Tutorial 8

1. Find the area under the normal curve

   a. To the right of $z = -0.85$.

   b. Between $z = 0.40$ and $z = 1.30$.

   c. Between $z = -0.30$ and $z = 0.90$.

   d. Outside $z = -1.50$ to $z = -0.45$.

1. (a) Using Table A.2: $1 - 0.1977 = 0.8023$

   (b) Using Table A.2: $0.9032 - 0.6554 = 0.2478$

   (c) Using Table A.2: $0.8159 - 0.3821 = 0.4338$

   (d) Using Table A.2: $0.0668 + (1 - 0.3264) = 0.7404$

Exercise

3. Let $Z \sim N(0, 1)$. Find a constant c for which

a. $P(Z \geq c) = 0.1587$

b. $P(c \leq Z \leq 0) = 0.4772$

c. $P(-c \leq Z \leq c) = 0.8664$

d. $P(0 \leq Z \leq c) = 0.2967$

e. $P(|Z| \leq c) = 0.1470$

3. (a) Using Table A.2: $c = 1$

(b) Using Table A.2: $c = -2.00$

(c) Using Table A.2: $c = 1.50$

(d) Using Table A.2: $c = 0.83$

(e) Using Table A.2: $c = 1.45$

5. A process manufactures ball bearings with diameters that are normally distributed with mean 25.1 mm and standard deviation 0.08 mm.

    a.   What proportion of the diameters are less than 25.0 mm?

    b.   Find the 10th percentile of the diameters.

    c.   A particular ball bearing has a diameter of 25.2 mm. What percentile is its diameter on?

    d.   To meet a certain specification, a ball bearing must have a diameter between 25.0 and 25.3 millimeters. What proportion of the ball bearings meet the specification?

(a) $z = (25.0 - 25.1)/0.08 = -1.25$. The area to the right of $z = -1.25$ is 0.1056.

(b) The z-score of the 10th percentile is ≈ -1.28.

The 10th percentile is therefore $\approx 25.1 - 1.28(0.08) = 24.9976$.

(c) $z = (25.2 - 25.1)/0.08 = 1.25$. The area to the left of $z = 1.25$ is 0.8944.

Therefore a diameter of 25.2 is on the 89th percentile, approximately.

(d) For 25.0, $z = (25.0 - 25.1)/0.08 = -1.25$. For 25.3, $z = (25.3 - 25.1)/0.08 = 2.50$.

The area between $z = -1.25$ and $z = 2.50$ is $0.9938 - 0.1056 = 0.8882$.

7. In a recent study, the Centers for Disease Control reported that diastolic blood pressures (in mmHg) of adult women in the U.S. are approximately normally distributed with mean 80.5 and standard deviation 9.9.

    a.   What proportion of women have blood pressures lower than 70?

    b.   What is the 80th percentile of blood pressures?

    c.   A woman has a blood pressure of 84. What percentile is her blood pressure on?

    d.   A diastolic blood pressure greater than 90 is classified as hypertension (high blood pressure). What proportion of women have hypertension?

7. (a) $z = (70 - 80.5)/9.9 = -1.06$. The area to the right of $z = -1.06$ is 0.1446.

(b) The z-score of the 80th percentile is ≈ 0.84.

    The 80th percentile is therefore $\approx 80.5 + 0.84(9.9) = 88.8$.

(c) $z = (84 - 80.5)/9.9 = 0.35$. The area to the left of $z = 0.35$ is 0.6368.

    Therefore a score of 84 is on the 64th percentile, approximately.

(d) $z = (90 - 80.5)/9.9 = 0.96$. The area to the left of $z = 0.96$ is 0.8315.

The weights of population of woman are normally distributed with mean 60 kg and standard deviation 5 kg. what is probability that the weight of women chosen at random is

1- Less than 61 kg

2- Less than 65 kg

3- Between 61 and 65 kg

4- Greater than 70 kg

5- Less than 57 kg

Sol $\mu = 60$, $\sigma = 5$

$Z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{61 - 60}{5} = 0.2$ $P(x < 61) = p(z < 0.2) = \phi(0.2) = 0.5793$

2- $z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{65 - 60}{5} = 1$ $P(x < 65) = p(z < 1) = \phi(1) = 0.8413$

3- $p(61 < x < 65) = p(0.2 < z < 1)$ $= \phi(1) - \phi(0.2) = 0.8413 - 0.5793$ #

4- $Z = \dfrac{70 - 60}{5} = 2$ $P(x > 70) = p(z > 2) = 1 - p(z < 2)$

$= 1 - \phi(2) = 1 - 0.9772 = 0.0228$

5- $Z = \dfrac{57 - 60}{5} = -0.6$ $P(x < 57) = p(z < -0.6) =$

$= \phi(-0.6) = 1 - \phi(0.6)$

The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 millimeters and a standard deviation of 0.2 millimeter.

i. What is the probability that a covering thickness is greater than 5.5 millimeters?

ii. If the specifications require the thickness to be between 4.5 and 5.5 millimeters, what is the proportion of coverings do not meet specifications?

iii. The covering thickness of 95 % of samples is below what value?

The diameter of a shaft in an optical storage drive is normally distributed by mean 0.2508 inch and standard deviation 0.0005 inch. The specifications on the shaft are 0.2500 ± 0.0015 inch. What proportion of the shafts conforms to specifications.

The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 6000 hours.

i. What is the probability that a laser fails before 5000 hours?

ii. What is the life in hours that 95 % of the lasers exceed?

Thank You