



Tutorial 7

Resistors labeled 100Ω have true resistances that are between 80Ω and 120Ω . Let X be the resistance of a randomly chosen resistor. The probability density function of X is given by

$$f(x) = \begin{cases} \frac{x - 80}{800} & 80 < x < 120 \\ 0 & \text{otherwise} \end{cases}$$

- a. What proportion of resistors have resistances less than 90Ω ?
- b. Find the mean resistance.
- c. Find the standard deviation of the resistances.
- d. Find the cumulative distribution function of the resistances.



$$a. p(x < 90) = \int_{80}^{90} \left(\frac{x - 80}{800} \right) dx = \left(\frac{x^2/2}{800} - \frac{80x}{800} \right)_{80}^{90} = 1/16$$

$$b. \mu_x = \int_{80}^{120} x \left(\frac{x - 80}{800} \right) dx = \int_{80}^{120} \left(\frac{x^2 - 80x}{800} \right) dx$$

$$= \left[\left(\frac{1}{800} \right) * \left(\frac{x^3}{3} - \frac{80x^2}{2} \right) \right]_{80}^{120} = 106.67$$

$$c. E(x^2) = \int_{80}^{120} x^2 \left(\frac{x - 80}{800} \right) dx = \int_{80}^{120} \left(\frac{x^3 - 80x^2}{800} \right) dx$$

$$= \left[\left(\frac{1}{800} \right) * \left(\frac{x^4}{4} - \frac{80x^3}{3} \right) \right]_{80}^{120} = 11466.67$$



$$V(x) = E(x^2) - \mu^2 = 11466.6 - (106.67)^2 = 88.11$$

$$\sigma = \sqrt{88.11} = 9.38$$

(d) $F(x) = \begin{cases} 0 & x < 80 \\ x^2/1600 - x/10 + 4 & 80 \leq x < 120 \\ 1 & x \geq 120 \end{cases}$

$$-\infty < x < 80$$

$$F(x) = F(-\infty) + \int_{-\infty}^x 0 dx = 0$$

$$80 < x < 120$$

$$\begin{aligned} F(x) &= F(80) + \int_{80}^x \frac{t - 80}{800} dt = 0 + \left(\frac{1}{800} \right) * \left(\frac{t^2}{2} - 80t \right) \Big|_{80}^x \\ &= \left(\frac{1}{800} \right) * \left(\left(\frac{x^2}{2} - 80x \right) - \frac{80^2}{2} - 80 * 80 \right) = \frac{x^2}{1600} - \frac{x}{10} + 4 \end{aligned}$$


$$F(x) = F(120) + \int_{120}^x 0 dt = \left(\frac{120^2}{1600} - \frac{120}{10} + 4 \right) + 0 = 1$$

(d)
$$F(x) = \begin{cases} 0 & x < 80 \\ x^2/1600 - x/10 + 4 & 80 \leq x < 120 \\ 1 & x \geq 120 \end{cases}$$

The lifetime of a transistor in a certain application is random with probability density function

$$f(t) = \begin{cases} 0.1e^{-0.1t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

- a. Find the mean lifetime.
- b. Find the standard deviation of the lifetimes.
- c. Find the cumulative distribution function of the lifetime.
- d. Find the probability that the lifetime will be less than 12 months.





$$\begin{aligned}
 \text{(a)} \quad \mu &= \int_0^{\infty} 0.1te^{-0.1t} dt \\
 &= -te^{-0.1t} \Big|_0^{\infty} - \int_0^{\infty} -e^{-0.1t} dt \\
 &= 0 - 10e^{-0.1t} \Big|_0^{\infty} \\
 &= 10
 \end{aligned}$$

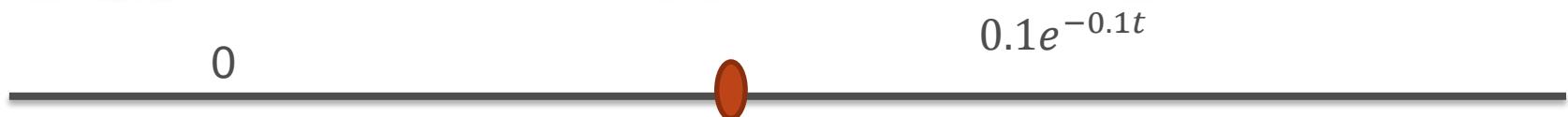
$$e^0 = 1$$

$$e^{\infty} = \infty$$

$$e^{-\infty} = 0$$

diff	int
t	$0.1e^{-0.1t}$
1	$-e^{-0.1t} = \frac{0.1e^{-0.1t}}{-0.1}$
0	$\frac{e^{-0.1t}}{0.1}$

$$\begin{aligned}(b) \sigma^2 &= \int_0^\infty 0.1t^2 e^{-0.1t} dt - \mu^2 \\&= -t^2 e^{-0.1t} \Big|_0^\infty - \int_0^\infty -2te^{-0.1t} dt - 100 \\&= 0 + 20 \int_0^\infty 0.1te^{-0.1t} dt - 100 \\&= 0 + 20(10) - 100 \\&= 100 \\&\sigma_x = \sqrt{100} = 10\end{aligned}$$



$$(c) F(x) = \int_{-\infty}^x f(t)dt.$$

$$\text{If } x \leq 0, \quad F(x) = \int_{-\infty}^x 0 dt = 0.$$

$$\text{If } x > 0, \quad F(x) = \int_{-\infty}^x 0 dt + \int_0^x 0.1e^{-0.1t} dt = 1 - e^{-0.1x}.$$

$$(d) \text{ Let } T \text{ represent the lifetime. } P(T < 12) = P(T \leq 12) = F(12) = 1 - e^{-1.2} = 0.6988.$$

$$0 < x < \infty$$

$$F(x) = F(0) + \int_0^x 0.1e^{-0.1t} dt = 0.1e^{-0.1t}/(-0.1)$$

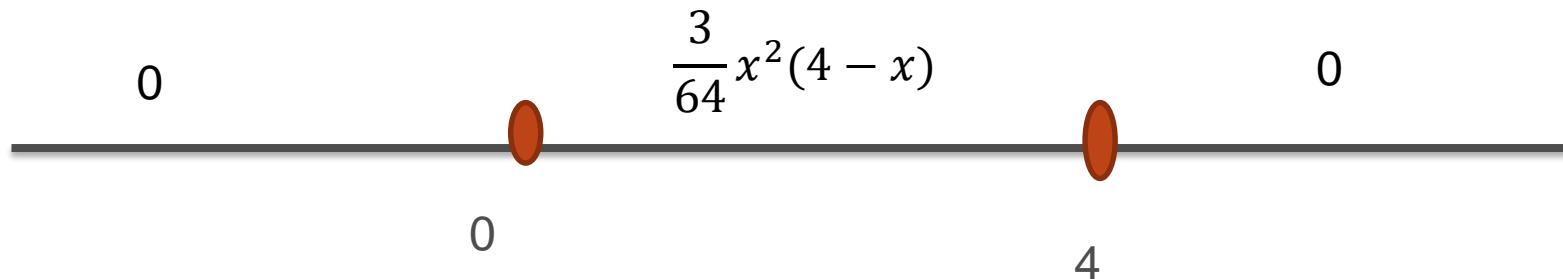
$$= -e^{-0.1t} \Big|_0^x = -e^{-0.1x} - (-e^0) = 1 - e^{-0.1x}$$



The level of impurity (in percent) in the product of a certain chemical process is a random variable with probability density function

$$\begin{cases} \frac{3}{64}x^2(4-x) & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

- a. What is the probability that the impurity level is greater than 3%?
- b. What is the probability that the impurity level is between 2% and 3%?
- c. Find the mean impurity level.
- d. Find the variance of the impurity levels.
- e. Find the cumulative distribution function of the impurity level.





$$(a) P(X > 3) = \int_3^4 (3/64)x^2(4-x)dx = \left(\frac{x^3}{16} - \frac{3x^4}{256} \right) \Big|_3^4 = 67/256$$

$$(b) P(2 < X < 3) = \int_2^3 (3/64)x^2(4-x)dx = \left(\frac{x^3}{16} - \frac{3x^4}{256} \right) \Big|_2^3 = 109/256$$

$$(c) \mu = \int_0^4 (3/64)x^3(4-x)dx = \left(\frac{3x^4}{64} - \frac{3x^5}{320} \right) \Big|_0^4 = 2.4$$



$$(d) \sigma^2 = \int_0^4 (3/64)x^4(4-x)dx - \mu^2 = \left(\frac{3x^5}{80} - \frac{x^6}{128} \right) \Big|_0^4 - 2.4^2 = 0.64$$

$$(e) F(x) = \int_{-\infty}^x f(t)dt$$

$$\text{If } x \leq 0, \quad F(x) = \int_{-\infty}^x 0 dt = 0$$

$$\text{If } 0 < x < 4, \quad F(x) = \int_0^x (3/64)t^4(4-t)dt = (16x^3 - 3x^4)/256$$

$$\text{If } x \geq 4, \quad F(x) = \int_0^4 (3/64)t^4(4-t)dt = 1$$

24. Particles are a major component of air pollution in many areas. It is of interest to study the sizes of contaminating particles. Let X represent the diameter, in micrometers, of a randomly chosen particle. Assume that in a certain area, the probability density function of X is inversely proportional to the volume of the particle; that is, assume that

$$f(x) = \begin{cases} \frac{c}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

where c is a constant.

- a. Find the value of c so that $f(x)$ is a probability density function.
- b. Find the mean particle diameter.
- c. Find the cumulative distribution function of the particle diameter.
- d. Find the median particle diameter.
- e. The term PM_{10} refers to particles $10 \mu\text{m}$ or less in diameter. What proportion of the contaminating particles are PM_{10} ?
- f. The term PM_{25} refers to particles $2.5 \mu\text{m}$ or less in diameter. What proportion of the contaminating particles are PM_{25} ?
- g. What proportion of the PM_{10} particles are PM_{25} ?

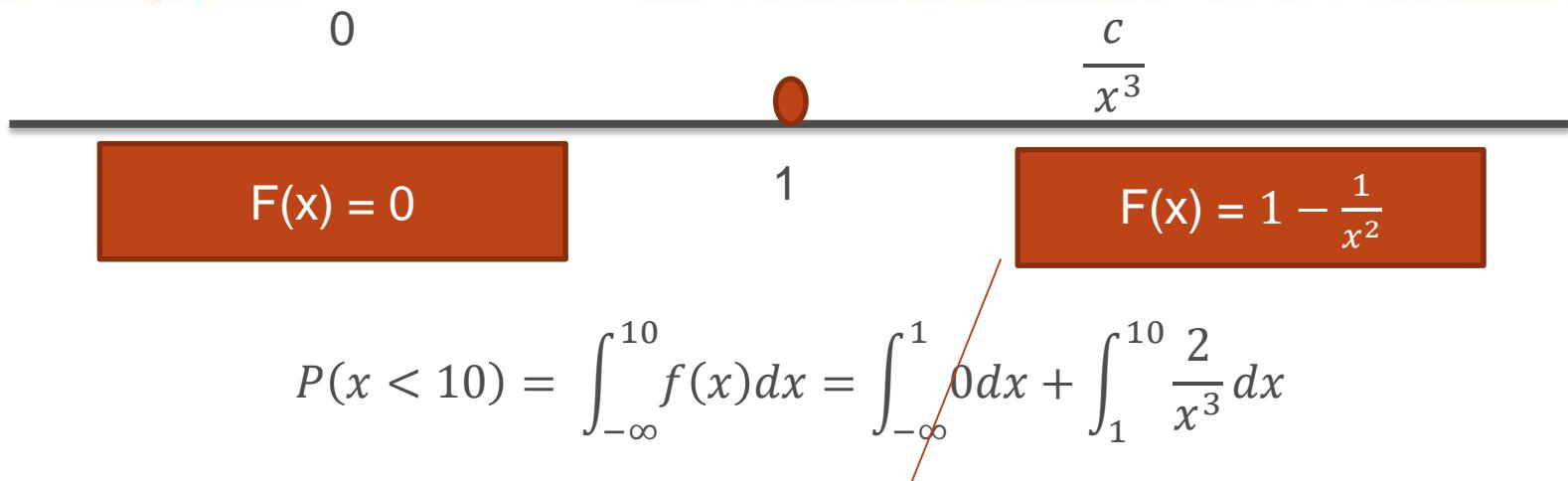


$$1. \int_{-\infty}^{\infty} f(x) = 1 = \int_1^{\infty} \frac{c}{x^3} dx = c \int_1^{\infty} x^{-3} dx = c \left(\frac{x^{-2}}{-2} \right) \Big|_1^{\infty} = 1$$

$$\begin{aligned} -\frac{c}{2} \left(\frac{1}{\infty} - 1 \right) &= 1 \\ \frac{c}{2} &= 1 \dots \dots \Rightarrow c = 2 \end{aligned}$$

$$2. \text{ Median} \dots \dots \int_{-\infty}^m f(x) = 0.5 \dots \dots \Rightarrow \int_1^m 2x^{-3} dx = 0.5$$

$$\begin{aligned} \frac{2x^{-2}}{-2} \Big|_1^m &= 0.5 \dots \dots \Rightarrow -\left(\frac{1}{m^2} - 1 \right) = 0.5 \\ -\frac{1}{m^2} + 1 &= 0.5 \dots \dots -\frac{1}{m^2} = -0.5 \dots \dots m = \sqrt{2} \end{aligned}$$



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$$P(x < 10) = F(10) = 1 - \frac{1}{10^2} = 1 - 0.01 = 0.99$$



(a) c solves the equation $\int_1^\infty c/x^3 dx = 1$. Therefore $-0.5c/x^2 \Big|_1^\infty = 1$, so $c = 2$.

$$(b) \mu_X = \int_1^\infty cx/x^3 dx = \int_1^\infty 2/x^2 dx = -\frac{2}{x} \Big|_1^\infty = 2$$

$$(c) F(x) = \int_{-\infty}^x f(t)dt$$

$$\text{If } x < 1, \quad F(x) = \int_{-\infty}^x 0 dt = 0.$$

$$\text{If } x \geq 1, \quad F(x) = \int_{-\infty}^1 0 dt + \int_1^x 2/t^3 dt = -\frac{1}{t^2} \Big|_1^x = 1 - 1/x^2.$$

(d) The median x_m solves $F(x_m) = 0.5$. Therefore $1 - 1/x_m^2 = 0.5$, so $x_m = 1.414$.

$$(e) P(X \leq 10) = F(10) = 1 - 1/10^2 = 0.99$$

$$(f) P(X \leq 2.5) = F(2.5) = 1 - 1/2.5^2 = 0.84$$

$$(g) P(X \leq 2.5 | X \leq 10) = \frac{P(X \leq 2.5 \text{ and } X \leq 10)}{P(X \leq 10)} = \frac{P(X \leq 2.5)}{P(X \leq 10)} = \frac{0.84}{0.99} = 0.85$$



H.W

21. The error in the length of a part (absolute value of the difference between the actual length and the target length), in mm, is a random variable with probability density function

$$f(x) = \begin{cases} 12(x^2 - x^3) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- What is the probability that the error is less than 0.2 mm?
- Find the mean error.
- Find the variance of the error.
- Find the cumulative distribution function of the error.
- The specification for the error is 0 to 0.8 mm. What is the probability that the specification is met?



H.W

25. The repair time (in hours) for a certain machine is a random variable with probability density function

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- a. What is the probability that the repair time is less than 2 hours?
- b. What is the probability that the repair time is between 1.5 and 3 hours?
- c. Find the mean repair time.
- d. Find the cumulative distribution function of the repair times.





Thank You