New Mansoura University

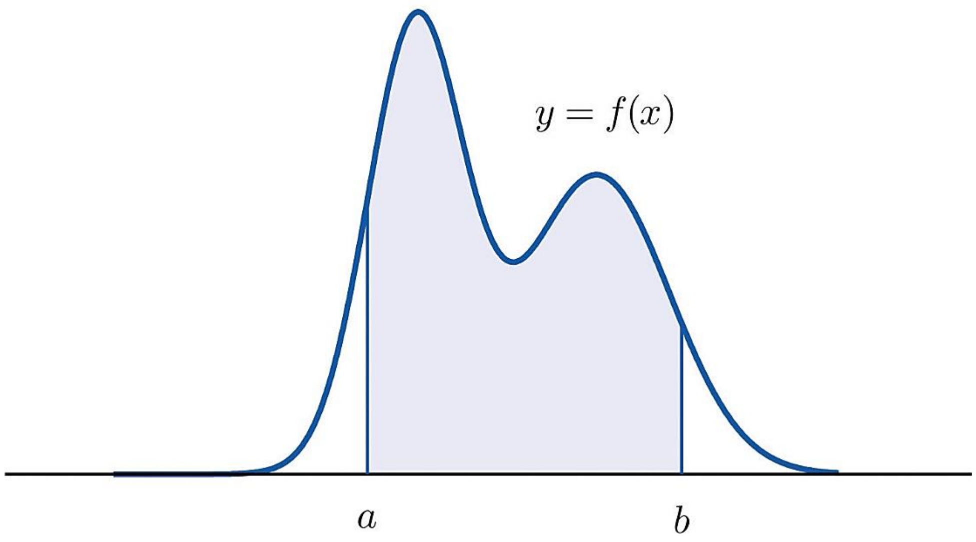
Lecture 8

Normal distribution

Previously in Lecture 7 Continuous R.V

$$P(a < X \leq b) = \int_a^b f(x)dx.$$

$P(a < X < b) =$ area of shaded region



- The mean of *X* is given by

$$\mu_X = \int_{-\infty}^{\infty} xf(x)dx.$$

- The variance of *X* is given by

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x)dx$$

$$= \int_{-\infty}^{\infty} x^2 f(x)dx - \mu_X^2.$$

Cumulative function

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt.$$

The Normal Distribution

- The **normal distribution** (also called the Gaussian distribution) is by far the most used distribution in statistics. This distribution provides a good model for many, although not all, continuous populations.

Normal RV: pdf, mean, and variance

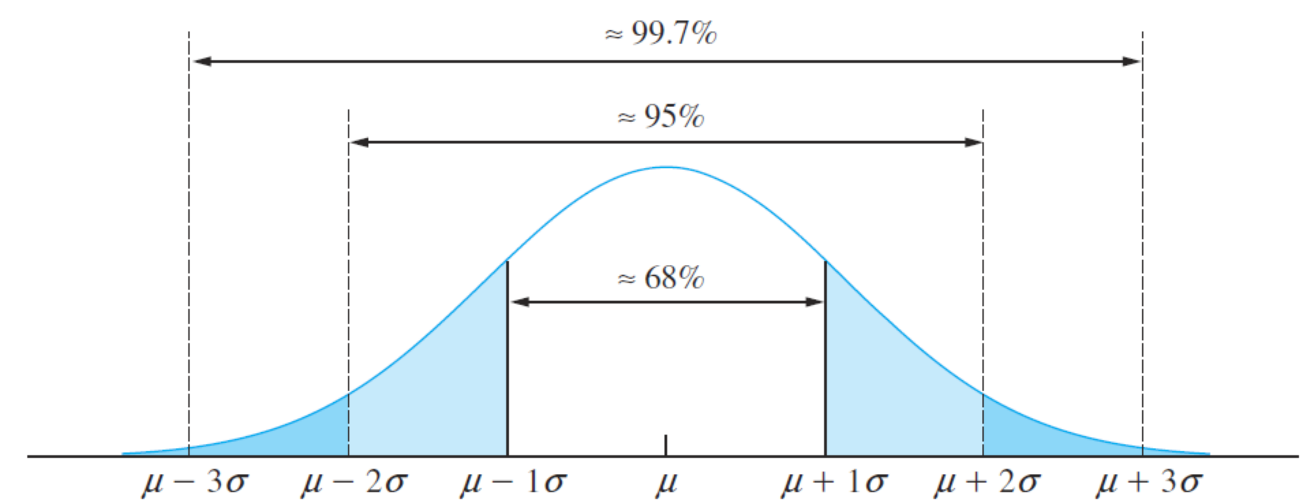
- The probability density function of a normal population with mean μ and variance σ^2 is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

- If $X \sim N(\mu, \sigma^2)$, then the mean and variance of *X* are given by

$$\mu_X = \mu$$
$$\sigma^2_X = \sigma^2$$

68-95-99.7% Rule



This figure represents a plot of the normal probability density function with mean μ and standard deviation σ. Note that the curve is symmetric about μ, so that μ is the median as well as the mean. It is also the case for the normal population.

- About 68% of the population is in the interval $\mu \pm \sigma$.
- About 95% of the population is in the interval $\mu \pm 2\sigma$.
- About 99.7% of the population is in the interval $\mu \pm 3\sigma$.

Standard Normal Distribution

- In general, we convert to standard units by subtracting the mean and dividing by the standard deviation. Thus, if *x* is an item sampled from a normal population with mean μ and variance σ^2, the standard unit equivalent of *x* is the number *z*, where

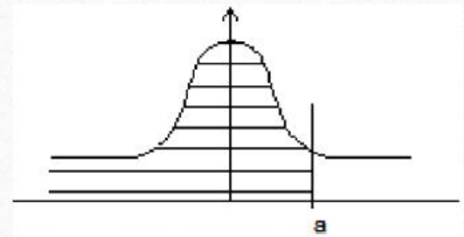
$$z = \frac{x - \mu}{\sigma}$$

- The number *z* is sometimes called the "*z*-score" of *x*. The *z*-score is an item sampled from a normal population with mean 0 and standard deviation of 1. This normal population is called the **standard normal population**.

Finding Areas Under the Normal Curve 1

Cumulative Function

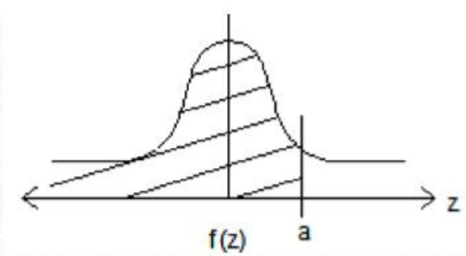
$$\boxed{\phi(z) = p(z \le z)}$$



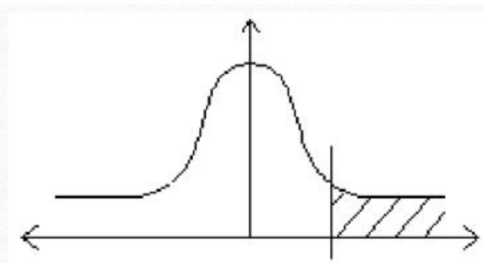
يتم حساب الاحتمالات عن طريق جدول (N . D) بعد تحويل المسأله

The normal distribution table gives values of the commulative function $\Phi(z) = p(z \le z)$. Any other probability value can be derived by using the following relations

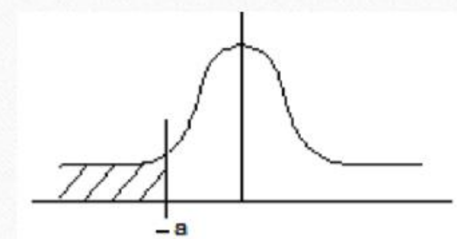
1- $P(z \le a) = \phi(a)$



2- $p(z \ge a) = 1 - \phi(z)$

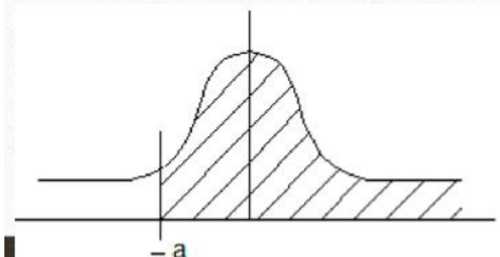


3- $p(z \le -a) = 1 - \phi(a)$

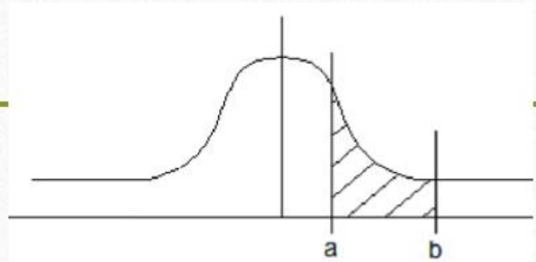


Finding Areas Under the Normal Curve 1

4- $p (z \geq -a) = \phi (a)$

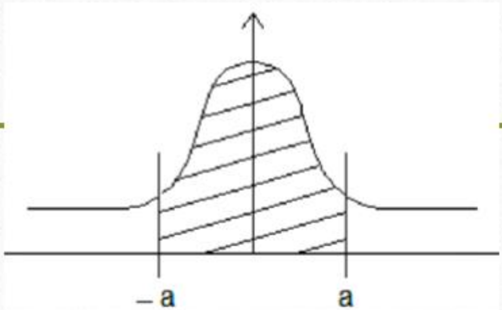


5- $p (a \leq z \leq b) = \phi (b) - \phi (a)$



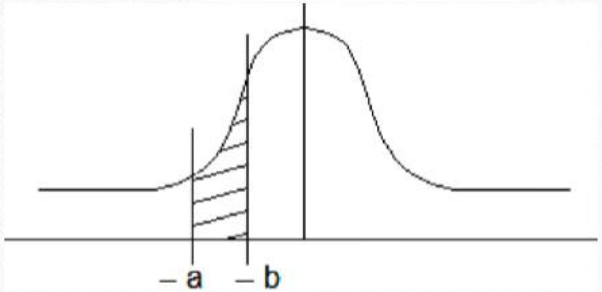
6- $p (-a \leq z \leq a) = \phi (a) - \phi (-a)$

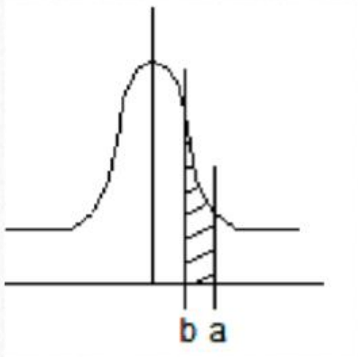
$= \phi (a) - [1 - \phi (a)]$



7- $p (-a \leq z \leq -b) = \phi (-b) - \phi (-a)$

$= \phi (b) - \phi (a)$





Example

Find the area under normal curve to the left of $z = 0.47$.

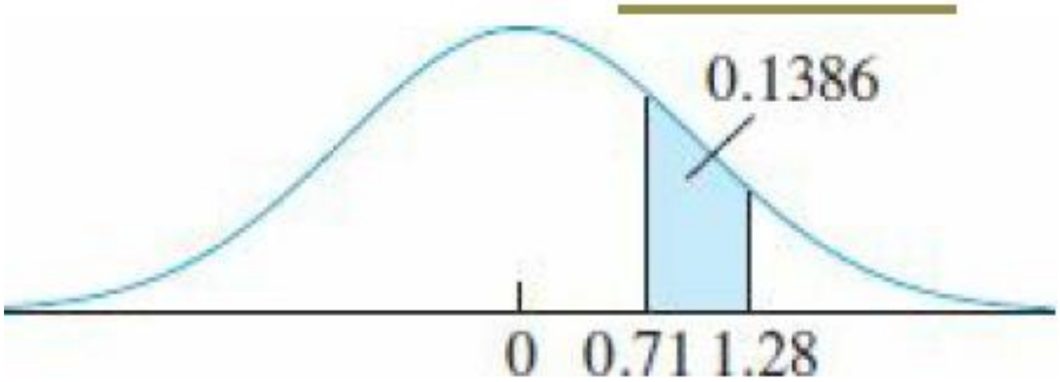
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	$\boxed{.6808}$	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133

Example

Find the area under the normal curve between $z = 0.71$ and $z = 1.28$.

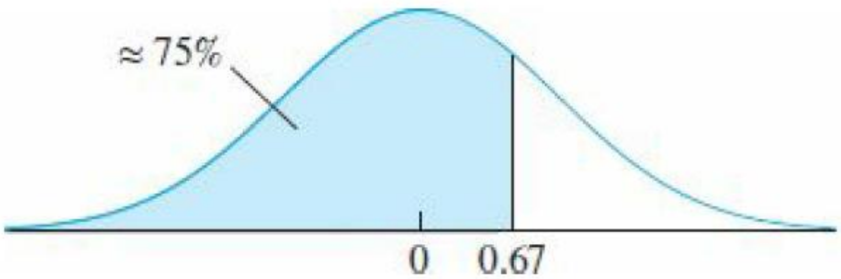
The area between $z = 0.71$ and $z = 1.28$ is therefore

$0.8997 - 0.7611 = 0.1386$.



Example

What z-score corresponds to the 75th percentile of a normal curve?



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133

Thank You