

CALCULUS

EARLY TRANSCENDENTAL FUNCTIONS

5th EDITION

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Trigonometric functions

1.

$$\int f' \sin f dx = -\cos f + c$$

Example

$$\int \sin x dx = -\cos x + c$$

Example

$$\int \sin 2x dx = \frac{1}{2} \int 2 \sin 2x dx$$

$$= -\frac{1}{2} \cos 2x + c$$

2.

$$\int f' \cos f dx = \sin f + c$$

Example

$$\int \cos x dx = \sin x + c$$

Example

$$\int \cos 2x \, dx = \frac{1}{2} \int 2 \cos 2x \, dx$$

$$= \frac{1}{2} \sin 2x + c$$

3.

$$\int f' \tan f \, dx = -\ln |\cos f| + c$$

Example

$$\int \tan x \, dx = -\ln |\cos x| + c$$

Example

$$\int \tan 3x \, dx = \frac{1}{3} \int 3 \tan 3x \, dx$$

$$= -\frac{1}{3} \ln |\cos 3x| + c$$

4. $\int f' \cot f dx = \ln|\sin f| + c$

Example $\int \cot x dx = \ln|\sin x| + c$

Example $\int \cot 3x dx = \frac{1}{3} \int 3 \cot 3x dx$

$$= \frac{1}{3} \ln|\sin 3x| + c$$

5. $\int f' \sec f dx = \ln|\sec f + \tan f| + c$

Example $\int \sec x dx = \ln|\sec x + \tan x| + c$

Example

$$\int \sec 3x \, dx = \frac{1}{3} \int 3 \sec 3x \, dx$$

$$= \frac{1}{3} \ln |\sec 3x + \tan 3x| + c$$

6.

$$\int f' \csc f \, dx = \ln |\csc f - \cot f| + c$$

Example

$$\int \csc x \, dx = \ln |\csc x - \cot x| + c$$

Example

$$\int \csc 3x \, dx = \frac{1}{3} \int 3 \csc 3x \, dx$$

$$= \frac{1}{3} \ln |\csc 3x - \cot 3x| + c$$

7. $\int f' \sec f \tan f dx = \sec f + c$

Example $\int \sec x \tan x dx = \sec x + c$

Example $\int \sec 3x \tan 3x dx$

$$= \frac{1}{3} \int 3 \sec 3x \tan 3x dx$$

$$= \frac{1}{3} \sec 3x + c$$

8. $\int f' \csc f \cot f dx = -\csc f + c$

Example $\int \csc x \cot x dx = -\csc x + c$

Example $\int \csc 3x \cot 3x \, dx$

$$= \frac{1}{3} \int 3 \csc 3x \cot 3x \, dx$$

$$= -\frac{1}{3} \csc 3x + c$$

9. $\int f' \sec^2 f \, dx = \tan f + c$

Example $\int \sec^2 x \, dx = \tan x + c$

Example $\int \sec^2 3x \, dx$

$$= \frac{1}{3} \int 3 \sec^2 3x \, dx$$

$$= \frac{1}{3} \tan 3x + c$$

10. $\int f' \csc^2 f dx = -\cot f + c$

Example $\int \csc^2 x dx = -\cot x + c$

Example $\int \csc^2 3x dx = \frac{1}{3} \int 3 \csc^2 3x dx$

$$= -\frac{1}{3} \cot 3x + c$$

Remark

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

Example $\int \tan^2 x dx = \int (\sec^2 x - 1) dx$

$$= \int \sec^2 x dx - \int dx = \tan x - x + c$$

Remark

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

Example

$$\int \cot^2 x \, dx$$

$$= \int (\csc^2 x - 1) \, dx$$

$$= \int \csc^2 x \, dx - \int dx$$

$$= -\cot x - x + c$$

Remark

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

Example

$$\begin{aligned}
 & \int \sin^2 x \, dx \\
 &= \int \frac{1}{2} (1 - \cos 2x) \, dx \\
 &= \frac{1}{2} \int (1 - \cos 2x) \, dx \\
 &= \frac{1}{2} \left(\int dx - \int \cos 2x \, dx \right) \\
 &= \frac{1}{2} \left(\int dx - \frac{1}{2} \int 2 \cos 2x \, dx \right) \\
 &= \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + c
 \end{aligned}$$

Remark

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

Example

$$\begin{aligned} & \int \cos^2 x \, dx \\ &= \frac{1}{2} \int (1 + \cos 2x) \, dx \\ &= \frac{1}{2} \left(\int dx + \int \cos 2x \, dx \right) \\ &= \frac{1}{2} \left(x + \frac{1}{2} \int 2 \cos 2x \, dx \right) \\ &= \frac{1}{2} \left(x + \frac{1}{2} \sin 2x \right) + c \end{aligned}$$

Inverse Trigonometric functions

1.

$$\int \frac{f'}{\sqrt{a^2 - f^2}} dx = \sin^{-1} \frac{f}{a} + c$$

or $\int \frac{f'}{\sqrt{a^2 - f^2}} dx = -\cos^{-1} \frac{f}{a} + c$

Example

$$\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$$

Example

$$\int \frac{1}{\sqrt{9 - x^2}} dx = \sin^{-1} \frac{x}{3} + c$$

Example $\int \frac{e^x}{\sqrt{9 - e^{2x}}} dx$

$$= \int \frac{e^x}{\sqrt{9 - (e^x)^2}} dx$$

$$= \sin^{-1} \frac{e^x}{3} + c$$

2. $\int \frac{f'}{a^2 + f^2} dx = \frac{1}{a} \tan^{-1} \frac{f}{a} + c$

or $\int \frac{f'}{a^2 + f^2} dx = -\frac{1}{a} \cot^{-1} \frac{f}{a} + c$

Example $\int \frac{1}{1 + x^2} dx = \tan^{-1} x + c$

Example $\int \frac{1}{16 + x^2} dx = \frac{1}{4} \tan^{-1} \frac{x}{4} + c$

Example $\int \frac{e^x}{16 + e^{2x}} dx$

$$= \int \frac{e^x}{16 + (e^x)^2} dx$$

$$= \frac{1}{4} \tan^{-1} \frac{e^x}{4} + c$$

3. $\int \frac{f'}{f\sqrt{f^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{f}{a} + c$

or $\int \frac{f'}{f\sqrt{f^2 - a^2}} dx = -\frac{1}{a} \csc^{-1} \frac{f}{a} + c$

Example $\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c$

Example $\int \frac{1}{x\sqrt{x^2 - 25}} dx = \frac{1}{5} \sec^{-1} \frac{x}{5} + c$