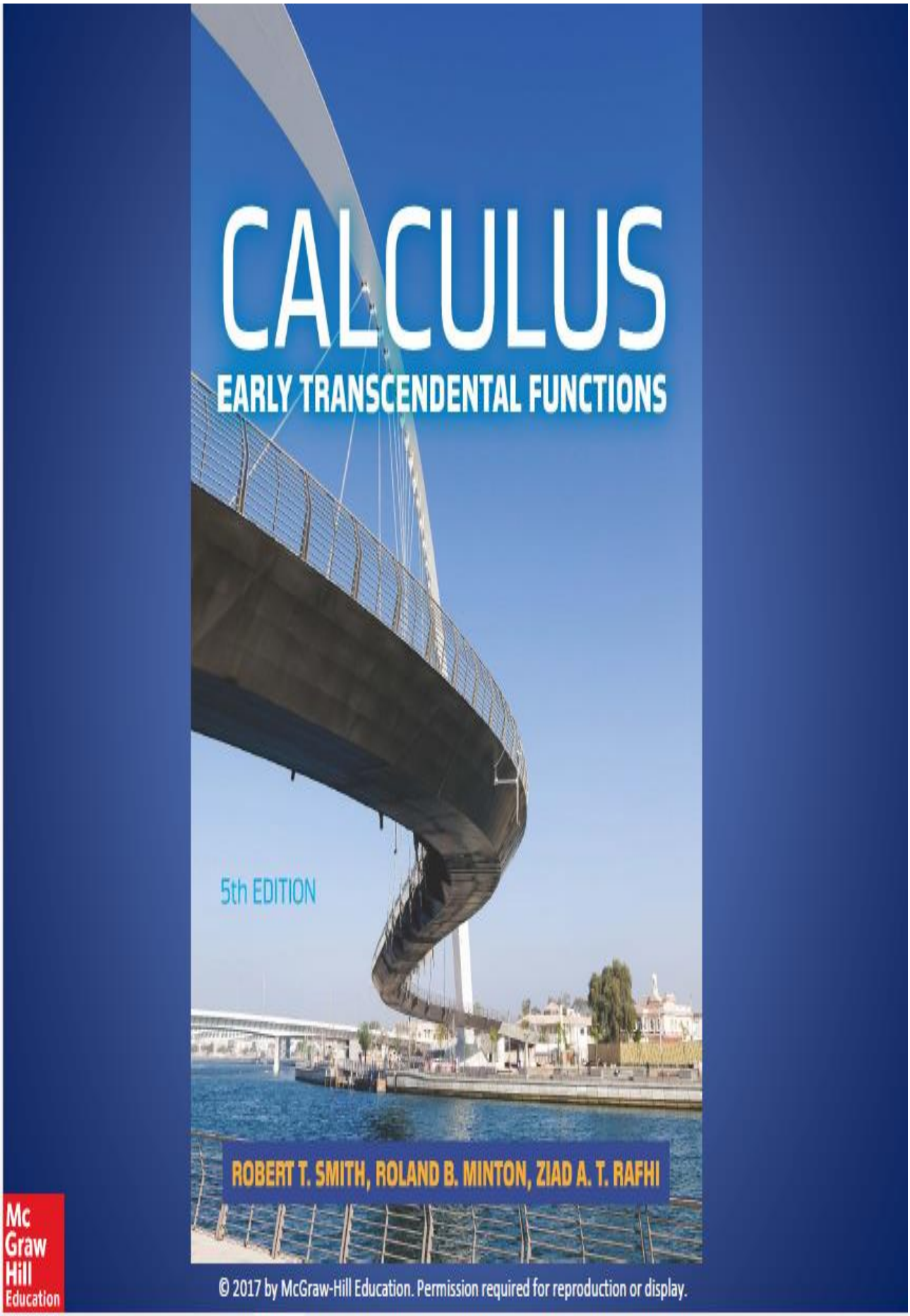

Integration

$$f \quad \underset{\text{Integration(Antiderivative)}}{\overset{\text{Differentiation}}{\rightleftarrows}} \quad f'$$

Types of Integral

Indefinite Integral:

$$\int f(x)\,dx$$

Definite Integral:

$$\int_a^b f(x)\,dx$$

Rules of Definite Integrals

1. $$\int dx = x + c$$

2. $$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

Example $$\int x^3\,dx = \frac{x^4}{4} + c$$

3. $$\int a\, f(x)\, dx = a \int f(x)\, dx$$

Example $$\int 4\, x^3\, dx = 4 \int x^3\, dx$$

$$= 4\left(\frac{x^4}{4}\right) + c = x^4 + c$$

4. $$\int (a\, f(x) \pm b\, g(x))\, dx = a \int f(x)\, dx \pm b \int h(x)\, dx$$

Example $$\int (x^2 - 2x + 3) dx$$

$$= \int x^2\, dx - 2 \int x\, dx + 3 \int dx$$

$$= \frac{x^3}{3} - x^2 + 3x + c$$

5. $$\int f'(f)^n\, dx = \frac{f^{n+1}}{n+1} + c$$

Example $$\int (x+5)^3\, dx = \frac{(x+5)^4}{4} + c$$

Example $\int x(x^2+5)^3\,dx$

$$= \frac{1}{2}\int 2x(x^2+5)^3\,dx$$

$$= \frac{1}{2}\frac{(x^2+5)^4}{4} + c = \frac{(x^2+5)^4}{8} + c$$

Example $\int \frac{(\sqrt{x}+1)^4}{\sqrt{x}}\,dx$

$$= \int \frac{1}{\sqrt{x}}(\sqrt{x}+1)^4\,dx = 2\int \frac{1}{2\sqrt{x}}(\sqrt{x}+1)^4\,dx$$

$$= (2)\frac{(\sqrt{x}+1)^5}{5} + c = \frac{2(\sqrt{x}+1)^5}{5} + c$$

6. $\int ff'\,dx = \frac{f^2}{2} + c$

Example $\int sinx\,cosx\,dx = \frac{sin^2x}{2} + c$

Example $\int \frac{lnx}{x}\,dx = \int \frac{1}{x}lnx\,dx$

$$= \frac{(lnx)^2}{2} + c$$

7. $$\int \frac{f'}{f}\,dx = ln|f| + c$$

Example $$\int \frac{1}{x}\,dx = ln|x| + c$$

Example $$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$$

$$= \frac{1}{2} ln|x^2+1| + c$$

8. $$\int \frac{f'}{\sqrt{f}}dx = 2\sqrt{f} + c$$

Example $$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$$

Example $$\int \frac{x}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}\,dx$$

$$= \frac{1}{2}\left(2\sqrt{x^2+1}\right) + c = \sqrt{x^2+1} + c$$

9. $$\int \frac{f'}{(f)^n}\,dx = \int f'(f)^{-n}dx$$

$$= \frac{f^{-n+1}}{-n+1} + c$$

Example $$\int \frac{1}{(x+2)^3}\,dx = \int (x+2)^{-3}dx$$

$$= \frac{(x+2)^{-2}}{-2} + c$$

$$= \frac{-1}{2(x+2)^2} + c$$

Exponential functions

1. $$\int f' e^f \, dx = e^f + c$$

Example $$\int e^x dx = e^x + c$$

Example $$\int e^{2x} dx = \frac{1}{2}\int 2e^{2x} dx$$

$$= \frac{1}{2} e^{2x} + c$$

Example $$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$$

$$= 2\int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = 2e^{\sqrt{x}} + c$$

2. $$\int f' a^f dx = \frac{a^f}{lna} + c$$

Example $$\int 3^x dx = \frac{3^x}{ln3} + c$$

Example $$\int 3^{2x} dx = \frac{1}{2}\int 2(3^{2x}) dx$$

$$= \frac{1}{2}\frac{3^{2x}}{ln3} + c = \frac{3^{2x}}{2ln3} + c$$