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Tutorial 5

Exercise 1

A Bernoulli random variable has variance 0.21. What are the possible values for its success probability?

The possible values of p are the solutions to $p(1-p) = 0.21$. The solutions are $p = 0.3$ and $p = 0.7$.

Exercise 2

Let $X \sim \text{Bin}(7, 0.3)$. Find

a. $P(X = 1)$

b. $P(X = 2)$

c. $P(X < 1)$

d. $P(X > 4)$

e. μ_X

f. σ_X^2

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Exercise 2

1. (a) $P(X = 1) = \dfrac{7!}{1!(7-1)!}(0.3)^1(1-0.3)^{7-1} = 0.2471$

(b) $P(X = 2) = \dfrac{7!}{2!(7-2)!}(0.3)^2(1-0.3)^{7-2} = 0.3177$

(c) $P(X < 1) = P(X = 0) = \dfrac{7!}{0!(7-0)!}(0.3)^0(1-0.3)^{7-0} = 0.0824$

(d) $$\begin{aligned} P(X > 4) &= P(X = 5) + P(X = 6) + P(X = 7) \\ &= \frac{7!}{5!(7-5)!}(0.3)^5(1-0.3)^{7-5} + \frac{7!}{6!(7-6)!}(0.3)^6(1-0.3)^{7-6} + \frac{7!}{7!(7-7)!}(0.3)^7(1-0.3)^{7-7} \\ &= 0.002500 + 0.003572 + 0.0002187 \\ &= 0.0288 \end{aligned}$$

(e) $\mu_X = (7)(0.3) = 2.1$

(f) $\sigma_X^2 = (7)(0.3)(1-0.3) = 1.47$

Exercise 3

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4. At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

a. Find the probability that all 10 of the flights were on time.

b. Find the probability that exactly eight of the flights were on time.

c. Find the probability that eight or more of the flights were on time.

4. Let X be the number of flights in the sample that are on time. Then $X \sim \text{Bin}(10, 0.75)$.

(a) $P(X = 10) = \dfrac{10!}{10!(10-10)!}(0.75)^{10}(1-0.75)^{10-10} = 0.0563$

(b) $P(X = 8) = \dfrac{10!}{8!(10-8)!}(0.75)^{8}(1-0.75)^{10-8} = 0.2816$

(c) $P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10) = \dfrac{10!}{8!(10-8)!}(0.75)^{8}(1-0.75)^{10-8} + \dfrac{10!}{9!(10-9)!}(0.75)^{9}(1-0.75)^{10-9} + \dfrac{10!}{10!(10-10)!}(0.75)^{10}(1-0.75)^{10-10} = 0.2816 + 0.1877 + 0.0563 = 0.5256$

Exercise 4

6. A fair die is rolled 8 times.

a. What is the probability that the die comes up 6 exactly twice?

b. What is the probability that the die comes up an odd number exactly five times?

c. Find the mean number of times a 6 comes up.

d. Find the mean number of times an odd number comes up.

e. Find the standard deviation of the number of times a 6 comes up.

f. Find the standard deviation of the number of times an odd number comes up.

Exercise 4

6. Let X be the number of times the die comes up 6, and let Y be the number of times the die comes up an odd number. Then $X \sim \text{Bin}(8, 1/6)$ and $Y \sim \text{Bin}(8, 1/2)$.

(a) $P(X = 2) = \dfrac{8!}{2!(8-2)!}(1/6)^2(1-1/6)^{8-2} = 0.2605$

(b) $P(Y = 5) = \dfrac{8!}{5!(8-5)!}(1/2)^5(1-1/2)^{8-5} = 0.2188$

(c) $\mu_X = 8(1/6) = 1.333$

(d) $\mu_Y = 8(1/2) = 4$

(e) $\sigma_X = \sqrt{(8)(1/6)(5/6)} = 1.0541$

(f) $\sigma_Y = \sqrt{(8)(1/2)(1/2)} = 1.4142$

Exercise 5

9. Several million lottery tickets are sold, and 60% of the tickets are held by women. Five winning tickets will be drawn at random.

a. What is the probability that three or fewer of the winners will be women?

b. What is the probability that three of the winners will be of one gender and two of the winners will be of the other gender?

9. Let X be the number of women among the five winners. Then $X \sim \text{Bin}(5, 0.6)$.

(a) $P(X \le 3) = 1 - P(X > 3) = 1 - P(X = 4) - P(X = 5) = 1 - \frac{5!}{4!(5-4)!}(0.6)^4(1-0.6)^{5-4} - \frac{5!}{5!(5-5)!}(0.6)^5(1-0.6)^{5-5} = 1 - 0.25920 - 0.07776 = 0.6630$

(b) $P(\text{3 of one gender and 2 of another}) = P(X = 2) + P(X = 3) = \frac{5!}{2!(5-2)!}(0.6)^2(1-0.6)^{5-2} + \frac{5!}{3!(5-3)!}(0.6)^3(1-0.6)^{5-3} = 0.2304 + 0.3456 = 0.5760$

Exercise 6

15. A commuter must pass through three traffic lights on her way to work. For each light, the probability that it is green when she arrives is 0.6. The lights are independent.

a. What is the probability that all three lights are green?

b. The commuter goes to work five days per week. Let X be the number of times out of the five days in a given week that all three lights are green. Assume the days are independent of one another. What is the distribution of X?

c. Find $P(X = 3)$.

15. (a) Let Y be the number of lights that are green. Then $Y \sim \text{Bin}(3, 0.6)$.

$$P(Y = 3) = \frac{3!}{3!(3-3)!}(0.6)^3(1-0.6)^{3-3} = 0.216$$

(b) $X \sim \text{Bin}(5, 0.216)$

(c) $$P(X = 3) = \frac{5!}{3!(5-3)!}(0.216)^3(1-0.216)^{5-3} = 0.0619$$

Homework

1 The number of cars arriving at a given intersection follows a Poisson distribution with a mean rate of 4 per second.

a. What is the probability that 3 cars arrive in a given second?

b. What is the probability that 8 cars arrive in three seconds?

c. What is the probability that more than 3 cars arrive in a period of two seconds?

2 Of all the registered automobiles in a certain state, 10% violate the state emissions standard. Twelve automobiles are selected at random to undergo an emissions test.

a. Find the probability that exactly three of them violate the standard.

b. Find the probability that fewer than three of them violate the standard.

c. Find the probability that none of them violate the standard.

Homework

3. Let $X \sim \text{Bin}(9, 0.4)$. Find

a. $P(X > 6)$

b. $P(X \geq 2)$

c. $P(2 \leq X < 5)$

d. $P(2 < X \leq 5)$

e. $P(X = 0)$

f. $P(X = 7)$

g. μ_X

h. σ_X^2

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Thank You