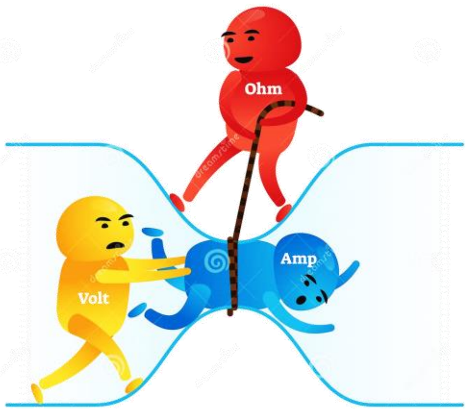
Circuits and Circuit Elements II

CSE 113



Physics Department
Faculty of Science
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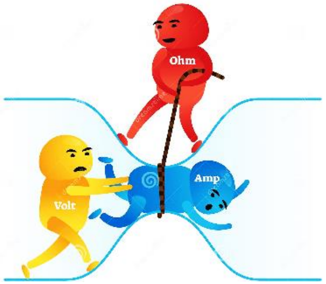
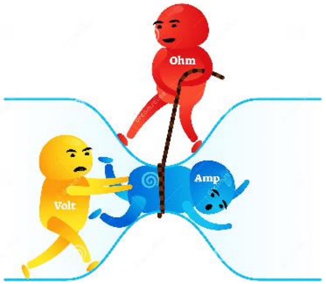
24/02/2024

Overview



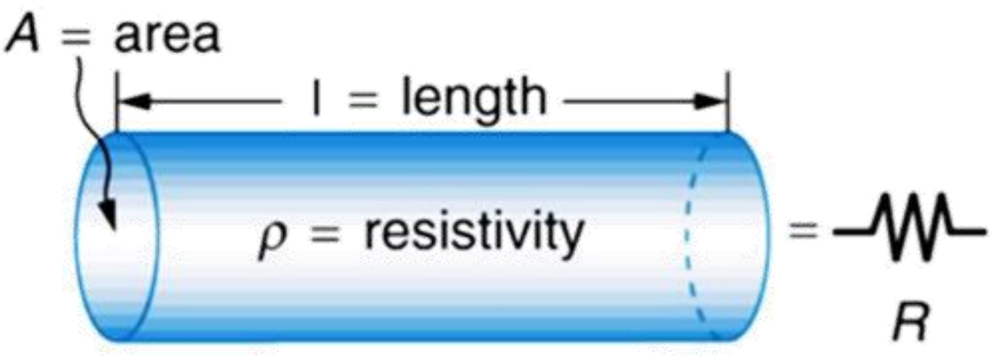
- Introduce Ohm’s law: a central concept in electric circuits.
- Resistors will be discussed in more detail.
- Circuit topology and the voltage and current laws will be introduced.
- Finally, meters for measuring voltage, current, and resistivity will be presented.

Resistivity

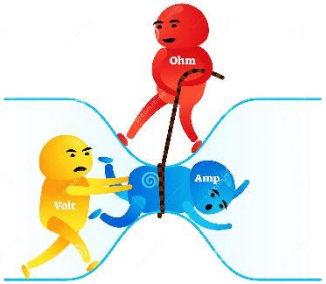


- Materials tend to resist the flow of electricity through them.
- This property is called "resistance".
- The resistance of an object is a function of its length, l, and cross-sectional area, A, and the material's resistivity:

$$R = \rho \frac{l}{A}$$



Ohm’s Law



- In a resistor, the voltage across a resistor is directly proportional to the current flowing through it.

$$V = IR$$

- The resistance of an element is measured in units of Ohms,

$$\Omega,\ (V/A).$$

- The higher the resistance, the less current will flow through for a given voltage.
- Ohm’s law requires conforming to the passive sign convention.

Resistivity of Common Materials

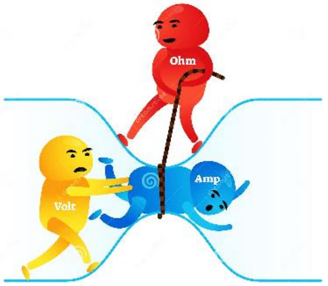


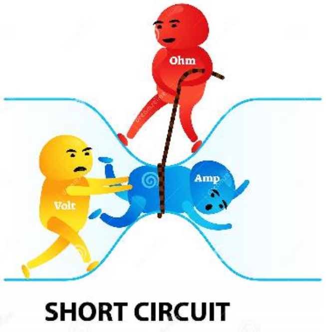
TABLE 2.1

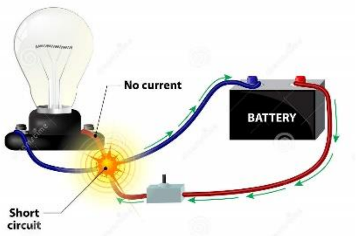
Resistivities of common materials.

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

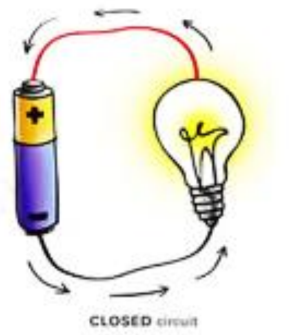
Short and Open Circuits

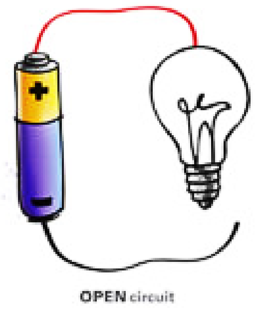
- A connection with almost zero resistance is called a short circuit.
- Ideally, any current may flow through the short.
- In practice this is a connecting wire.



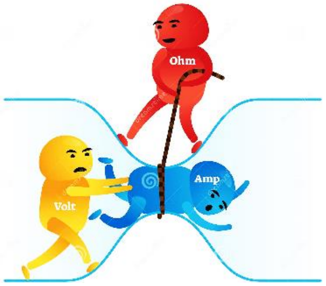


- A connection with infinite resistance is called an open circuit.
- Here no matter the voltage, no current flows.





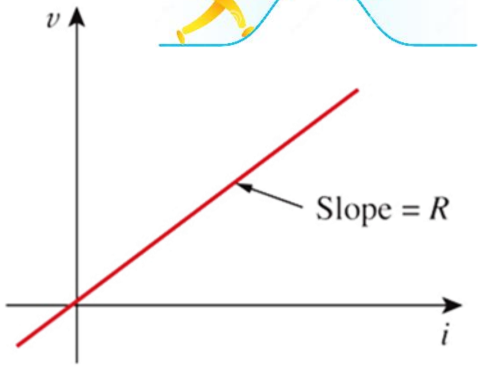
Linearity



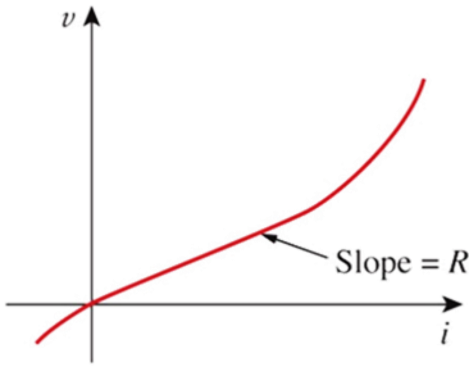
- Not all materials obey Ohm's Law.

- Resistors that do are called linear resistors because their current voltage relationship is always linearly proportional.

- Diodes and light bulbs are examples of non-linear elements.

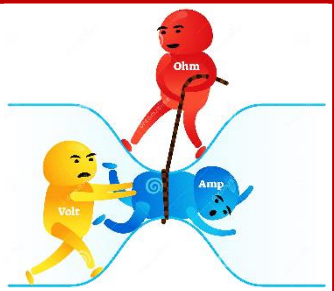


(a)



(b)

Power Dissipation

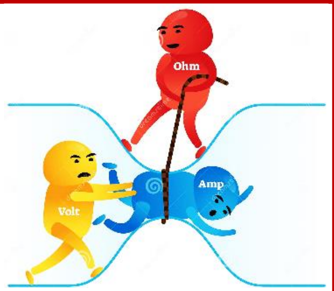


- Running current through a resistor dissipates power.

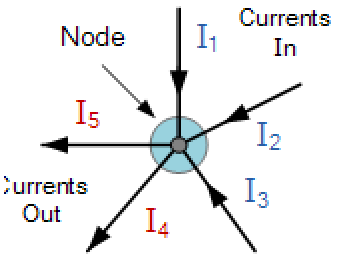
$$p = vi = i^2 R = \frac{v^2}{R}$$

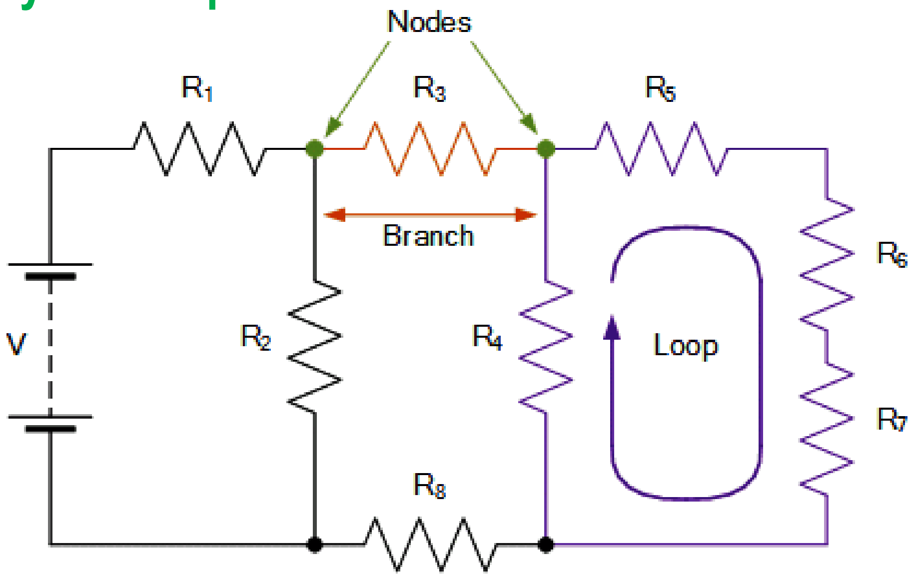
- The power dissipated is a non-linear function of current or voltage.
- Power dissipated is always positive.
- A resistor can never generate power.

Nodes Branches and Loops

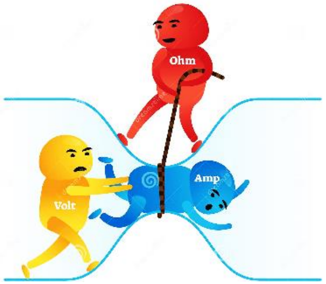


- Circuit elements can be interconnected in multiple ways.
- To understand this, we need to be familiar with some network topology concepts.
- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

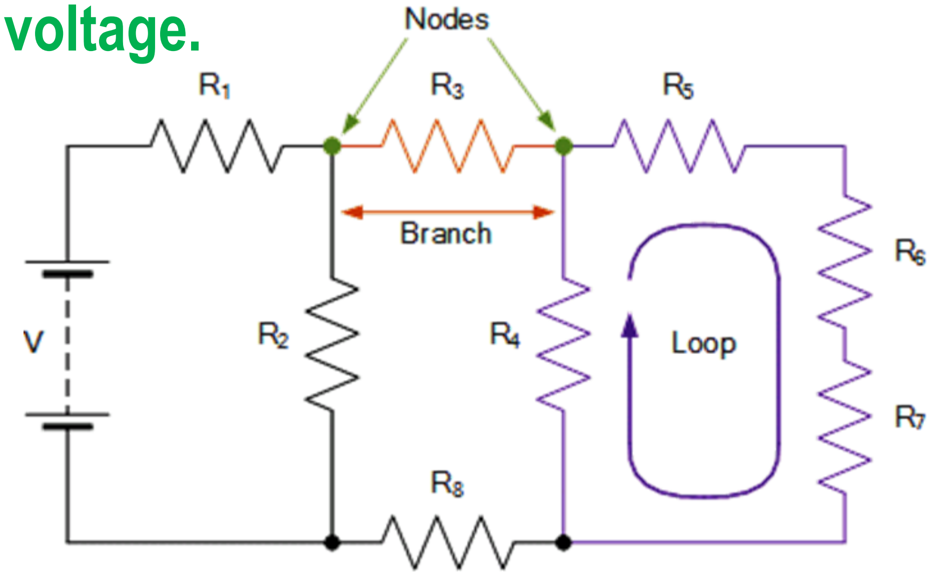




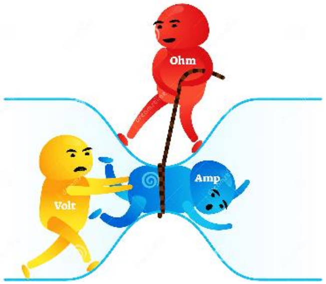
Network Topology



- A loop is independent if it contains at least one branch not shared by any other independent loops.
- Two or more elements are in series if they share a single node and thus carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and thus have the same voltage.



Kirchhoff's Laws

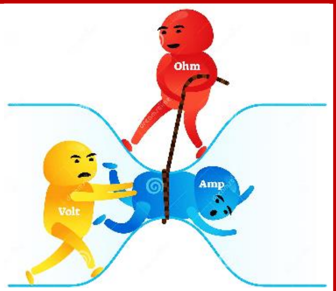


- Ohm’s law is not sufficient for circuit analysis.
- Kirchhoff's laws complete the needed tools.

There are two laws:

- Current law.
- Voltage law.

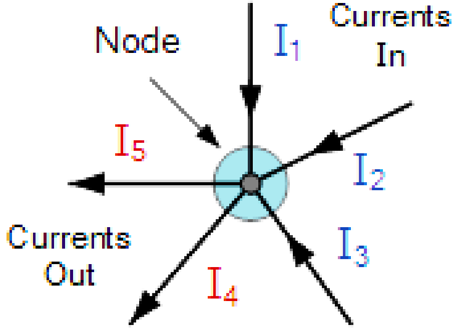
Kirchhoff's current law (KCL)



- Kirchhoff's current law is based on conservation of charge.
- It states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- It can be expressed as:

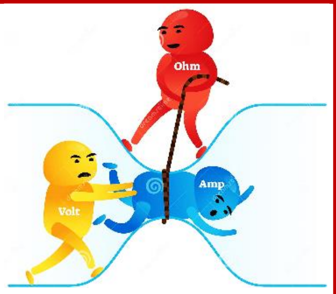
$$\sum_{n=1}^{N} i_n = 0$$

Currents Entering the Node
Equals
Currents Leaving the Node



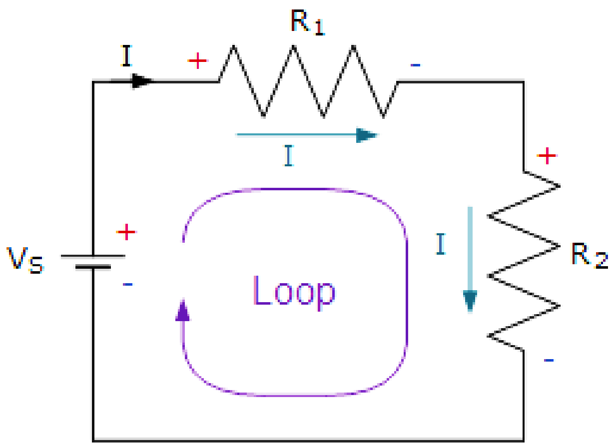
$$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$$

Kirchhoff's voltage law (KVL)

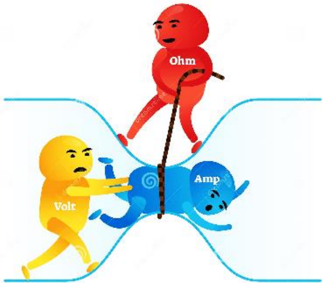


- Kirchhoff's voltage law is based on conservation of energy.
- It states that the algebraic sum of currents around a closed path (or loop) is zero.
- It can be expressed as

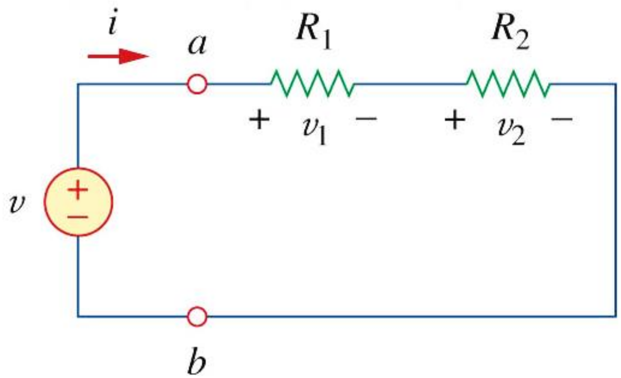
$$\sum_{m=1}^{M} v_m = 0$$



Series Resistors



- Two resistors are considered in series if the same current pass through them.
- Take the circuit shown:



- Applying Ohm's law to both resistors.

$$v_1 = iR_1 \quad v_2 = iR_2$$

- If we apply K V L to the loop, we have:

$$-v + v_1 + v_2 = 0$$

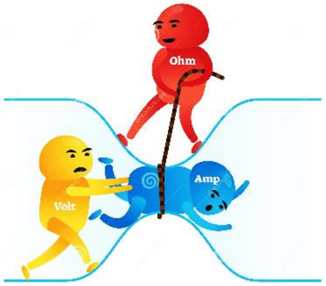
- Combining the two equations:

$$v = v_1 + v_2 = i\left(R_1 + R_2\right)$$

- From this we can see there is an equivalent resistance of the two resistors: $R_{eq} = R_1 + R_2$
- For N resistors in series:

$$R_{eq} = \sum_{n=1}^{N} R_n$$

Voltage Division

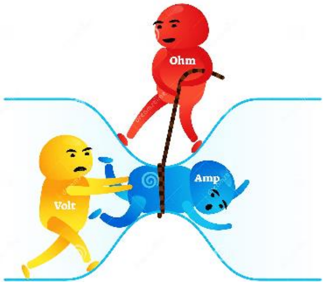


- The voltage drop across any one resistor can be known.
- The current through all the resistors is the same, so using Ohm's law:

$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

- This is the principle of voltage division.

Parallel Resistors

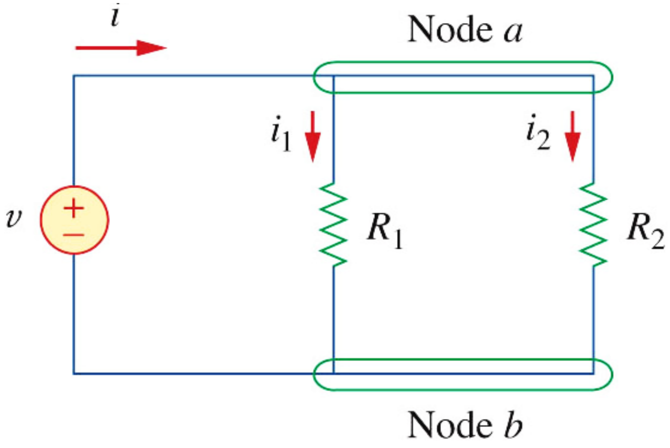


- **When resistors are in parallel, the voltage drop across them is the same.**

$$v = i_1 R_1 = i_2 R_2$$

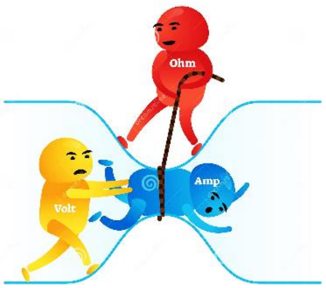
- **By K C L, the current at node a is**

$$i = i_1 + i_2$$



- **The equivalent resistance is:**

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



Current Division

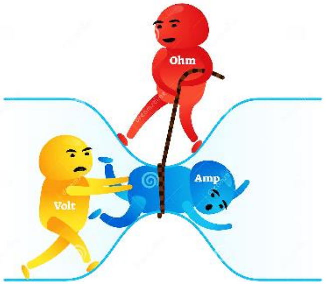
- Given the current entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors.

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

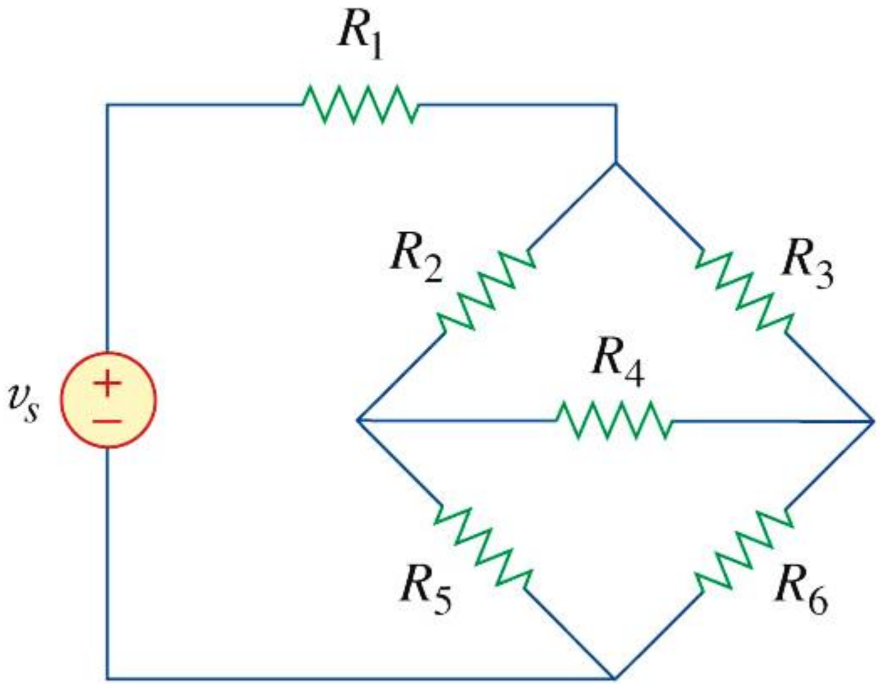
- This can be used in combination with Ohm's law to get the current through each resistor:

$$i_1 = \frac{iR_2}{R_1 + R_2} \qquad i_2 = \frac{iR_1}{R_1 + R_2}$$

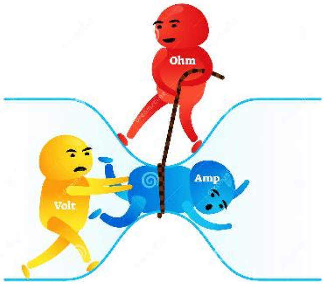
Wye-Delta Transformations



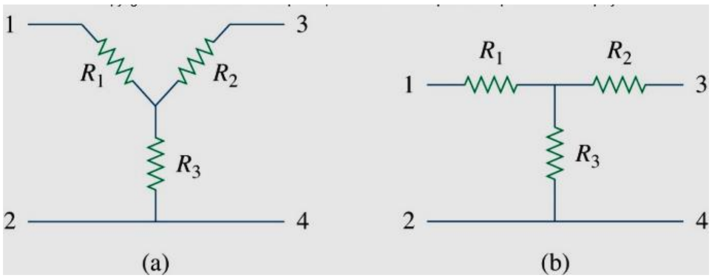
- There are cases where resistors are neither parallel nor series.
- Consider the bridge circuit shown here.
- This circuit can be simplified to a three-terminal equivalent.

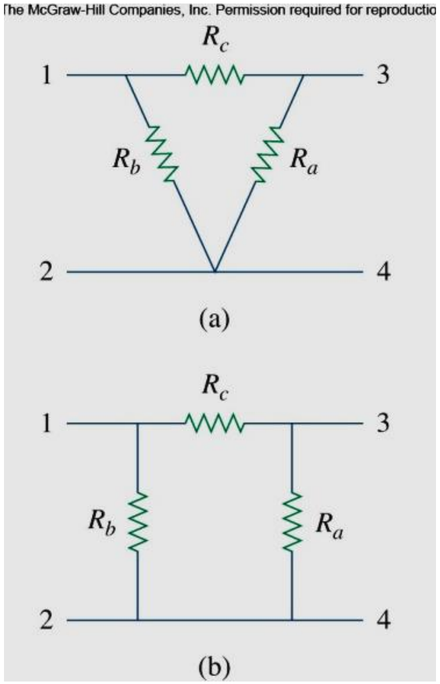


Wye-Delta Transformations II

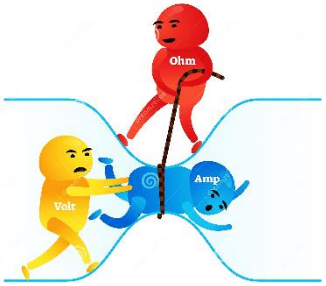


- Two topologies can be interchanged:
  - Wye (Y) or tee (T) networks.
  - Delta (Δ) or pi (Π) networks.
- Transforming between these two topologies often makes the solution of a circuit easier.

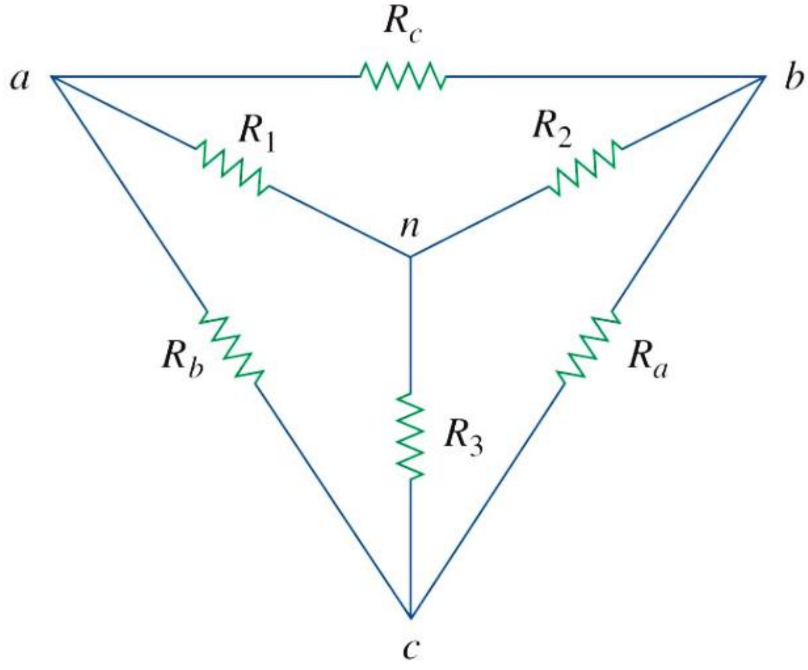




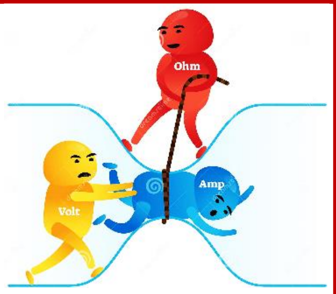
Wye-Delta Transformations III



- The superimposed wye and delta circuits shown here will used for reference.
- The delta consists of the outer resistors, labeled a,b, and c.
- The wye network are the inside resistors, labeled 1,2, and 3.



Delta to Wye

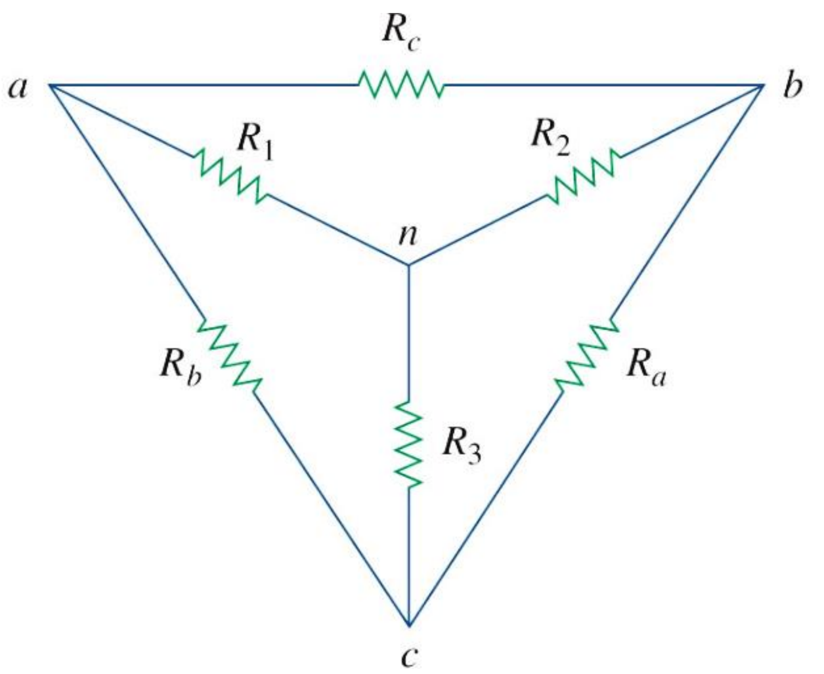


- The conversion formula for a delta to wye transformation are:

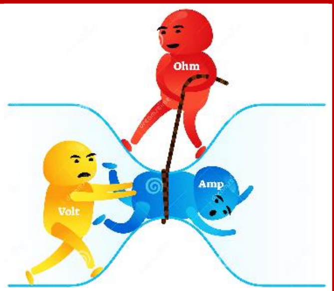
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Wye to Delta



- The conversion formula for a wye to delta transformation are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$