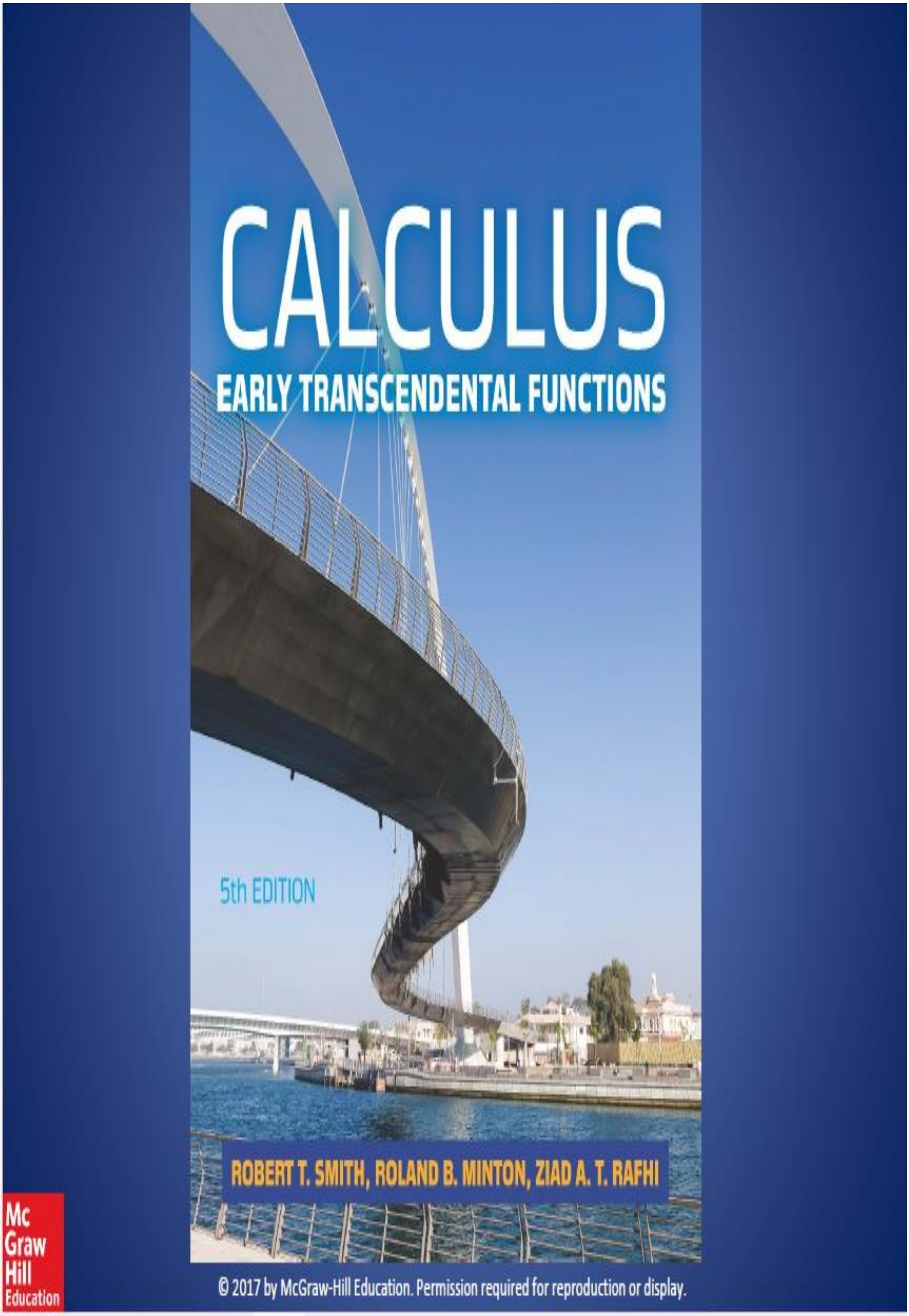
CALCULUS
EARLY TRANSCENDENTAL FUNCTIONS
5th EDITION
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Mc Graw Hill Education
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Definite Integral:

$$\int_a^b f(x)\,dx$$

Example

$$\int_0^1 x^3\,dx = \frac{x^4}{4}\,\Big|_0^1$$

$$= \left(\frac{1^4}{4}\right) - \left(\frac{0^4}{4}\right) = \frac{1}{4} - 0 = \frac{1}{4}$$

Example

$$\int_1^2 (3x^2 + 2x)\,dx = x^3 + x^2\,\Big|_1^2$$

$$= (2^3 + 2^2) - (1^3 + 1^2)$$

$$= 12 - 2 = 10$$

Example

$$\int_0^{\frac{\pi}{4}} \cos(2x)\,dx = \frac{\sin(2x)}{2} \Big|_0^{\frac{\pi}{4}}$$

$$= \left(\frac{\sin(\frac{\pi}{2})}{2}\right) - \left(\frac{\sin(0)}{2}\right) = \frac{1}{2} - 0 = \frac{1}{2}$$

Example

$$\int_0^1 xe^x\,dx$$

	$differentiation$	$Integration$
$+$	x	e^x
$-$	1	e^x
$+$	0	e^x

$$\rightarrow \int x\,e^x\,dx = x\,e^x - e^x \Big|_0^1$$

$$= (e - e) - (0 - 1) = 0 + 1 = 1$$

Example $\int_0^1 \sqrt{1-x^2}\,dx$

$$x = sin\theta$$

$$dx = cos\theta\ d\theta$$

$$\rightarrow \int_0^1 \sqrt{1-x^2}\,dx =$$

$$= \int \sqrt{1-sin^2\theta}\ \ cos\theta\ d\theta$$

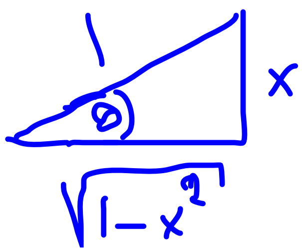
$$= \int cos^2\theta\ d\theta$$

$$= \frac{1}{2}\int (1+\cos(2\theta))\ d\theta$$

$$= \frac{1}{2}(\theta + \frac{\sin(2\theta)}{2})$$

$$= \frac{1}{2}(\theta + sin\theta\ cos\theta)$$

$$sin\theta = x = \frac{x}{1},\ \ \theta = sin^{-1}x$$



$$cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

$$\rightarrow \int_0^1 \sqrt{1-x^2}\,dx =$$

$$= \frac{1}{2}\left(sin^{-1}x + x\sqrt{1-x^2}\right) \Big|_0^1$$

$$= \frac{1}{2}(sin^{-1}1 + 0) - \frac{1}{2}(sin^{-1}0 + 0)$$

$$= \frac{\pi}{4}$$

Proberties of Definite Integrals

1. $$\int_a^a f(x)\,dx = 0$$

Example:

$$\int_\pi^\pi cos^{100}x\,dx = 0$$

Example:

If $\int_2^h \frac{1+cosx}{1+sinx}\,dx = 0$, then $h = 2$

2. $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Example:

$$\int_0^1 \sqrt{1-x^2}\,dx = -\int_1^0 \sqrt{1-x^2}\,dx$$

Example:

If $\int_0^2 h(x)\,dx = k\int_2^0 h(x)\,dx$, then $k = -1$

3. $$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

Example:

If $\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$, then $\int_0^1 \sqrt{1-t^2}\,dt = \frac{\pi}{4}$

4. $$\int_{-a}^{a} f(x)\,dx = 0 \quad if \quad f(x)\ is\ odd$$

Example:

$$\int_{-\pi}^{\pi} sinx\,dx = 0$$

5. $$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \quad if \quad f(x)\ is\ even$$

Example:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx\,dx = 2\int_0^{\frac{\pi}{2}} cosx\,dx$$

6. $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Example:

If $\int_0^3 f(x)dx = 6$, $\int_3^7 f(x)dx = 4$, then

$$\int_0^7 f(x)dx = \int_0^3 f(x)dx + \int_3^7 f(x)dx =$$

$$= 6 + 4 = 10$$

Example:

If $\int_0^3 f(x)dx = 6$, $\int_0^7 f(x)dx = 4$, then

$$\int_3^7 f(x)dx = \int_3^0 f(x)dx + \int_0^7 f(x)dx =$$

$$= -\int_0^3 f(x)dx + \int_0^7 f(x)dx$$

$$= -6 + 4 = -2$$

Example:

If $\int_2^7 f(x)dx = 6$, $\int_5^7 f(x)dx = 4$, then

$$\int_2^5 f(x)dx = \int_2^7 f(x)dx + \int_7^5 f(x)dx =$$

$$= \int_2^7 f(x)dx - \int_5^7 f(x)dx$$

$$= 6 - 4 = 2$$

7. $If\ m \leq f(x) \leq M\ , then$

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

Example:

If $3 \leq f(x) \leq 6$, then

$$3(8-2) \leq \int_2^8 f(x)dx \leq 6(8-2)$$

$$18 \leq \int_2^8 f(x)dx \leq 36$$