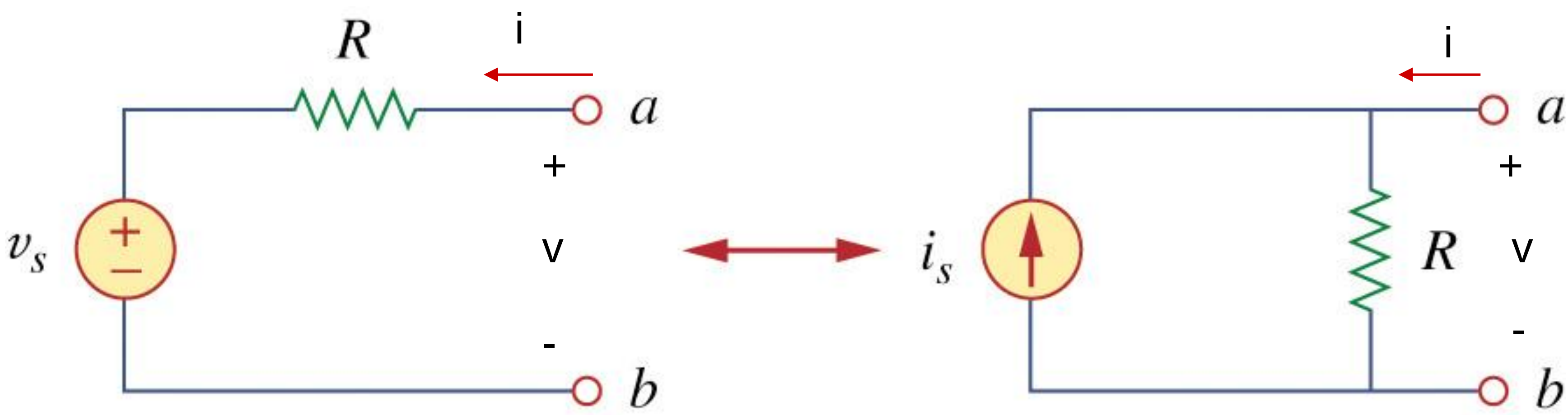
Source Transformation

- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa
- a source transformation does not affect the remaining part of the circuit. When applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis.

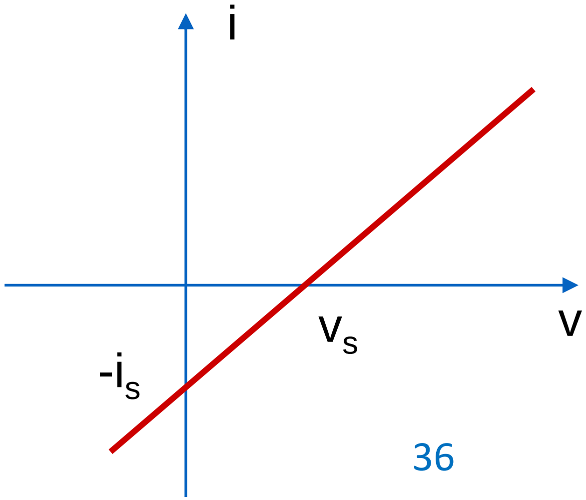


$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Equivalent Circuits



- Arrow of the current source ⟺ positive terminal of voltage source
- Impossible source Transformation
  - ideal voltage source (R = 0)
  - ideal current source (R=∞)

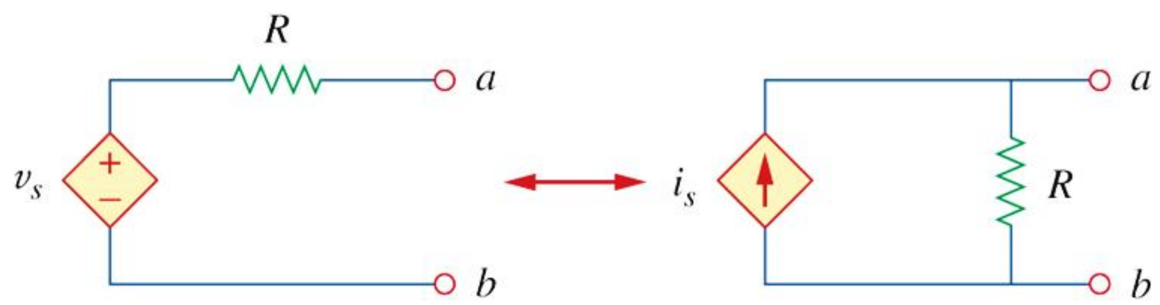


Source transformation rules

- Arrow of the current source ⟺ positive terminal of voltage source
- Impossible source Transformation
  - ideal voltage source (R = 0)
  - ideal current source (R=∞)

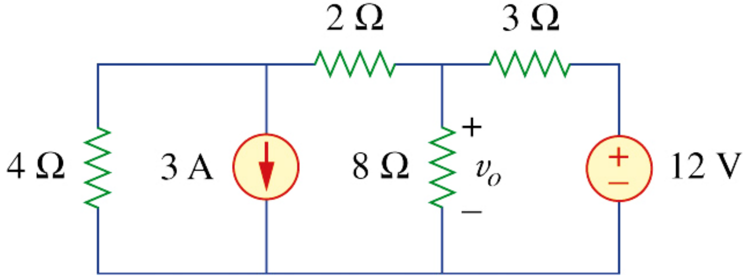
Dependent Sources

- Source transformation also applies to dependent sources.
- But, the dependent variable must be handled carefully.
- The same relationship between the voltage and current holds here:

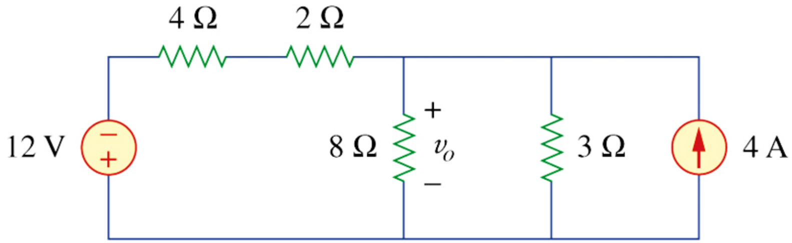


Example 5

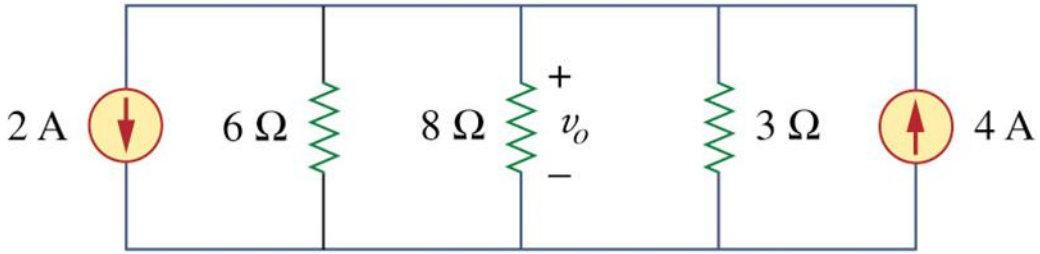
Use source transformation to find vo in the circuit in Fig



- We first transform the current and voltage sources to obtain

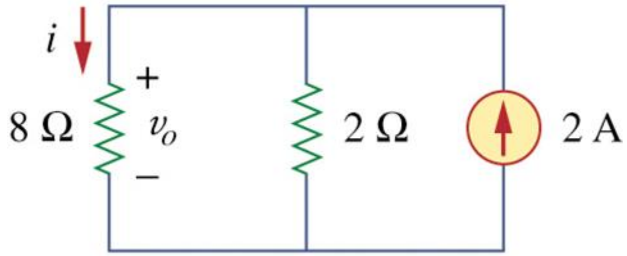


- Combining the 2Ω and 6 Ω resistors in series and transforming the 12-V voltage source gives



Example 5

- We now combine the **6Ω** and **3Ω** resistors in parallel to get **2Ω** . We also
- combine the 2-A and 4-A current sources to get a 2-A source



- we use current division in above Fig. to get

$$i = \frac{2}{2+8}(2) = 0.4\text{A}$$

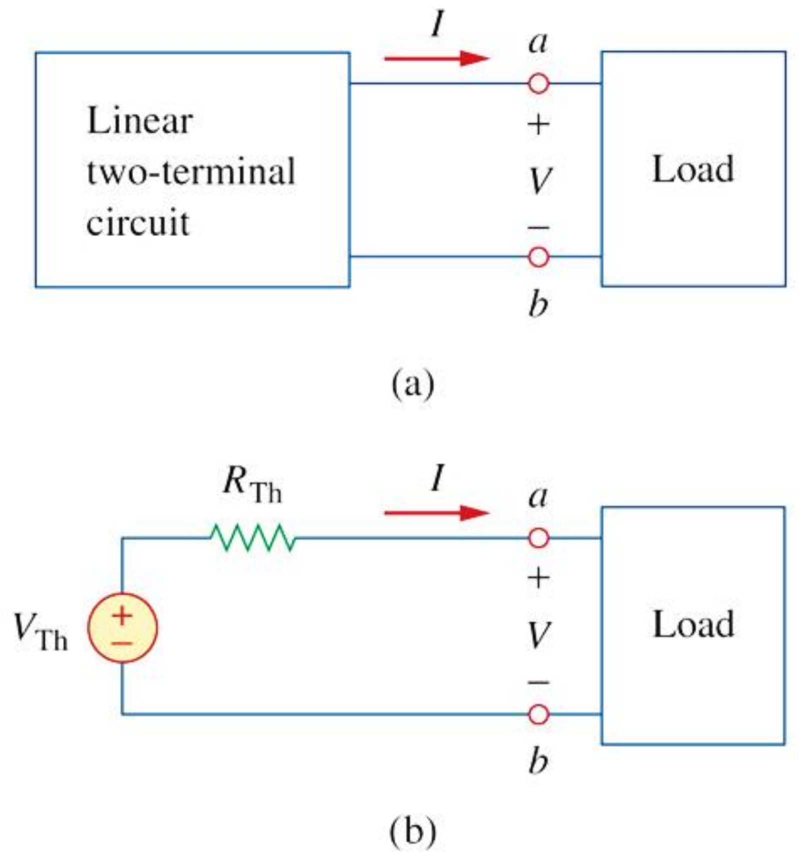
$$v_o = 8i = 8(0.4) = 3.2\text{V}$$

Thevenin's Theorem

- In many circuits, one element will be variable.
- An example of this is mains power; many different appliances may be plugged into the outlet, each presenting a different resistance.
- This variable element is called the load.
- Ordinarily one would have to reanalyze the circuit for each change in the load.

Thevenin's Theorem II

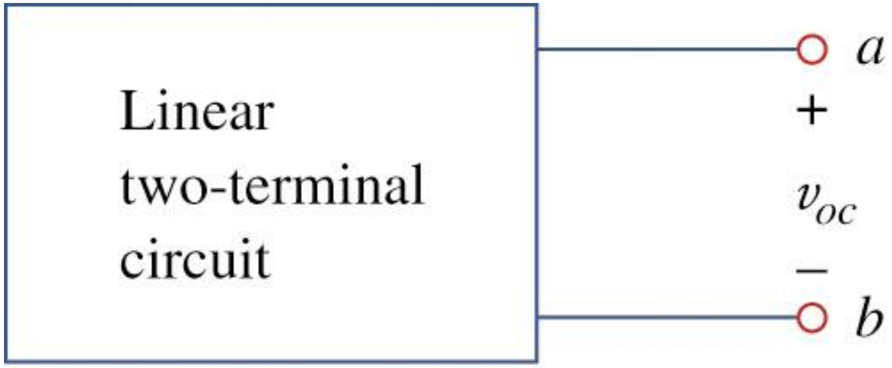
- Thevenin's theorem states that a linear two terminal circuit may be replaced with a voltage source and resistor.
- The voltage source's V_{Th} value is equal to the open circuit voltage at the terminals.
- The resistance R_{Th} is equal to the resistance measured at the terminals when the independent sources are turned off.



How to Find Thevenin's Voltage

- **Equivalent circuit: same voltage-current relation at the terminals.**

$$V_{\mathrm{Th}} = v_{oc}: \quad \text{open circuit voltage at } a-b$$



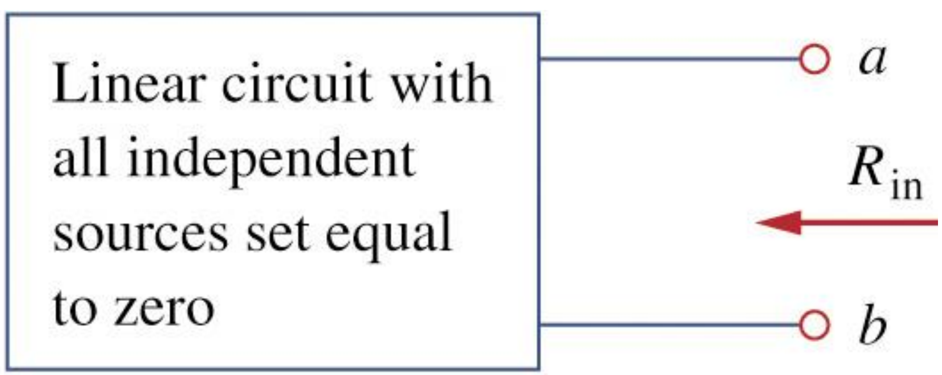
$$V_{\mathrm{Th}} = v_{oc}$$

How to Find Thevenin's Resistance

$$R_{\text{Th}} = R_{\text{in}} :$$

$$\text{input} - \text{resistance of the dead circuit at } a - b.$$

- $a - b$ open circuited
- Turn off all independent sources

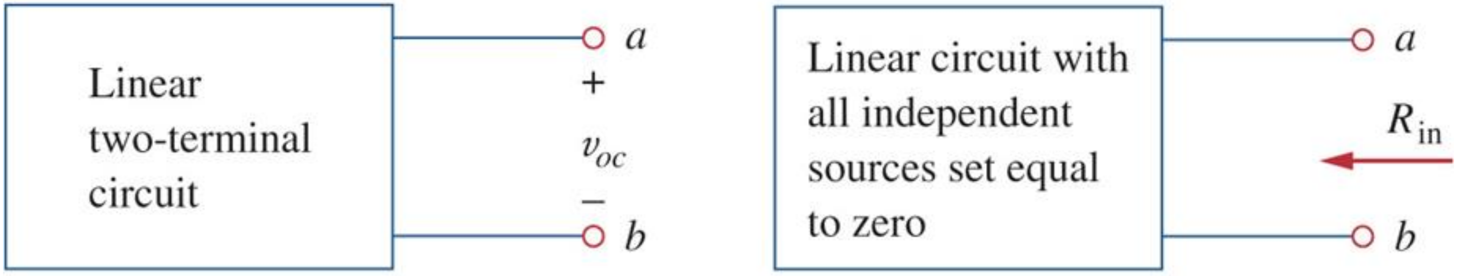


$R_{\text{Th}} = R_{\text{in}}$

CASE 1

If the network has no dependent sources:

- Turn off all independent source.
- R_{Th}: can be obtained via simplification of either parallel or series connection seen from a-b



$V_{Th} = v_{oc}$ (a)

$R_{Th} = R_{in}$ (b)

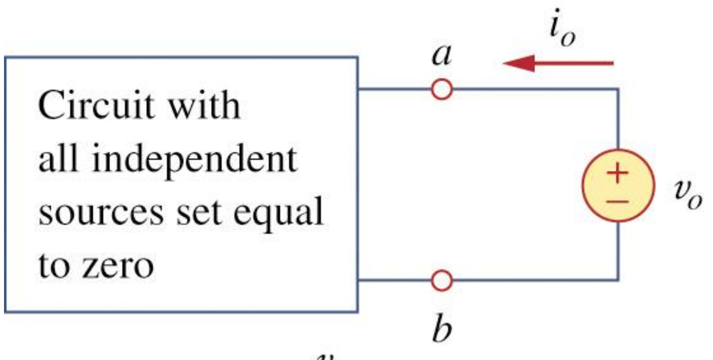
CASE 2

- If the network has dependent sources
- Turn off all independent sources.
- Apply a voltage source v_o at a-b

$$R_{\text{Th}} = \frac{v_o}{i_o}$$

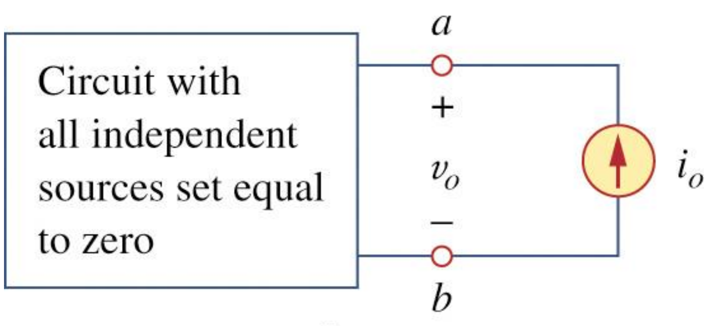
- Alternatively, apply a current source i_o at a-b

$$R_{Th} = \frac{v_o}{i_o}$$



$$R_{\text{Th}} = \frac{v_o}{i_o}$$

(a)



$$R_{\text{Th}} = \frac{v_o}{i_o}$$

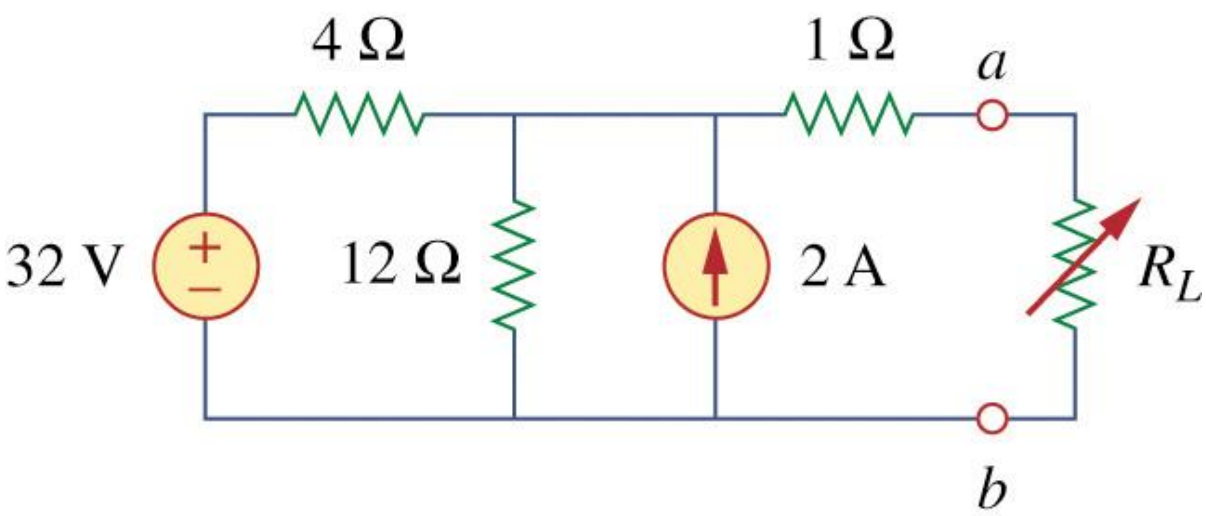
(b)

Thevenin’s Theorem

- Thevenin’s theorem is very powerful in circuit analysis.
- It allows one to simplify a circuit.
- A large circuit may be replaced by a single independent voltage source and a single resistor.
- The equivalent circuit behaves externally exactly the same as the original circuit.
- The Thevenin’s resistance may be negative, indicating that the circuit has ability providing power

Example 6

- Find the Thevenin's equivalent circuit of the circuit shown in Fig, to the left of the terminals a-b. Then find the current through R_L = 6,16,and 36 Ω.

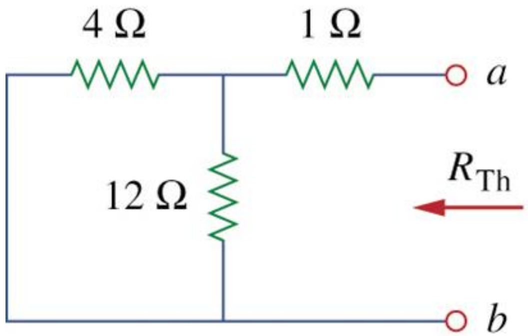


Example 6

- Find R_{Th}

$$R_{\mathrm{Th}}: 32\mathrm{V}\ \text{voltage source} \rightarrow \text{short}$$

$$R_{\mathrm{Th}} = 4 \,\|\, 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



(a)

Example 6

- **Find V_{Th}**

(1) *Mesh analysis*

$$-32+4i_1+12(i_1-i_2)=0\ ,\ \ i_2=-2\text{A}$$

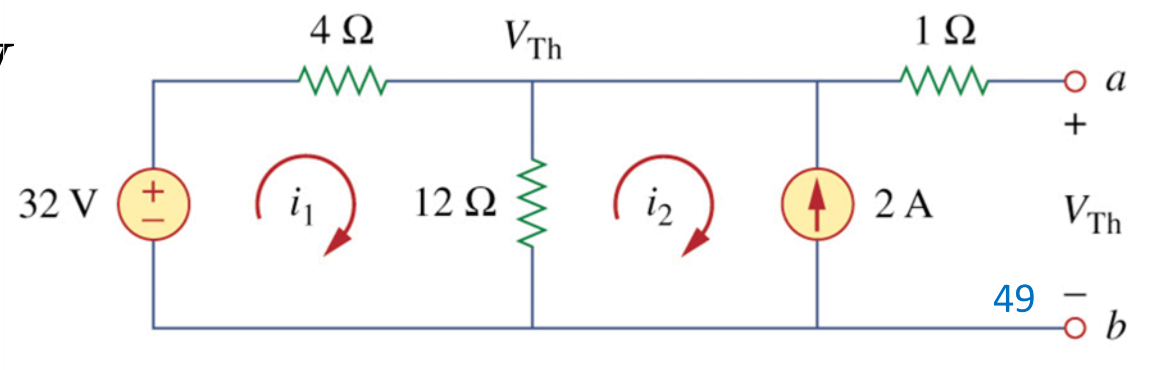
$$\therefore i_1=0.5\text{A}$$

$$V_{\text{Th}}=12(i_1-i_2)=12(0.5+2.0)=30\text{V}$$

(2) Alternatively, Nodal Analysis

$$(32-V_{\text{Th}})/4+2=V_{\text{Th}}/12$$

$$\therefore V_{\text{Th}}=30\text{V}$$

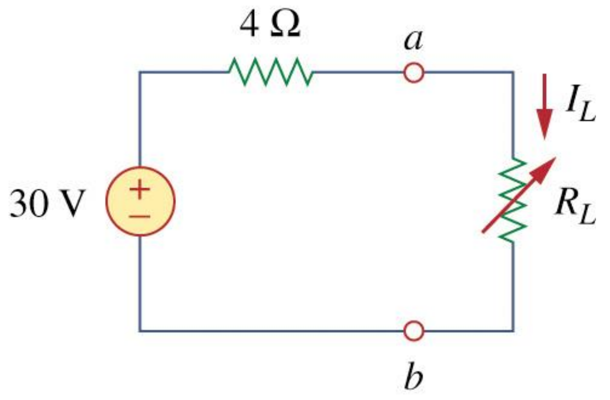


(b)

Find I_L

$\text{To get } i_L :$

$$i_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$



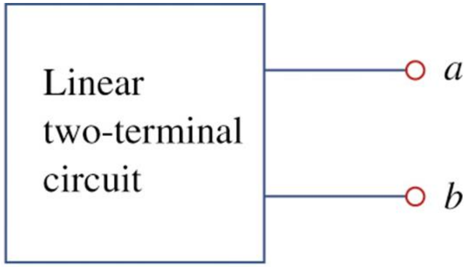
$$R_L = 6 \rightarrow I_L = 30/10 = 3\text{A}$$

$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\text{A}$$

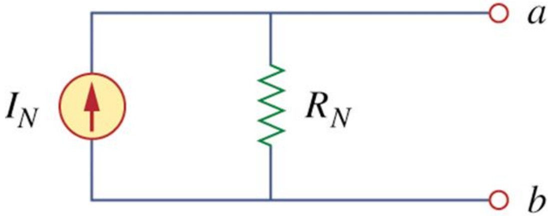
$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\text{A}$$

Norton's Theorem

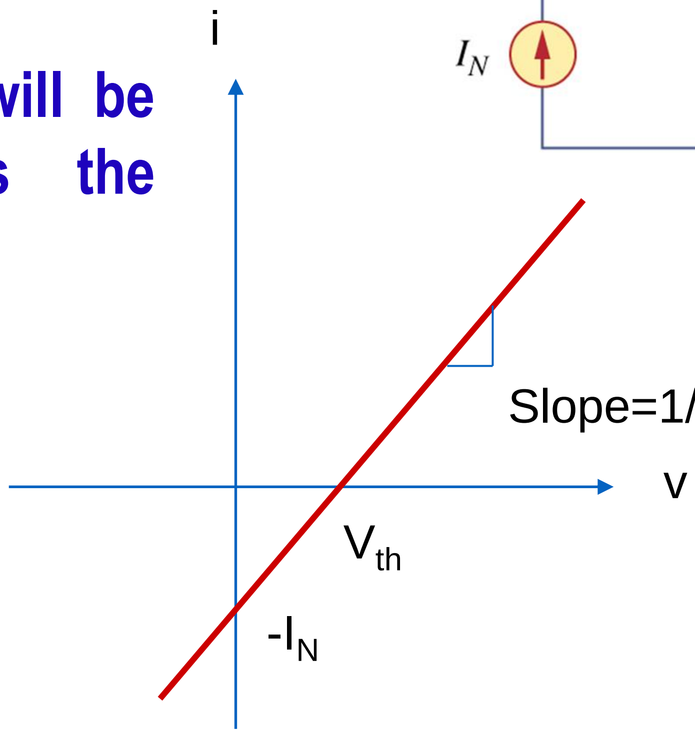
- Similar to Thevenin's theorem, Norton's theorem states that a linear two terminal circuit may be replaced with an equivalent circuit containing a resistor and a current source.
- The Norton resistance will be exactly the same as the Thevenin.



(a)



(b)



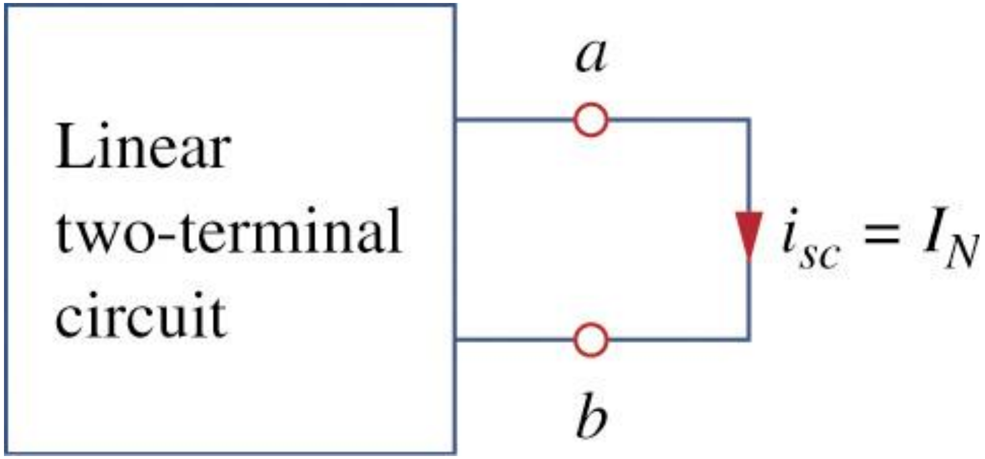
How to Find Norton Current

- Thevenin and Norton resistances are equal:

$$\boxed{R_N = R_{\text{Th}}}$$

- Short circuit current from a to b :

$$\boxed{I_N = i_{sc} = \frac{V_{\text{Th}}}{R_{\text{Th}}}}$$



Thevenin or Norton equivalent circuit :

- The open circuit voltage v_{oc} across terminals a and b
- The short circuit current i_{sc} at terminals a and b
- The equivalent or input resistance R_{in} at terminals a and b when all independent source are turn off.

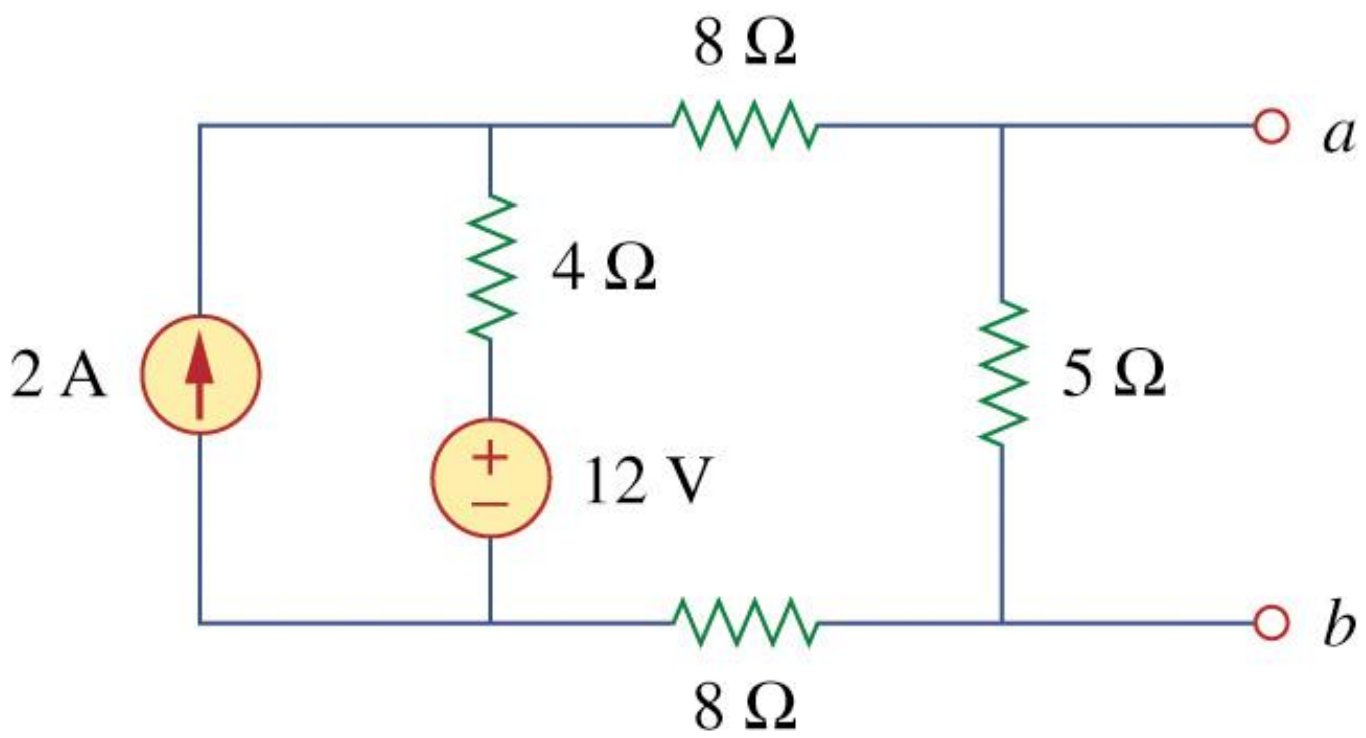
$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{Th} = \frac{V_{Th}}{R_{Th}} = R_N$$

Example 7

- Find the Norton equivalent circuit of the circuit in Fig.



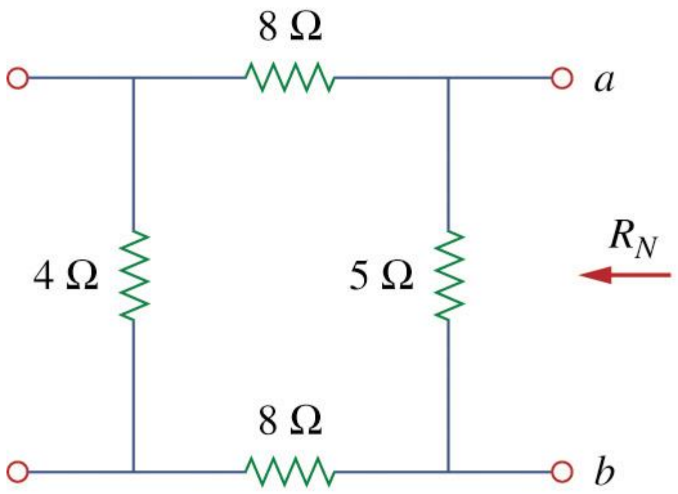
Example 7

- Determine the Thevenin or Norton equivalent circuit requires that we find:
  - The open-circuit voltage across terminals a and b.
  - The short-circuit current at terminals a and b.
  - The equivalent or input resistance at terminals a and b when
  - all independent sources are turned off.

To find R_N

$$R_N = 5 \| (8+4+8)$$

$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$



(a)

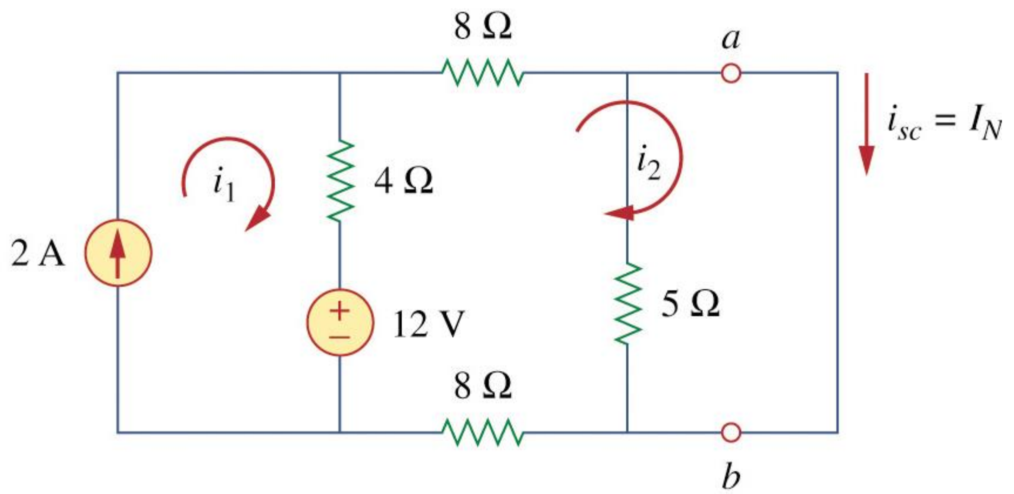
To Find I_N

- To find we short-circuit terminals a and b, as shown in Fig.
- We ignore the resistor 5Ω because it has been short-circuited

$$\text{short} - \text{circuit terminals } a \text{ and } b.$$

$$\text{Mesh}: \quad i_1 = 2A, \quad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1\text{A} = i_{sc} = I_N$$



(b)

Alternative Method for I_N

$$I_N = \frac{V_{Th}}{R_{Th}}$$

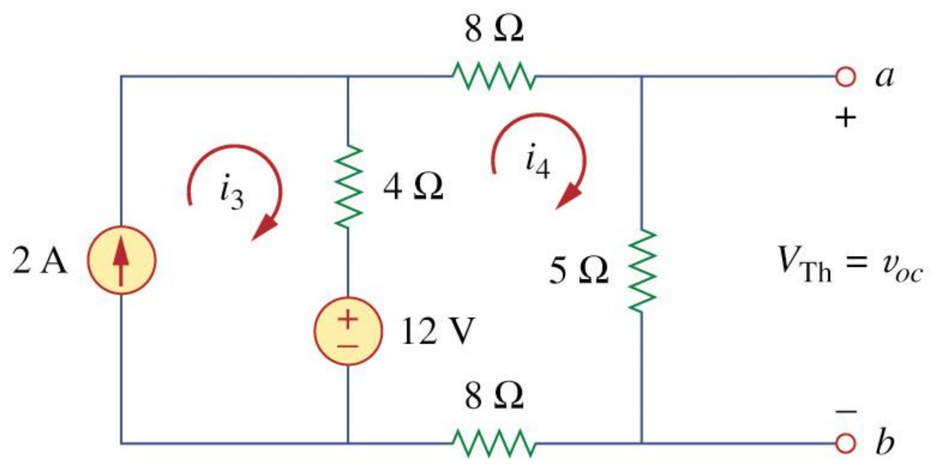
V_{Th}: open – circuit voltage across terminals a and b

Mesh analysis:

$$i_3 = 2A, \quad 25i_4 - 4i_3 - 12 = 0$$

$$\therefore \; i_4 = 0.8\text{A}$$

$$\therefore v_{oc} = V_{Th} = 5i_4 = 4\text{V}$$



(c)