New Mansoura University

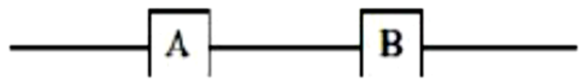
بسم الله الرحمن الرحيم

Statistics

Tutorial 2

Ex 1

A system contains two components, A and B, connected in series as shown in the following diagram.



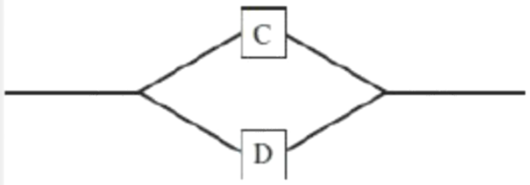
The system will function only if both components function. The probability that A functions is given by $P(A) = 0.98$, and the probability that B functions is given by $P(B) = 0.95$. Assume that A and B function independently. Find the probability that the system functions.

$$
\begin{aligned}
P(\text{system functions}) &= P(A \cap B) \\
&= P(A)P(B) \text{ by the assumption of independence} \\
&= (0.98)(0.95) \\
&= 0.931
\end{aligned}
$$

Ex 2

A system contains two components, C and D, connected in parallel as shown in the following

The system will function if either C or D functions. The probability that C functions is 0.90, and the probability that D functions is 0.85. Assume C and D function independently. Find the probability that the system functions.



$$
\begin{aligned}
P(\text{system functions}) &= P(C \cup D) \\
&= P(C) + P(D) - P(C \cap D) \\
&= P(C) + P(D) - P(C)P(D) \\
&\qquad \text{by the assumption of independence} \\
&= 0.90 + 0.85 - (0.90)(0.85) \\
&= 0.985
\end{aligned}
$$

Ex 3

Let A and B be events with $P(A) = 0.8$and $P(A \cap B) = 0.2$. For what value of $P(B)$ will A and B be independent?

A and B are independent if $P(A \cap B) = P(A)P(B)$. Therefore $P(B) = 0.25$.

Ex 4

A group of 18 people have gotten together to play baseball. They will divide themselves into two teams of 9 players each, with one team wearing green uniforms and the other wearing yellow uniforms. In how many ways can this be done?

$$\binom{18}{9} = \frac{18!}{9!9!} = 48,620$$

Ex 5

A computer password consists of eight characters. How many different passwords are possible if each character may be any lowercase letter or digit?

يوجد لدينا 26 حرف و 10 ارقام (من 0 الي 9) وبالتالي كل خانه من الخانات ال 8 يكون لها 36 اختيار

No of ways = 36*36*…..*36

$36^8 = 2.8211 \times 10^{12}$

Ex 6

Ex 17:- A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women . find the probability that the committee is

1- All women

2- 2 men and 3 women

3- At most one man

Sol

n (S) = $\binom{15}{5}$

5

5 men

10 women

1- $\in_1 \Rightarrow$ all women n $(\in_1)$ = $\binom{10}{5}$

$$P(\in_1) = \frac{\binom{10}{5}}{\binom{15}{5}} \qquad \frac{n(\in_1)}{n(S)}$$

2- $\in_2 \Rightarrow$ 2 men and 3 women $\quad n(\in_2) = \binom{5}{2}\binom{10}{3}$

$$P(\in_2) = \frac{\binom{5}{2}\binom{10}{3}}{\binom{15}{5}}$$

3- $\in_3$ at most one man

معناها احتمال احد افراد الفريق رجل واحد او لا يوجد اى رجل

$n(\in_3) = n$ (one man or no man)

$$= \binom{5}{1}\binom{10}{4} + \binom{5}{0}\binom{10}{5}$$

$$P(\in_2) = \frac{\binom{5}{1}\binom{10}{4}+\binom{5}{0}\binom{10}{5}}{\binom{15}{5}}$$

Ex 7

Three designer work independently , the probability that the first solve a problem is 0.6 the second solve it 0.7 and the third solve it is 0.75 .what the probability that the problem will be solved.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= P(A) + P(B) + P(C) - P(A)\,P(B) - P(A)\,P(C) - P(B)\,P(C) +$$

$P(A) \,.\, P(B) \,.\, P(C)$

$= 0.6 + 0.7 + 0.75 - 0.6 * 0.7 - 0.6 * 0.75 - 0.7 * 0.75 +$

$0.6 * 0.7 * 0.75 = 0.97$

حل اخر

$P(A \cup B \cup C) = 1 - P(A \cup B \cup C)^c$

$= 1 - P(A^c \cap B^c \cap C^c)$

$= 1 - P(A)^c \,.\, P(B)^c \,.\, P(C)^c = 1 - 0.4 * 0.3 * 0.2$

$= 0.97$

Ex 8

If A and B are independent events, prove that the following pairs of events are independent: A^c and B, A and B^c, and A^c and B^c.

We will prove that A^c *and* B^c are independent the student tries, in the same manner, to prove the other two cases.

$$
\begin{aligned}
p(A^c \cap B^c) = p(A \cup B)^c &= 1 - p(A \cup B) \\
&= 1 - \left(p(A) + p(B) - p(A \cap B)\right) \\
&= 1 - p(A) - p(B) + p(A \cap B) \\
&= 1 - p(A) - p(B) + p(A).p(B) \\
&= \left(1 - p(A)\right) - p(B)\left(1 - p(A)\right) \\
&= \left(1 - p(A)\right)\left(1 - p(B)\right) \\
&= p\left(A^c\right) \ \ p\left(B^c\right)
\end{aligned}
$$

Try yourself

All the fourth-graders in a certain elementary school took a standardized test. A total of 85% of the students were found to be proficient in reading, 78% were found to be proficient in mathematics, and 65% were found to be proficient in both reading and mathematics. A student is chosen at random.

a. What is the probability that the student is proficient in mathematics but not in reading?
b. What is the probability that the student is proficient in reading but not in mathematics?
c. What is the probability that the student is proficient in neither reading nor mathematics?

A fair die is rolled twice. Let A be the event that the number on the first die is odd, let B be the event that the number on the second die is odd, and let C be the event that the sum of the two rolls is equal to 7.

a. Show that A and B are independent, A and C are independent, and B and C are independent. This property is known as *pairwise independence*.

b. Show that A, B, and C are not independent. Conclude that it is possible for a set of events to be pairwise independent but not independent. (In this context, independence is sometimes referred to as *mutual independence*.)

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Thank you