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Two dice are shown on a light-colored surface. One die is white with black pips, and the other is yellow with black pips. They are positioned diagonally, with the white die slightly behind and to the left of the yellow die. The background is a soft, out-of-focus gradient of light colors.

## Lecture 5

### Special Discrete distributions



# The Bernoulli Distribution

We use the Bernoulli distribution when we have an experiment which can result in one of two outcomes. One outcome is labeled “success,” and the other outcome is labeled “failure.”

- The probability of a success is denoted by  $p$ .
- The probability of a failure is then  $1 - p$ .

Such a trial is called a **Bernoulli trial** with success probability  $p$ .

Two yellow dice are shown in the top left corner. One die is standing upright showing faces with 1, 2, and 3 dots. The other die is lying on its side showing faces with 4, 5, and 6 dots.

## Examples 1 and 2

1. The simplest Bernoulli trial is the toss of a coin. The two outcomes are heads and tails. If we define heads to be the success outcome, then  $p$  is the probability that the coin comes up heads. For a fair coin,  $p = 0.5$ .
2. Another Bernoulli trial is a selection of a component from a population of components, some of which are defective. If we define “success” to be a defective component, then  $p$  is the proportion of defective components in the population.



# Mean and Variance

For any Bernoulli trial, we define a random variable  $X$  as follows: If the experiment results in success, then  $X = 1$ . Otherwise  $X = 0$ . It follows that  $X$  is a discrete random variable, with probability mass function  $p(x)$  defined by

$$p(0) = P(X = 0) = 1 - p$$

$$p(1) = P(X = 1) = p$$

$$p(x) = 0 \text{ for any value of } x \text{ other than } 0 \text{ or } 1$$

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## Summary

If  $X \sim \text{Bernoulli}(p)$ , then

$$\mu_X = p \tag{4.1}$$

$$\sigma_X^2 = p(1 - p) \tag{4.2}$$



# The Binomial Distribution

If a total of  $n$  Bernoulli trials are conducted, and,

- **The trials are independent.**
- **Each trial has the same success probability  $p$ .**
- **$X$  is the number of successes in the  $n$  trials.**

then  $X$  has the **binomial distribution** with parameters  $n$  and  $p$ , denoted  $X \sim \text{Bin}(n, p)$ .



# Binomial RV: pmf, mean, and variance

4-6

- If  $X \sim \text{Bin}(n, p)$ , the probability mass function of  $X$  is

$$p(x) = P(X = x) = \begin{cases} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, & x = 0, 1, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

- Mean:  $\mu_X = np$
- Variance:  $\sigma_X^2 = np(1-p)$

## Example

Find the probability mass function of the random variable  $X$  if  $X \sim \text{Bin}(10, 0.4)$ . Find  $P(X = 5)$ .

### Solution

with  $n = 10$  and  $p = 0.4$ . The probability mass function is

$$p(x) = \begin{cases} \frac{10!}{x!(10-x)!} (0.4)^x (0.6)^{10-x} & x = 0, 1, \dots, 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} P(X = 5) &= p(5) = \frac{10!}{5!(10-5)!} (0.4)^5 (0.6)^{10-5} \\ &= 0.2007 \end{aligned}$$

## Example

A fair die is rolled eight times. Find the probability that no more than 2 sixes come up.

### Solution

Each roll of the die is a Bernoulli trial with success probability  $1/6$ . Let  $X$  denote the number of sixes in 8 rolls. Then  $X \sim \text{Bin}(8, 1/6)$ . We need to find  $P(X \leq 2)$ . Using the probability mass function,

$$\begin{aligned}
 P(X \leq 2) &= P(X = 0 \text{ or } X = 1 \text{ or } X = 2) \\
 &= P(X = 0) + P(X = 1) + P(X = 2) \\
 &= \frac{8!}{0!(8-0)!} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{8-0} + \frac{8!}{1!(8-1)!} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{8-1} \\
 &\quad + \frac{8!}{2!(8-2)!} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{8-2} \\
 &= 0.2326 + 0.3721 + 0.2605 \\
 &= 0.8652
 \end{aligned}$$



# Thank You