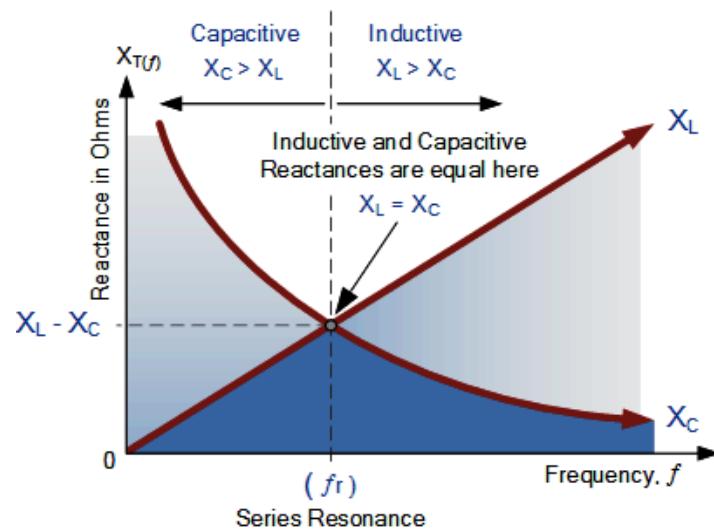


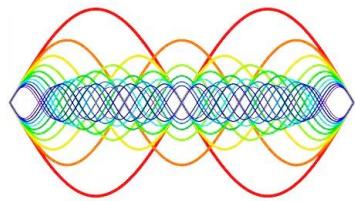
Alternating Current (AC) Circuits II

Resonance

CSE 113

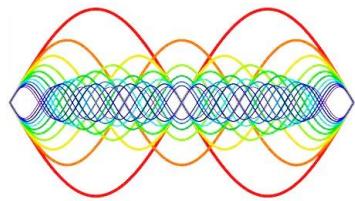


OUTLINES



- AC Power
- Power Triangle
- AC Power Factor
- Power Factor Correction
- Resonance.
 - Series Resonant Circuit
 - Paralell Resonant Circuit

AC Power



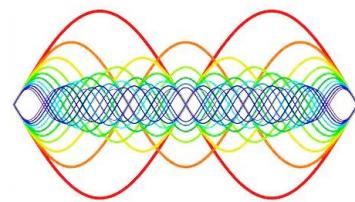
- AC Impedance (Z) is a **complex** quantity made up of real **resistance** (R) and imaginary **reactance** (X):

$$\vec{Z} = R + jX \quad (\Omega)$$

- AC Apparent Power (S) is a **complex** quantity made up of real **active** (P) power and imaginary **reactive** (Q) power:

$$\bar{S} = P + jQ \quad (VA)$$

AC Power



- The Active power (**P**) is the power that is dissipated in the resistance of the load.
- It uses the same formula used for DC (**V** & **I** are the *magnitudes*, not the phasors):

$$P = I^2 R = \frac{V^2}{R} \quad [\text{watts, W}]$$

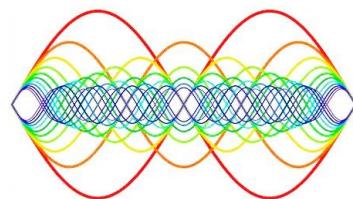
WARNING! #1 mistake with AC power calculations!

The **Voltage** in the above equation is the Voltage drop across the resistor, not across the entire circuit!

CAUTION!

REAL value of resistance (**R**) is used in REAL power calculations, not IMPEDANCE (**Z**)!

AC Imaginary (Reactive) Power (Q)



- The reactive power (Q) is the power that is exchanged between reactive components (inductors and capacitors).
- The formulas look similar to those used by the active power, but use reactance instead of resistances.

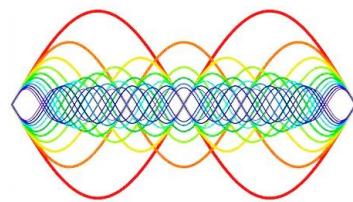
$$Q = I^2 X = \frac{V^2}{X^*} \quad [\text{VAR}]$$

WARNING! #1 mistake with AC power calculations!

The **Voltage** in the above equation is the Voltage drop across the reactance, not across the entire circuit!

- Units: Volts-Amps-Reactive (VAR)
- Q is negative for a capacitor by convention and positive for inductor.
- Just like X is negative for a capacitor! (-jXc)

AC Apparent Power (S)

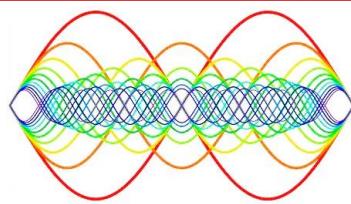


- The Apparent Power (S) is the power that is “appears” to flow to the load.
- The magnitude of apparent power can be calculated using similar formulas to those for active or reactive power:

$$S = VI^* = I^2 Z = \frac{V^2}{Z^*} \quad [\text{VA}]$$

- Units: Volts-Amps (VA)
- V & I are the magnitudes, not the phasors.

AC Power Relationships

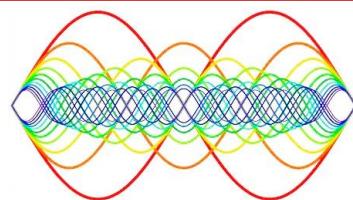


Notice the relationship between Z and S :

$$\boxed{\mathbf{Z}} = \boxed{R} + j \boxed{X} \quad (\Omega)$$

$$\boxed{\mathbf{S}} \text{ Apparent power calculated with } Z = \boxed{P} + j \boxed{Q} \quad (VA)$$

Power Triangle

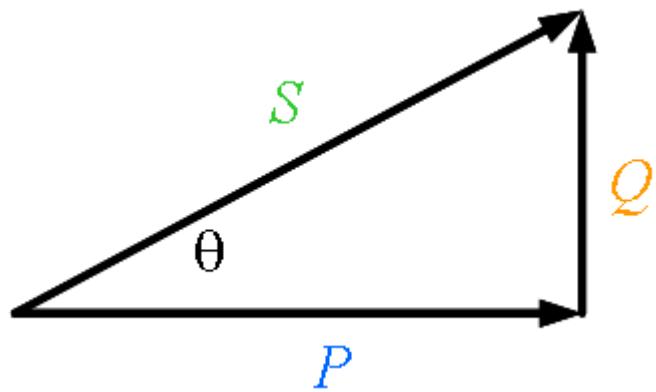
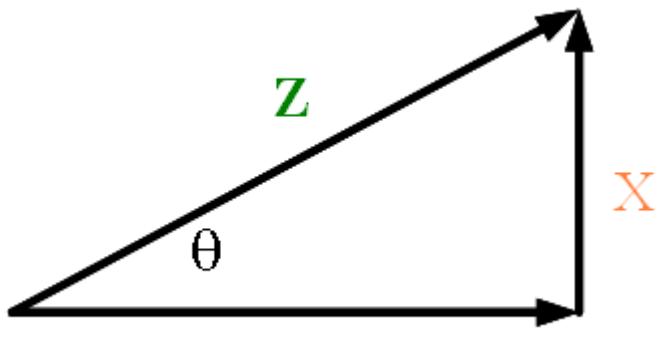


- The power triangle graphically shows the relationship between real (P), reactive (Q) and apparent power (S).

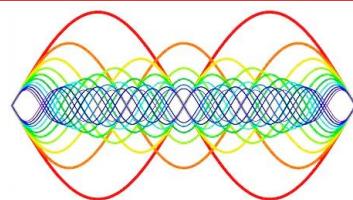
$$S = \sqrt{P^2 + Q^2}$$

$$\bar{S} = P + jQ_L$$

$$\bar{S} = S \angle \theta$$

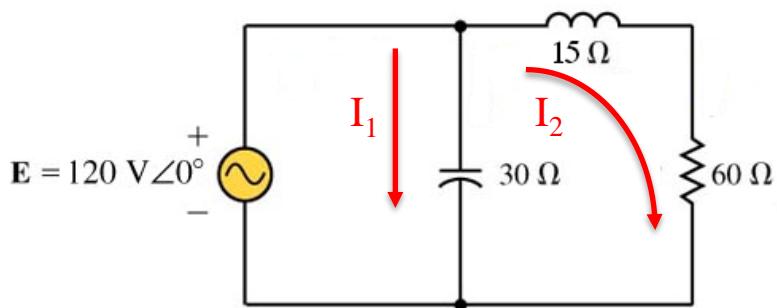


Example Problem 1



Determine the real (P) and reactive (Q) power of each component.

Determine the apparent (S) power delivered by the source.



$$P = I^2 R = \frac{V^2}{R}$$

$$P_{R60} = I_2^2 R = (1.94A)^2 * (60\Omega) = 225.8W$$

$$Q = I^2 X^* = \frac{V^2}{X^*}$$

$$Q_{L15} = I_2^2 X_{L15} = (1.94A)^2 * (15\Omega) = 56.5VAR$$

$$Q_{C30} = I_1^2 X_{C30} = (4A)^2 * (-30\Omega) = -480VAR$$

$$Q_T = Q_{L15} + Q_{C30} = (56.5 - 480)VAR = -423.5VAR$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(225.8)^2 + (-423.5)^2} = 480VA$$

$$\vec{S} = (\vec{V})(\vec{I}^*) = (120V\angle 0)(4A\angle -62) = 480VA\angle -62^\circ \Rightarrow 225.35 - j423.81$$

$$Z_T = Z_{C30} / (Z_{L15} + Z_{R60}) = \left(\frac{1}{30\Omega\angle -90^\circ} + \frac{1}{(15\Omega\angle 90^\circ) + (60\Omega\angle 0^\circ)} \right)^{-1}$$

$$Z_T = 30\Omega\angle -62^\circ$$

$$I_S = \frac{E_S}{Z_T} = \frac{120V\angle 0^\circ}{30\Omega\angle -62^\circ} = 4A\angle 62^\circ$$

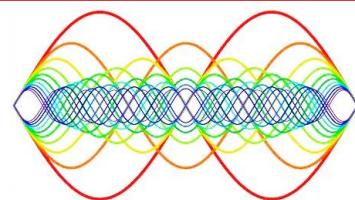
$$I_1 = I_S \frac{Z_T}{Z_{C30}} = (4A\angle 62^\circ) \frac{30\Omega\angle -62^\circ}{30\Omega\angle -90^\circ} = 4A\angle 90^\circ$$

$$I_2 = I_S \frac{Z_T}{Z_{L15} + Z_{R60}} = (4A\angle 62^\circ) \left(\frac{30\Omega\angle -62^\circ}{60\Omega + j15\Omega} \right) = 1.94A\angle -14^\circ$$

θ is from Z_T

Real Power Reactive Power

Real and Reactive Power



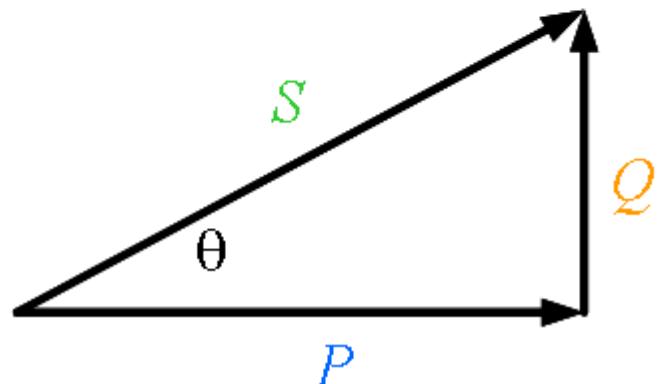
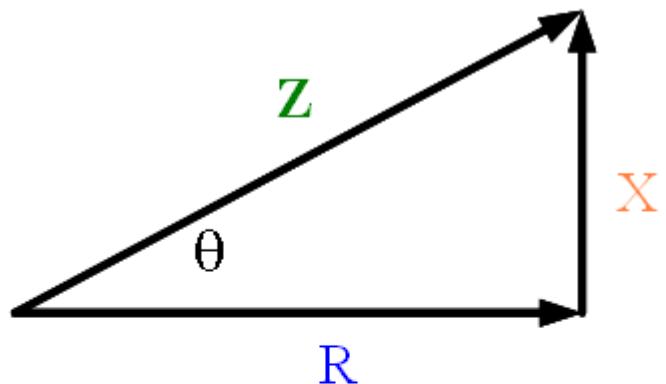
- The power triangle also shows that we can find real (P) and reactive (Q) power.

$$S = IV \quad (\text{VA})$$

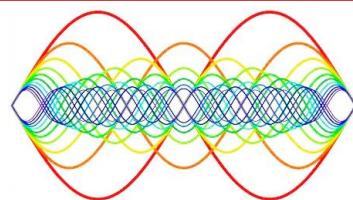
$$P = S \cos \theta \quad (\text{W})$$

$$Q = S \sin \theta \quad (\text{VAR})$$

NOTE: The impedance angle and the “power factor angle” are the same value!



Example Problem 2 [TUTORIAL]

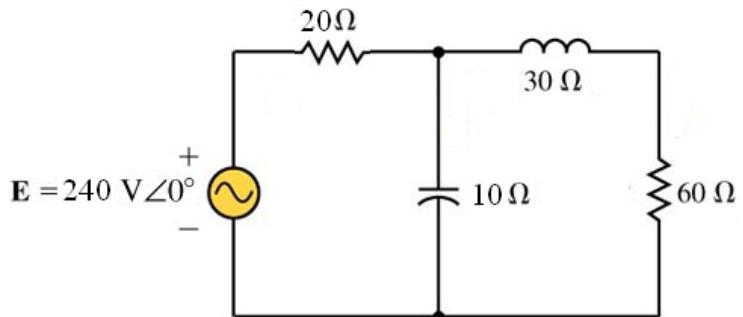


- Determine the apparent (S) power, total real (P) and reactive (Q) power using the following equations:

$$S = VI \quad (\text{VA})$$

$$P = S \cos \theta \quad (\text{W})$$

$$Q = S \sin \theta \quad (\text{VAR})$$



$$Z_T = Z_{R20} + [Z_{C10} / (Z_{L30} + Z_{R60})] = 20\Omega \angle 0^\circ + \left(\frac{1}{10\Omega \angle -90^\circ} + \frac{1}{(30\Omega \angle 90^\circ) + (60\Omega \angle 0^\circ)} \right)^{-1}$$

$$Z_T = 23.9\Omega \angle -26^\circ$$

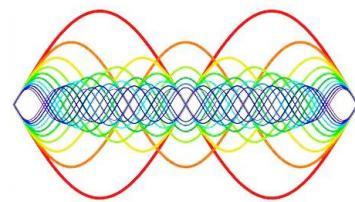
$$I_s = \frac{E_s}{Z_T} = \frac{240V \angle 0^\circ}{23.9\Omega \angle -26^\circ} = 10A \angle 26^\circ$$

$$S_T = |VI^*| = |(240V \angle 0^\circ) * (10A \angle -26^\circ)| = 2.4kVA$$

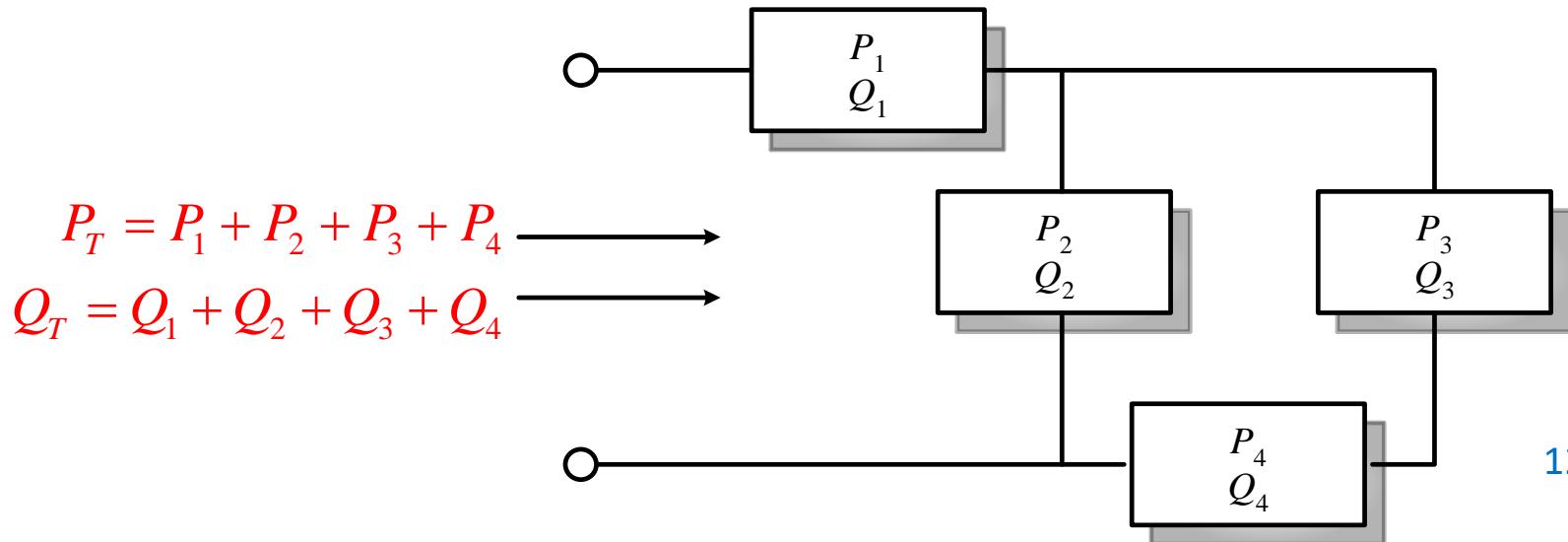
$$P_T = S_T \cos \theta = 2.4kVA * \cos(-26^\circ) = 2157W$$

$$Q_T = S_T \sin \theta = 2.4kVA * \sin(-26^\circ) = -1052VAR$$

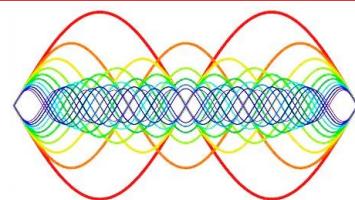
Total Power in AC Circuits



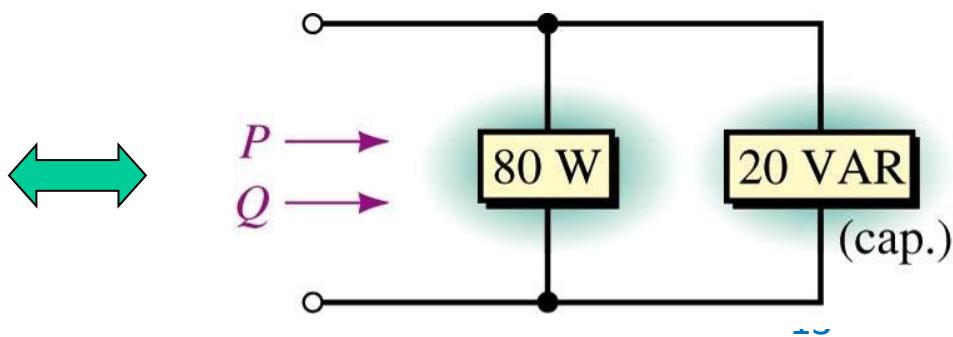
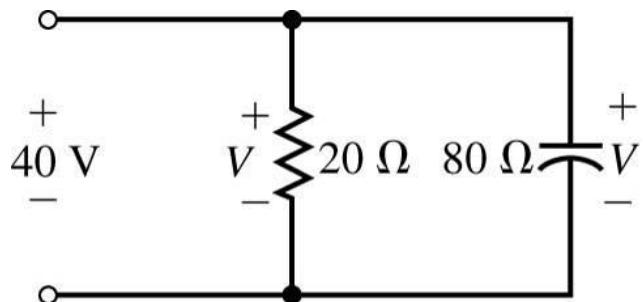
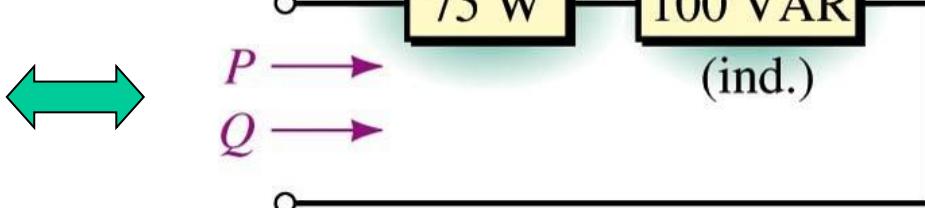
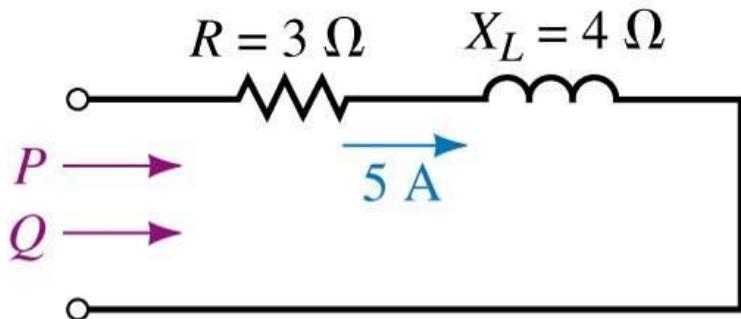
- The total power real (P_T) and reactive power (Q_T) is simply the sum of the real and reactive power for each individual circuit elements.
- *How elements are connected does not matter for computation of total power.*



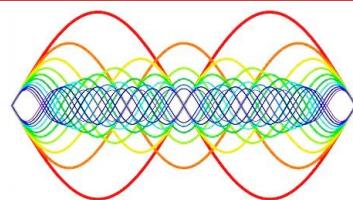
Total Power in AC Circuits



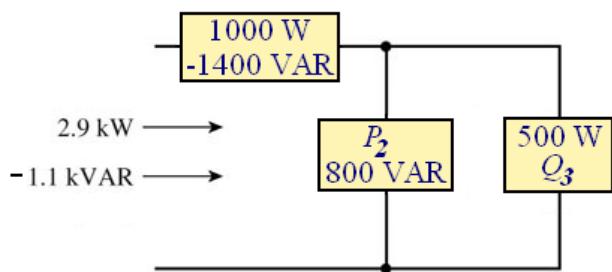
- Sometimes it is useful to redraw the circuit to symbolically express the real and reactive power loads.



Example Problem 3



- a. Determine the unknown real (P_2) and reactive powers (Q_3) in the circuit below.
- b. Determine total apparent power.
- c. Draw the power triangle.
- d. Is the unknown element in Load #3 an inductor or capacitor?



a) $P_T = P_1 + P_2 + \dots + P_N$

Rearrange to find P_2 :

$$P_2 = 2900W - 1000W - 500W = 1400W$$

a) $Q_T = Q_1 + Q_2 + \dots + Q_N$

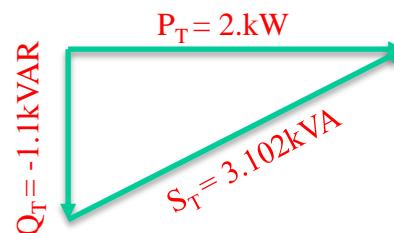
Like wise, for Q_3 ; rearrange to find Q_3 :

$$Q_3 = -1100VAR - (-1400VAR) - (800VAR) = -500VAR$$

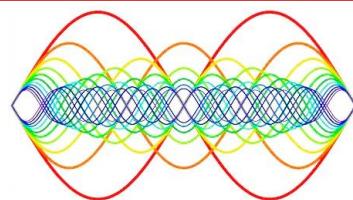
b) $S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(2900)^2 + (-1100)^2} = 3102VA$

d) Because Q_3 is negative, it is capacitive

c) Power Triangle



Example Problem 4 [TUTORIAL]



- a. Determine the value of R, P_T and Q_T
- b. Draw the power triangle and determine S.

$$S_T = V * I = 600V * 30A = 18kVA$$

$$Q_T = 2750\text{VAR} - 17880\text{VAR} + 12960\text{VAR} = -2170\text{VAR}$$

$$P_T = \sqrt{S_T^2 - Q_T^2} = \sqrt{18kVA^2 - (-2170\text{VAR})^2} = 17.87kW$$

$$P_T = P_R + P_{L1} + P_{L2} + P_C$$

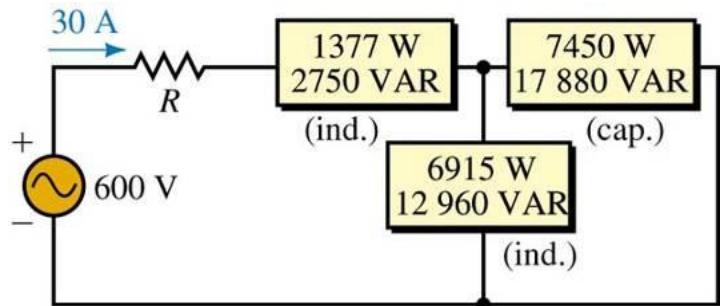
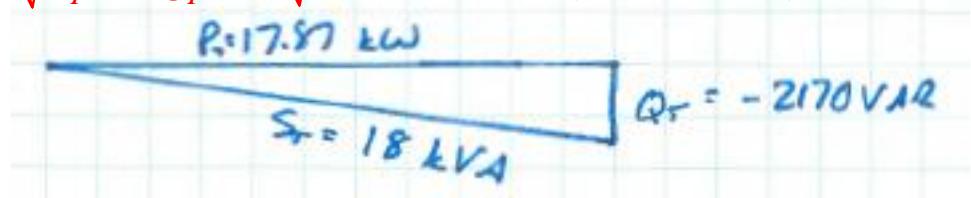
$$\Rightarrow P_R = P_T - P_{L1} - P_{L2} - P_C$$

$$P_R = 17.87kW - 1377W - 7450W - 6915W = 2127W$$

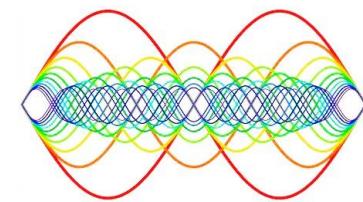
$$P_R = I_T^2 * R$$

$$\Rightarrow R = \frac{P^R}{I^2_T} = \frac{2127W}{30A^2} = 2.36\Omega$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{17.87kVA^2 + (-2170\text{VAR})^2} = 18kVA$$



Use of Complex Numbers in Power Calculations

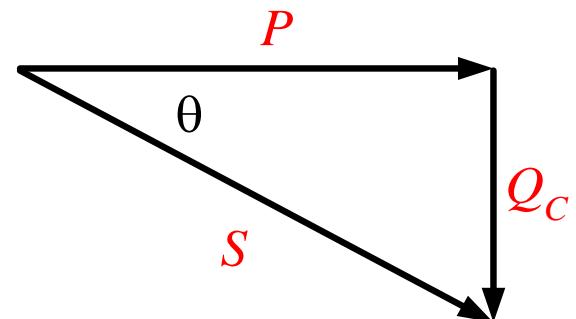


- AC power can be calculated using complex equations.
- Apparent Power can be represented as a complex number.
- The resultant can be used to determine real and reactive power by changing it to rectangular form.

\bar{I}^* is complex conjugate of \bar{I}

$$\bar{S} = \bar{V}\bar{I}^* = P + jQ$$

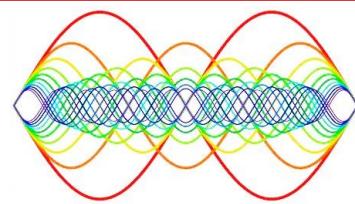
$$\bar{S} = \frac{|\bar{V}|^2}{\bar{Z}^*} = |\bar{I}|^2 \bar{Z}$$



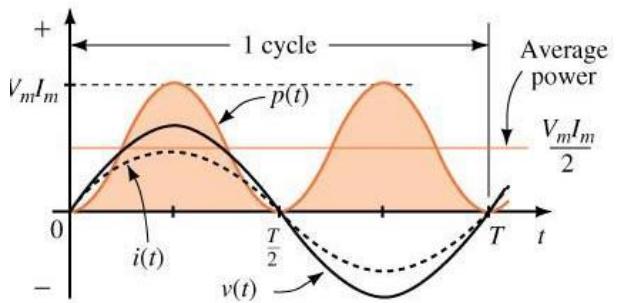
NOTE!

The complex conjugate of Current is used to make the power angle the same as ₁₆ the impedance angle!

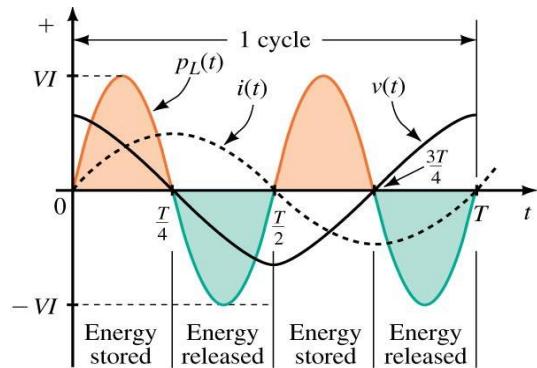
Summary



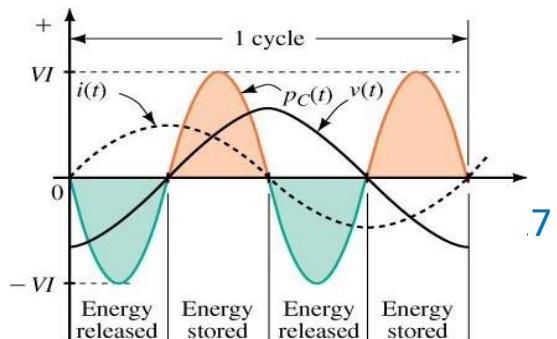
AC Power to a Resistive Load



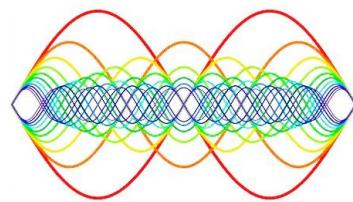
AC Power to a Inductive Load



AC Power to a Capacitive Load



AC Power Factor

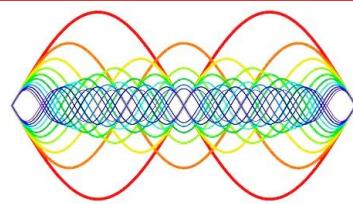


- Power factor (F_P) tells us what portion (or ratio) of the apparent power (S) is actually real power (P).
- Power factor is a ratio given by:

$$F_P = \cos \theta = \frac{\overline{P}}{\overline{S}}$$

- Power factor is expressed as a number between 0 to 1.0 (or as a percent from 0% to 100%).
- The closer to 1.0 the power factor gets, the more resistive.
- The closer to 0.0 the power factor gets, the more reactive.

Power Factor



- From the power triangle it can be seen that:

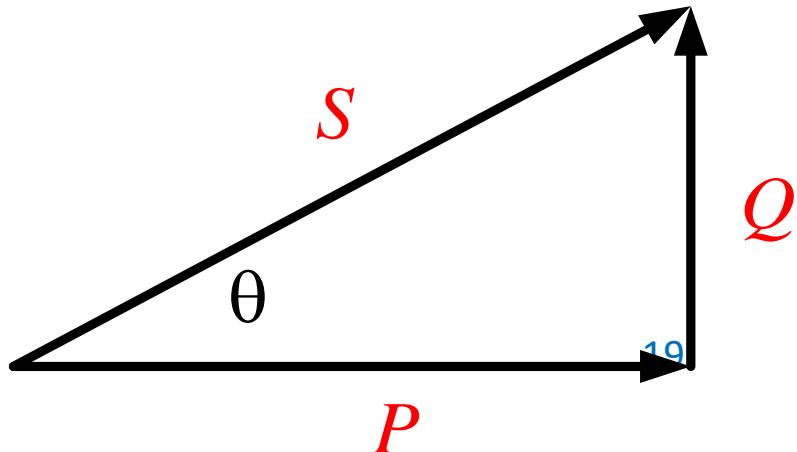
$$F_P = P / S = \cos \theta$$

- Power factor angle is thus given:

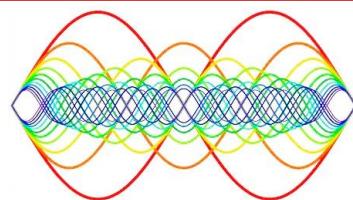
$$\theta = \cos^{-1}(P / S)$$

- For a **pure** resistance: $\theta = 0^\circ$
- For a **pure** inductance: $\theta = 90^\circ$
- For a **pure** capacitance: $\theta = -90^\circ$

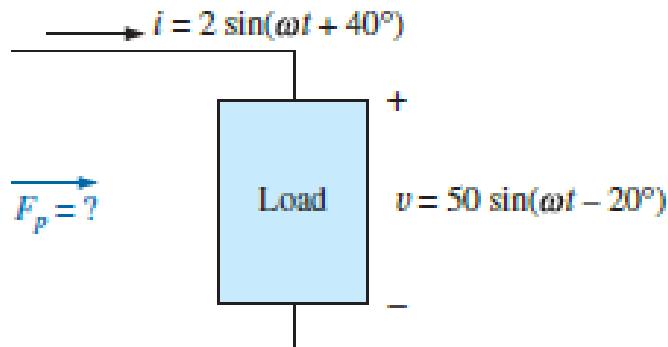
NOTE: θ is the phase angle of Z_T , not the current or voltage.



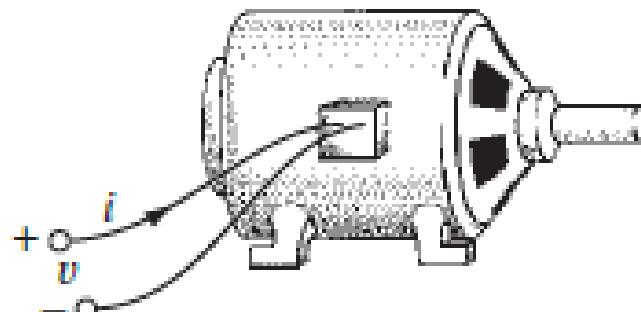
Power Factor Leading or Lagging



- The term leading and lagging are defined in **reference to the current** through the load.
 - If the **current leads** the voltage across the load then the load has a **leading power factor**.
 - If the **current lags** the voltage across the load then the load has a **lagging power factor**.



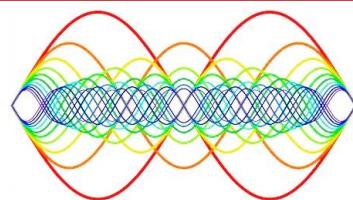
$$\begin{aligned}F_P &= \cos \theta \\F_P &= \cos(-20^\circ) - 40^\circ \\F_P &= \cos 60^\circ = 0.5 \text{ leading}\end{aligned}$$



$$\begin{aligned}v &= 120 \sin(\omega t + 80^\circ) \\i &= 5 \sin(\omega t + 30^\circ)\end{aligned}$$

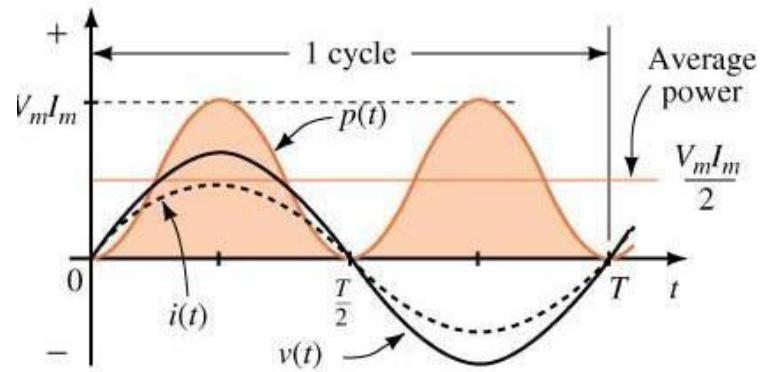
$$\begin{aligned}F_P &= \cos \theta \\F_P &= \cos(80^\circ - 30^\circ) \\F_P &= \cos 50^\circ = 0.64 \text{ lagging}\end{aligned}$$

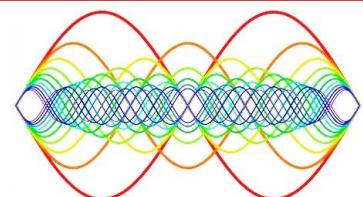
Unity Power Factor ($F_p = 1$)



- Unity Power Factor implies that *all* of a load's apparent power is real power ($S = P$).
- If $F_p = 1$, then $\theta = 0^\circ$.
- It could also be said that the load looks purely *resistive*.
- Load current and voltage are *in phase*.

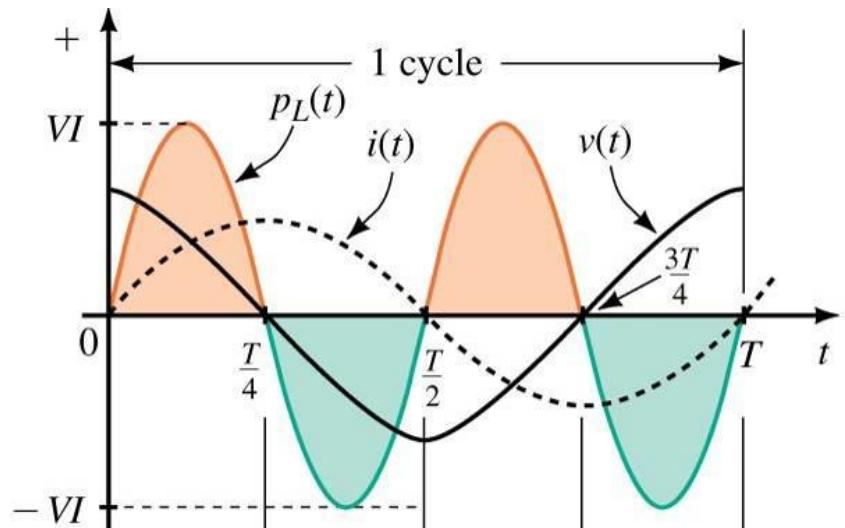
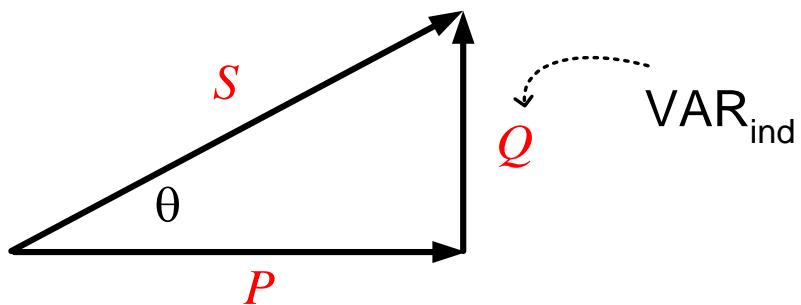
$$\begin{array}{c} \theta = 0^\circ \\ \xrightarrow{\hspace{1cm}} \\ P, S \end{array}$$
$$Q = 0$$



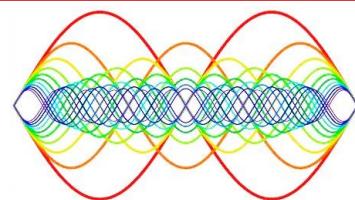


Lagging Power Factor ($\theta > 0^\circ$)

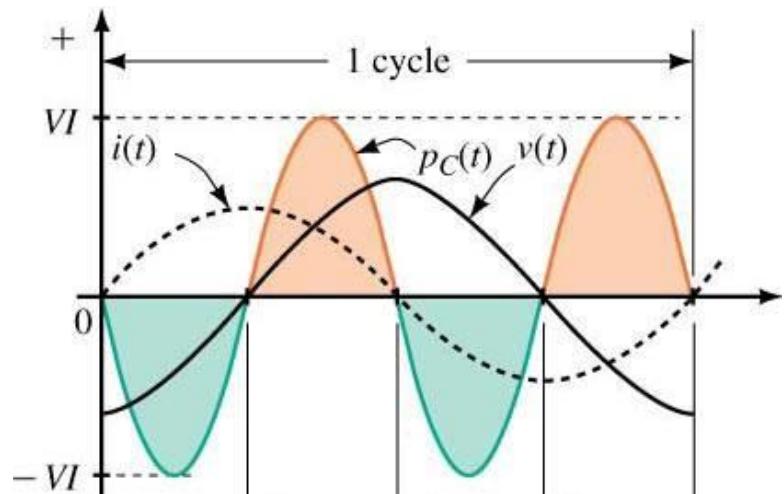
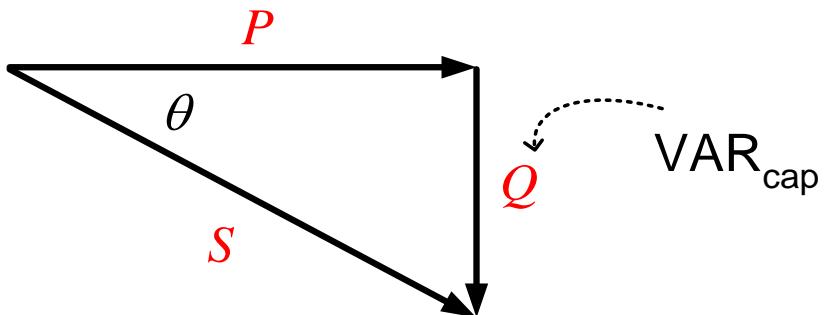
- The load current lags load voltage: ELI
- Implies that the load looks *inductive*.



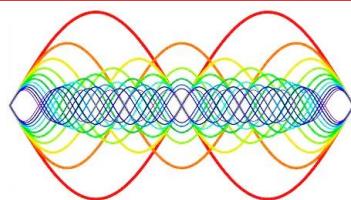
Leading Power Factor ($\theta < 0^\circ$)



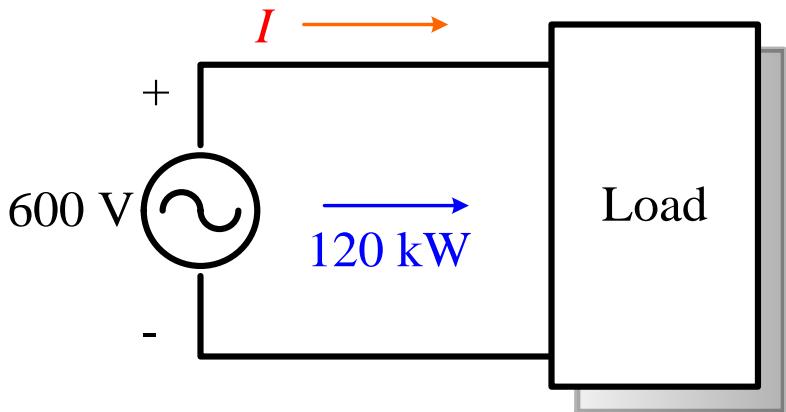
- The load current leads load voltage: **ICE**
- Implies that the load looks **capacitive**.



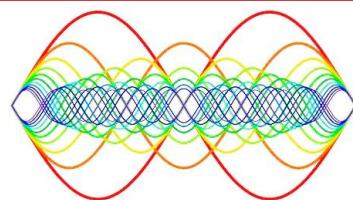
Why is Power Factor Important?



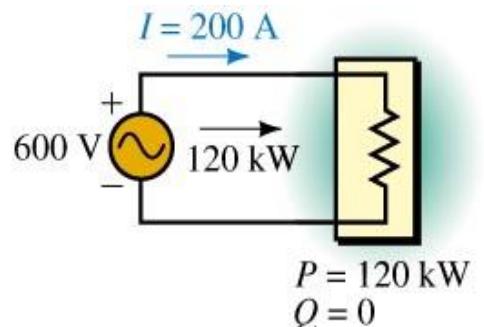
- Consider the following example: A generator is rated at 600 V and supplies one of two possible loads.
 - Load1: $P=120\text{ kW}$, $F_p=1$
 - Load 2: $P = 120 \text{ kW}$, $F_p = 0.6$
- Determining how much current (I) is required is one such reason...



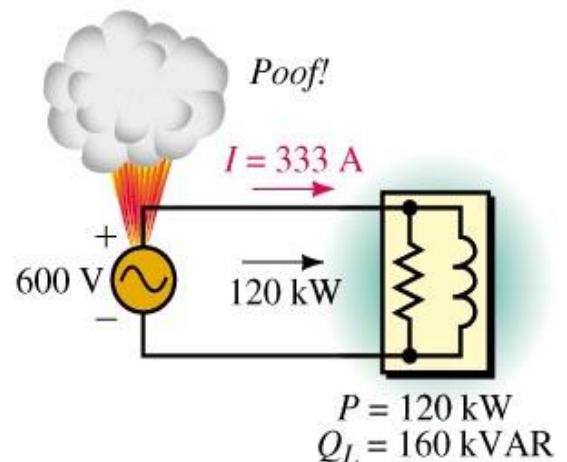
Why is Power Factor Important?



- For the load with $F_p = 1$, the generator had to supply 200 A
- For the load with $F_p = 0.6$, the generator had to supply 133 more amperes in order to do the same work (P)!
- Larger current means larger equipment (wires, transformers, generators) which cost more.
- Larger current also means larger transmission losses (think I^2R).

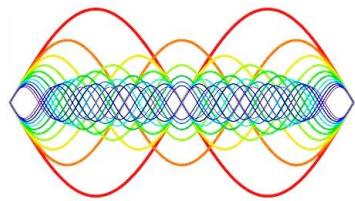


(a) $S = 120 \text{ kVA}$



(b) $S = \sqrt{(120)^2 + (160)^2} = 200 \text{ kVA}$
The generator is overloaded

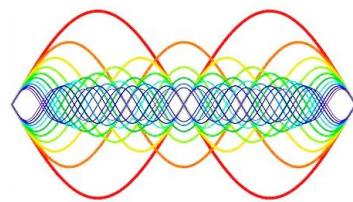
Why is Power Factor Important?



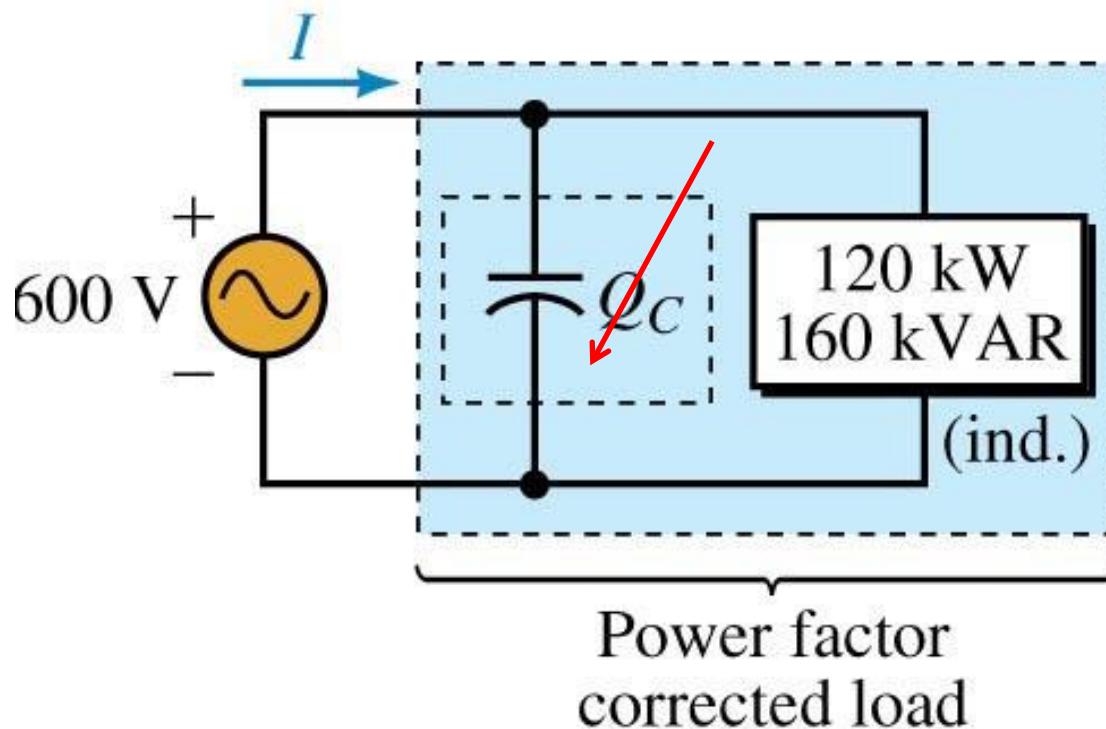
- Because of the wide variation in possible current requirements due to power factor, most large electrical equipment is rated using **apparent power (S)** in volt-amperes (VA) instead of **real power (P)** in watts (W).
- Is it possible to change the power factor of the load?

The answer is yes...through power factor correction...

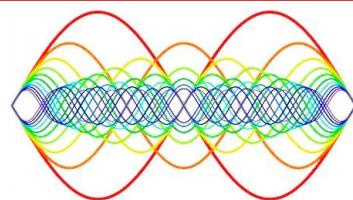
Power-Factor Correction



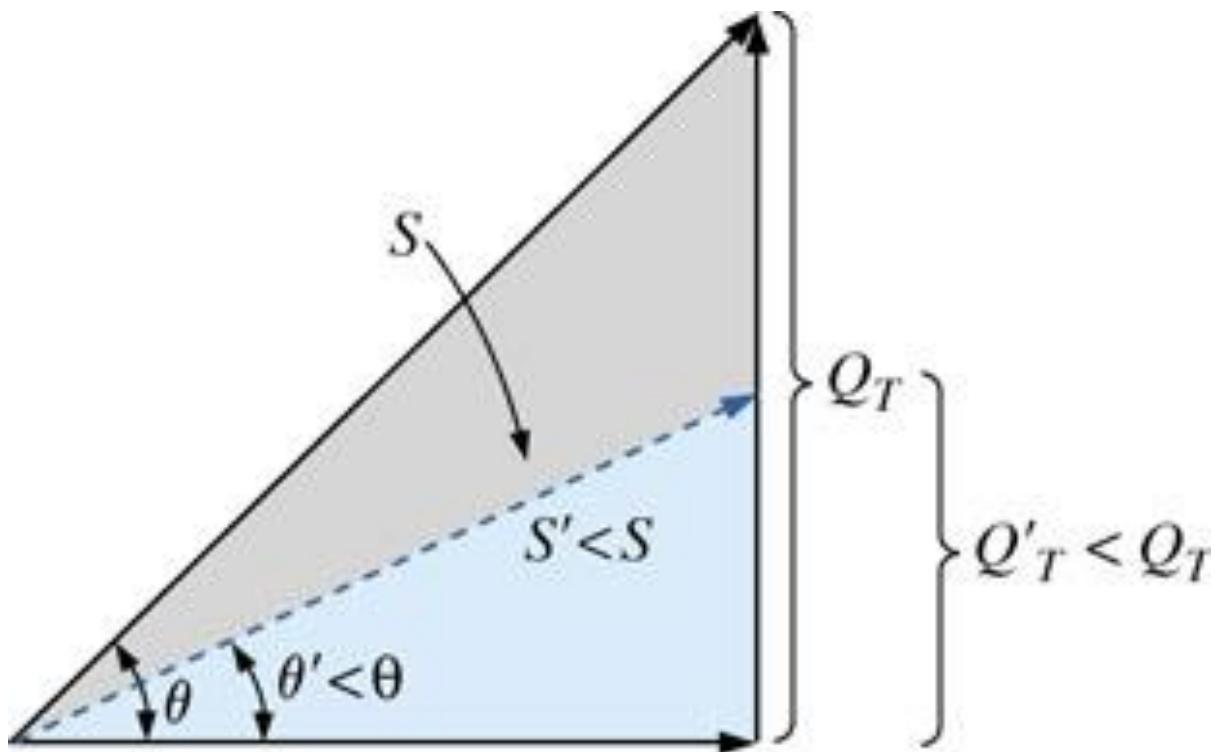
- In practice, almost all loads (commercial, industrial and residential) look **inductive** (due to motors, fluorescent lamp ballasts, etc.).
- Hence, almost all power factor correction consists of adding **capacitance**.



Power-Factor Correction

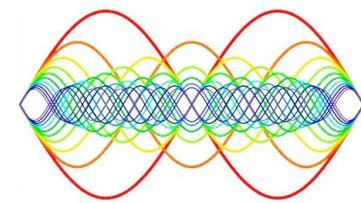


- How it changes the power triangle:



*Demonstrating the impact of power-factor correction
on the power triangle of a network.*

Power-Factor Correction Solution Steps

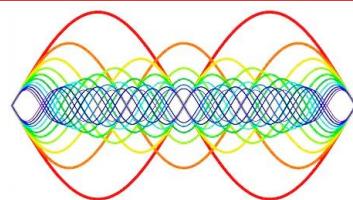


1. Calculate the reactive power (Q) of the load.
2. Insert a component in parallel of the load that will cancel out that reactive power.

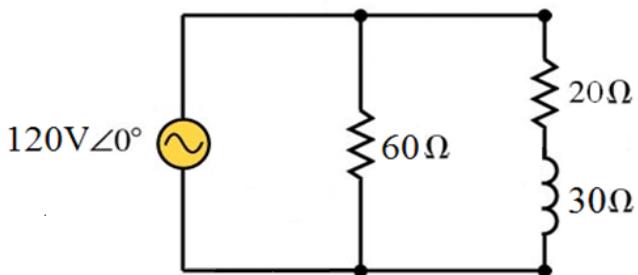
e.g. If the load has $Q_{LD} = 512 \text{ VAR}$, insert a capacitor with $Q_C = -512 \text{ VAR}$.

3. Calculate the reactance (X) that will give this value of Q Normally the $Q=V^2/X$ formula will work.
4. Calculate the component value (F or H) required to provide that reactance.

Example Problem 5



- Determine the value of the capacitance (in F) required to bring the power factor up to unity (freq of 60 Hz).
- Determine load current before and after correction.



a)

$$Z_T = \frac{1}{\frac{1}{60\Omega} * \frac{1}{20\Omega + j30\Omega}} = 25.3\Omega \angle 35.8^\circ$$

$$I_T = \frac{E_s}{Z_T} = \frac{120V \angle 0^\circ}{25.3\Omega \angle 35.8^\circ} = 4.74A \angle -35.8^\circ$$

↑ Sign Change

$$S_T = E_s * (I_T^*) = (120 \angle 0^\circ) * (4.74A \angle 35.8^\circ)$$

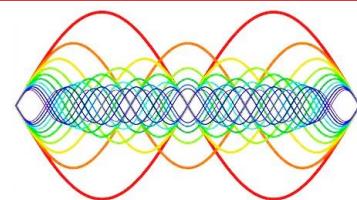
$$S_T = 568.7VA \angle 35.8 = 462W + j332VAR$$

P_T Q_T

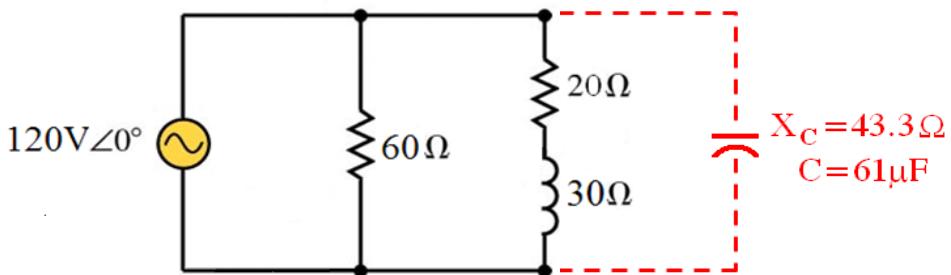
$$F_P = \frac{P}{S} = \frac{462W}{568.7VA} = 0.81 \quad \text{or} \quad 81\%$$

b) Because the $Q_T = 332$ VAR, we can insert a capacitor with $Q_C = -332$ VAR

Example Problem 5 cont...



- a. Determine the value of the capacitance (in F) required to bring the power factor up to unity (freq of 60 Hz).
- b. Determine load current before and after correction.



Notice that $X_C \neq X_L$!
 Notice that $X_C \neq X_{LD}$!

c) Adding a unity cap changes Z_T :

$$Z_{T_{Unity}} = \frac{1}{\frac{1}{60\Omega} * \frac{1}{20\Omega + j30\Omega} + \frac{1}{-j43.4\Omega}} = 31.2\Omega\angle 0^\circ$$

NEW Current:

$$I_T = \frac{E_S}{Z_T} = \frac{120V\angle 0^\circ}{31.2\Omega\angle 0^\circ} = 3.85A\angle 0^\circ$$

b) Because the $Q_T = 332 \text{ VAR}$, we can insert a capacitor with $Q_C = -332 \text{ VAR}$

$$Q_{T_{Unity}} = -332 \text{ VAR} \Rightarrow \frac{V^2}{X_C}$$

$$X_C = \frac{V^2}{|Q_{T_{Unity}}|} = \frac{(120V)^2}{332} = 43.3\Omega$$

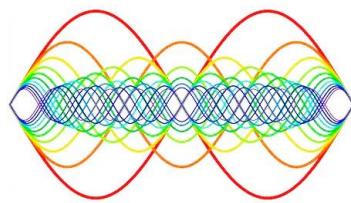
$$X_{C_{Unity}} = \frac{1}{2\pi f C_{Unity}}$$

$$C_{Unity} = \frac{1}{2\pi f X_{C_{Unity}}} = \frac{1}{2\pi(60Hz)(43.3\Omega)} = 61.1\mu F$$

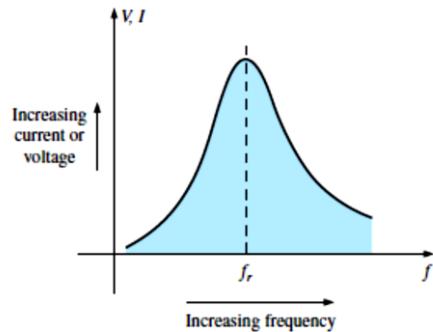
OLD Current:

$$I_T = \frac{E_S}{Z_T} = \frac{120V\angle 0^\circ}{25.3\Omega\angle 35.8^\circ} = 4.74A\angle -35.8^\circ$$

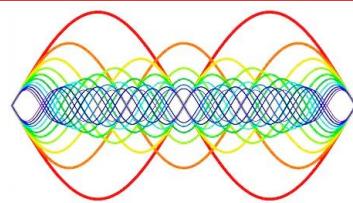
Resonance



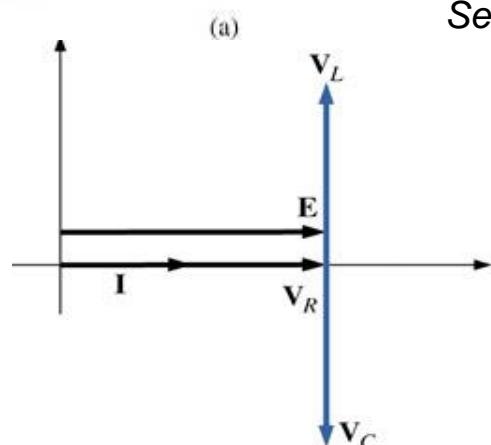
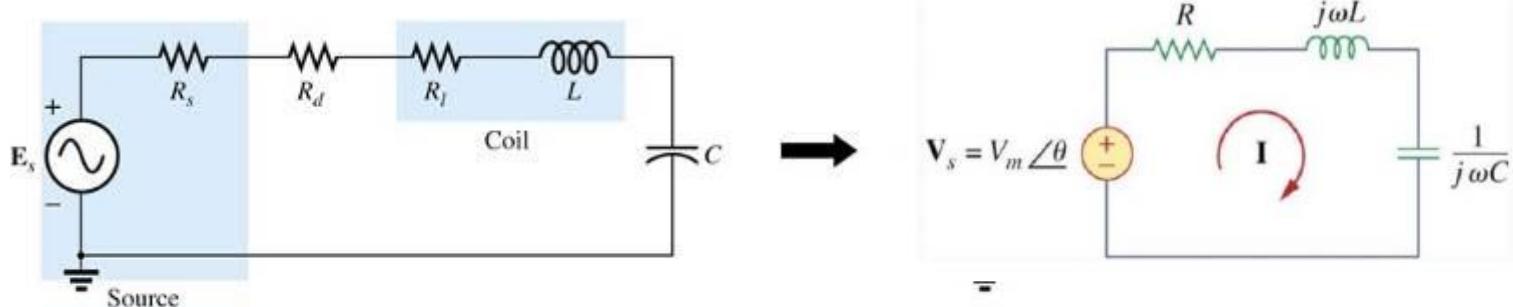
- **Resonant (or tuned) circuits**, are fundamental to the operation of a wide variety of electrical and electronic systems in use today.
- The resonant circuit is a combination of **R**, **L**, and **C** elements having a frequency response characteristic similar to the one below:
- **The resonant electrical circuit must have both inductance and capacitance.**
- In addition, resistance will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve.
- When resonance occurs due to the application of the proper frequency (f_r), the energy absorbed by one reactive element is the same as that released by another reactive element within the system.
- **Remember:** $X_C = \frac{1}{wC} = \frac{1}{2\pi fC} \Rightarrow Z_C = -jX_C$
 $X_L = wL = 2\pi fL \Rightarrow Z_L = jX_L$



Series Resonant Circuit

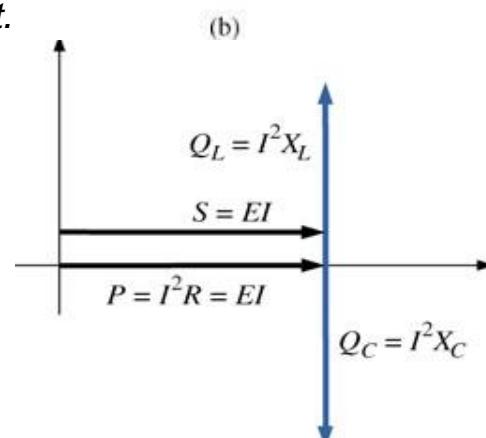


- The basic format of the series resonant circuit is a series R-L-C combination in series with an applied voltage source.



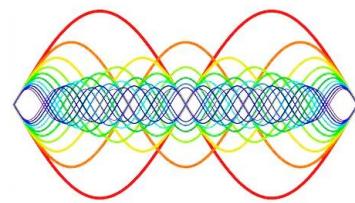
Phasor diagram for the series resonant circuit at resonance.

Series resonant circuit.



Power triangle for the series resonant circuit at resonance.

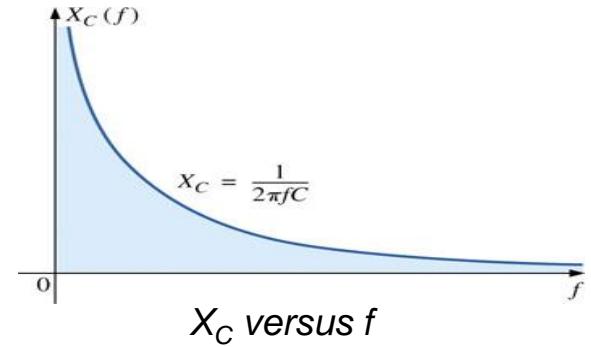
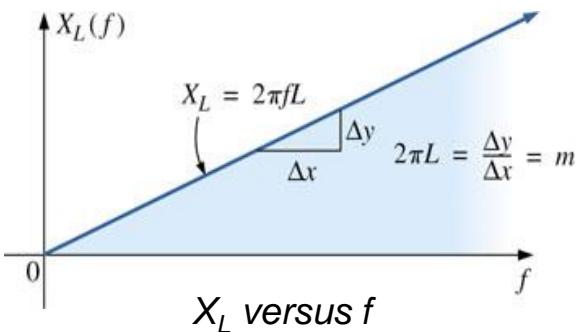
Z_T Versus Frequency



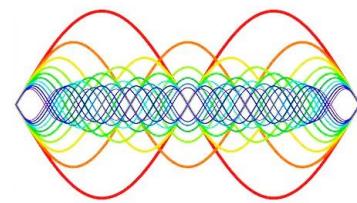
- Knowing that X_C and X_L are dependent upon frequency it can be stated:
 - Capacitor Impedance decreases as frequency increases.
 - Inductor Impedance increases as frequency increases.
- This implies that the total impedance of the series R-L-C circuit below, at any frequency, is determined by:

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z_T = R + jX_L - jX_C$$



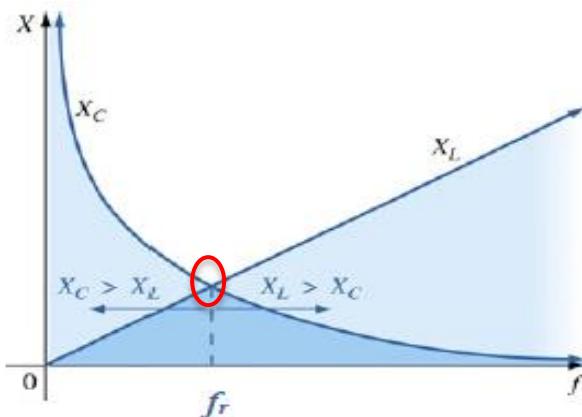
Z_T Versus Frequency cont.



- The total-impedance-versus-frequency curve for the series resonant circuit below can be found by applying the impedance-versus-frequency curve for each element of the equation previously shown, written in the following form:

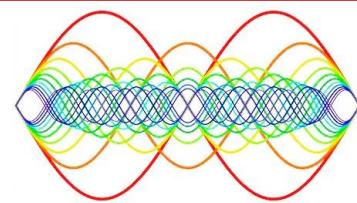
$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$

- When $X_L = X_C$ the resonant frequency (f_r) can be found.



Frequency response of X_L and X_C of a series R-L-C circuit on the same set of axes

The Resonant Frequency (f_r)



- To find f_r , set the impedances equal and solve:

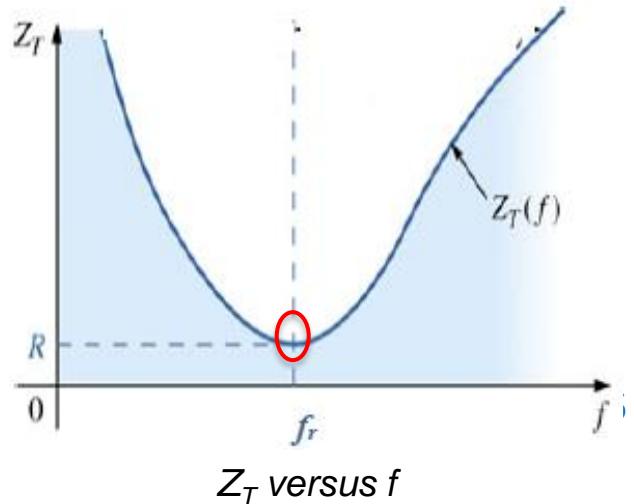
$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow \omega^2 = \frac{1}{LC}$$

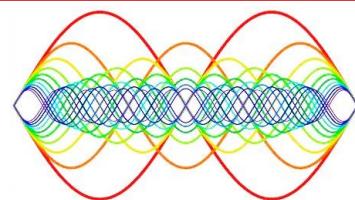
$$\omega = \frac{1}{\sqrt{LC}}, \text{ since } \omega = 2\pi f$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- This is the key equation for resonance. Total impedance at this point is shown to the right:

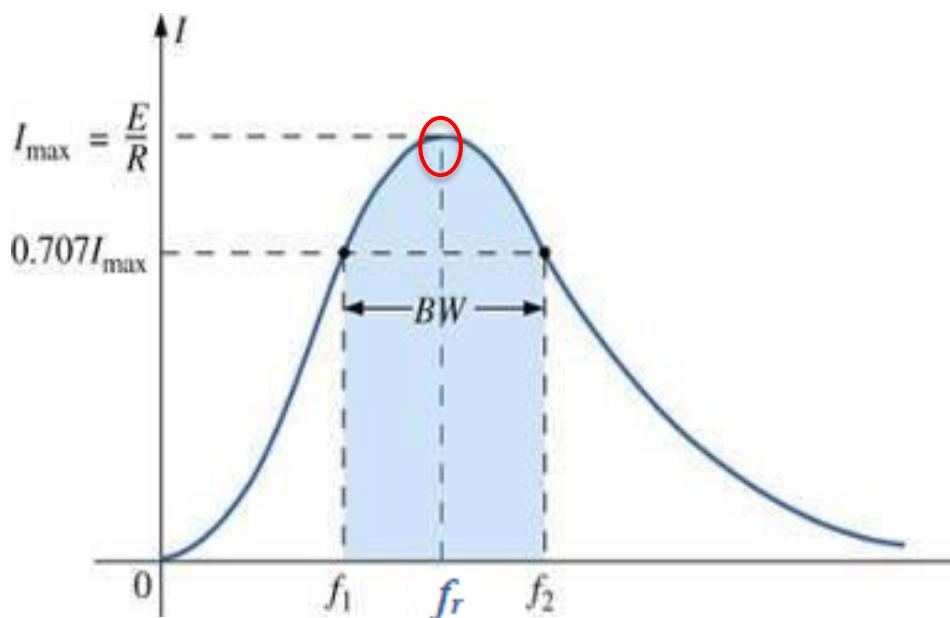


Current Versus Frequency



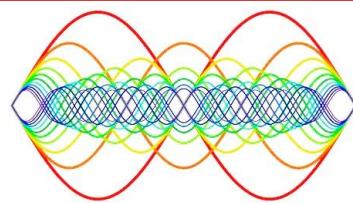
- If impedance is minimum at f_r , current will be at a maximum:
- If we now plot the magnitude of the current versus frequency for a *fixed* applied voltage E , we obtain the curve showing that current is maximum at f_r :

$$I = \frac{E}{Z_T}$$



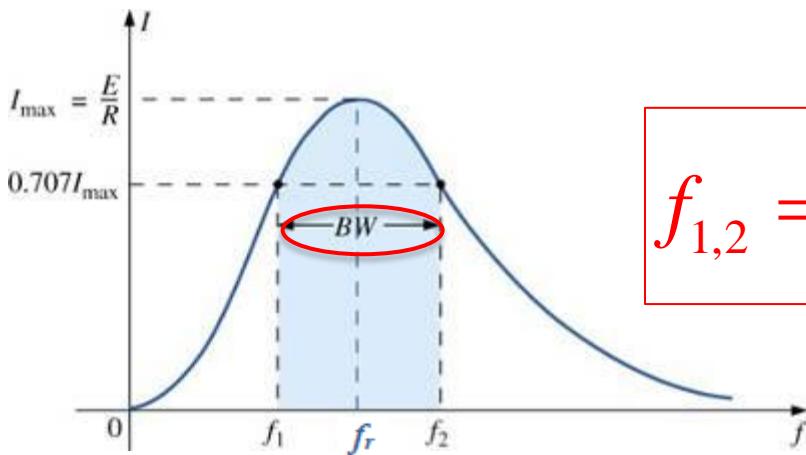
I versus f for the series resonant circuit

Bandwidth (BW)



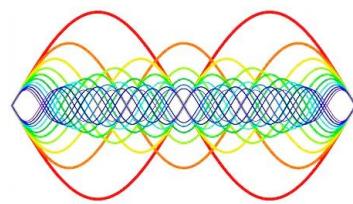
- Band frequencies are those that define the points on the resonance curve that are 0.707 ($\frac{1}{\sqrt{2}} = 0.707$) of the peak current or voltage.
- Bandwidth (BW) is the range of frequencies between the band, or $\frac{1}{2}$ power frequencies. Defined by:

$$BW = f_2 - f_1$$



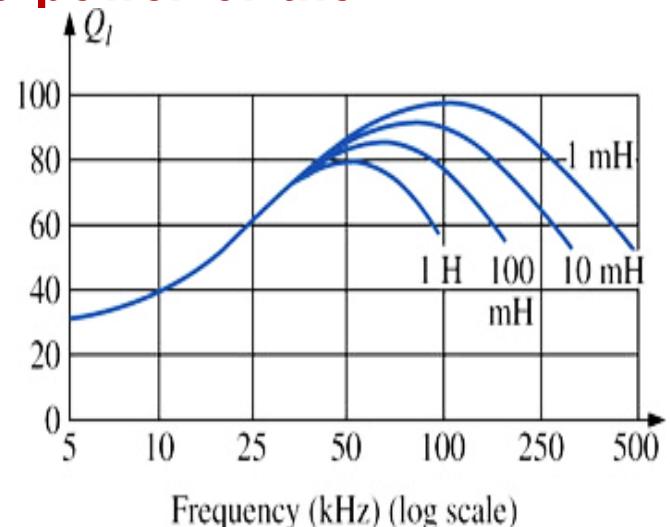
$$f_{1,2} = f_r \pm \frac{BW}{2}$$

The Quality Factor (Q)



- The *quality factor (Q)* of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance.
- Q can be found several ways:

$$Q = \frac{f_r}{BW} = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$$

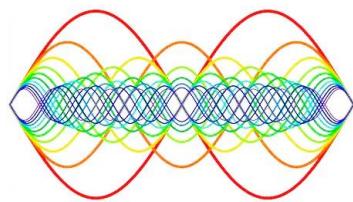


- This also gives an alternate way to find BW:

$$BW = \frac{f_r}{Q}$$

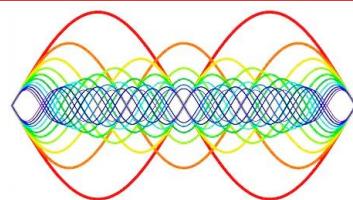
Q_l versus frequency for a series of inductors

OUTLINES



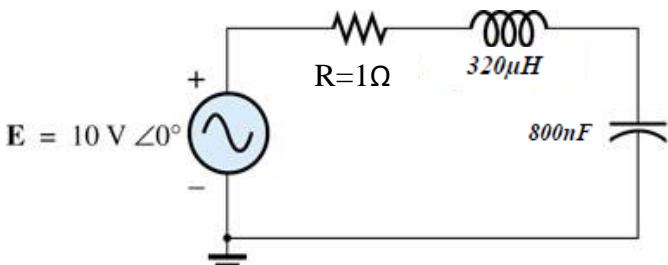
- AC Power and Power Triangle
- AC Power Factor and Power Factor Correction
- Resonance.
 - Series Resonant Circuit
 - Parallel Resonant Circuit

Example Problem 6



Determine f_r , Q , BW and the current (I) at resonance.

Plot the current vs. frequency and label f_r , f_1 , f_2 and BW .



We know X_L so we can find Q:

$$Q = \frac{X_L}{R} = \frac{20\Omega}{1\Omega} = 20$$

We know Q so we can find BW:

$$BW = \frac{f_r}{Q} = \frac{10kHz}{20} = 500Hz$$

Now find I_{max} :

$$I = \frac{E}{Z_T} = \frac{10V}{1\Omega} = 10A$$

Remember, since $X_L = X_C$ at resonance, they cancel out for Z_T and only R is left.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(320\mu H)(800nF)}} = 10kHz$$

$$X_L = \omega_r L = 2\pi f_r L = 2\pi(10kHz)(320\mu H) = 20\Omega$$

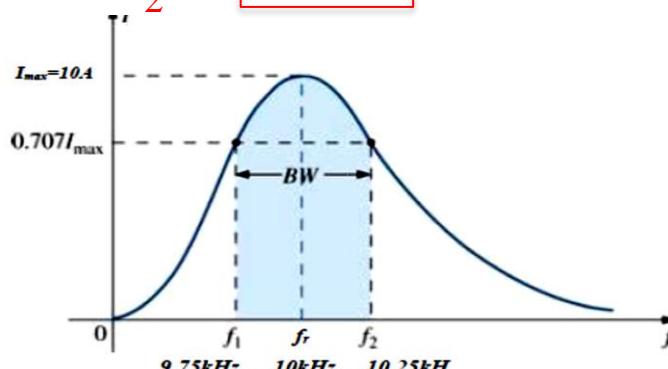
Because we are at f_r we know that $X_L = X_C$, but just to show it:

$$X_C = \frac{1}{\omega_r C} = \frac{1}{2\pi f_r C} = \frac{1}{2\pi(10kHz)(800nH)} = 20\Omega$$

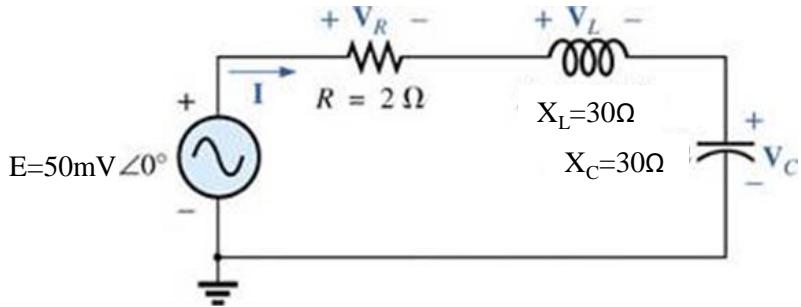
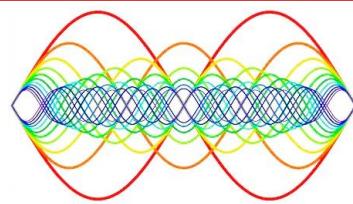
Now let's find f_1 and f_2 and then plot:

$$f_{1,2} = f_r \pm \frac{BW}{2} \Rightarrow f_2 = 10kHz + \frac{500Hz}{2} = 10.25kHz$$

$$\Rightarrow f_1 = 10kHz - \frac{500Hz}{2} = 9.75kHz$$



Example Problem 7 [TUTORIAL]



a) $Z_T = R + jX_L - jX_C = 2\Omega + j30\Omega - j30\Omega = 2\Omega$

$$I = \frac{E}{Z_T} = \frac{50mV\angle 0^\circ}{2\Omega\angle 0^\circ} = 25mA\angle 0^\circ$$

$$V_R = I * R = (25mA\angle 0^\circ) * (2\Omega\angle 0^\circ) = 50mV\angle 0^\circ$$

$$V_L = I * X_L = (25mA\angle 0^\circ) * (30\Omega\angle 90^\circ) = 750mV\angle 90^\circ$$

$$V_C = I * X_C = (25mA\angle 0^\circ) * (30\Omega\angle -90^\circ) = 750mV\angle -90^\circ$$

This shows that at resonance $V_C = V_L$

b) $Q = \frac{X_L}{R} = \frac{30\Omega}{2\Omega} = 15$

- a) Find I, V_R, V_C, V_L at resonance.
- b) Determine Q for the circuit.
- c) If the f_r is 5 kHz, what is the BW ?
- d) With $f_r = 5\text{kHz}$ what are the values of L and C ?
- e) What is the power dissipated in the circuit at the *half-power* frequency?

c) $BW = \frac{f_r}{Q} = \frac{5\text{kHz}}{15} = 333.3\text{Hz}$

d) $X_L = 2\pi f_r L \Rightarrow$

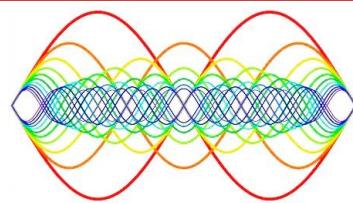
$$L = \frac{X_L}{2\pi f_r} = \frac{30\Omega}{2\pi(5\text{kHz})} = 955\mu\text{H}$$

$$X_C = \frac{1}{2\pi f_r C} \Rightarrow$$

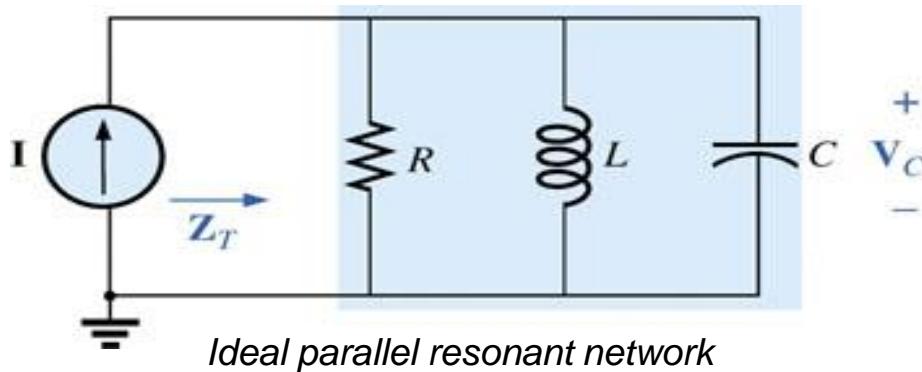
$$C = \frac{1}{2\pi f_r X_C} = \frac{1}{2\pi(5\text{kHz})(30\Omega)} = 1.06\mu\text{F}$$

e) $P = I_{0.707}^2 * R = (25mA * 0.707) * (2\Omega) = 35.4\text{mW}$

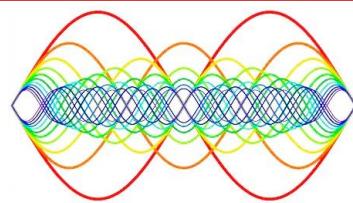
Parallel Resonant Circuit



- The basic format of the parallel resonant circuit is a parallel R-L-C combination with an applied current source.
- The parallel resonant circuit has the basic configuration shown below:



Parallel Resonant Circuit



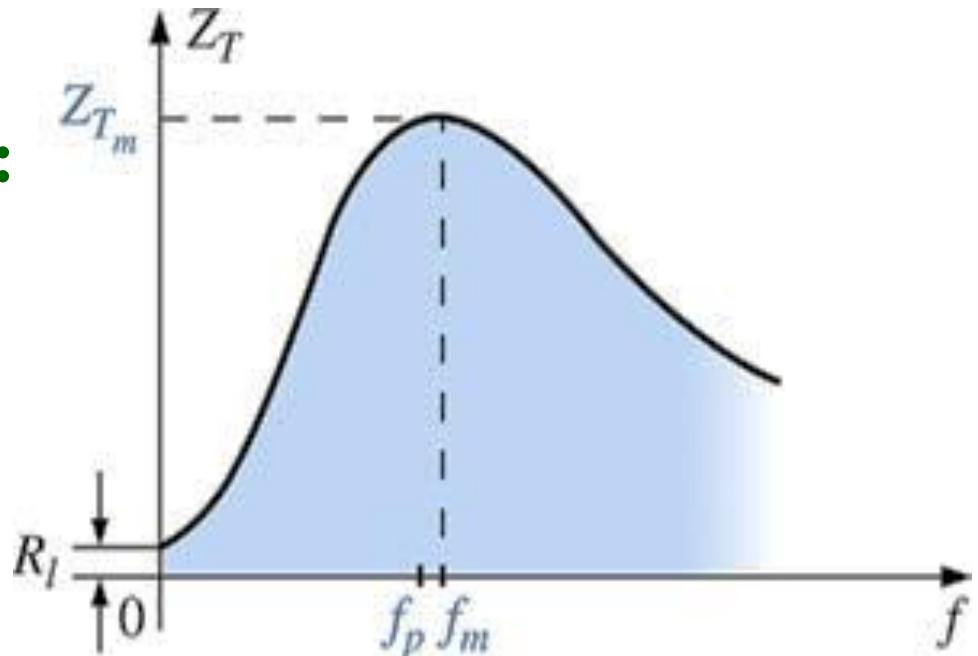
- Unity Power Factor, f_p :

$$f_p = f_r \sqrt{1 - \frac{R_l^2 C}{L}}$$

- Maximum Impedance, f_m :

$$f_m = f_r \sqrt{1 - \frac{1}{4} \left(\frac{R_l^2 C}{L} \right)}$$

$$f_r > f_p > f_m$$



Z_T versus frequency for the parallel resonant circuit