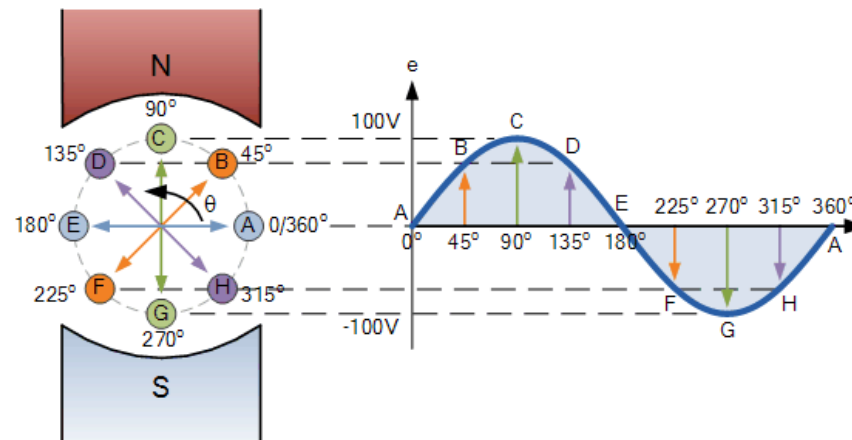


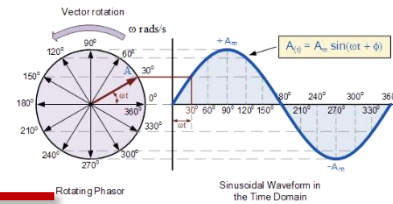
# Alternating Current & Phasors

CSE 113



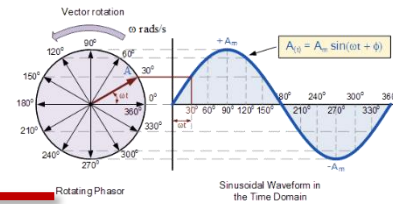
Physics Department  
Faculty of Science  
New Mansoura University

# OUTLINES



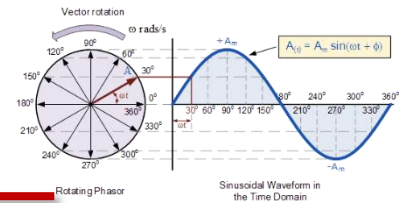
- ❑ Capacitors and Inductors
- ❑ Time Varying Signals
- ❑ Types of famous signals
- ❑ Sinusoidal waveform
- ❑ (Polarity, Period, Frequency, Peak, Peak to peak, Average value, RMS value)
- ❑ Introduction to Phasors, Phasors and the Sine Wave Formula and phasor diagram
- ❑ Resistance in ac circuits
- ❑ Capacitor in ac circuits
- ❑ Inductor in ac circuits

# Capacitors and Inductors



- ❑ **Resistor**: a passive element which dissipates energy only
- ❑ Two important passive linear circuit elements:
  - Capacitor
  - Inductor
- ❑ **Capacitor** and **inductor** can store energy only and they can neither generate nor dissipate energy.

# Capacitors



- A capacitor consists of two conducting plates separated by an insulator (or dielectric).

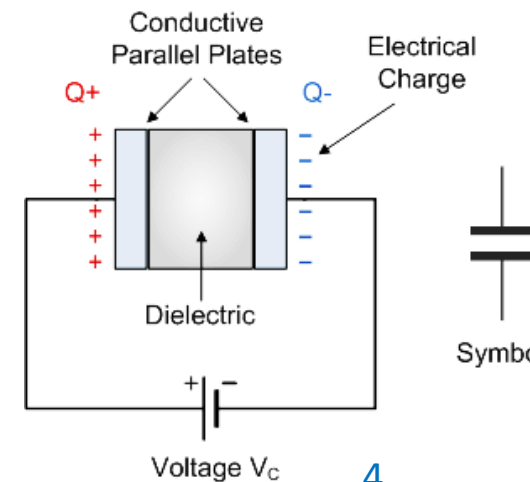
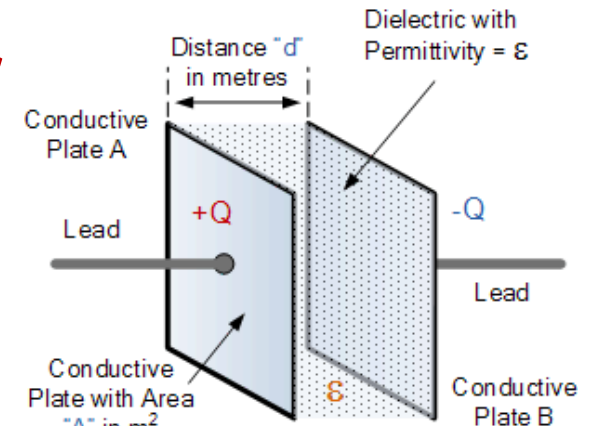
$$C = \frac{\epsilon A}{d}$$

$$\epsilon = \epsilon_r \epsilon_0$$

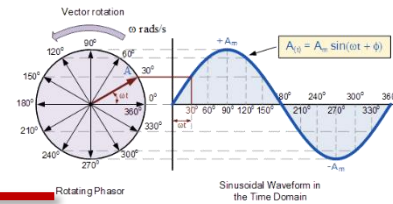
$$\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$$

- Three factors affecting the value of capacitance:

1. Area: the larger the area, the greater the capacitance.
2. Spacing between the plates: the smaller the spacing, the greater the capacitance.
3. Material permittivity: the higher the permittivity, the greater the capacitance.



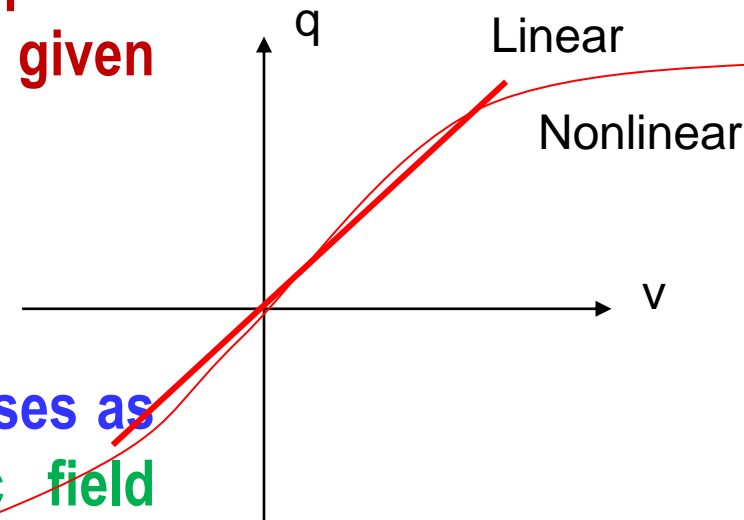
# Charge in Capacitors



- The relation between the charge in plates and the voltage across a capacitor is given below.

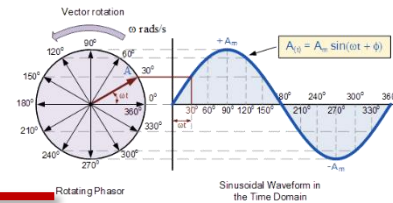
$$q = C v$$

$$1\text{F} = 1 \text{ C/V}$$



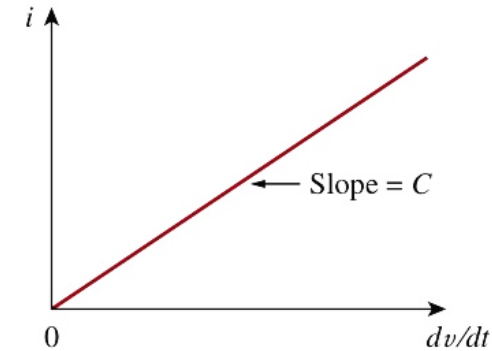
- Since  $q = C v$ , the plate charge increases as the voltage increases. The electric field intensity between two plates increases. If the voltage across the capacitor is so large that the field intensity is large enough to break down the insulation of the dielectric, the capacitor is out of work. Hence, every practical capacitor has a maximum limit on its operating voltage.

# I-V Relation of Capacitor

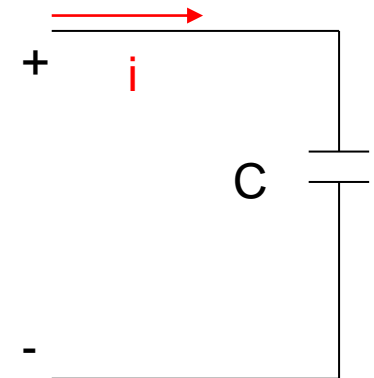


$$q = Cv, i = \frac{dq}{dt} = C \frac{dv}{dt}$$

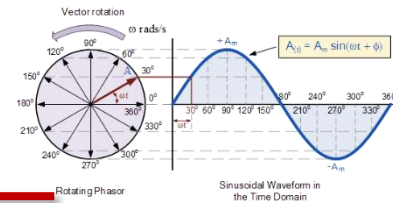
$$i = C \frac{dv}{dt}$$



- when **v** is a constant voltage, then **i=0**; a constant voltage across a capacitor creates no current through the capacitor, the capacitor in this case is the same as an open circuit.
- If **v** is abruptly changed, then the **current** will have an **infinite value** that is practically **impossible**. Hence, a capacitor is impossible to have an abrupt change in its voltage except an infinite current is applied.



# I-V Relation of Capacitor

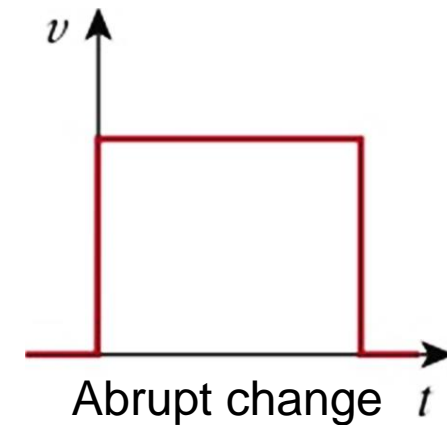
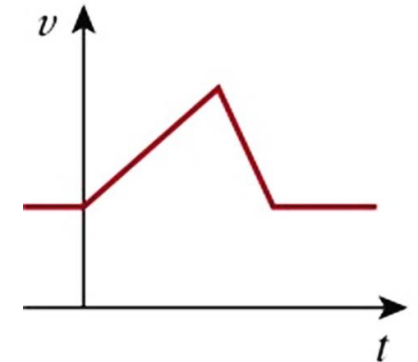


- A capacitor is an open circuit to dc.
- The voltage on a capacitor cannot change abruptly.

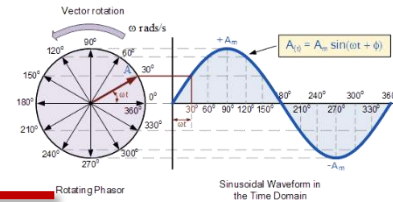
$$i = C \frac{dv}{dt} \quad v(t) = \frac{1}{C} \int_{-\infty}^t i dt \quad (v(-\infty) = 0)$$

$$v(t) = \frac{1}{C} \int_{t_o}^t i dt + v(t_o) \quad (v(t_o) = q(t_o) / C)$$

- The charge on a capacitor is an integration of current through the capacitor. Hence, the memory effect counts.



# Energy Storing in Capacitor

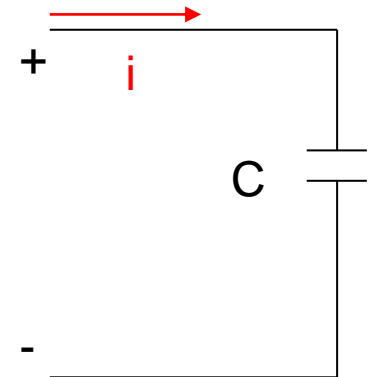


$$p = vi = Cv \frac{dv}{dt}$$

$$w = \int_{-\infty}^t p dt = C \int_{-\infty}^t v \frac{dv}{dt} dt = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} C v^2 \Big|_{v(-\infty)}^{v(t)}$$

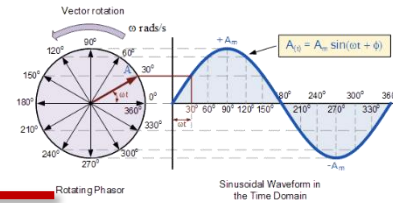
$$w(t) = \frac{1}{2} C v^2(t) \quad (v(-\infty) = 0)$$

$$w(t) = \frac{q^2(t)}{2C}$$





# Example 1



- A. Calculate the charge stored on a 3-pF capacitor with 20V across it.
- B. Find the energy stored in the capacitor.

**Solution:**

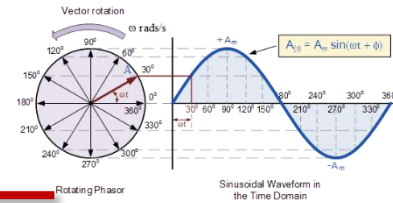
(a) Since  $q = Cv$ ,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

## Example 2



- The voltage across a 5-  $\mu\text{F}$  capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

- Calculate the current through it.

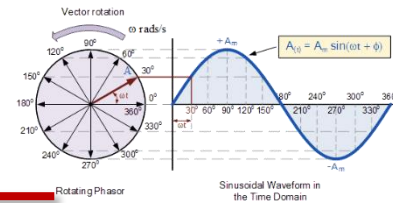
**Solution:**

- By definition, the current is

$$i = C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt} (10 \cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

## Example 3



- Obtain the energy stored in each capacitor in Fig. under dc condition.

### Solution:

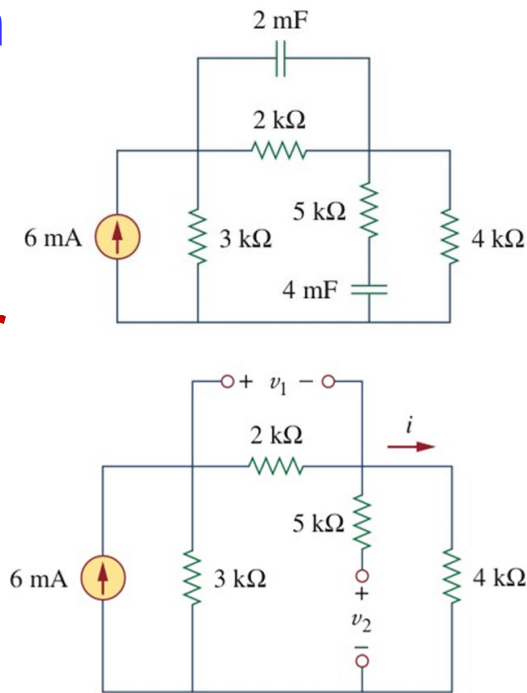
- Under dc condition, we replace each capacitor with an open circuit. By current division,

$$i = \frac{3}{3 + 2 + 4} (6\text{mA}) = 2\text{mA}$$

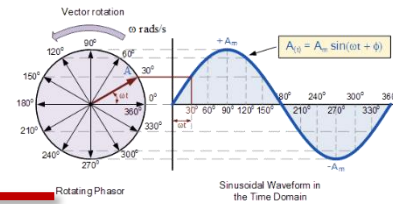
$$\therefore v_1 = 2000i = 4\text{ V}, \quad v_2 = 4000i = 8\text{ V}$$

$$\therefore w_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2} (2 \times 10^{-3}) (4)^2 = 16\text{mJ}$$

$$w_2 = \frac{1}{2} C_2 v_2^2 = \frac{1}{2} (4 \times 10^{-3}) (8)^2 = 128\text{mJ}$$



# Series and Parallel Capacitors

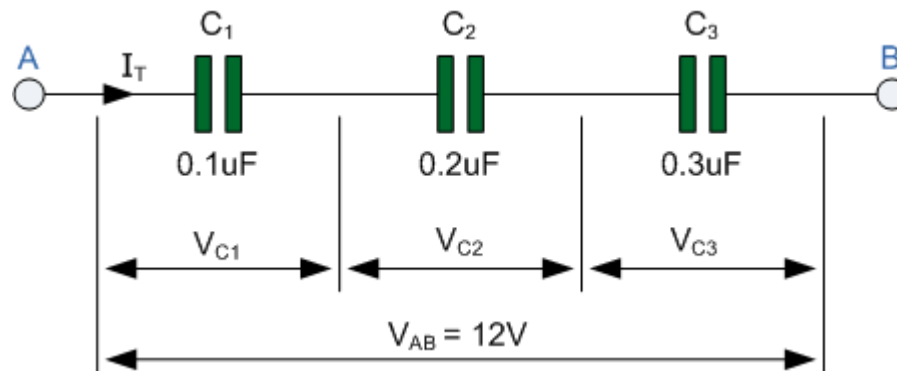


- The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

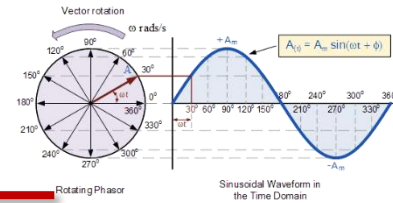
$$v(t) = v_1(t) + v_2(t) + \dots + v_N(t)$$

$$\frac{1}{C_{eq}} \int_{-\infty}^t i d\tau = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N} \right) \int_{-\infty}^t i d\tau$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$



# Series and Parallel Capacitors



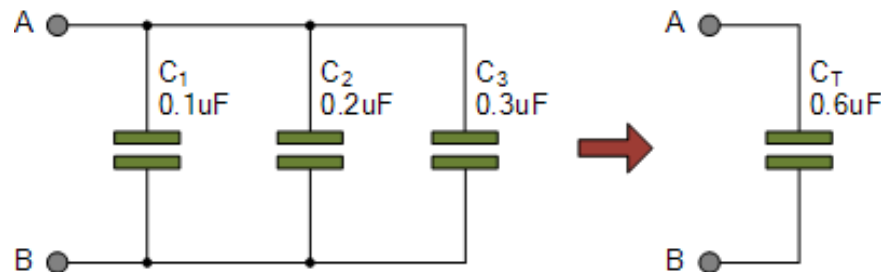
- The equivalent capacitance of  $N$  parallel-connected capacitors is the sum of the individual capacitance.

$$i = i_1 + i_2 + i_3 + \dots + i_N$$

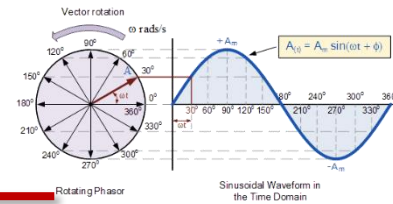
$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

$$= \left( \sum_{k=1}^N C_K \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$



# Inductors



- An inductor is made of a coil of conducting wire

$$L = \frac{N^2 \mu A}{l}$$

$$\mu = \mu_r \mu_0$$

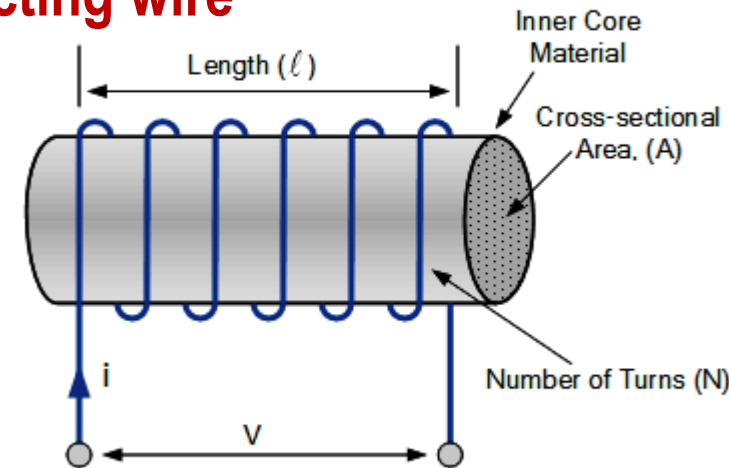
$$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$$

$N$ : number of turns.

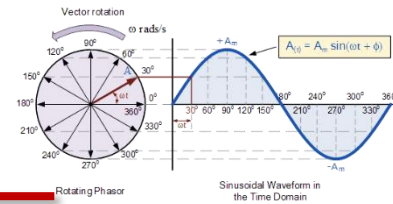
$l$ : length.

$A$ : cross – sectional area.

$\mu$ : permeability of the core



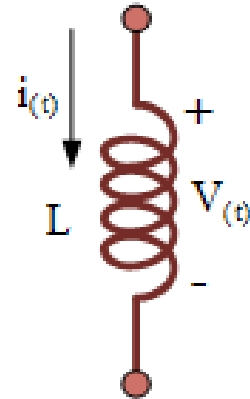
# I-V Relation of Inductor



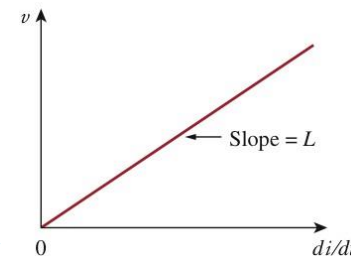
- The relation between the flux in inductor and the current through the inductor is given below.

$$\phi = Li \quad 1\text{H} = 1 \text{ Weber/A}$$

$$v = \frac{d\phi}{dt} = L \frac{di}{dt}$$



- When the current through an inductor is a constant, then the voltage across the inductor is zero, same as a short circuit.
- No abrupt change of the current through an inductor is possible except an infinite voltage across the inductor is applied.
- The inductor can be used to generate a high voltage, for example, used as an igniting element.



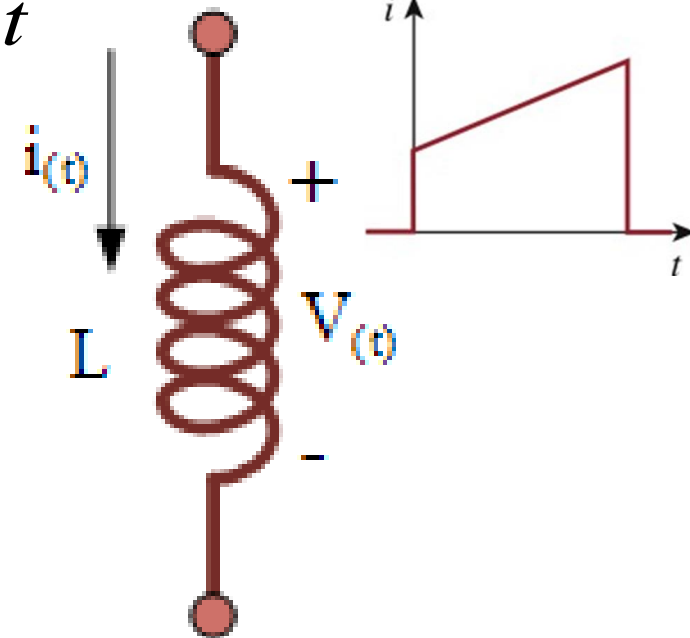
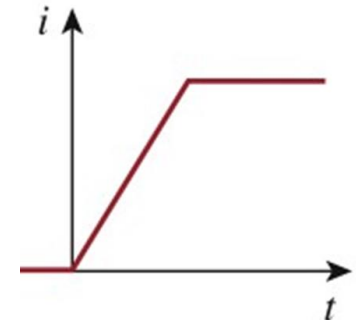
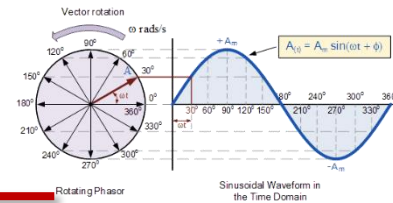
# I-V Relation of Inductor

- ❑ An inductor are like a short circuit to dc.
- ❑ The current through an inductor cannot change instantaneously.

$$di = \frac{1}{L} v dt \quad i = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

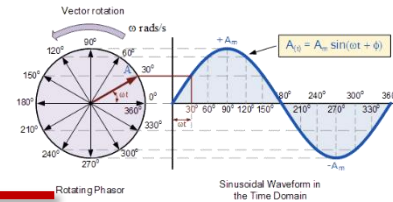
$$i = \frac{1}{L} \int_{t_o}^t v(t) dt + i(t_o)$$

The inductor has memory.





# Energy Stored in an Inductor



$$P = vi = \left( L \frac{di}{dt} \right) i$$

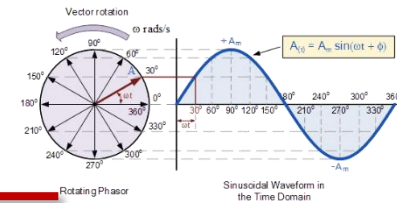
$$w = \int_{-\infty}^t p dt = \int_{-\infty}^t \left( L \frac{di}{dt} \right) i dt$$

$$= L \int_{i(-\infty)}^{i(t)} i di = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty) \quad i(-\infty) = 0,$$

★ The energy stored in an inductor

$$w(t) = \frac{1}{2} Li^2(t)$$

## Example 4



- ★ Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also find the energy stored within  $0 < t < 5\text{s}$ . Assume  $i(0)=0$ .

### Solution:

Since  $i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$  and  $L = 5\text{H}$ .

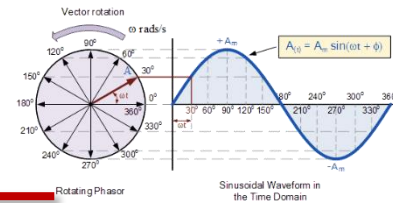
$$i = \frac{1}{5} \int_0^t 30t^2 dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power  $p = vi = 60t^5$ ,

and the energy stored is then

$$w = \int p dt = \int_0^5 60t^5 dt = 60 \frac{t^6}{6} \Big|_0^5 = 156.25 \text{ kJ}$$

# Example 5



□ Consider the circuit in Fig. Under dc conditions, find:

(a)  $i$ ,  $v_C$ , and  $i_L$ .

(b) the energy stored in the capacitor and inductor.

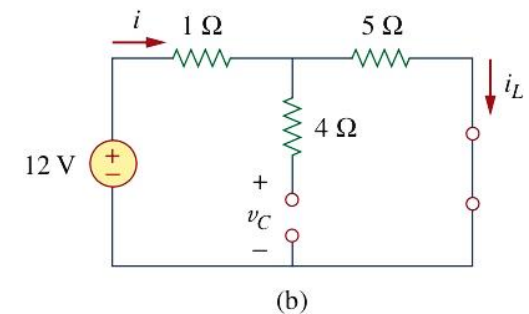
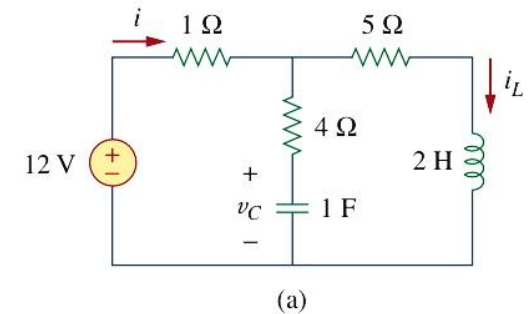
**Solution:**

(a) Under dc condition : capacitor  $\rightarrow$  open circuit  
inductor  $\rightarrow$  short circuit

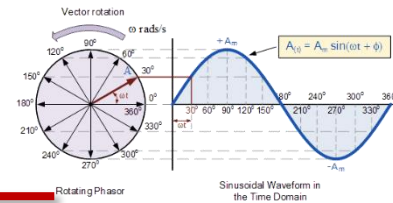
$$i = i_L = \frac{12}{1+5} = 2A, \quad v_C = 5i = 10V$$

$$w_C = \frac{1}{2} C v_C^2 = \frac{1}{2} (1)(10^2) = 50J,$$

$$w_L = \frac{1}{2} L i^2 = \frac{1}{2} (2)(2^2) = 4J$$



# Series and Parallel Inductors

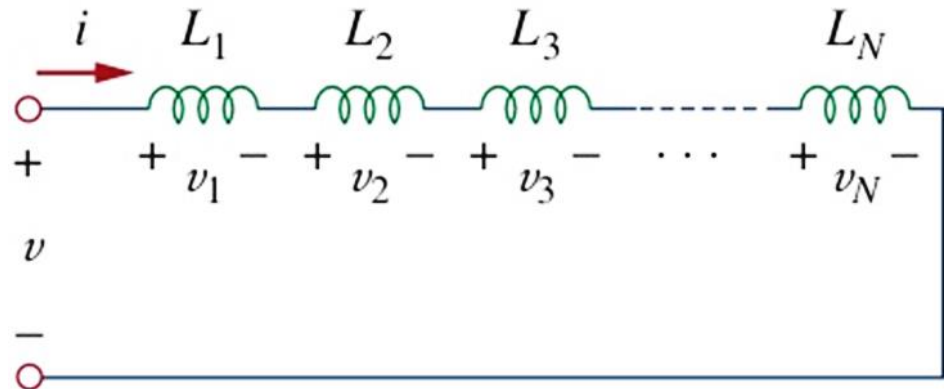


- Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \dots + v_N$$

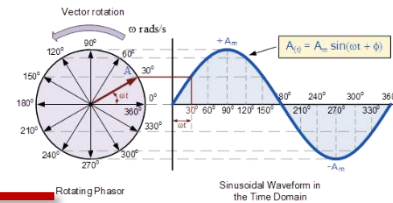
- Substituting  $v_k = L_k di/dt$  results in

$$\begin{aligned} v &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_N \frac{di}{dt} \\ &= (L_1 + L_2 + L_3 + \dots + L_N) \frac{di}{dt} \\ &= \left( \sum_{K=1}^N L_K \right) \frac{di}{dt} = L_{eq} \frac{di}{dt} \end{aligned}$$



$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_N$$

# Parallel Inductors



□ Using KCL,

$$i = i_1 + i_2 + i_3 + \dots + i_N$$

□ But

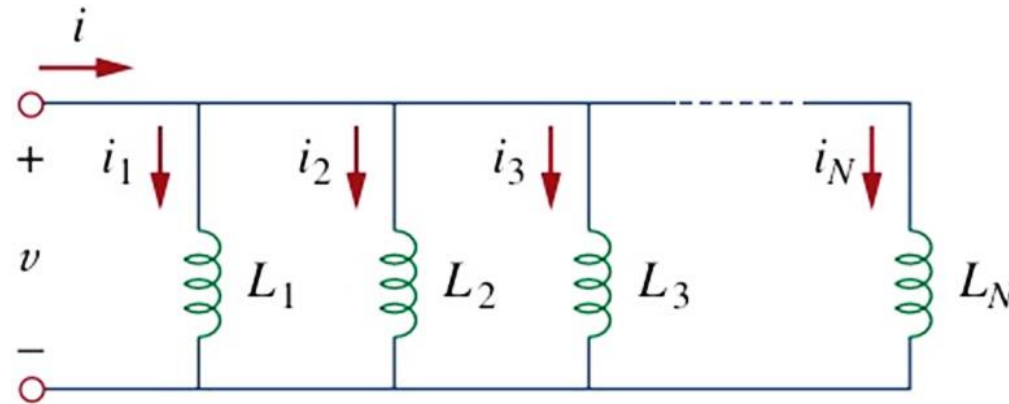
$$i_k = \frac{1}{L_k} \int_{t_0}^t v dt + i_k(t_0)$$

$$\therefore i = \frac{1}{L_1} \int_{t_0}^t v dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^t v dt + i_2(t_0) + \dots + \frac{1}{L_N} \int_{t_0}^t v dt + i_N(t_0)$$

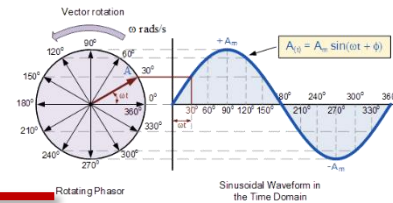
$$= \left( \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N} \right) \int_{t_0}^t v dt + i_1(t_0) + i_2(t_0) + \dots + i_N(t_0)$$

$$= \left( \sum_{k=1}^N \frac{1}{L_k} \right) \int_{t_0}^t v dt + \sum_{k=1}^N i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^t v dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

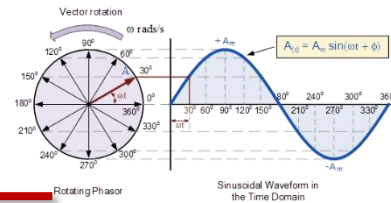


# Summary

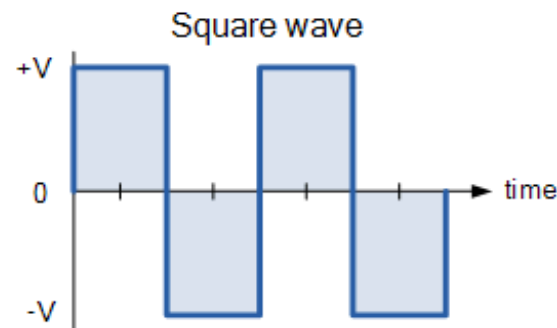
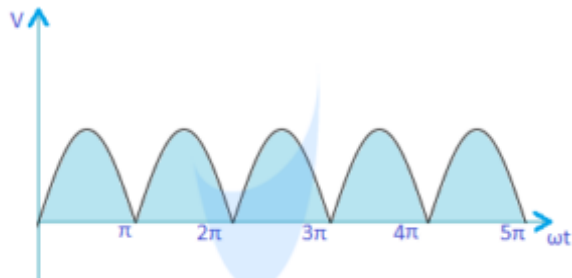
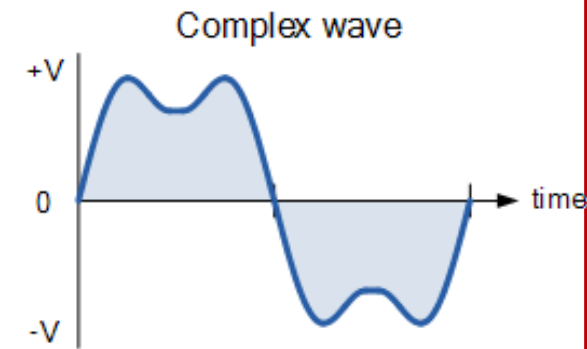
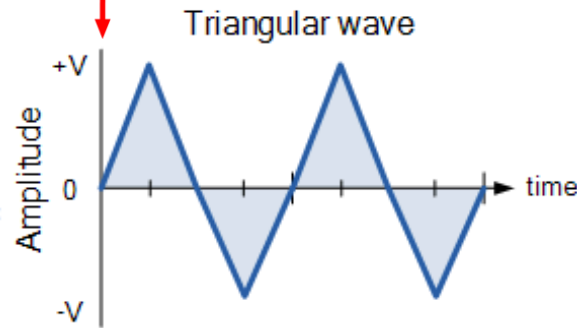
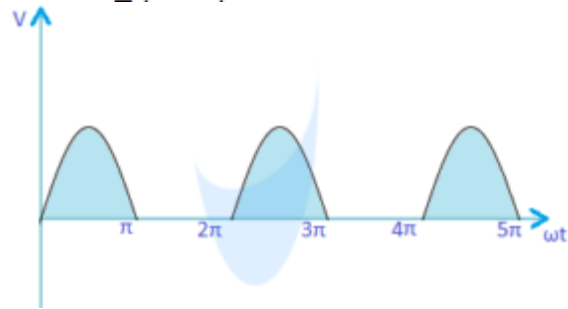
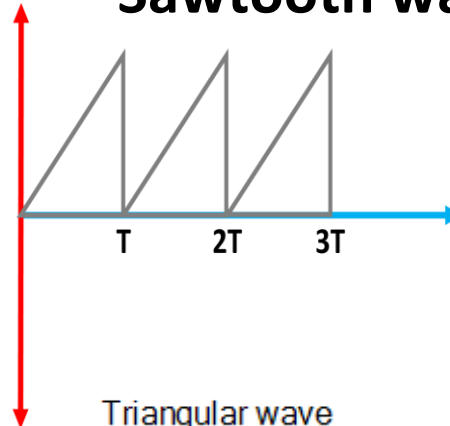
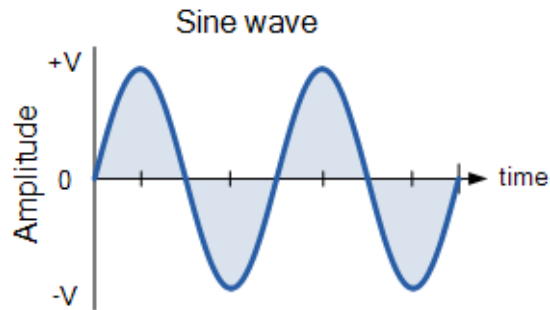


Relation	Resistor ( $R$ )	Capacitor ( $C$ )	Inductor ( $L$ )
$v-i$ :	$v = iR$	$v = \frac{1}{C} \int_{t_0}^t i \, dt + v(t_0)$	$v = L \frac{di}{dt}$
$i-v$ :	$i = v/R$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v \, dt + i(t_0)$
$p$ or $w$ :	$p = i^2 R = \frac{v^2}{R}$	$w = \frac{1}{2} C v^2$	$w = \frac{1}{2} L i^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	$v$	$i$

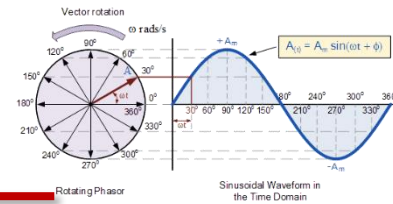
# Time Varying Signals:



## Sawtooth wave

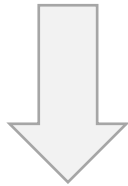


# Time Varying Signals

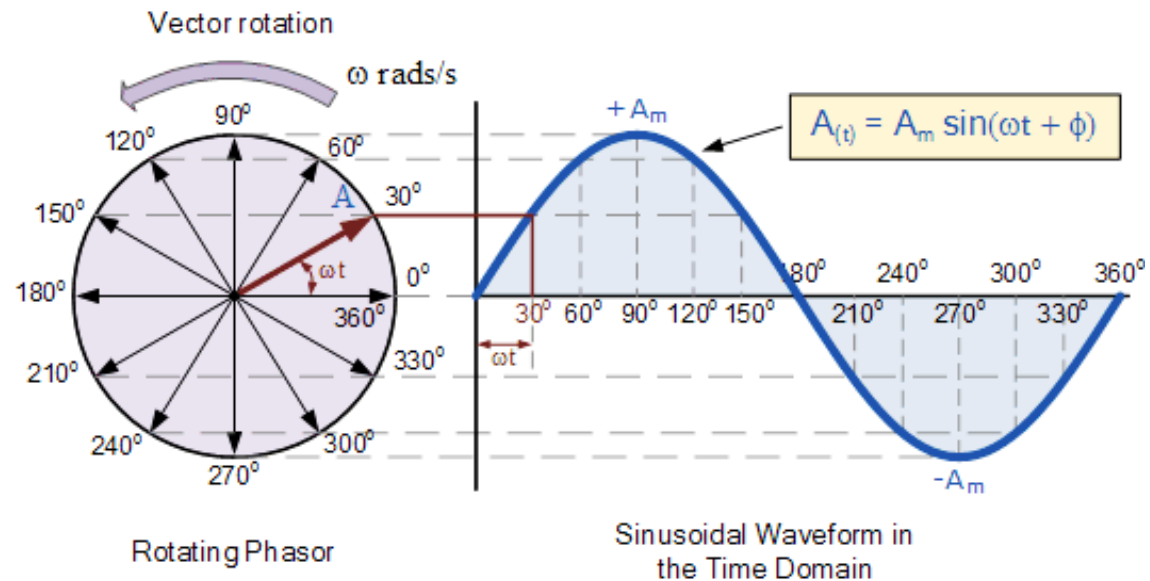


- How to obtain the wave function from the waveform
- Determine the period of the function,
  - Study the behavior of the wave of one single wave,
  - Some waveforms are famous, you can guess the wave function,

Wave Function

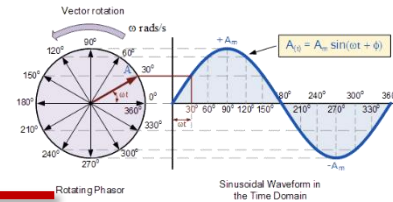


$$V = v_m \sin \omega t$$

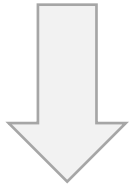




# Time Varying Signals



Wave Function

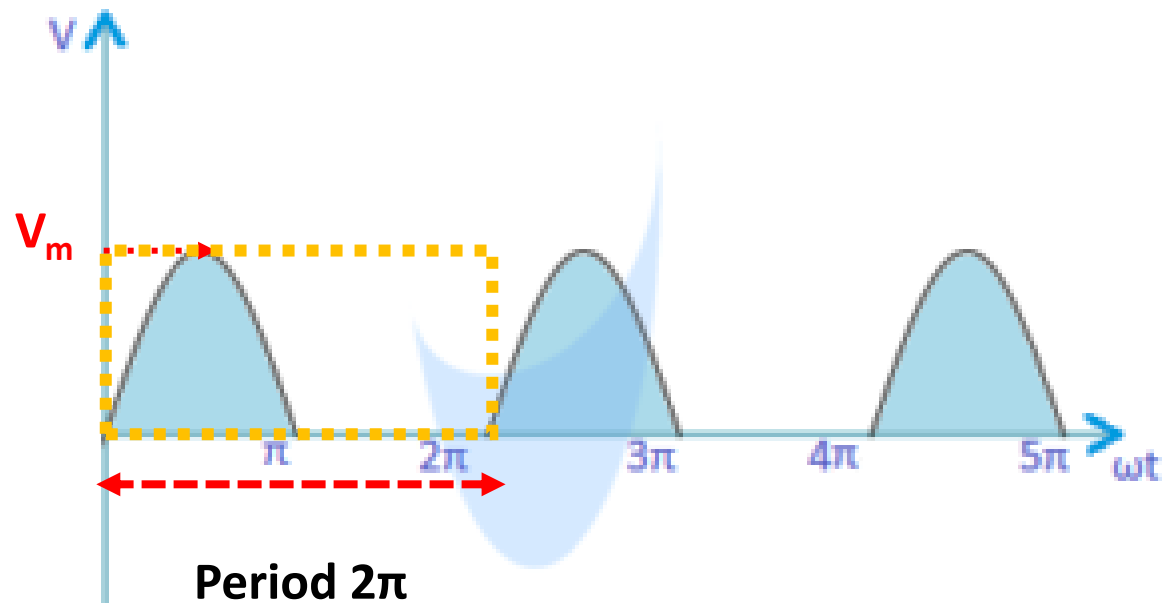


$$0 < \omega t < \pi$$

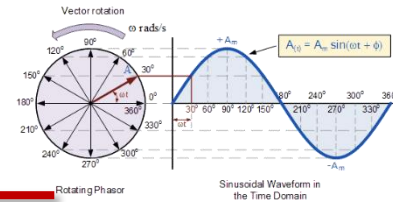
$$V = v_m \sin \omega t$$

$$\pi < \omega t < 2\pi$$

$$V = 0$$



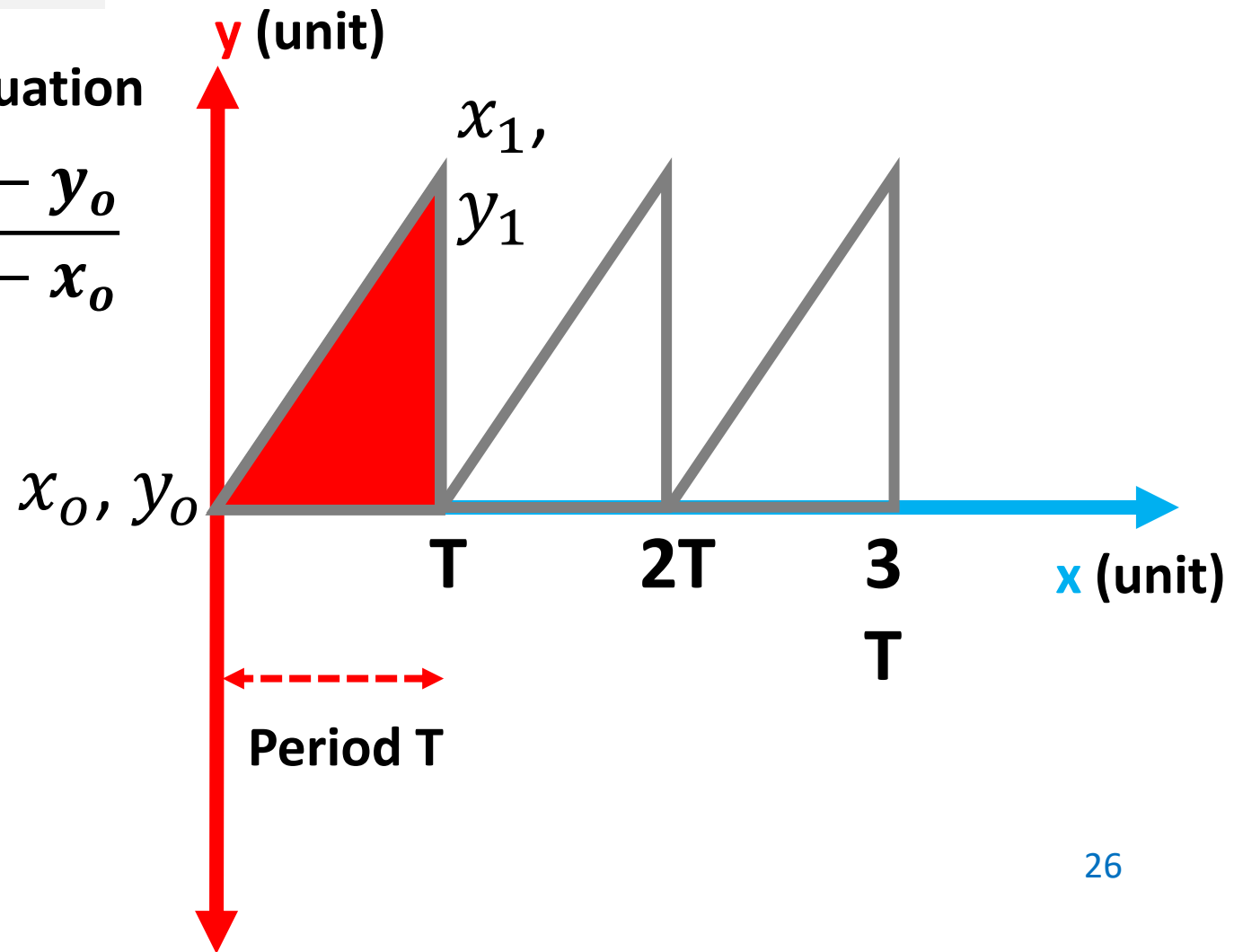
# Time Varying Signals



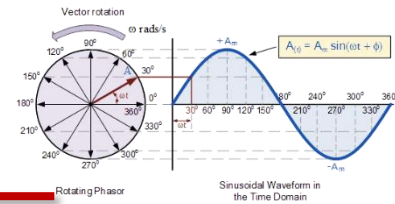
## Sawtooth wave

Straight line equation

$$\frac{y - y_o}{x - x_o} = \frac{y_1 - y_o}{x_1 - x_o}$$

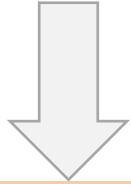


# Time Varying Signals



square wave

Wave Function

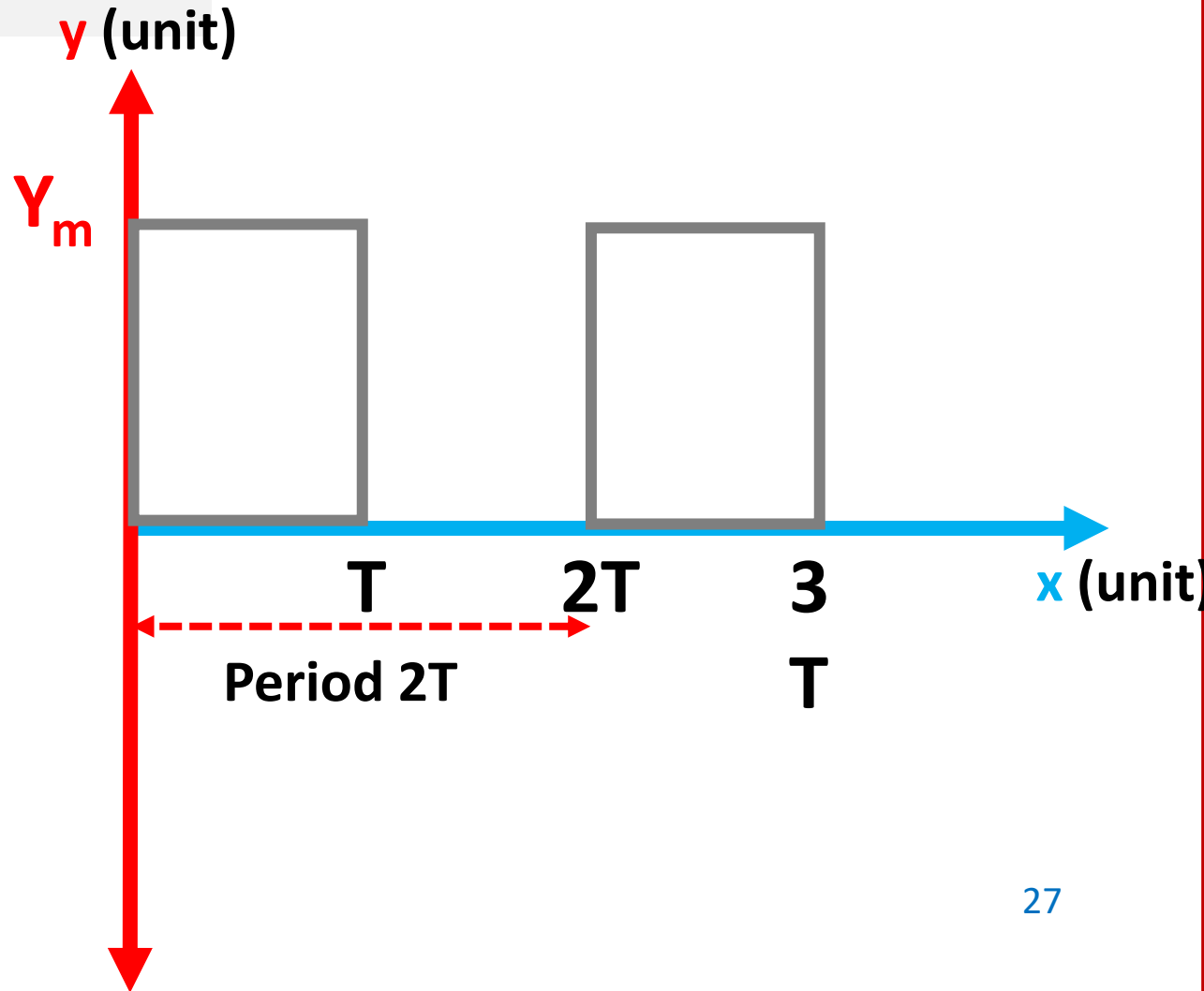


$$0 < x < T$$

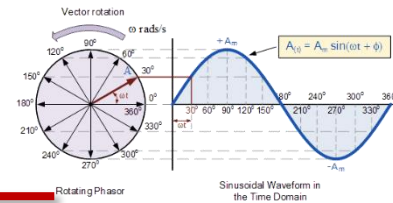
$$Y = Y_m$$

$$T < x < 2T$$

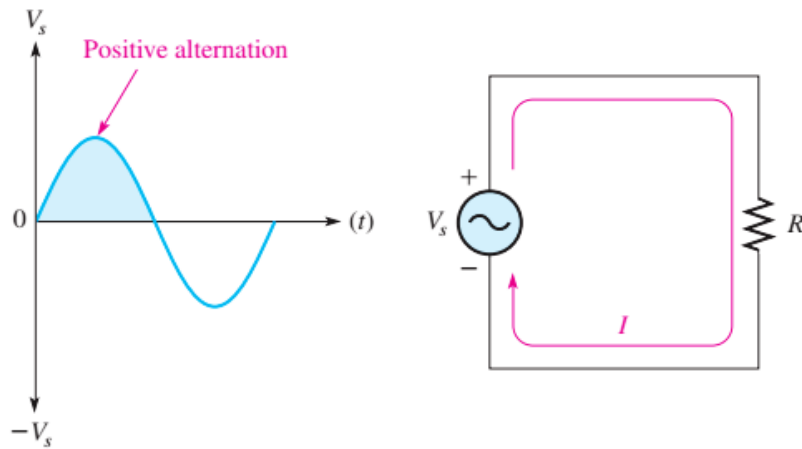
$$Y = 0$$



# Sinusoidal Waveform

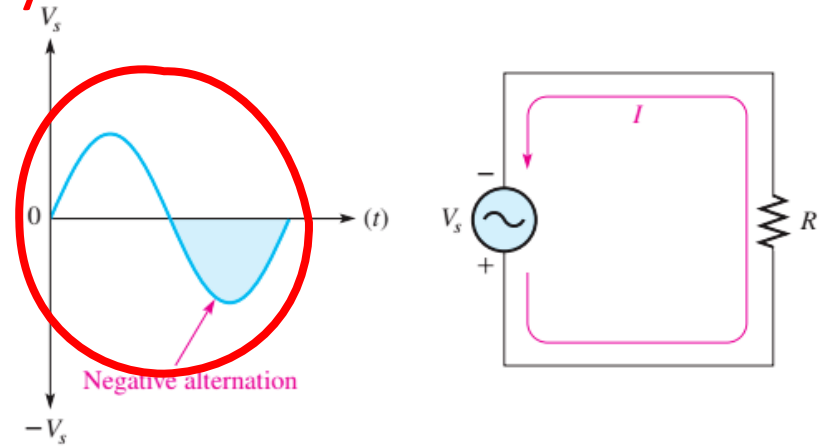


## □ Polarity of a Sine Wave



(a) During a positive alternation of voltage, current is in the direction shown.

## Cycle or Period

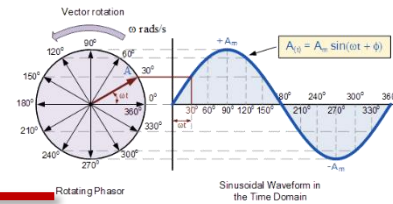


(b) During a negative alternation of voltage, current reverses direction, as shown.

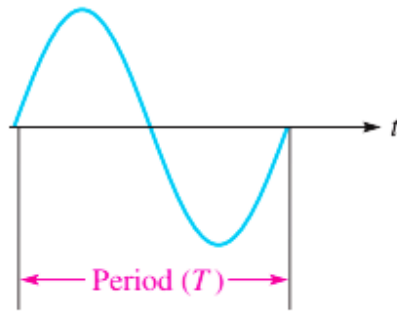
▲ FIGURE 11-3

Alternating current and voltage.

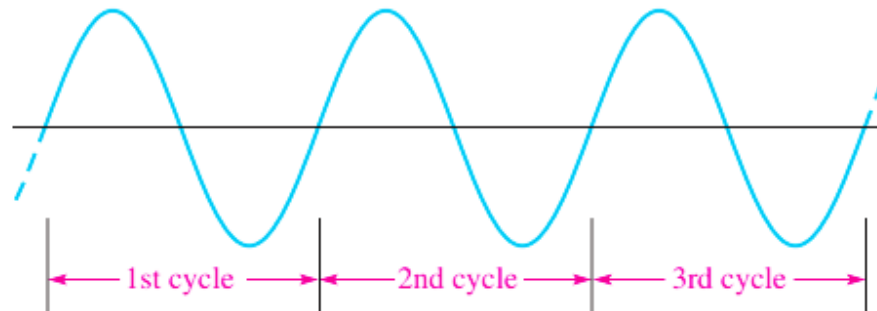
# Period of a Sine Wave



- The time required for a sine wave to complete one full cycle is called the period ( $T$ ).



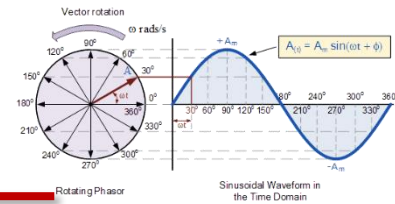
(a)



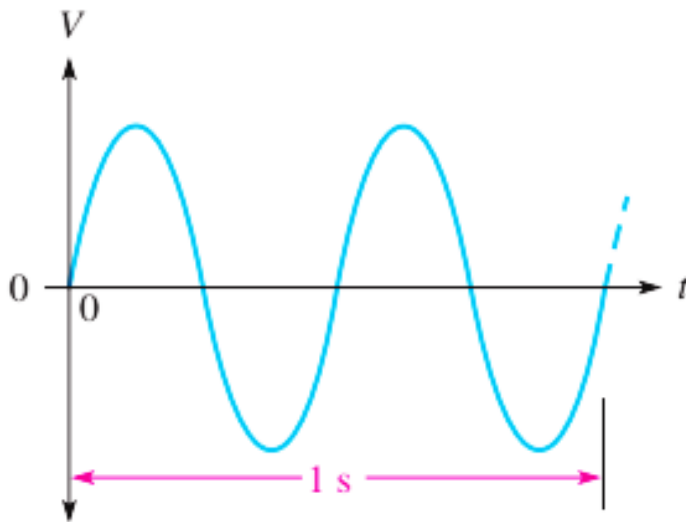
(b)

The period of a sine wave is the same for each cycle.

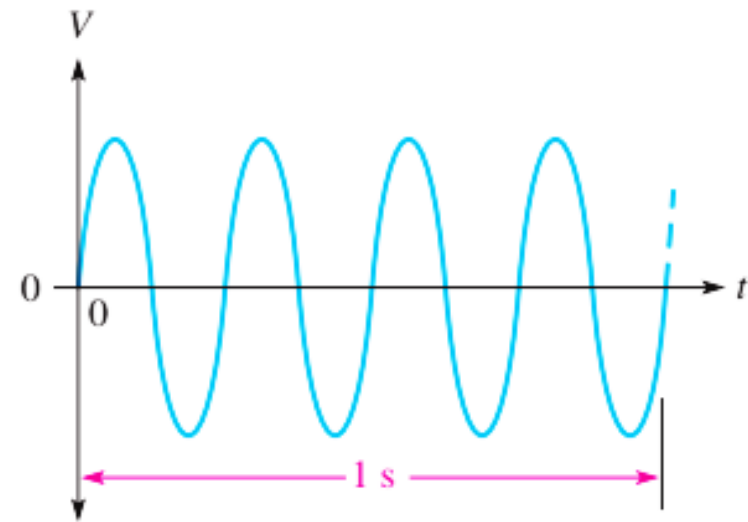
# Frequency of a Sine Wave



- Frequency (  $f$  ) is the number of cycles that a sine wave completes in one second.

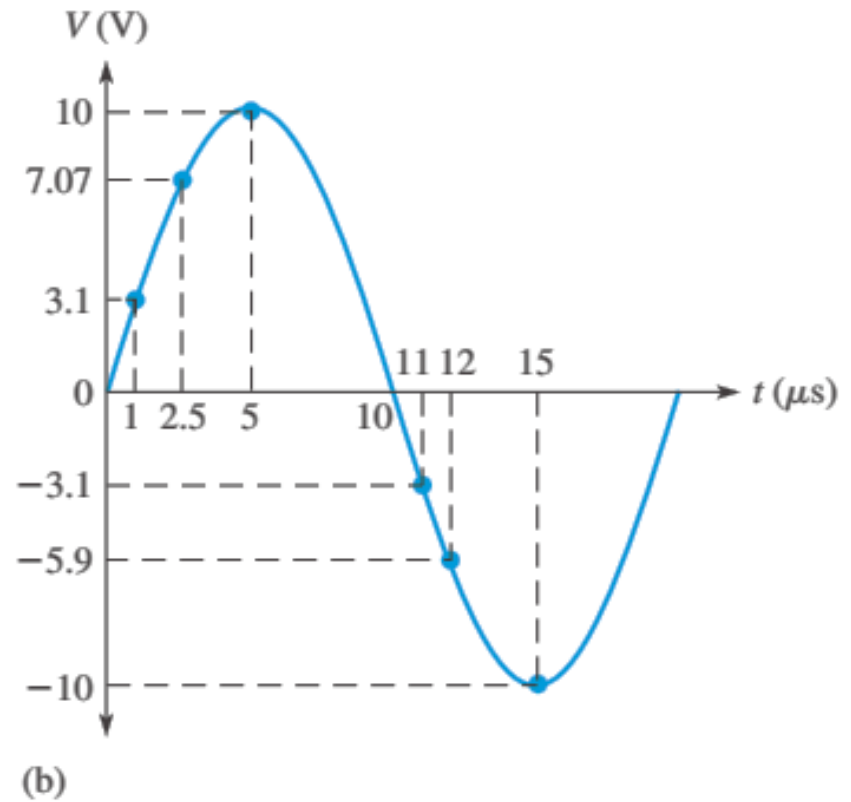
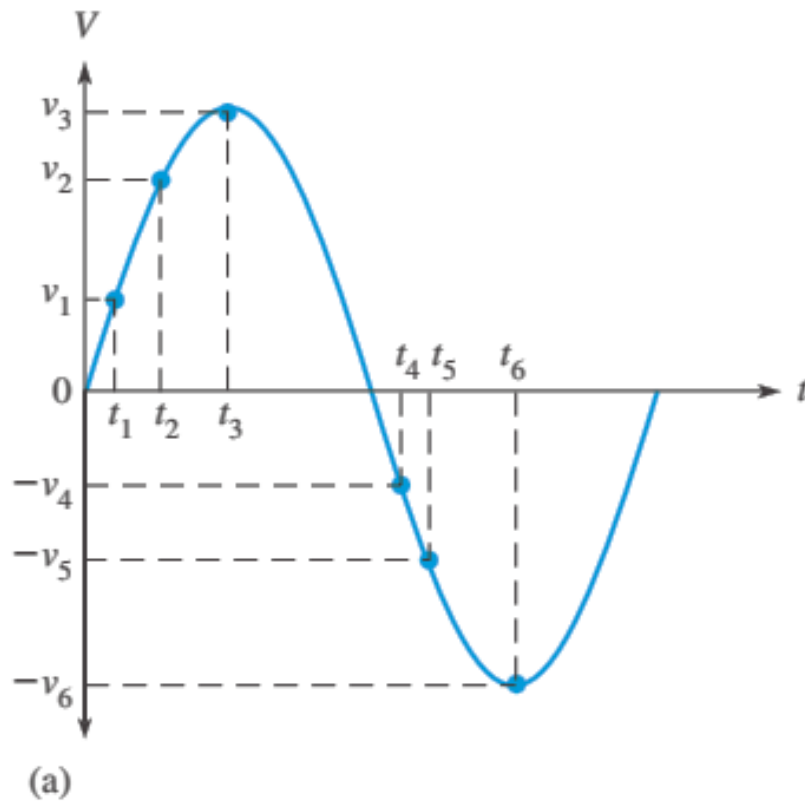
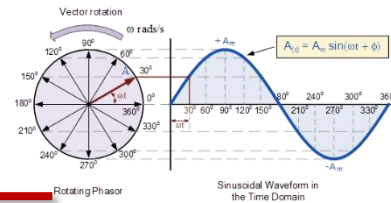


(a) Lower frequency: fewer cycles per second



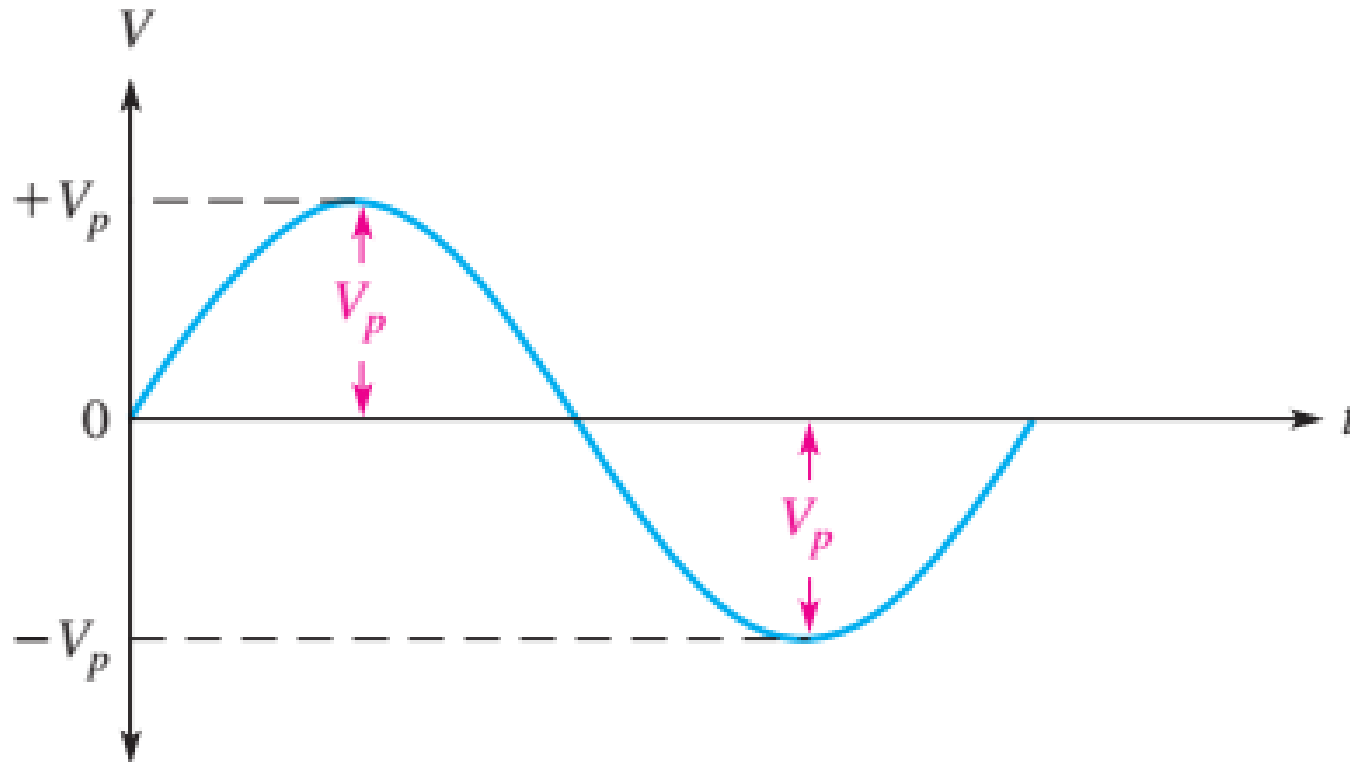
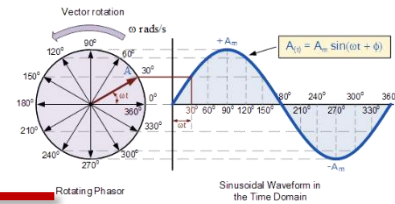
(b) Higher frequency: more cycles per second

# Instantaneous Value



Instantaneous values.

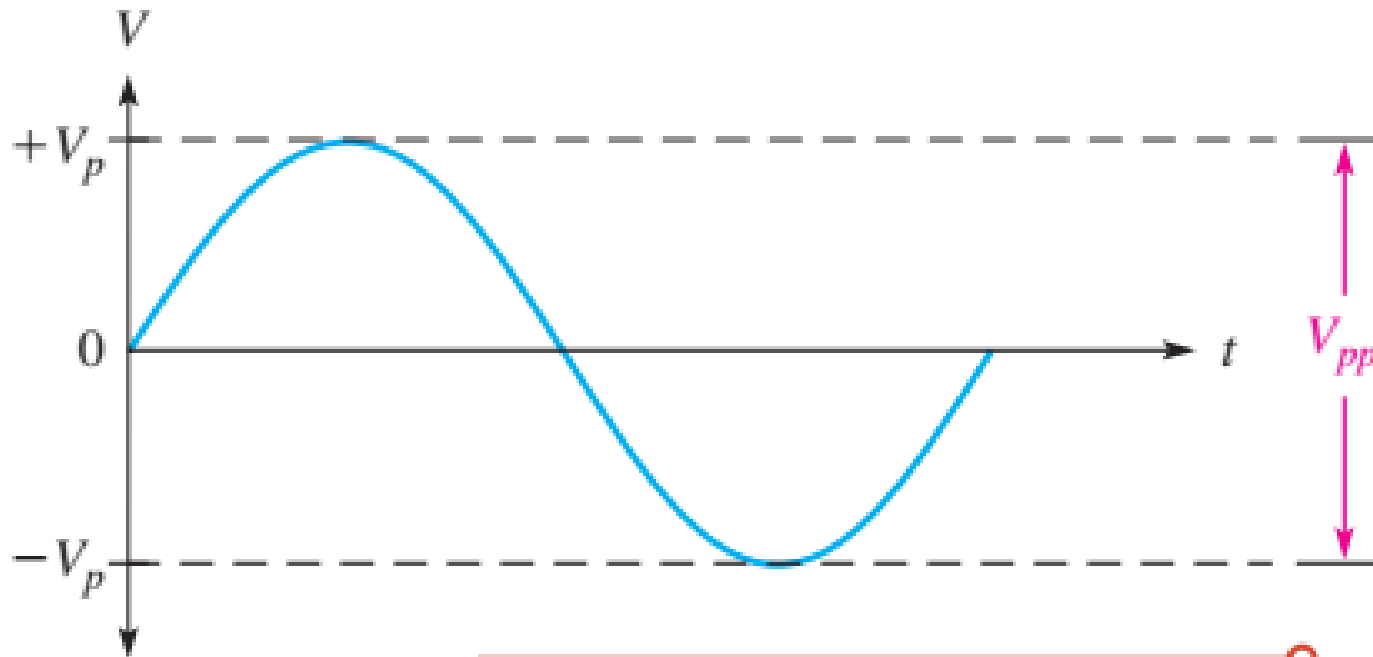
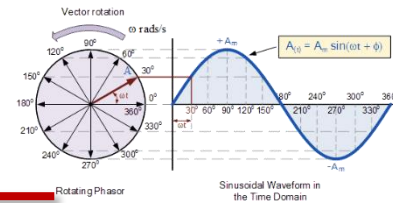
# Peak Value



Peak values.



# Peak-to-Peak Value

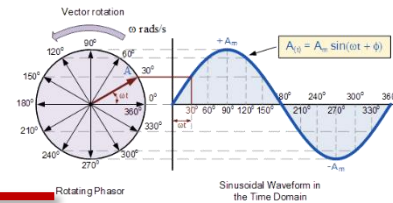


Peak-to-peak value.

$$V_{pp} = 2V_p$$

$$I_{pp} = 2I_p$$

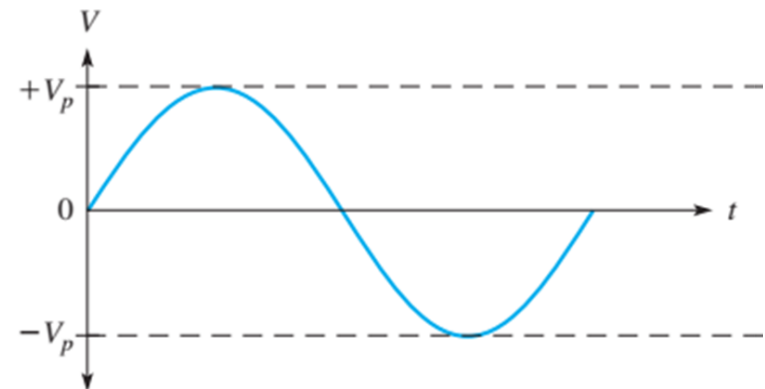
# Average Value



- The average value of a sine wave taken over one complete cycle is always zero because the positive values (above the zero crossing) offset the negative values (below the zero crossing).
- To be useful for certain purposes such as measuring types of voltages found in power supplies, the average value of a sine wave is defined over a half-cycle rather than over a full cycle.

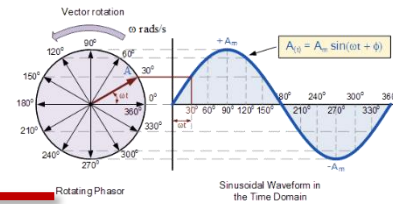
$$V_{\text{avg}} = \left( \frac{2}{\pi} \right) V_p$$

$$V_{\text{avg}} = 0.637 V_p$$



$$Y_{\text{av}} = \frac{1}{T} \int_0^T y(t) dt$$

# RMS Value

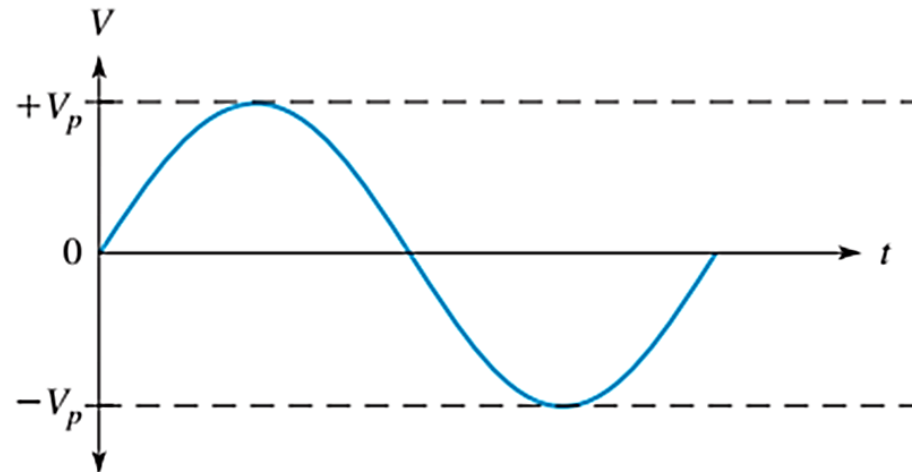


- The term **rms** stands for **root mean square**. Most ac voltmeters display rms voltage. The 220 V at your wall outlet is an rms value. The rms value, also referred to as the **effective value**,

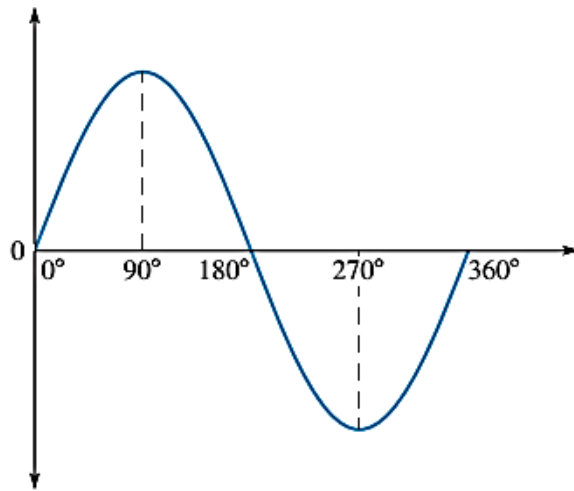
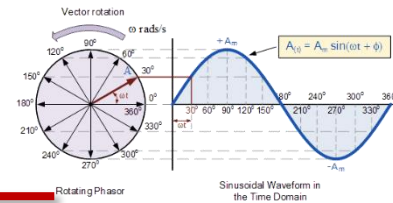
$$Y_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T \overline{y(t)}^2 dt}$$

$$V_{\text{rms}} = 0.707 V_p$$

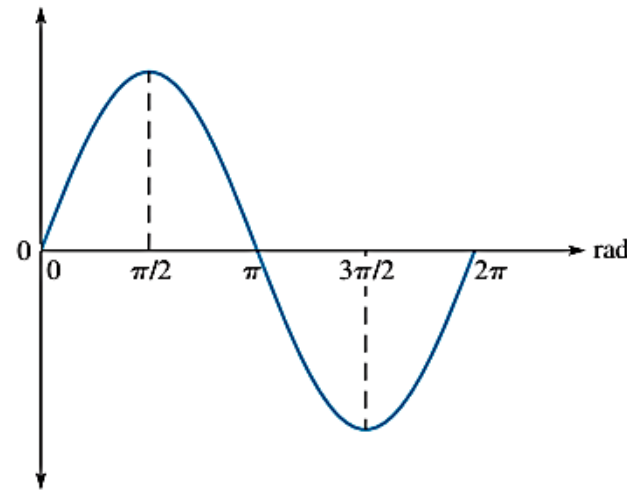
$$I_{\text{rms}} = 0.707 I_p$$



# Sine Wave Angles



(a) Degrees



(b) Radians

## Radian/Degree Conversion

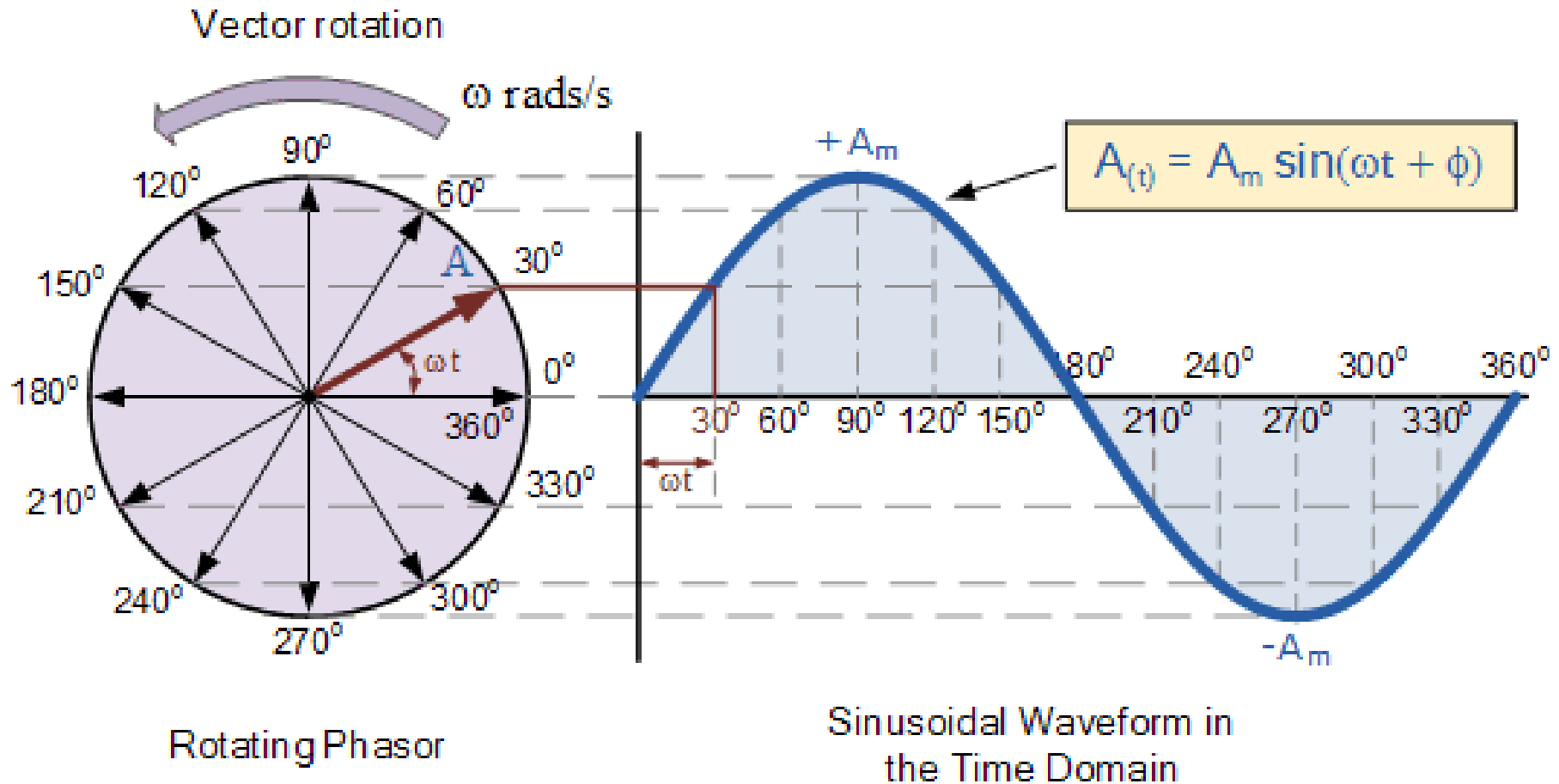
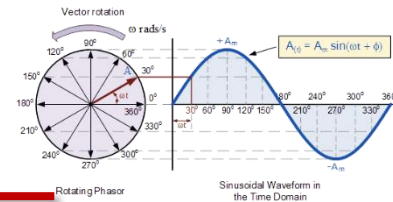
Degrees can be converted to radians.

$$\text{rad} = \left( \frac{\pi \text{ rad}}{180^\circ} \right) \times \text{degrees}$$

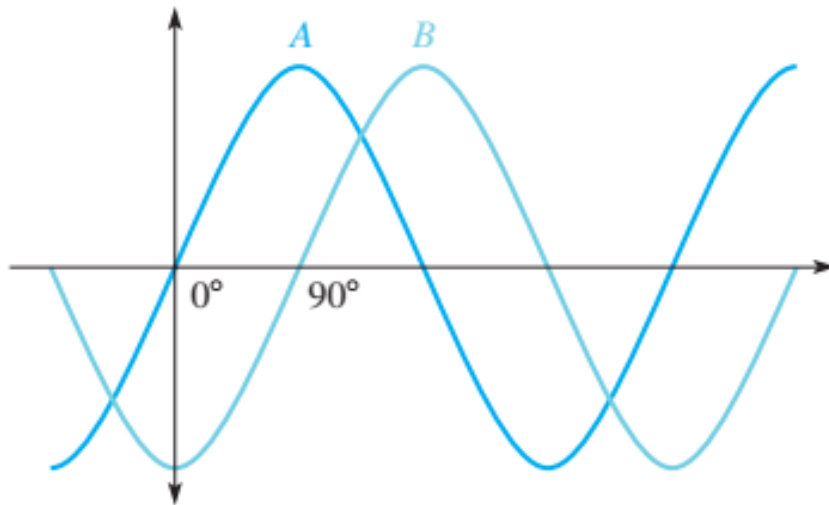
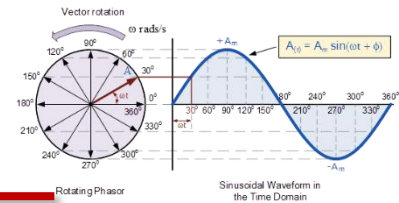
Similarly, radians can be converted to degrees.

$$\text{degrees} = \left( \frac{180^\circ}{\pi \text{ rad}} \right) \times \text{rad}$$

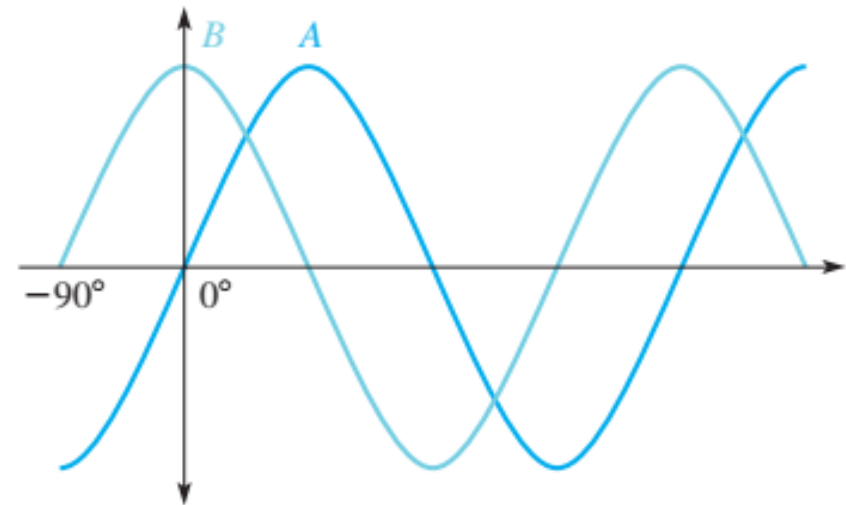
# Phase of a Sine Wave



# Phase of a Sine Wave

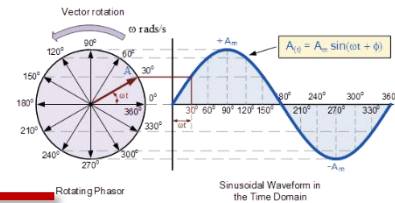


(a)  $A$  leads  $B$  by  $90^\circ$ , or  $B$  lags  $A$  by  $90^\circ$ .

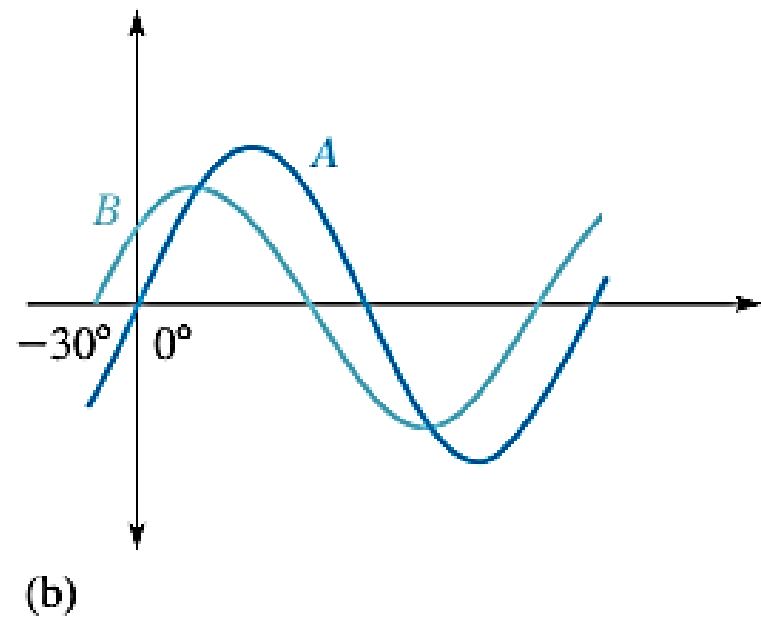
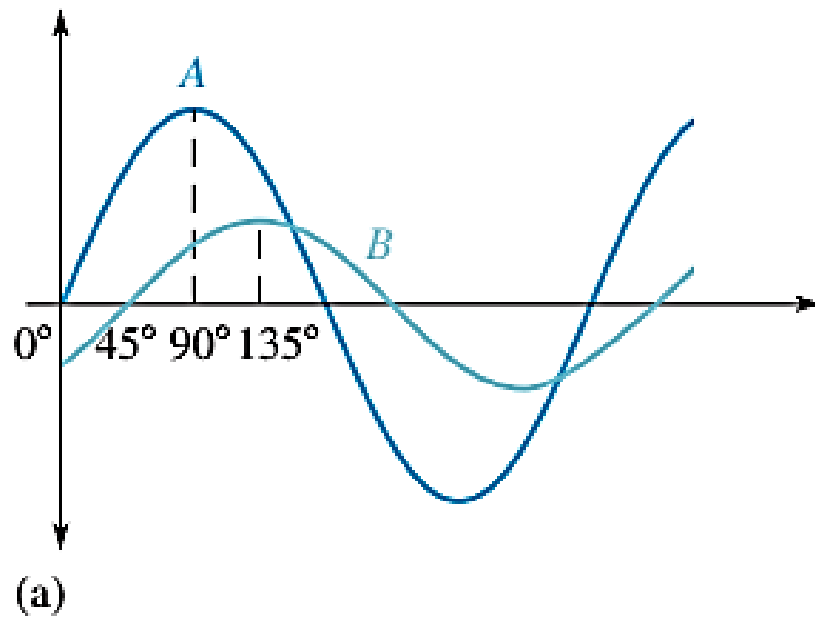


(b)  $B$  leads  $A$  by  $90^\circ$ , or  $A$  lags  $B$  by  $90^\circ$ .

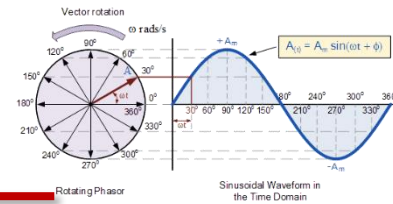
## Example 6



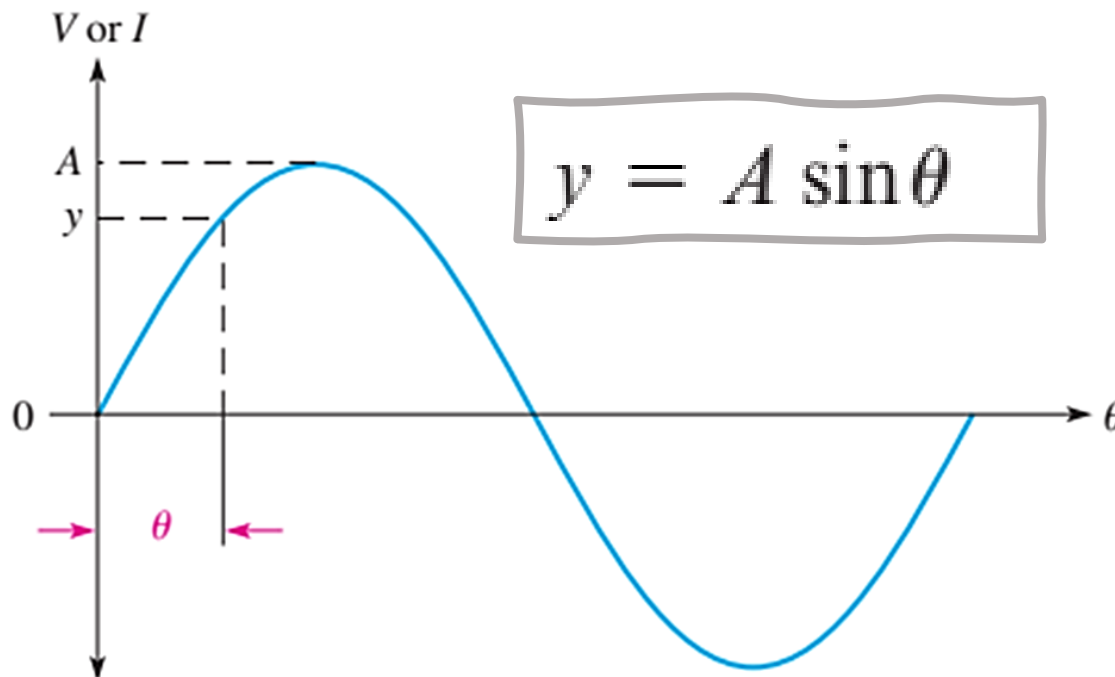
What are the phase angles between the two sine waves in parts (a) and (b) of Figure 11–21?



# Sine wave formula

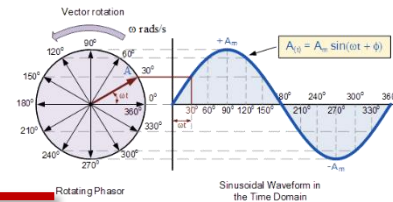


- This formula states that any point on the sine wave, represented by an instantaneous value ( $y$ ), is equal to the maximum value  $A$  times the sine ( $\sin$ ) of the angle  $\theta$  at that point.

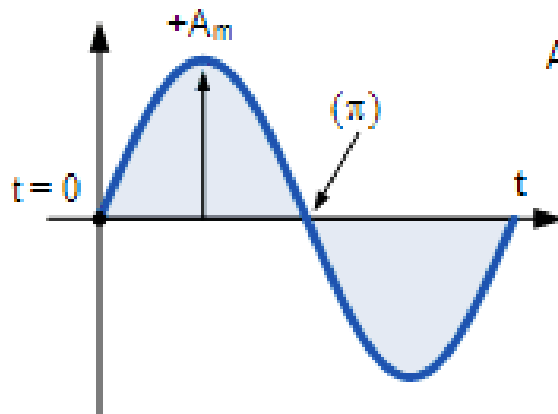




# Expressions for Phase-Shifted Sine Waves

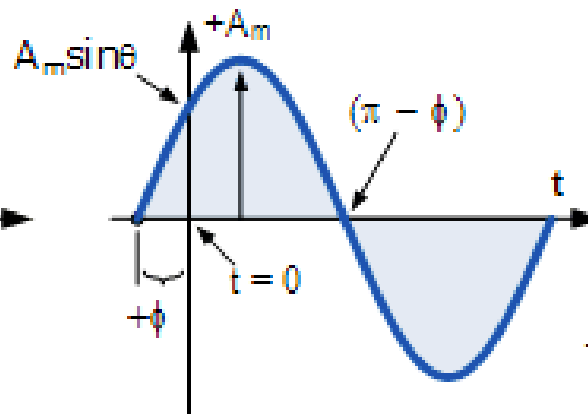


In-phase ( $\phi = 0^\circ$ )



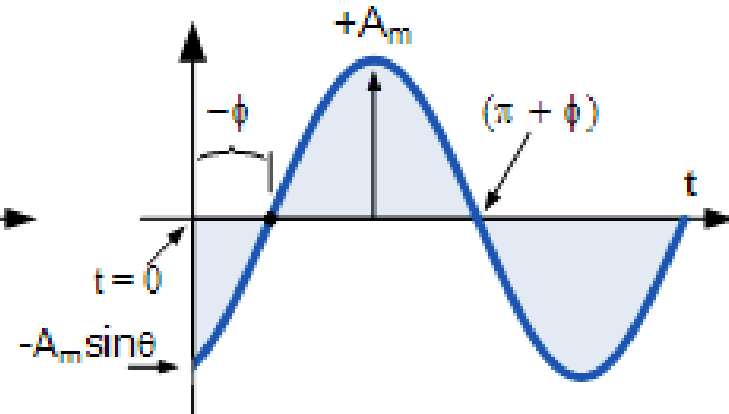
$$A_{(t)} = A_m \sin(\omega t)$$

Positive Phase ( $+\phi$ )

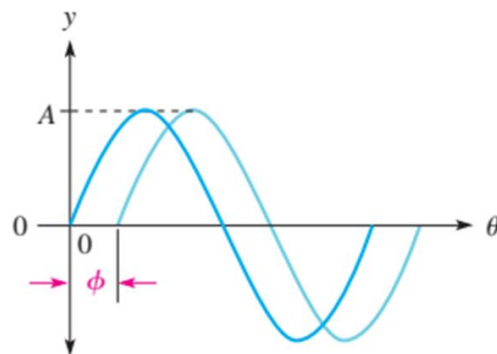


$$A_{(t)} = A_m \sin(\omega t + \phi)$$

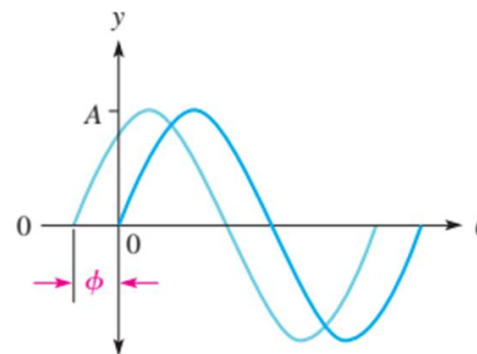
Negative Phase ( $-\phi$ )



$$A_{(t)} = A_m \sin(\omega t - \phi)$$

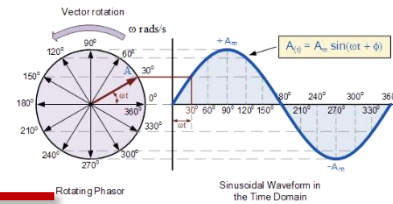


(a)  $y = A \sin(\theta - \phi)$

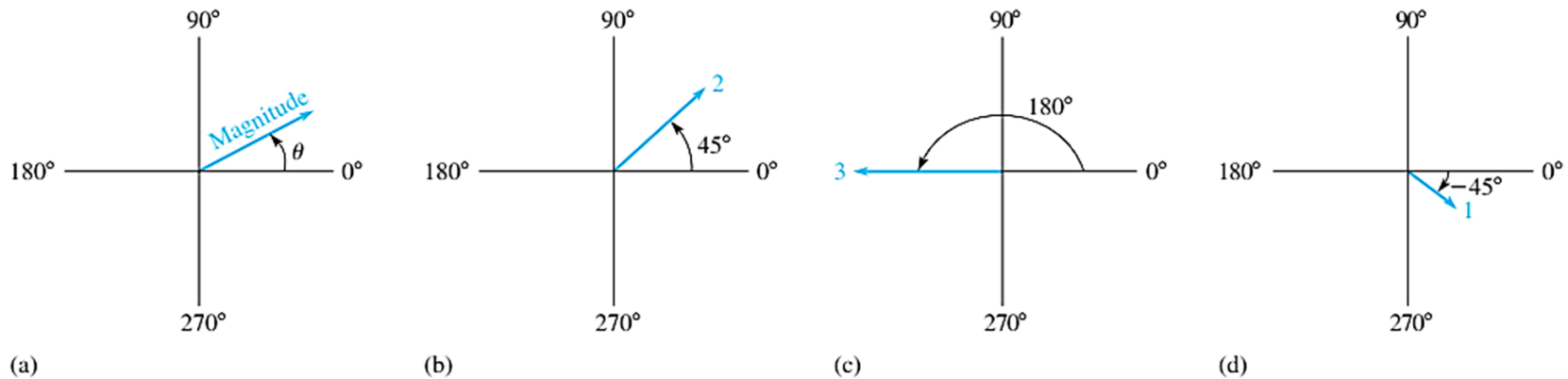


(b)  $y = A \sin(\theta + \phi)$

# Introduction to Phasors



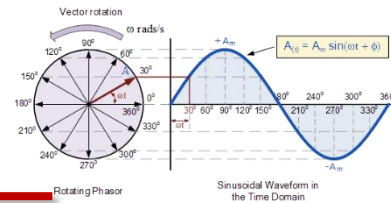
- ❑ In electronics, a phasor is a rotating vector. Examples of phasors are shown in Figure.
- ❑ The length of the phasor “arrow” represents the magnitude of a quantity.
- ❑ The angle,  $\theta$  (relative to  $0^\circ$ ), represents the angular position,



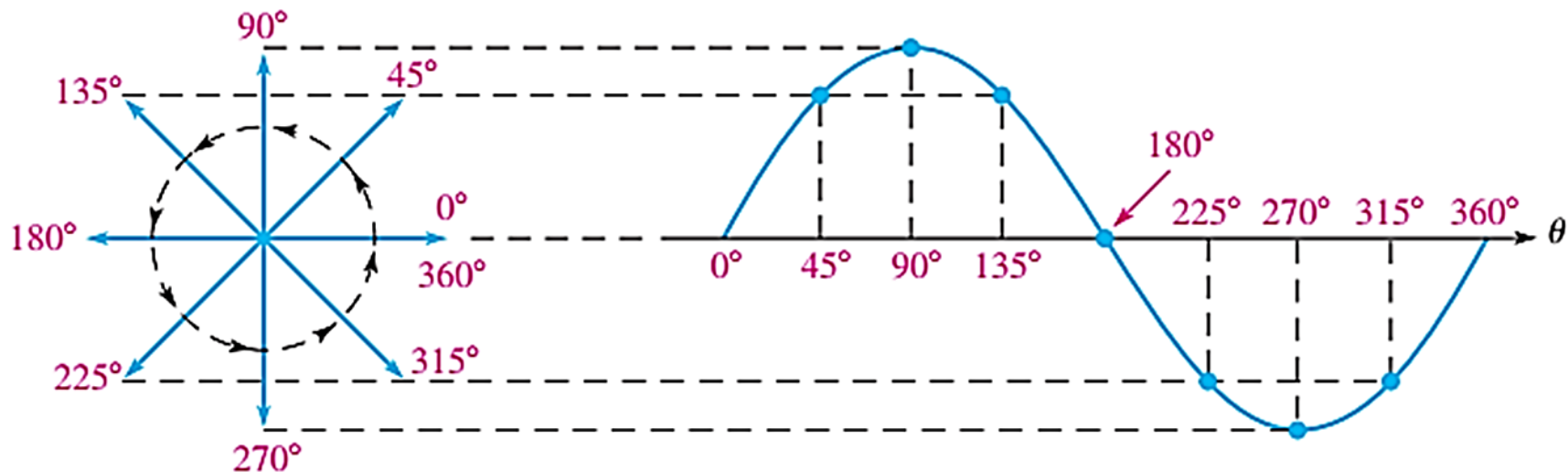
▲ FIGURE 11-28

Examples of phasors.

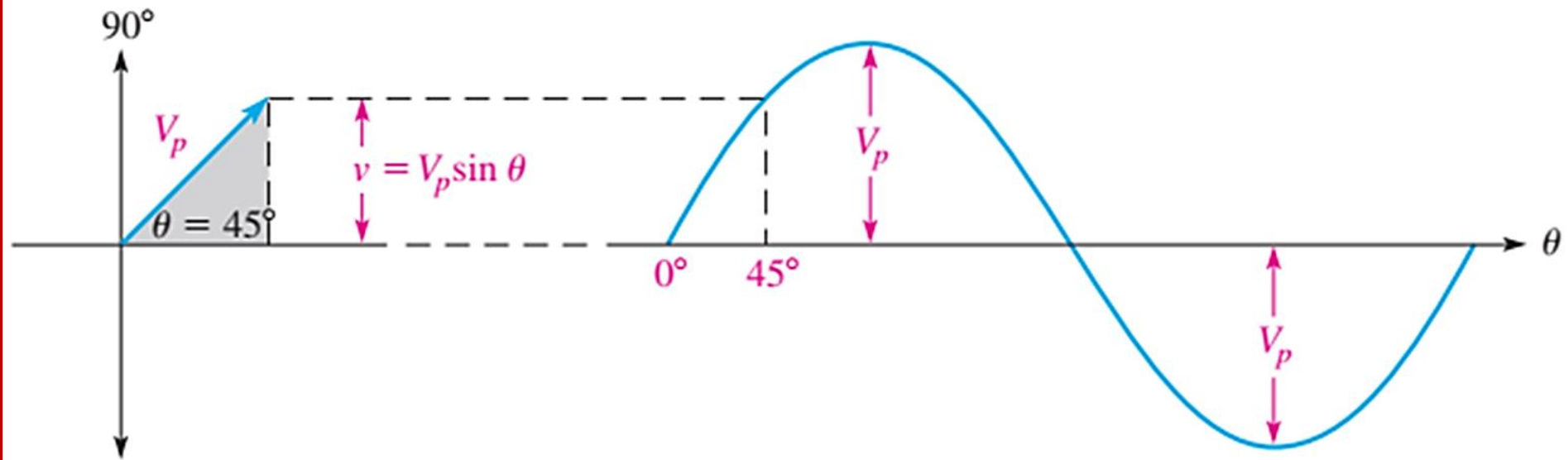
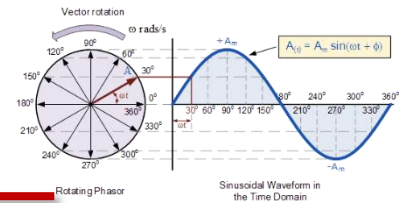
# Phasor Representation of a Sine Wave



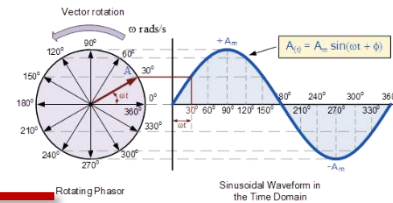
- The instantaneous value of the sine wave at any point is equal to the vertical distance from the tip of the phasor to the horizontal axis.



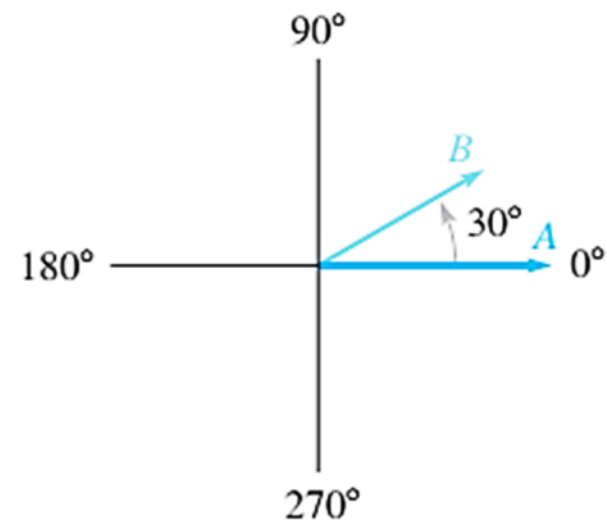
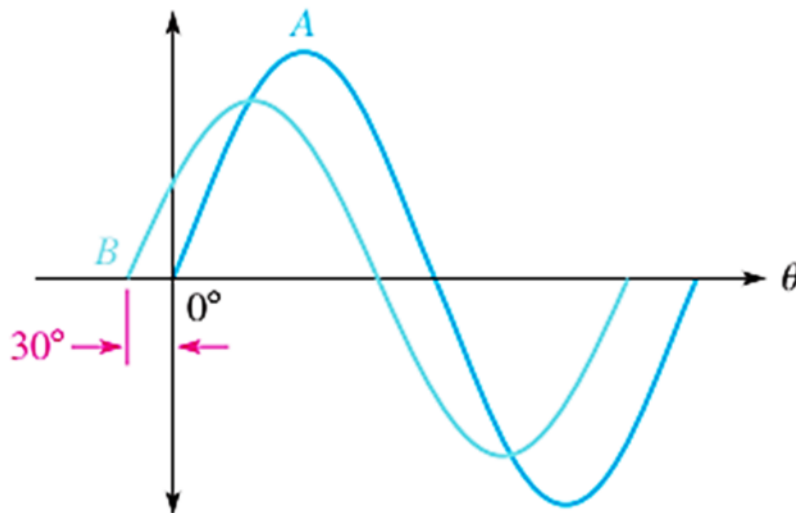
# Phasors and the Sine Wave Formula



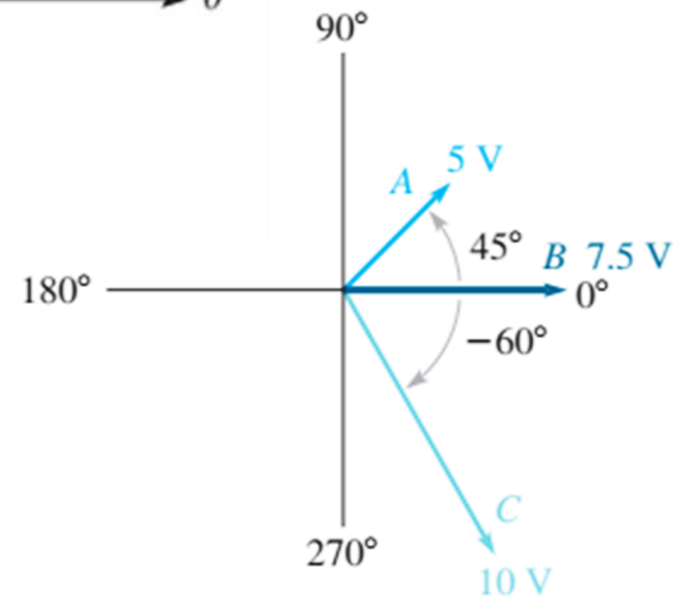
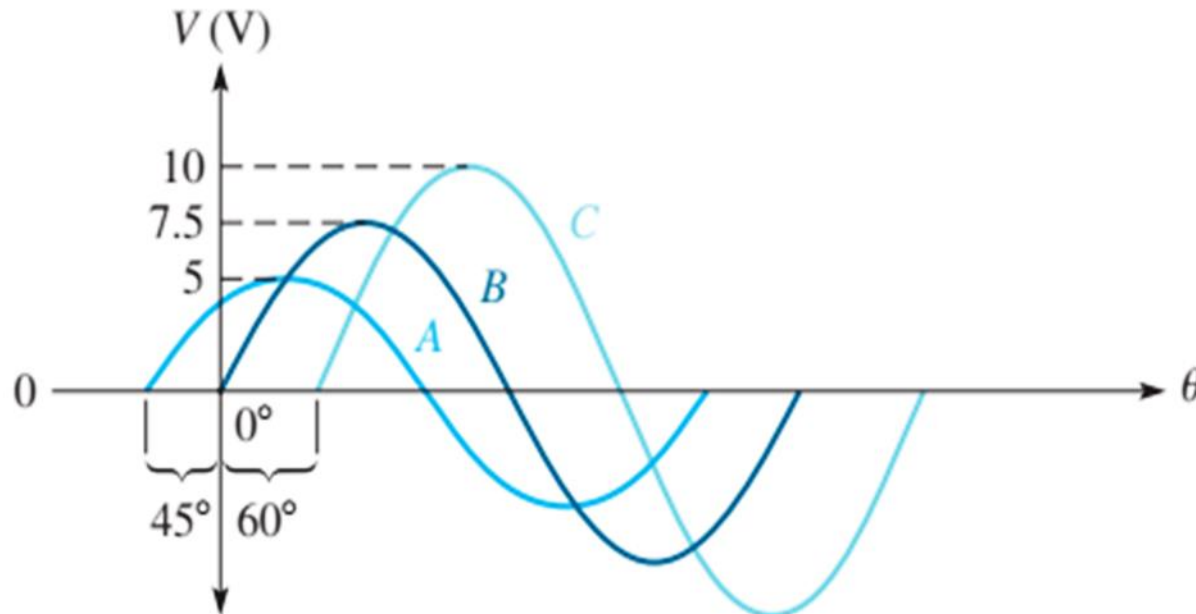
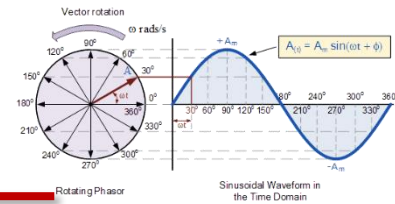
# Phasor Diagrams



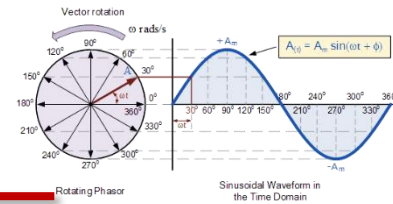
- A phasor diagram can be used to show the relative relationship of two or more sine waves of the same frequency.
- A phasor in a fixed position is used to represent a complete sine wave because once the phase angle between two or more sine waves of the same frequency or between the sine wave and a reference is established, the phase angle remains constant throughout the cycles.



# Example 7



# Angular Velocity of a Phasor



$$\omega = \frac{2\pi}{T}$$

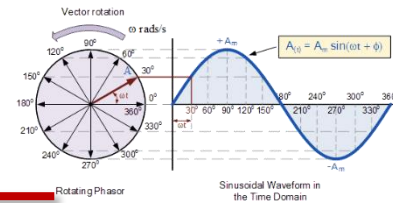
Since  $f = 1/T$ ,

$$\omega = 2\pi f$$

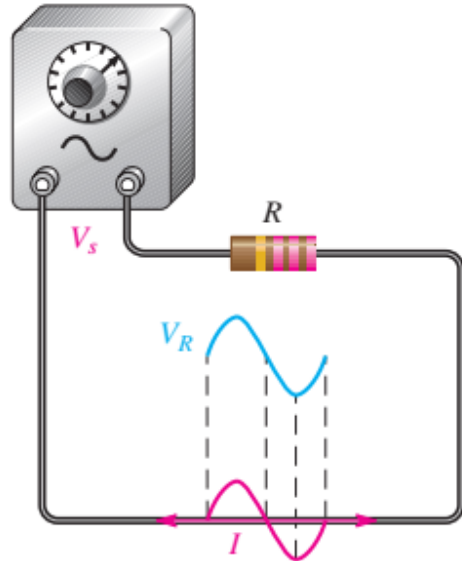
$$\theta = \omega t$$

$$v = V_p \sin 2\pi f t$$

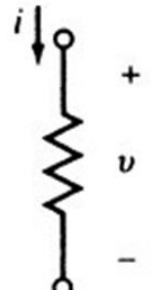
# Resistance in ac circuits



Sine wave generator

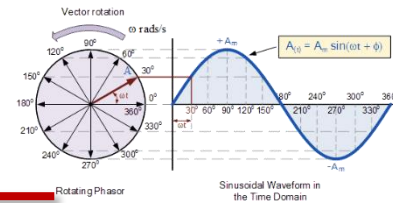


A sinusoidal voltage produces a sinusoidal current.

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms ( $\Omega$ )	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2 R$



# Power in a resistance



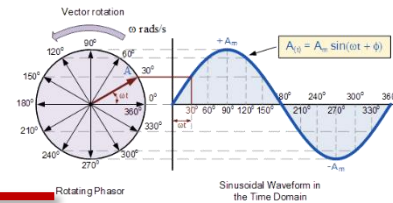
- ❑ Power in resistive ac circuits is determined the same as for dc circuits except that you must use rms values of current and voltage.
- ❑ Recall that the rms value of a sine wave voltage is equivalent to a dc voltage of the same value in terms of its heating effect.
- ❑ The general power formulas are restated for a resistive ac circuit as

$$P = V_{\text{rms}} I_{\text{rms}}$$

$$P = \frac{V_{\text{rms}}^2}{R}$$

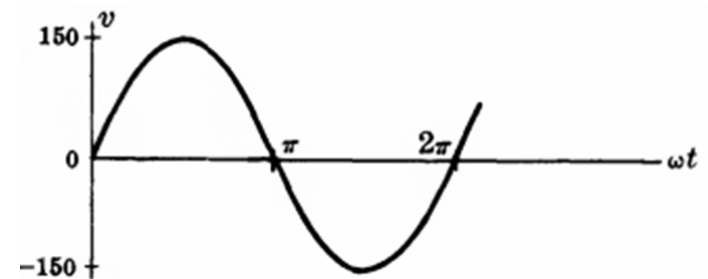
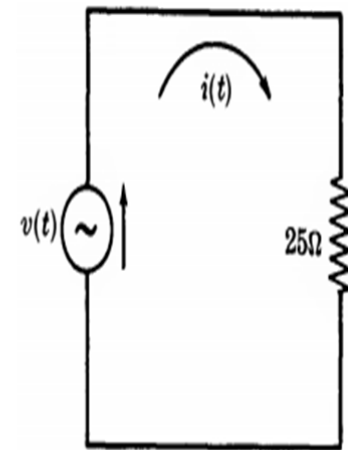
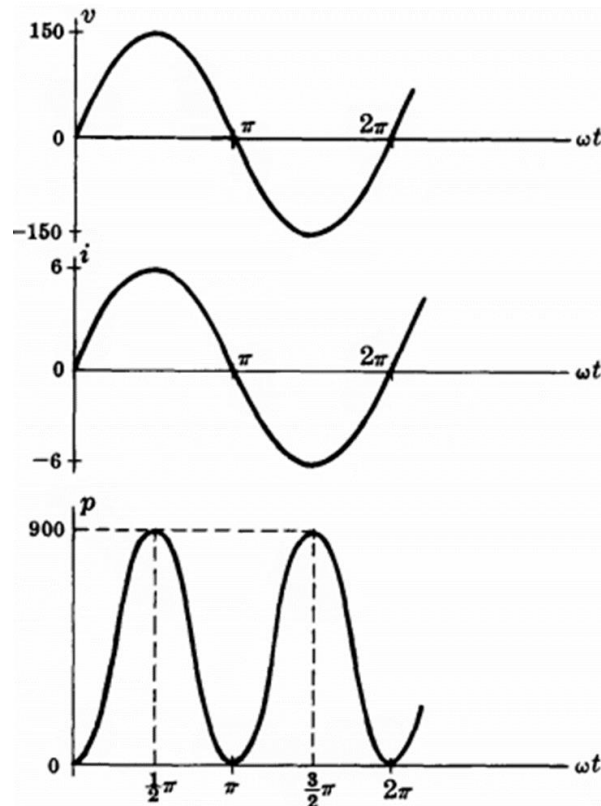
$$P = I_{\text{rms}}^2 R$$

# Example 7

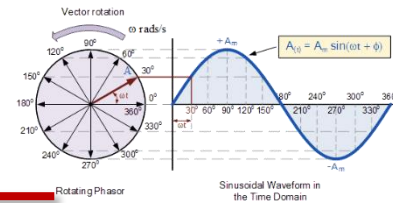


- In the circuit shown in Fig. the voltage function is  $v(t) = 150 \sin \omega t$ . Find the current  $i(t)$ , the instantaneous power  $p(t)$ , and the average power  $P$ .

**Solution:**



## Example 8



Determine the rms voltage across each resistor and the rms current in Figure 11–39. The source voltage is given as an rms value. Also determine the total power.

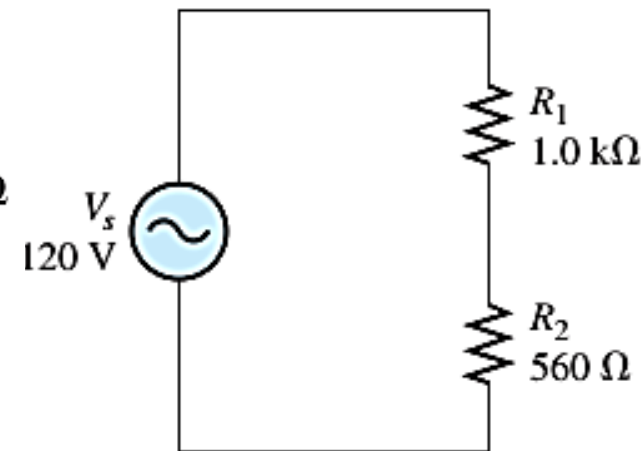
### Solution:

The total resistance of the circuit is

$$R_{tot} = R_1 + R_2 = 1.0 \text{ k}\Omega + 560 \text{ }\Omega = 1.56 \text{ k}\Omega$$

Use Ohm's law to find the rms current.

$$I_{\text{rms}} = \frac{V_{s(\text{rms})}}{R_{tot}} = \frac{120 \text{ V}}{1.56 \text{ k}\Omega} = \mathbf{76.9 \text{ mA}}$$



The rms voltage drop across each resistor is

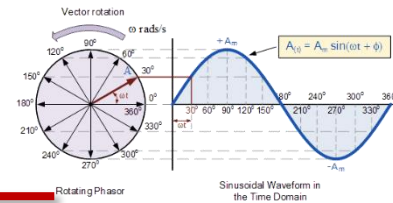
$$V_{1(\text{rms})} = I_{\text{rms}} R_1 = (76.9 \text{ mA})(1.0 \text{ k}\Omega) = \mathbf{76.9 \text{ V}}$$

$$V_{2(\text{rms})} = I_{\text{rms}} R_2 = (76.9 \text{ mA})(560 \text{ }\Omega) = \mathbf{43.1 \text{ V}}$$

The total power is

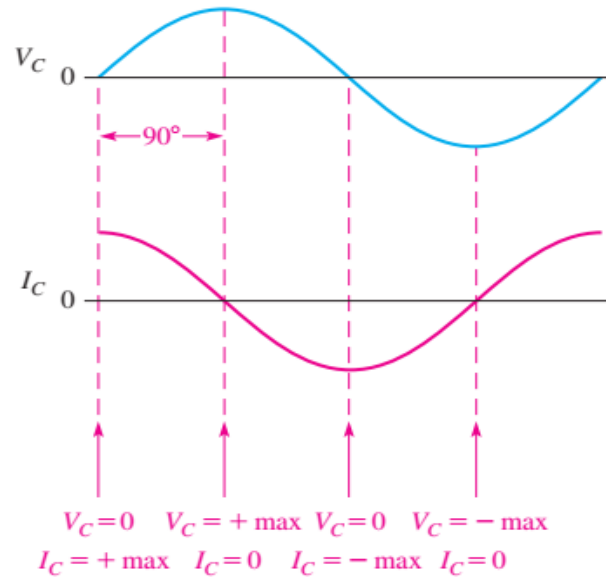
$$P_{tot} = I_{\text{rms}}^2 R_{tot} = (76.9 \text{ mA})^2 (1.56 \text{ k}\Omega) = \mathbf{9.23 \text{ W}}$$

# capacitors in ac circuits

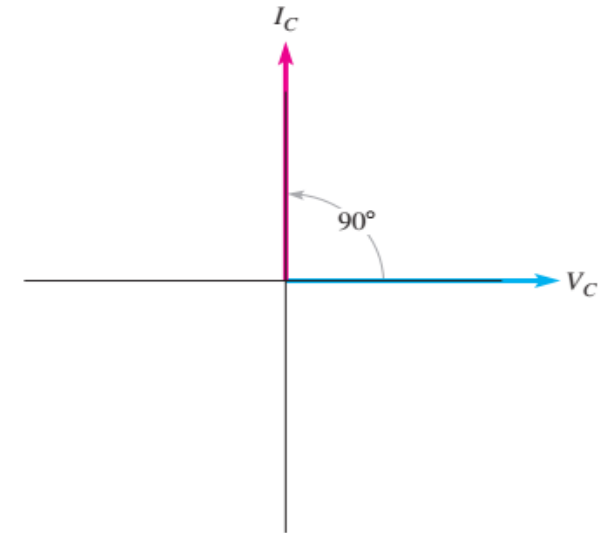


## Phase Relationship of Current and Voltage in a capacitor

$$X_C = \frac{1}{2\pi fC}$$

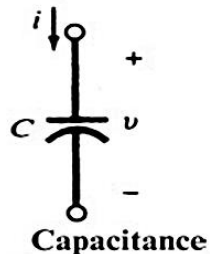


(a) Waveforms



(b) Phasor diagram

Phase relation of  $V_C$  and  $I_C$  in a capacitor. Current always leads the capacitor voltage by  $90^\circ$ .



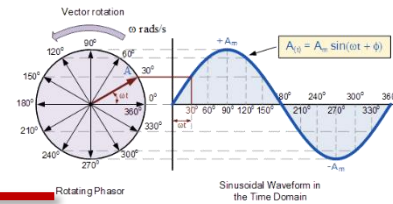
farads (F)

$$v = \frac{1}{C} \int i dt$$

$$i = C \frac{dv}{dt}$$

$$p = vi = Cv \frac{dv}{dt}$$

# Power in a Capacitor



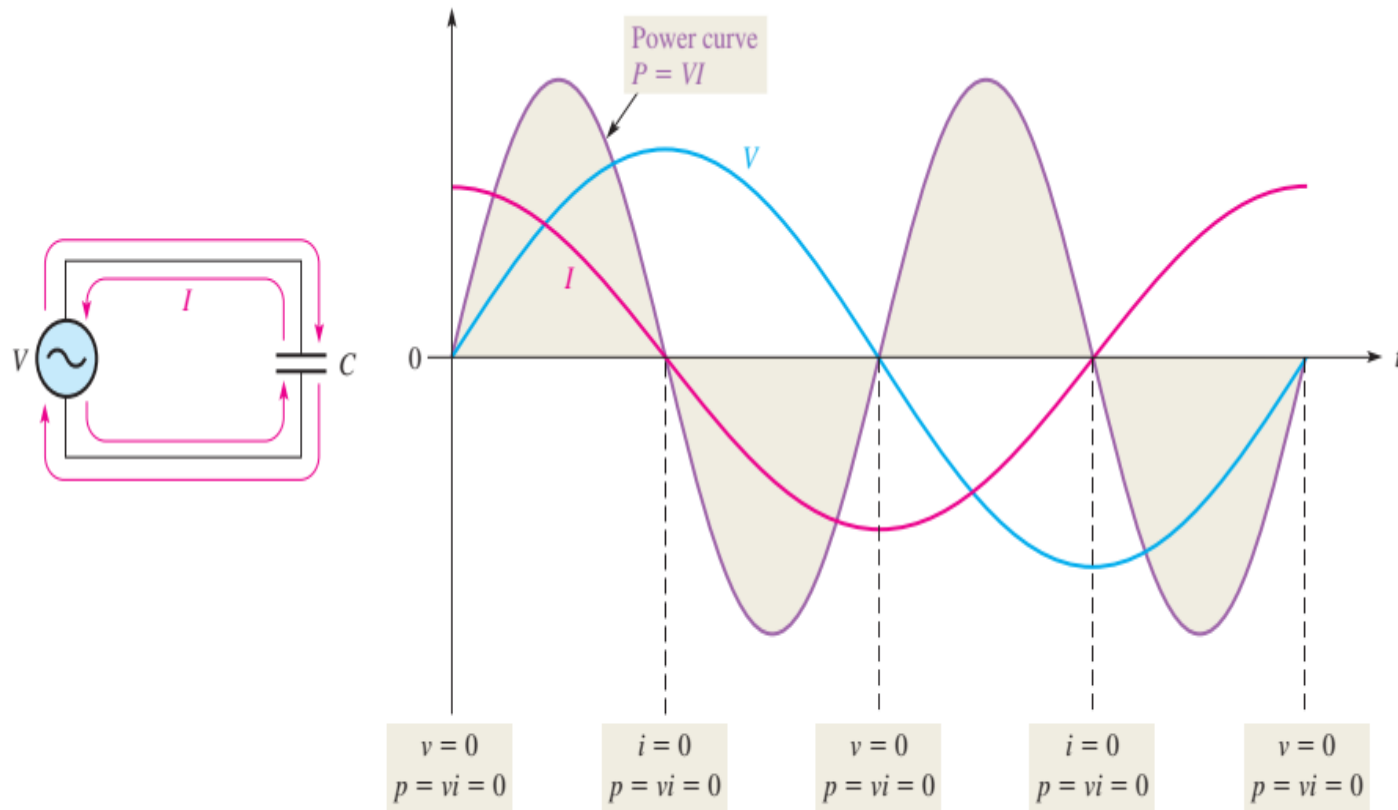
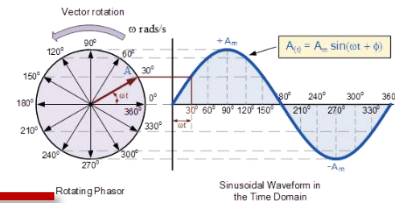
- ❑ **Reactive Power ( $P_r$ )** The rate at which a capacitor stores or returns energy is called its reactive power.
- ❑ The reactive power is a nonzero quantity, because at any instant in time, the capacitor is actually taking energy from the source or returning energy to it.
- ❑ Reactive power does not represent an energy loss.

$$P_r = V_{\text{rms}} I_{\text{rms}}$$

$$P_r = \frac{V_{\text{rms}}^2}{X_C}$$

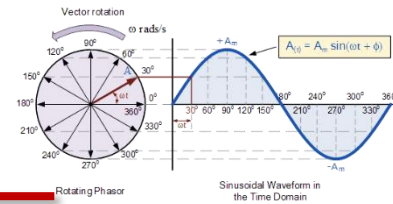
$$P_r = I_{\text{rms}}^2 X_C$$

# Power in a Capacitor



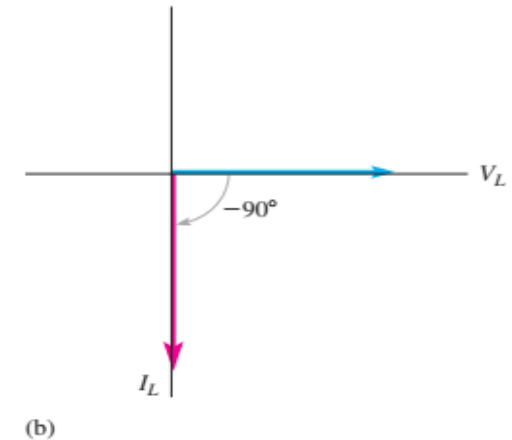
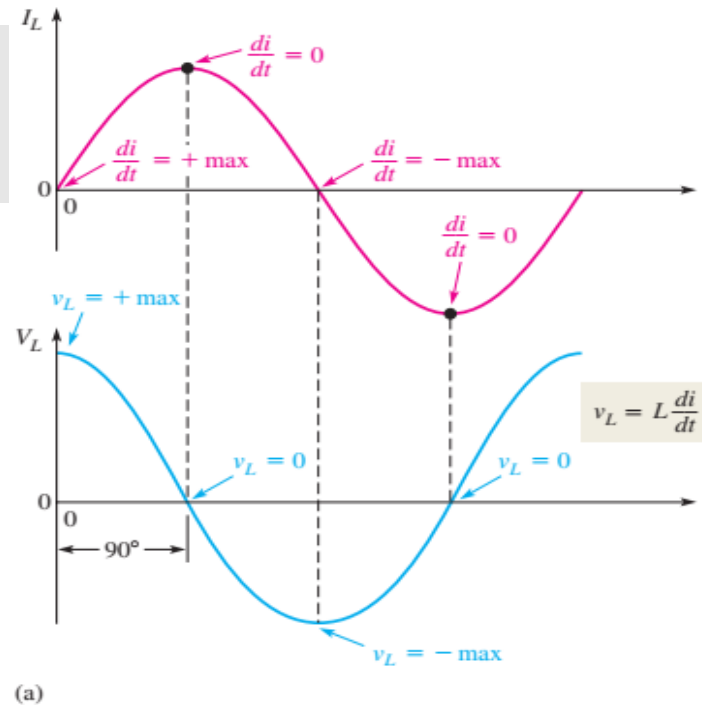
▲  
Power curve.

# Inductors in AC circuits



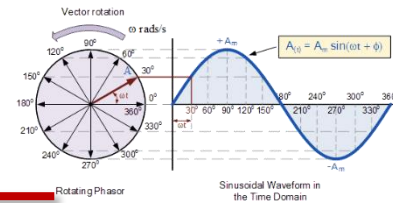
## Phase Relationship of Current and Voltage in an Inductor

$$X_L = 2\pi fL$$



Phase relation of  $V_L$  and  $I_L$  in an inductor. Current always lags the inductor voltage by  $90^\circ$ .

# Power in an Inductor



- ❑ **Reactive Power ( $P_r$ )** The rate at which an inductor stores or returns energy is called its reactive power, with the unit of VAR (volt-ampere reactive).
- ❑ The reactive power is a nonzero quantity because at any instant in time the inductor is actually taking energy from the source or returning energy to it.
- ❑ Reactive power does not represent an energy loss due to conversion to heat.

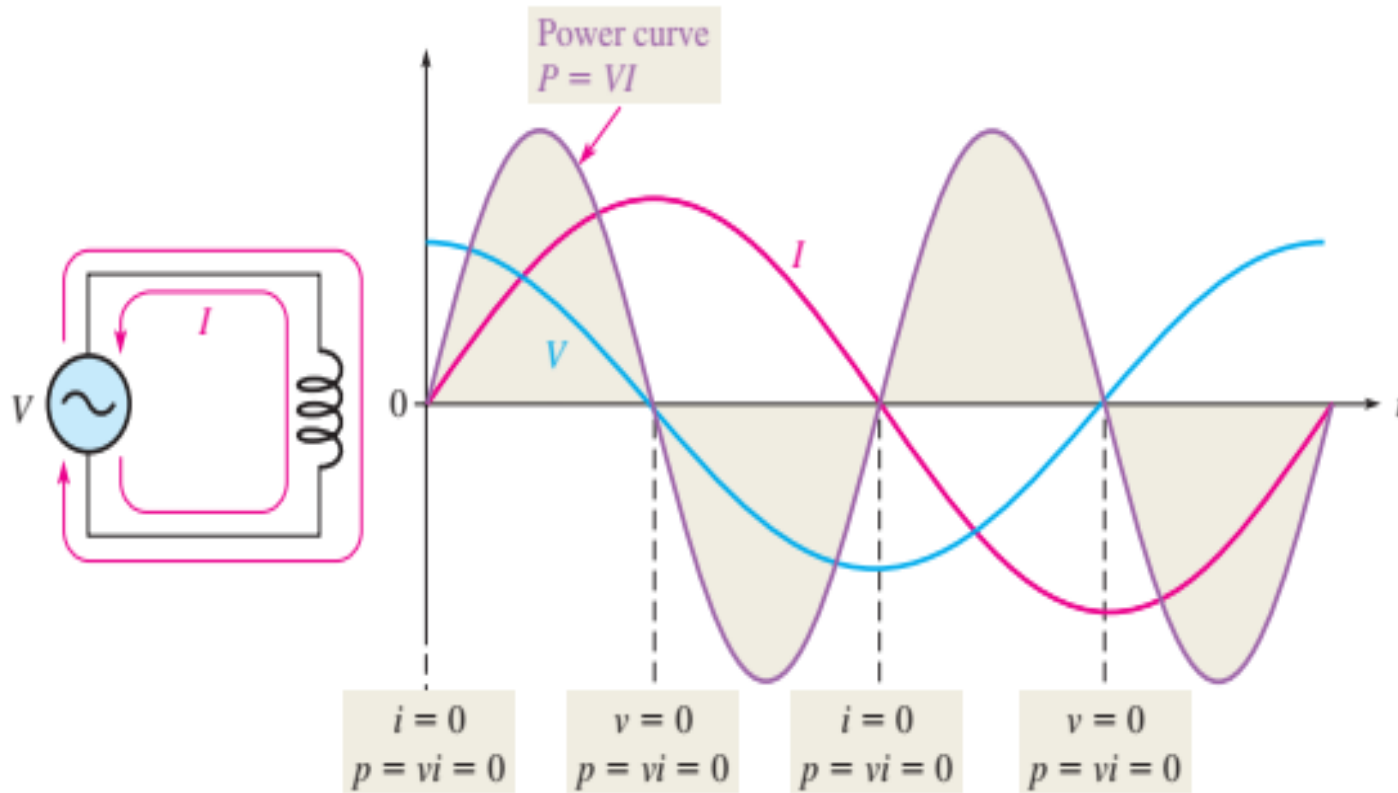
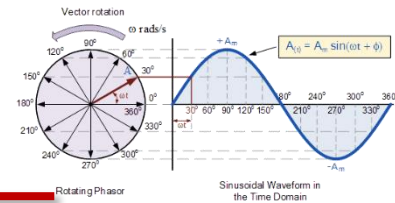
$$P_r = V_{\text{rms}} I_{\text{rms}}$$

$$P_r = \frac{V_{\text{rms}}^2}{X_L}$$

$$P_r = I_{\text{rms}}^2 X_L$$



# Power in an Inductor



Power curve.