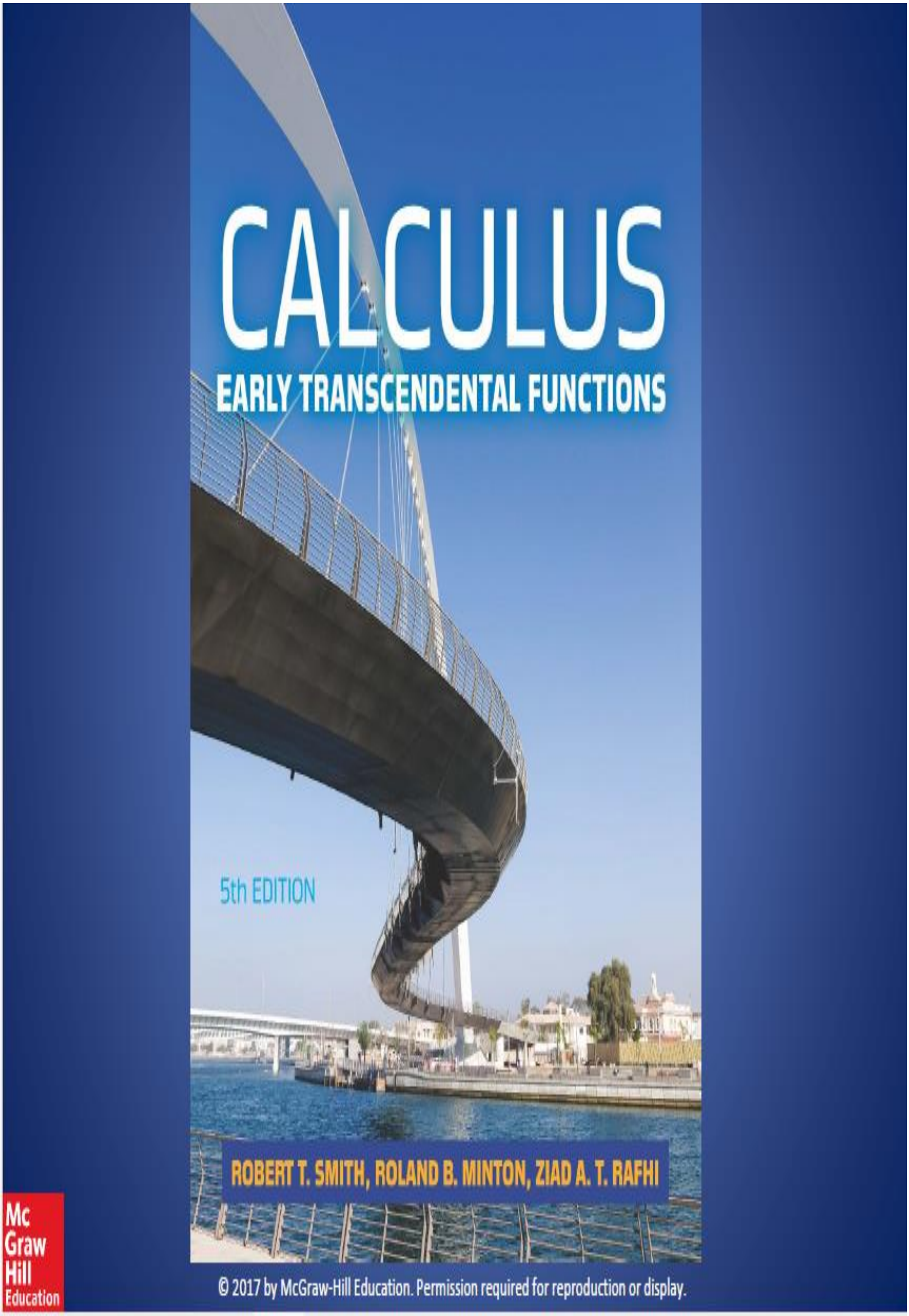
CALCULUS
EARLY TRANSCENDENTAL FUNCTIONS
5th EDITION
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Integrals involving powers of trigonometric functions

1- Integrals on the form

$$\int sin^n x\, dx \quad or \quad \int cos^n x\, dx$$

$\boldsymbol{a})$ $\quad$ $\boldsymbol{n\ is\ an\ odd\ positive\ integer}$

$\boldsymbol{Use}$ $\qquad sin^2 x + cos^2 x = 1$

$\boldsymbol{Example}$ $\qquad \int sin^3 x\, dx$

$$\int sin^3 x\, dx =$$

$$= \int sin^2 x\, sinx\, dx$$

$$= \int (1 - cos^2x)\, sinx\, dx$$

$$= \int sinx\, dx - \int cos^2x\, sinx\, dx$$

$$= \int sinx\, dx + \int (cosx)^2(-sinx)\, dx$$

$$= -cosx + \frac{1}{3}cos^3x + c$$

$\boldsymbol{b)}$ $\boldsymbol{n\ is\ an\ even\ positive\ integer}$

$\boldsymbol{Use}$ $$sin^2x = \frac{1}{2}(1 - cos2x)$$

, $$cos^2x = \frac{1}{2}(1 + cos2x)$$

$\boldsymbol{Example}$ $\int sin^2x\, dx\ , \int cos^2x\, dx$ $\quad Done$

Example $\int cos^4x\, dx$

$$\int cos^4x\, dx =$$

$$= \int (cos^2x)^2\, dx$$

$$= \int (\frac{1}{2}(1 + cos2x))^2\, dx$$

$$= \frac{1}{4}\int (1 + cos2x)^2\, dx$$

$$= \frac{1}{4}\int (1 + 2cos2x + cos^2 2x)\, dx$$

$$= \frac{1}{4}\int (1 + 2cos2x + \frac{1}{2}(1 + cos4x))\, dx$$

$$= \frac{1}{4}\int (1 + 2cos2x + \frac{1}{2} + \frac{1}{2}cos4x)\, dx$$

$$= \frac{1}{4}\int (\frac{3}{2} + 2cos2x + \frac{1}{2}cos4x)\, dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x + sin2x + \frac{1}{8}sin4x\right) + c$$

2- <u>Integrals on the form</u>

$$\int sin^m x\, cos^n x \;\; dx$$

a*) *m or n is an odd positive integer

Use $sin^2x + cos^2x = 1$

Example $\int cos^4x\, sin^3x\, dx$

$$\int cos^4x \; sin^3x \; dx =$$

$$= \int cos^4x \; sin^2x \; sinx \; dx$$

$$= \int cos^4x \; (1 - cos^2x) \; sinx \; dx$$

$$= \int cos^4x \; sinx \; dx - \int cos^6x \; sinx \; dx$$

$$= -\int (cosx)^4 \; (-sinx) dx + \int (cosx)^6 \; (-sinx) \; dx$$

$$= -\frac{1}{5} cos^5x + \frac{1}{7} cos^7x + c$$

$\boldsymbol{b)}$ $\boldsymbol{m \; and \; n \; are \; even \; positive \; integers}$

$\boldsymbol{Use}$ $$sin^2x = \frac{1}{2}(1 - cos2x)$$

, $$cos^2x = \frac{1}{2}(1 + cos2x)$$

Example $\int cos^2x\, sin^2x\, dx$

$$\int cos^2x\, sin^2x\, dx =$$

$$= \int \frac{1}{2}(1 + cos2x)\ \frac{1}{2}(1 - cos2x)\, dx$$

$$= \frac{1}{4}\int\ (1 - cos^2 2x)\, dx$$

$$= \frac{1}{4}\int\ sin^2 2x\, dx$$

$$= \frac{1}{4}\int \frac{1}{2}(1 - cos4x)\, dx$$

$$= \frac{1}{8}\int (1 - cos4x)\, dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}sin4x\right) + c$$

3- Integrals on the form

$$\int tan^m x \; sec^n x \;\; dx$$

a) ***n is an even positive integer***

Use $sec^2 x \; = 1 + tan^2 x$

Example $$\int tan^2 x \; sec^4 x \; dx$$

$$\int tan^2 x \; sec^4 x \; dx =$$

$$= \int tan^2 x \; sec^2 x \; sec^2 x \; dx$$

$$= \int tan^2 x \; (1 + tan^2 x) \; sec^2 x \; dx$$

$$= \int tan^2 x \; sec^2 x \; dx + \int tan^4 x \; sec^2 x \; dx$$

$$= \int (tanx)^2\, sec^2x\, dx + \int (tanx)^4\, sec^2x\, dx$$

$$= \frac{1}{3} tan^3x + \frac{1}{5} tan^5x + c$$

$\boldsymbol{b)}$ $\quad$ $\boldsymbol{m\ and\ n\ are\ odd\ positive\ integers}$

$\boldsymbol{Use}$ $\qquad tan^2x = sec^2x - 1$

$\boldsymbol{Example}$ $\qquad \int tan^3x\, sec^3x\, dx$

$$\int tan^3x\, sec^3x\, dx =$$

$$= \int tan^2x\, sec^2x\, tanx\, secx\, dx$$

$$= \int (sec^2x - 1)\, sec^2x\, tanx\, secx\, dx$$

$$= \int sec^4x\ \ tanx\, secx\, dx - \int sec^2x\, tanx\, secx\, dx$$

$$= \int (secx)^4 \ \ tanx\ secx\ dx - \int (secx)^2\ tanx\ secx\ dx$$

$$= \frac{1}{5} sec^5x - \frac{1}{3} sec^3x + c$$

$\boldsymbol{c)\ n\ is\ an\ odd\ positive\ integer\ and\ m\ is\ an\ even\ positive\ integer}$

$\boldsymbol{Use}$ $\qquad tan^2x\ = sec^2x - 1$

Remark

$$\int sec^3x\ dx = \frac{1}{2}(secx\ tanx + ln|secx + tanx|) + c$$

$\boldsymbol{Example}$ $\qquad \int tan^2x\ secx\ dx$

$$\int tan^2x\ secx\ dx =$$

$$= \int (sec^2x - 1)\ secx\ dx$$

$$= \int sec^3x\ dx - \int secx\ dx$$

$$= \frac{1}{2}(secx\ tanx + ln|secx + tanx|) - ln|secx + tanx| + c$$

$$= \frac{1}{2}secx\ tanx + \frac{1}{2}ln|secx + tanx| - ln|secx + tanx| + c$$

$$= \frac{1}{2}secx\ tanx - \frac{1}{2}ln|secx + tanx| + c$$

$$= \frac{1}{2}(secx\ tanx - ln|secx + tanx|) + c$$