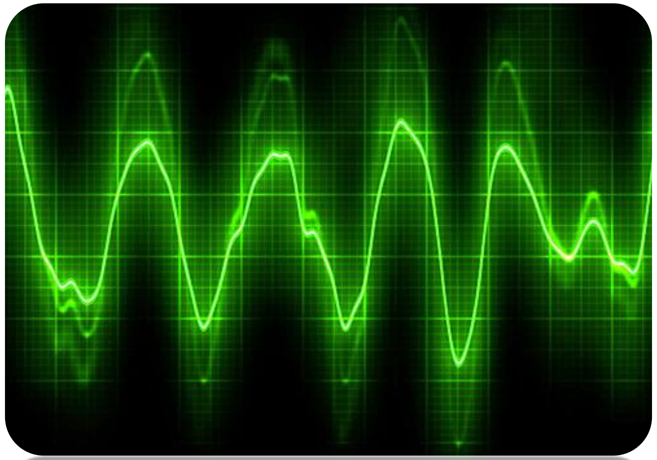
Alternating Current (AC) Circuits

CSE 113



OUTLINES

- Complex Numbers and Phasors
- Analysis of ac circuits.
  - AC Resistance and Impedance
  - AC Inductance and Inductive Reactance
  - AC Capacitance and Capacitive Reactance
- Impedance and Complex Impedance
  - Series AC Circuits
  - Parallel AC Circuit
- Admittance

Complex Numbers

- A complex number (C) is a number of the form: $\boxed{C = a + jb}$, which is known as the rectangular form.
  - where a and b are real and $j = \sqrt{-1}$
- a is the real part of C and jb is the imaginary part.
- Complex numbers are merely an invention designed to allow us to talk about the quantity j.
- Solving AC circuits is simplified through the use of phasor transforms, which we will now discuss at length…

Properties of j

$$j = \sqrt{-1}$$

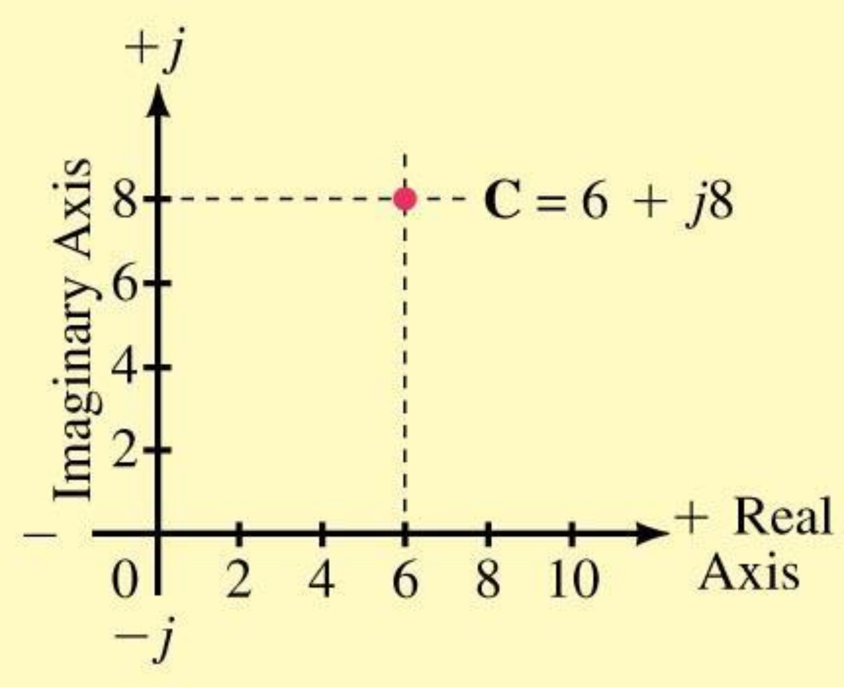
$$j^2 = (\sqrt{-1})(\sqrt{-1}) = -1$$

$$\frac{1}{j} = \frac{1}{j} \times \left(\frac{j}{j}\right) = \frac{j}{j^2} = -j$$

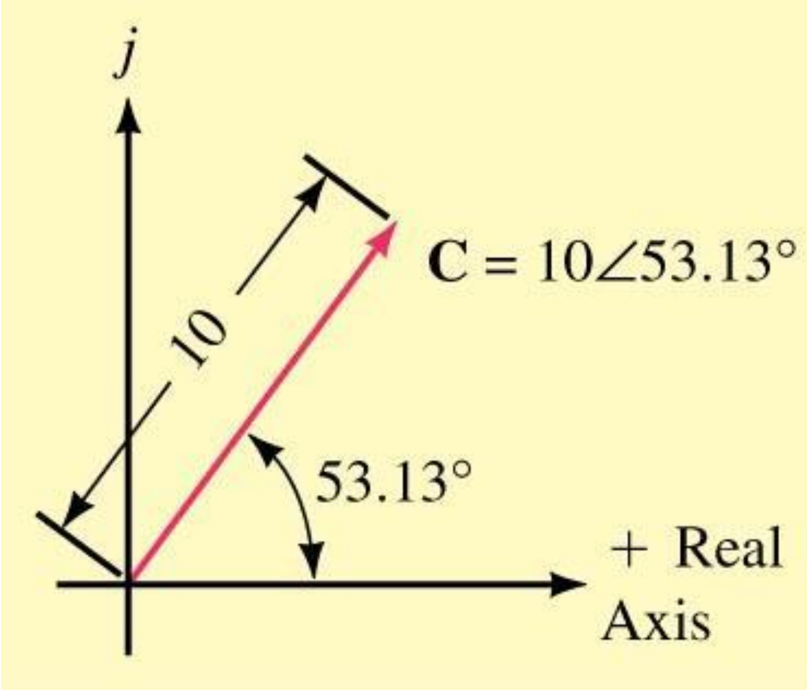
Geometric Representation

- In the rectangular form (C=a+jb), the x-axis is the real axis and the y-axis is the imaginary (j) axis.

- The polar form (C=Z ∠ ө), where Z is the distance (magnitude) from the origin and ө is the angle measured counterclockwise (CCW) from the positive, x (or real) axis (the y-axis is still the imaginary (j) axis).



C = 6 + j8
(rectangular form)



C = 10∠53.13º
(polar form)

Conversion Between Forms

- To convert between forms where:

$$\mathbf{C} = a + jb \qquad \text{(rectangular form)}$$

$$\mathbf{C} = C\angle\theta \qquad \text{(polar form)}$$

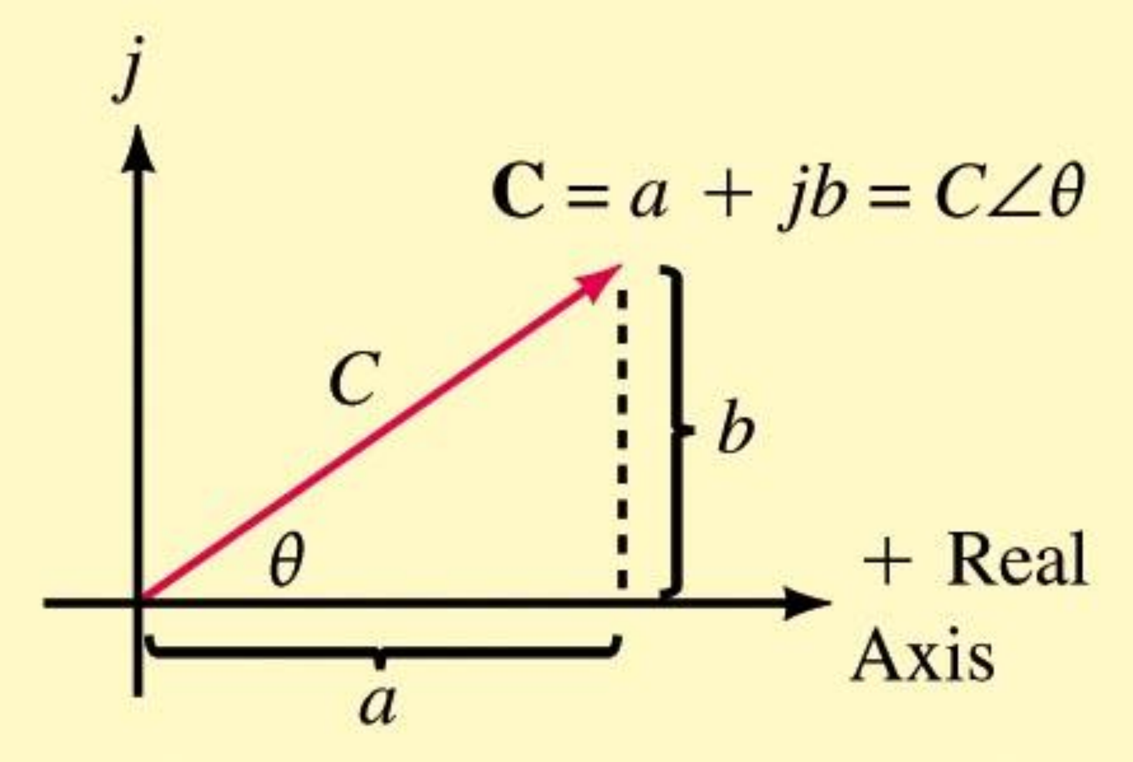
- apply the following relations:

$$a = C\cos\theta$$

$$b = C\sin\theta$$

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$



Example Problem 1

$\mathbf{C} = a + jb$ (rectangular form)

$\mathbf{C} = C\angle\theta$ (polar form)

$$a = C\cos\theta$$

$$b = C\sin\theta$$

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

- Convert (5∠60) to rectangular form.

$$a = C\cos\theta = 5\cos(60°) = 2.5$$

$$b = C\sin\theta = 5\sin(60°) = 4.3$$

$$\boxed{\mathbf{C} = 2.5 + j4.3 \quad \text{(rectangular form)}}$$

- Convert 6 + j 7 to polar form.

$$C = \sqrt{a^2 + b^2} = \sqrt{(6^2 + 7^2)} = 9.22$$

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{7}{6} = 49.4°$$

$$\boxed{\mathbf{C} = 9.22\angle 49.4° \quad \text{(polar form)}}$$

Addition and Subtraction of Complex Numbers

- **Easiest to perform in rectangular form.**

Example: Given $\mathbf{A} = 6 + j12$ and $\mathbf{B} = 7 + j2$

ADDITION:

- Add the real and imaginary parts separately.

$$(6 + j12) + (7 + j2) = (6 + 7) + j(12 + 2) = \boxed{13 + j14}$$

SUBTRACTION:

- Subtract the real and imaginary parts separately.

$$(6 + j12) - (7 + j2) = (6 - 7) + j(12 - 2) = \boxed{-1 + j10}$$

Multiplication and Division of Complex Numbers

- Multiplication and Division is easiest to perform in polar form:
- Multiplication: multiply magnitudes and add the angles:

$$(6\angle 70°)\cdot(2\angle 30°) = 6\cdot 2\angle(70°+30°) = 12\angle 100°$$

- Division: Divide the magnitudes and subtract the angles:

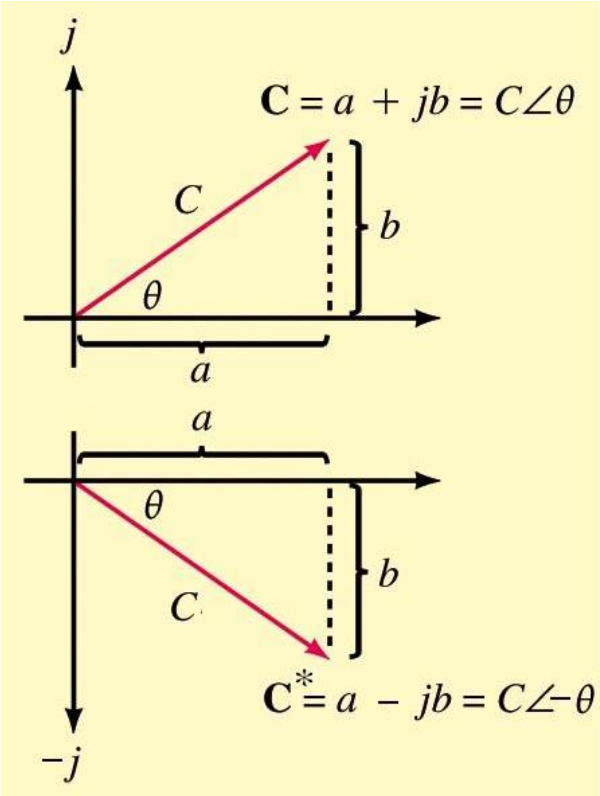
$$\frac{(6\angle 70°)}{(2\angle 30°)} = \frac{6}{2}\angle(70°-30°) = 3\angle 40°$$

$$C = C\angle\theta \Rightarrow \frac{1}{C\angle\theta} = \frac{1}{C}\angle(-\theta)$$

- Also: the **reciprocal** of
- The **conjugate of C is C*** and has the same real value but the OPPOSITE imaginary part:

$$C = a + jb = C\angle(\theta)$$

$$C^{*} = a - jb = C\angle(-\theta)$$



Example Problem 2

Given A =1 +*j*1 and B =2 – *j*3

- Determine A+B:

$$\boxed{(1+j1)\ +\ (2-j3) = (1+2)\ +j(1+(-3)) = 3-j2}$$

- and A-B:

$$\boxed{(1+j1)\ -\ (2-j3) = (1-2)\ +j(1-(-3)) = -1+j4}$$

Given A =1.41∠45° and B =3.61∠-56°

- Determine A/B:

$$\boxed{\frac{(1.41\angle 45°)}{(3.61\angle -56°)} = \frac{1.41}{3.61}\angle(45°-(-56°)) = 0.391\angle 101°}$$

- and A*B:

$$\boxed{(1.41\angle 45°)\cdot(3.61\angle -56°) = (1.41\cdot 3.61)\angle(45°+(-56°)) = 5.09\angle -11°}$$

Example Problem 3

Now, make sure you know how to use your calculator:

1. (3-*i*4) + (10∠44) then convert to rectangular:

$$a = C\cos\theta = 10\cos(44°) = 7.19$$

$$b = C\sin\theta = 10\sin(44°) = 6.95$$

$$(3 - j4) \; + \; (7.19 + j6.95) = (3 + 7.19) \; + j(-4 + 6.95) = 10.19 + j2.95$$

ANS: 10.6∠16.1 *(polar)*
ANS: 10.2 + *j*2.9 *(rectangular)*

2. (22000+*i*13)/(3∠-17) then convert to rectangular:

$$C = \sqrt{a^2 + b^2} = \sqrt{22000^2 + 13^2)} = 22k$$

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{13}{20000} = 0.339°$$

$$\frac{(22000\angle 0.034)}{(3\angle -17)} = \frac{22000}{3}\angle(0.034 - (-17)) = 7333\angle 17.034°$$

ANS: $7.3 * 10^3$∠17.0 *(polar)*
ANS: $7.01 * 10^3$ + *j*2.15 *(rectangular)*

2. Convert 95-12j to polar:

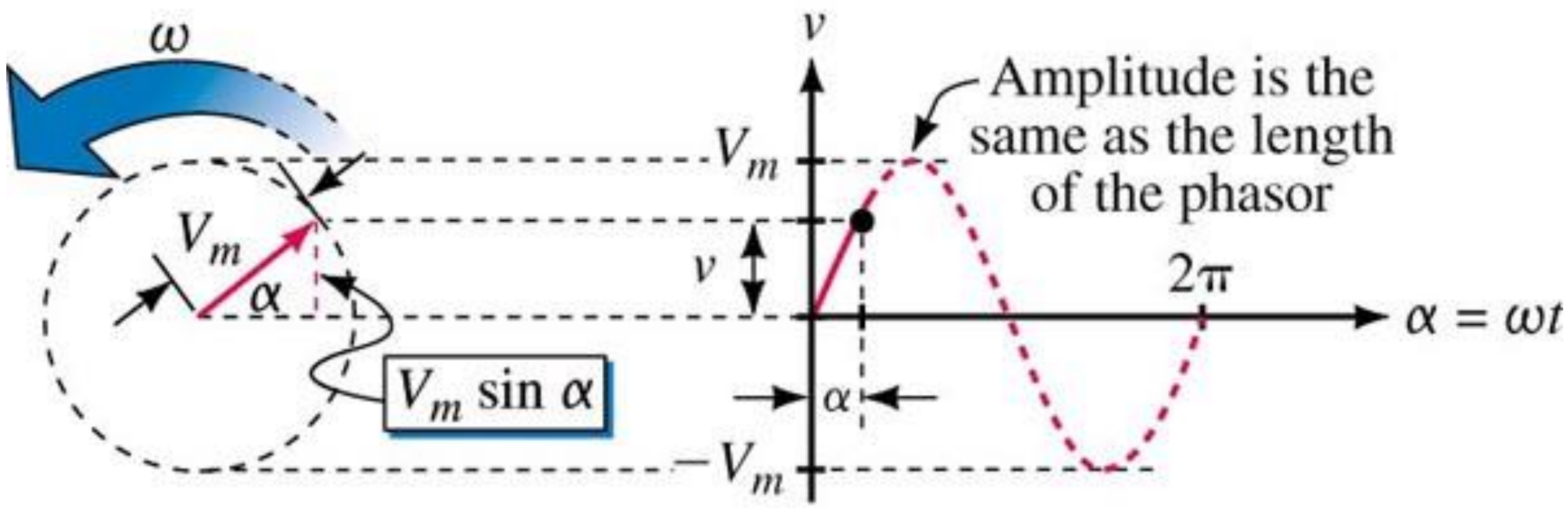
ANS: 95.8∠-7.2 *(polar)*

Phasor Transform

- **To solve problems that involve sinusoids (such as AC voltages and currents) we use the phasor transform:**
  - **We transform sinusoids into complex numbers in polar form…**
  - **solve the problem using complex arithmetic (as described previously)…**
  - **Then transform the result back to a sinusoid.**

Phasors

- A **phasor** is a rotating **vector** whose projection on the vertical axis can be used to represent a sinusoid.
- The length of the phasor is the amplitude of the sinusoid (V_m)
- The angular velocity of the phasor is ω .



(a) Phasor

(b) Resulting sine wave

Using Phasors to Represent AC Voltage and Current

- Looking at the sinusoid equation, determine V_m and the phase offset θ:

$$v(t) = \mathrm{V}_m \sin(\omega t + 30^\circ) \ \ \mathrm{V}$$

- Using V_m, determine V_{RMS} using the formula:

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

- The voltage phasor is then:

$$V_{RMS} \angle \theta$$

- The same holds true for current:

$$i(t) = \mathrm{I}_m \sin(\omega t + 30^\circ) \qquad I_{RMS} = \frac{I_m}{\sqrt{2}} \qquad I_{RMS} \angle \theta$$

Example Problem 4

1. **Express 100 sin (ωt) as a voltage phasor.**

$$V_{RMS} = \frac{V_m}{\sqrt{2}} \rightarrow V_{RMS} = \frac{100V}{\sqrt{2}} = 70.7V \rightarrow \boxed{V_{RMS}\angle\theta = 70.7\angle 0°V}$$

2. **Express 50sin(ωt+45^o) as a voltage phasor.**

$$V_{RMS} = \frac{V_m}{\sqrt{2}} \rightarrow V_{RMS} = \frac{50V}{\sqrt{2}} = 35.35V \rightarrow \boxed{V_{RMS}\angle\theta = 35.35\angle 45°V}$$

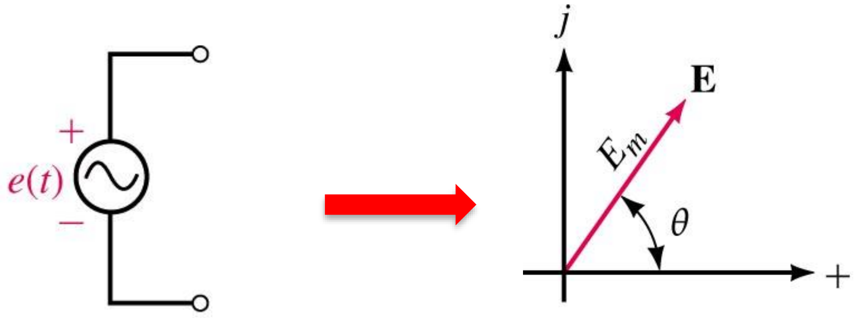
3. **Express 50cos(ωt+45^o) as a voltage phasor.**
   - **Remember, we are representing the sinusoid as a sin function, therefore we need to use one of the trig conversions:**

$\cos(\omega t+45^o) = \sin(\omega t+45^o+90^o)$

$$V_{RMS} = \frac{V_m}{\sqrt{2}} \rightarrow V_{RMS} = \frac{50V}{\sqrt{2}} = 35.35V \rightarrow \boxed{\begin{array}{l} V_{RMS}\angle\theta = 35.35\angle 45° + 90°V \\ V_{RMS}\angle\theta = 35.35\angle 135°V \end{array}}$$

Representing AC Signals with Complex Numbers

- **Phasor representations can be viewed as a complex number in polar form:**



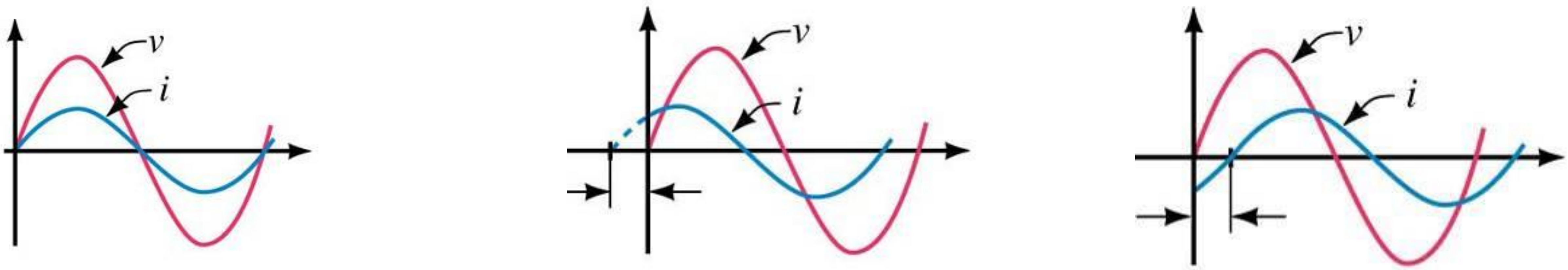
(a) $e(t) = E_m \sin(\omega t + \theta)$ (b) $\mathbf{E} = E_m \angle \theta$

$$\boxed{E = V_{RMS} \angle \theta \rightarrow E_m = V_{RMS} * \sqrt{2} \rightarrow e(t) = E_m \sin(2\pi ft + \theta)}$$

- ***NOTE:* *Before writing the sinusoid equation after finding the phasor, you need to convert the phasor magnitude back to the sinusoid representation (i.e. multiplying the phasor magnitude by*** $\sqrt{2}$.)

Phase Difference

- Phase difference is angular displacement between waveforms of same frequency.
- If angular displacement is 0° then the waveforms are in phase.
- If angular displacement is not 0º, they are out of phase by amount of displacement.



(a) In phase

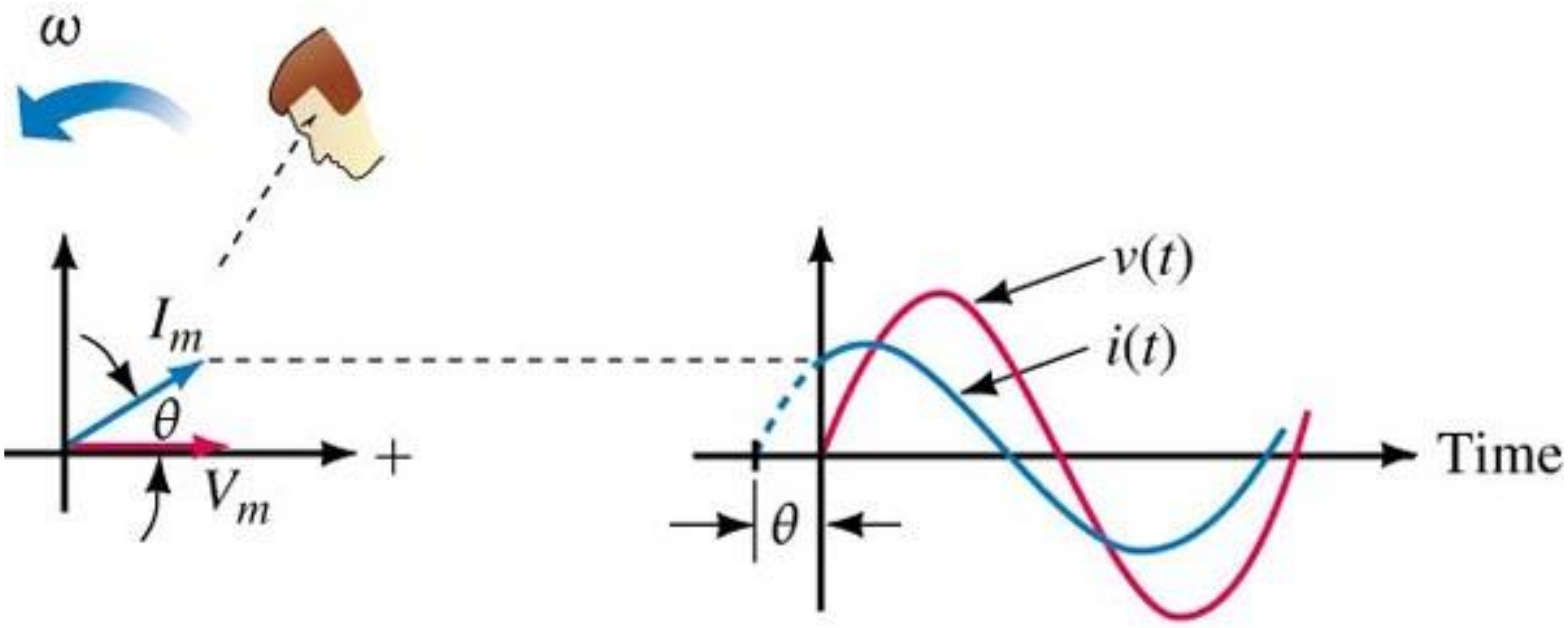
(b) Current leads

(c) Current lags

Example:
If $v_1 = 5 \sin(100t)$ and $v_2 = 3 \sin(100t - 30°)$, then v_1 leads v_2 by 30°

Phase Difference with Phasors

- The waveform generated by the leading phasor leads the waveform generated by the lagging phasor.



(a) I_m leads V_m

(b) Therefore, $i(t)$ leads $v(t)$

R, L and C circuits with Sinusoidal Excitation

- R, L, C have very different voltage-current relationships. Recall:

$$v_R = i_R R \qquad \text{(Ohm's law)}$$

$$i_C = C\frac{dv_C}{dt} \qquad \text{(Capacitor Current relationship)}$$

$$v_L = L\frac{di_L}{dt} \qquad \text{(Inductor Voltage relationship)}$$

- Sinusoidal (AC) sources are a special case!

The Impedance Concept

Impedance as a complex number

Resistance and Sinusoidal AC

Inductance and Sinusoidal AC

Capacitance and Sinusoidal AC

VLI & ICV

Series AC Circuits

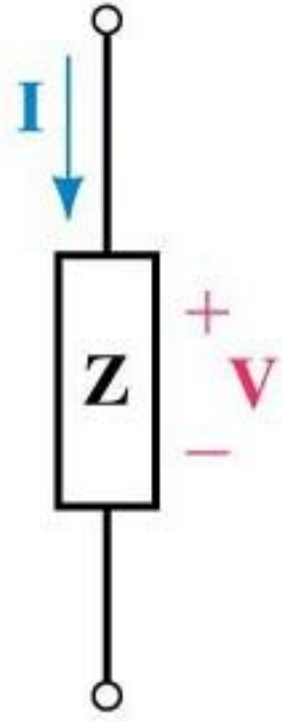
Parallel AC Circuits

Admittance

The Impedance Concept

- **Impedance (Z)** is the opposition (i.e. resistance) that a circuit element presents to current in the phasor domain. It is defined as:

$$\vec{\mathbf{Z}} = \frac{\vec{\mathbf{V}}}{\vec{\mathbf{I}}} = \frac{V}{I}\angle\theta = Z\angle\theta$$



- **Ohm's law for AC circuits:**

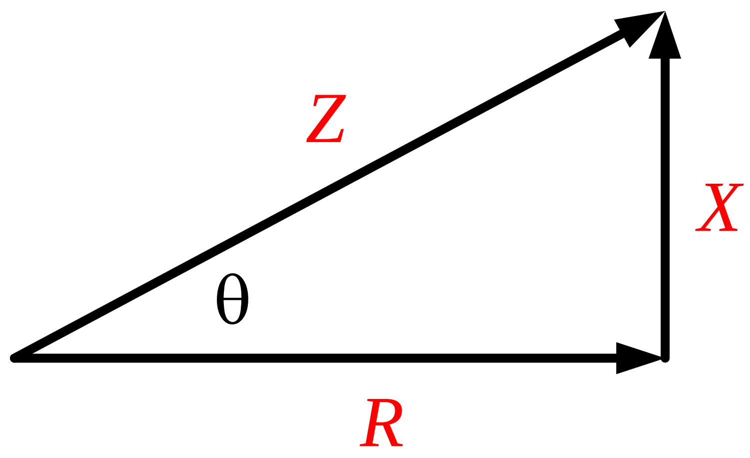
$$\boxed{V = IZ}$$

Impedance

- Impedance is a complex quantity that can be made up of Resistance (R) (real part) and Reactance (X) (imaginary part).

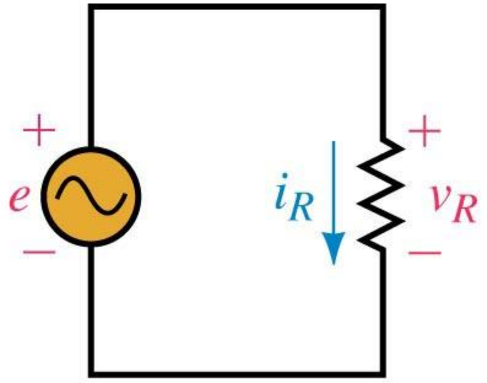
$$\vec{\mathbf{Z}} = R + jX \qquad (\Omega)$$

- Unit of impedance is Ohms (Ω).

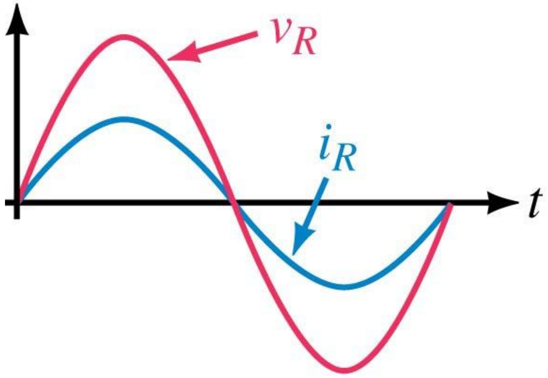


Resistance and Sinusoidal AC

- For a **purely resistive circuit**, current and voltage **are in phase**.



(a) Source voltage is a sine wave. Therefore, v_R is a sine wave

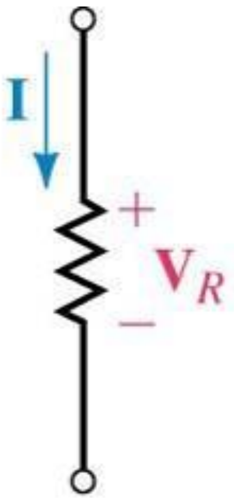


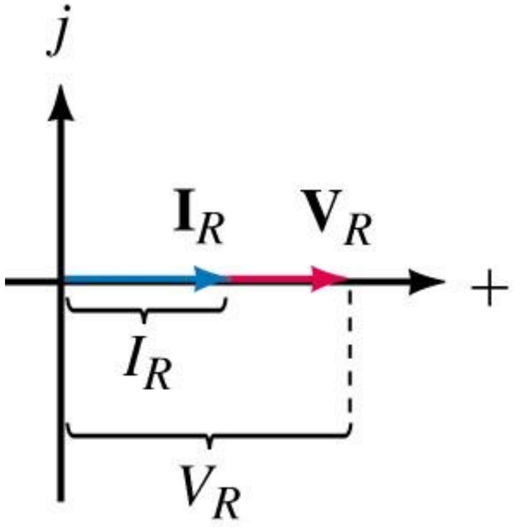
(b) $i_R = v_R/R$. Therefore i_R is a sine wave also

- **For resistors, voltage and current are in phase:**

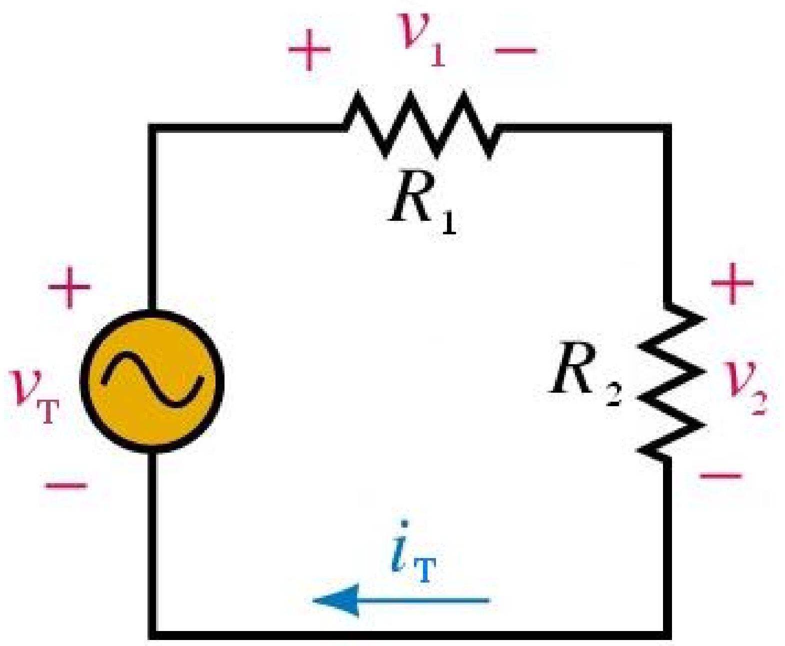
$$\mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{V_R\angle\theta}{I\angle\theta} = \frac{V_R}{I}\angle 0^\circ = R\angle 0^\circ = R$$

$$\boxed{\mathbf{Z}_R = R\angle 0^\circ}$$





Example Problem 6



Two resistors R_1=10 kΩ and R_2=12.5 kΩ are in series.

$i(t) = 14.7 \sin(\omega t + 39^\circ)$ mA

a) Compute $\mathbf{V}_{R1}$ and $\mathbf{V}_{R2}$
b) Compute $\mathbf{V}_T = \mathbf{V}_{R1} + \mathbf{V}_{R2}$
c) Calculate $\mathbf{Z}_T$
d) Compare $\mathbf{V}_T$ to the results of $\mathbf{V}_T = \mathbf{I}\,\mathbf{Z}_T$

$$I_T = \frac{I_m}{\sqrt{2}} = \frac{14.7mA}{\sqrt{2}} = 10.39\angle 39^\circ mA$$

$$V_{R1} = I_T * Z_{R1} = 10.39\angle 39^\circ mA * 10k\Omega\angle 0^\circ = \boxed{104\angle 39^\circ V}$$

$$V_{R2} = I_T * Z_{R2} = 10.39\angle 39^\circ mA * 12.5k\Omega\angle 0^\circ = \boxed{129.9\angle 39^\circ V}$$

$$V_T = V_{R1} + V_{R2} = 104\angle 39^\circ V + 129.9\angle 39^\circ V = \boxed{234\angle 39^\circ V}$$

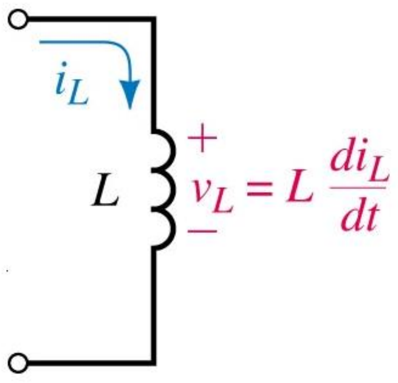
Note that these are in phase so we simply sum the magnitudes. May not always be this easy…

$$Z_T = Z_{R1} + Z_{R2} = 10k\angle 0^\circ\Omega + 12.5k\angle 0^\circ\Omega = \boxed{22.5k\angle 0^\circ\Omega}$$

$$V_T = I_T * Z_T = 10.39\angle 39^\circ mA * 22.5k\Omega\angle 0^\circ = \boxed{234\angle 39^\circ V}$$ => Same value calculated previously.

Inductance and Sinusoidal AC

- **Voltage-Current relationship for an inductor:**



$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}\left(I_m \sin\omega t\right)$$

$$= \omega L I_m \cos\omega t = \omega L I_m \sin\left(\omega t + 90^\circ\right)$$

$$Z_L = \frac{v_L}{i_L} = \frac{\omega L I_m \sin\left(\omega t + 90^\circ\right)}{I_m \sin\omega t}$$

$$= \frac{\dfrac{\omega L I_m}{\sqrt{2}}\angle 90^\circ}{\dfrac{I_m}{\sqrt{2}}\angle 0^\circ} = \omega L\angle 90^\circ \quad (\Omega)$$

- It should be noted that *for a purely inductive circuit voltage leads current by 90º.*

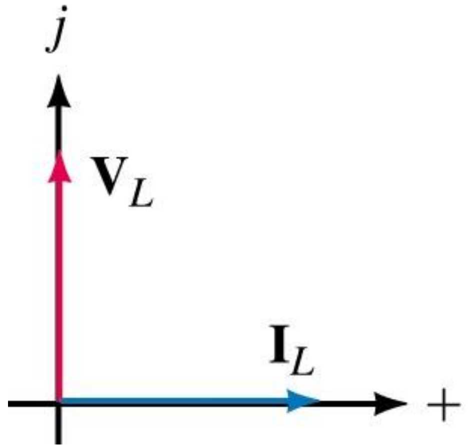
Inductive Impedance

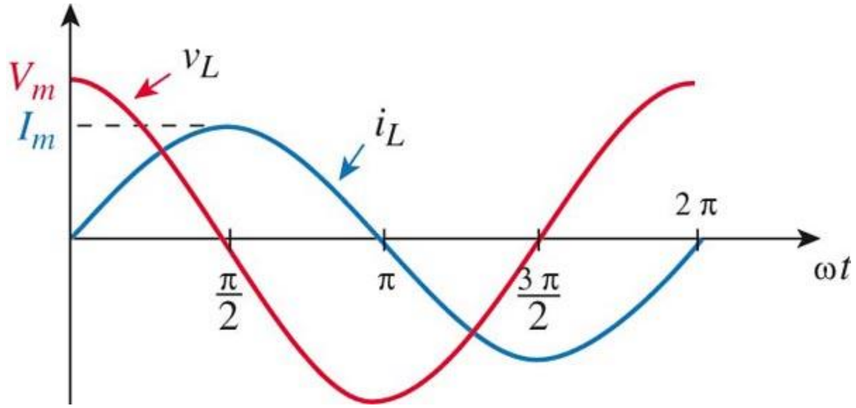
- Impedance can be written as a complex number (in rectangular or polar form):

$$Z_L = \omega L \angle 90^\circ = j\omega L \quad (\Omega)$$

- Since an ***ideal inductor*** has no real resistive component, this means the ***reactance*** (*X*) of an inductor is the pure imaginary part:

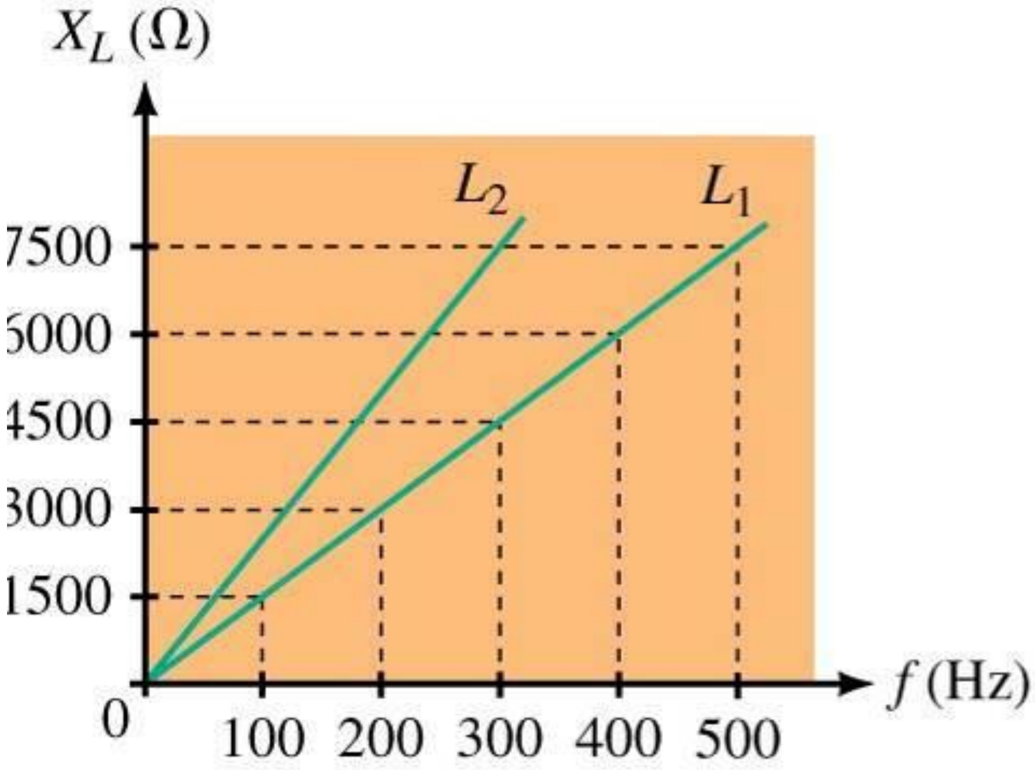
$$\boxed{X_L = \omega L}$$





Variation with Frequency

- Since $X_L = \omega L = 2\pi fL$, inductive reactance is ***directly proportional to frequency***.



- Extreme case $f = 0$ Hz (DC): inductor looks like a short circuit!

Impedance and AC Circuits

- Solution technique

1. Transform time domain currents and voltages into phasors.
2. Calculate impedances for circuit elements.
3. Perform all calculations using complex math.
4. Transform resulting phasors back to time domain (if reqd).

Important Notes

- Peak values are useful for **time domain** representations of signals.
- RMS values are the standard when dealing with **phasor domain** representations.
- If you need to represent something in the time domain, you will need to convert RMS->Peak voltage to obtain $\boldsymbol{V_m}$.

$$\boxed{V = V_{RMS}\angle\theta \quad \rightarrow \quad V_m = V_{RMS}\sqrt{2} \quad \rightarrow \quad V(t) = V_m \sin(2\pi f\, t + \theta)}$$

Example Problem 7

For the inductive circuit:

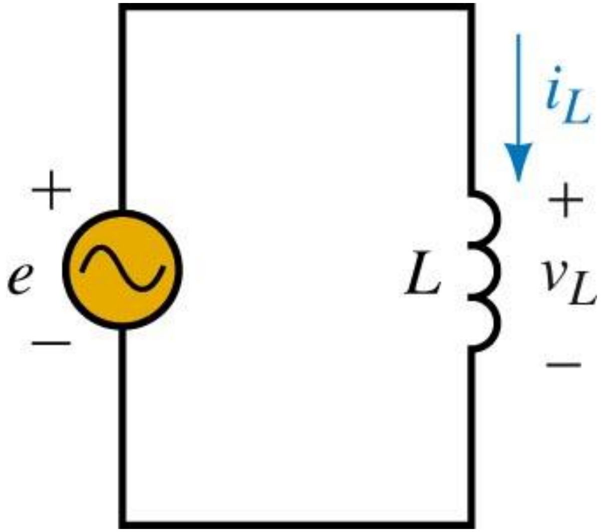
$v_L = 40 \sin(\omega t + 30^\circ)$ V

$f = 26.53$ kHz

L = 2 mH

Determine $\mathbf{V}_L$ and $\mathbf{I}_L$

Graph v_L and i_L



$$\mathbf{Z}_L = jX_L = X_L\angle 90^\circ$$

$$X_L = \omega L = 2\pi fL$$

$$Z_L = 2\Pi fL\angle 90^\circ\,\mathbf{\Omega}$$

$$Z_L = 2\Pi(26.3kHz)(2mH)\angle 90^\circ\,\mathbf{\Omega}$$

$$\boxed{Z_L = 330.5\angle 90^\circ\,\mathbf{\Omega}}$$

$$\boxed{V_L = \frac{40V\angle 90^\circ}{\sqrt{2}} = 28.3\angle 30^\circ V}$$

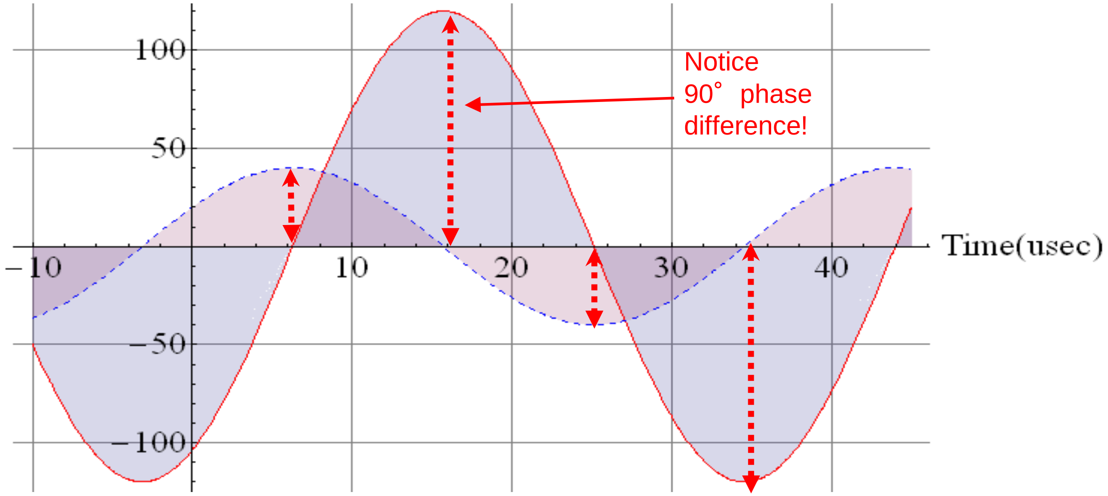
$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}} \Rightarrow I_L = \frac{V_L}{Z_L}\angle 90^\circ$$

$$I_L = \frac{28.3V\angle 30^\circ V}{333\angle 90^\circ\,\mathbf{\Omega}} = \boxed{85\angle -60mA}$$

$$I_L = \frac{i_L}{\sqrt{2}} \Rightarrow i_L = I_L * \sqrt{2}$$

$$i_L = 85\sqrt{2}\sin(\omega t - 60^\circ)$$

$$\boxed{i_L = 120 \sin(\omega t - 60^\circ) \text{ mA}}$$

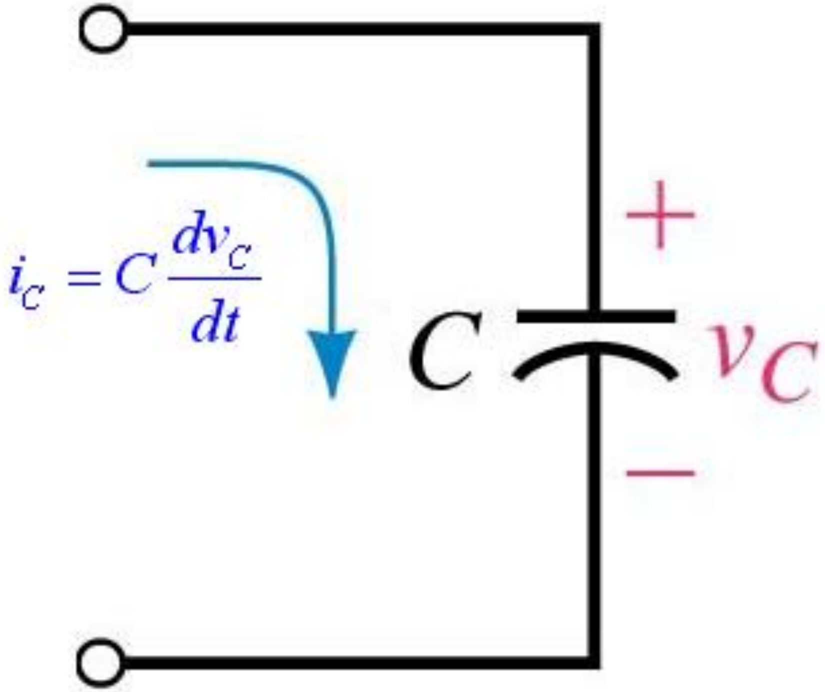


Capacitance and Sinusoidal AC

- **Current-voltage relationship for an capacitor:**

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}\left(V_m \sin\omega t\right)$$
$$= \omega C V_m \cos\omega t = \omega C V_m \sin\left(\omega t + 90^\circ\right)$$

$$Z_c = \frac{v_c}{i_c} = \frac{V_m \sin\omega t}{\omega C V_m \sin\left(\omega t + 90^\circ\right)}$$
$$= \frac{\frac{V_m}{\sqrt{2}}\angle 0^\circ}{\omega C\frac{V_m}{\sqrt{2}}\angle 90^\circ} = \frac{1}{\omega C}\angle -90^\circ\ (\Omega)$$



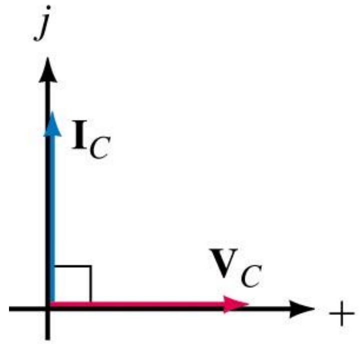
- **It should be noted that, for a purely capacitive circuit *current leads voltage* by 90º.**

Capacitive Impedance

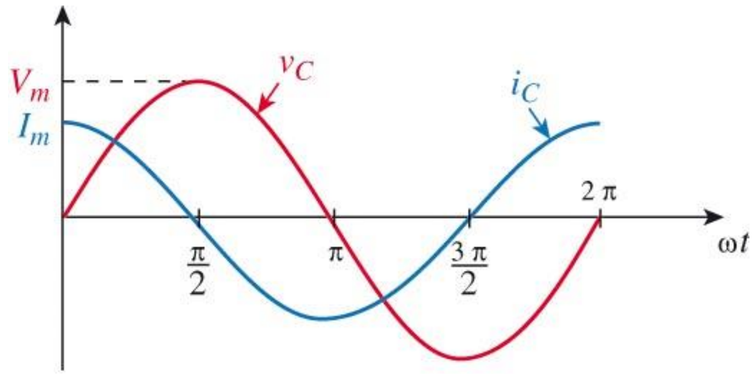
- Impedance can be written as a complex number (in rectangular or polar form):

$$Z_c = \left(\frac{1}{\omega C}\right)\angle -90^\circ = -\left(\frac{1}{\omega C}\right) j \ (\Omega)$$

- Since a capacitor has ***no real resistive*** component, this means ***the reactance of a capacitor*** is the **pure imaginary part**:



(c) $\mathbf{V}_C$ at 0°



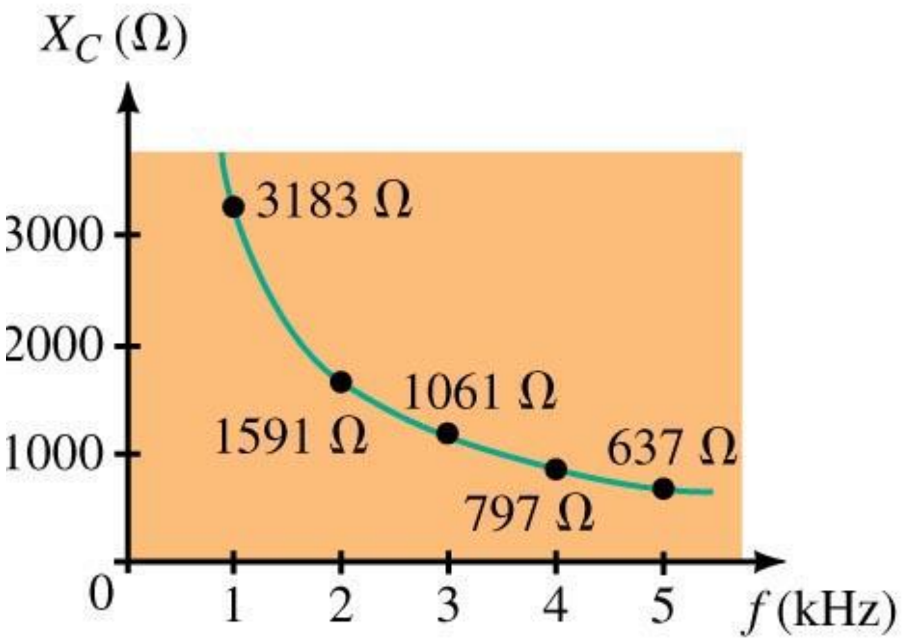
(b) Waveforms with v_C as reference

$$\boxed{X_c = \left(\frac{1}{\omega C}\right)}$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}} = \frac{V_C\angle 0^\circ}{I\angle 90^\circ} = \frac{V_C}{I}\angle -90^\circ = \frac{1}{\omega C}\angle -90^\circ = -\left(\frac{1}{\omega C}\right) j$$

Variation with Frequency

❑ Since $X_C = \left(\frac{1}{\omega C}\right) = \left(\frac{1}{2\pi fC}\right)$, capacitive reactance *is **inversely proportional to frequency***.



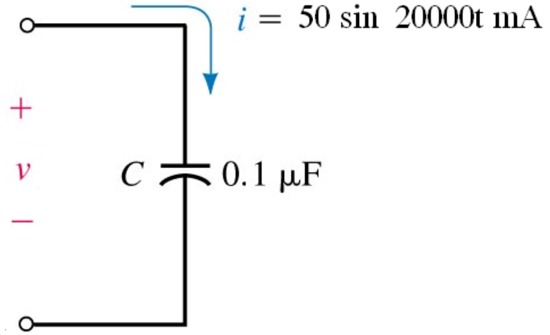
- Extreme case f = 0 Hz (DC): capacitor looks like an open circuit!

Example Problem 8

For the circuit below:

a) Find the Time Domain voltage and current *v*(t) and *i*(t).

b) Draw the sine waveforms for *v* and *i.*



$$\mathbf{Z}_C == \left(\frac{1}{\omega C}\right)\angle -90^{\circ}\,\mathbf{\Omega} = \left(\frac{1}{20000 * 0.1\mu F}\right)\angle -90^{\circ}\,\mathbf{\Omega}$$

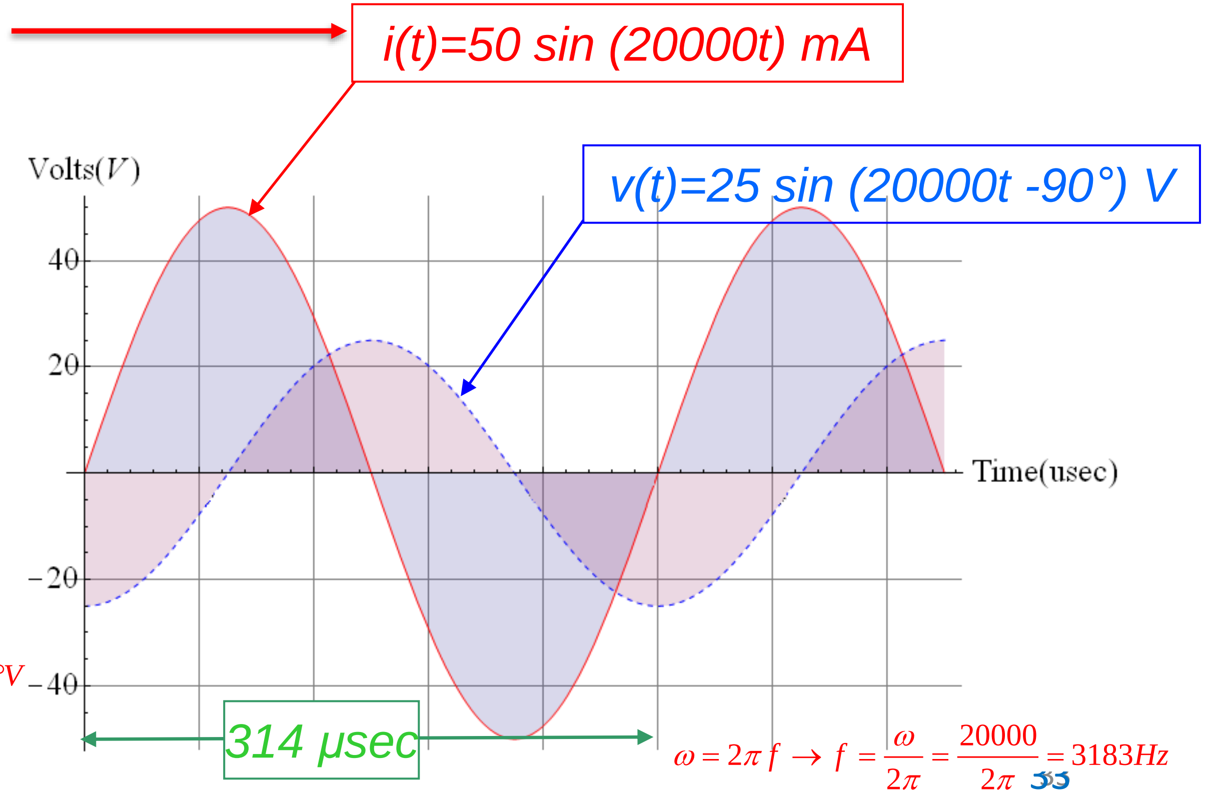
$$\mathbf{Z}_C = 500\angle -90^{\circ}\,\mathbf{\Omega}$$

$$I_{RMS} = \frac{(50mA)}{\sqrt{2}} = 35.4mA\angle 0°$$

$$V_{RMS} = I_{RMS} * Z_C = (35.4mA\angle 0°)(500\angle -90°\mathbf{\Omega})$$

$$V_{RMS} = (35.4mA * 500\mathbf{\Omega})\angle(0° + (-90°)) = 17.7\angle -90°V$$

$$\boxed{\begin{aligned} v(t) &= 17.7 * \sqrt{2}\sin(\omega t - 90°)V \\ v(t) &= 25\sin(\omega t - 90°)V \end{aligned}}$$



$$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} = \frac{20000}{2\pi} = 3183Hz$$

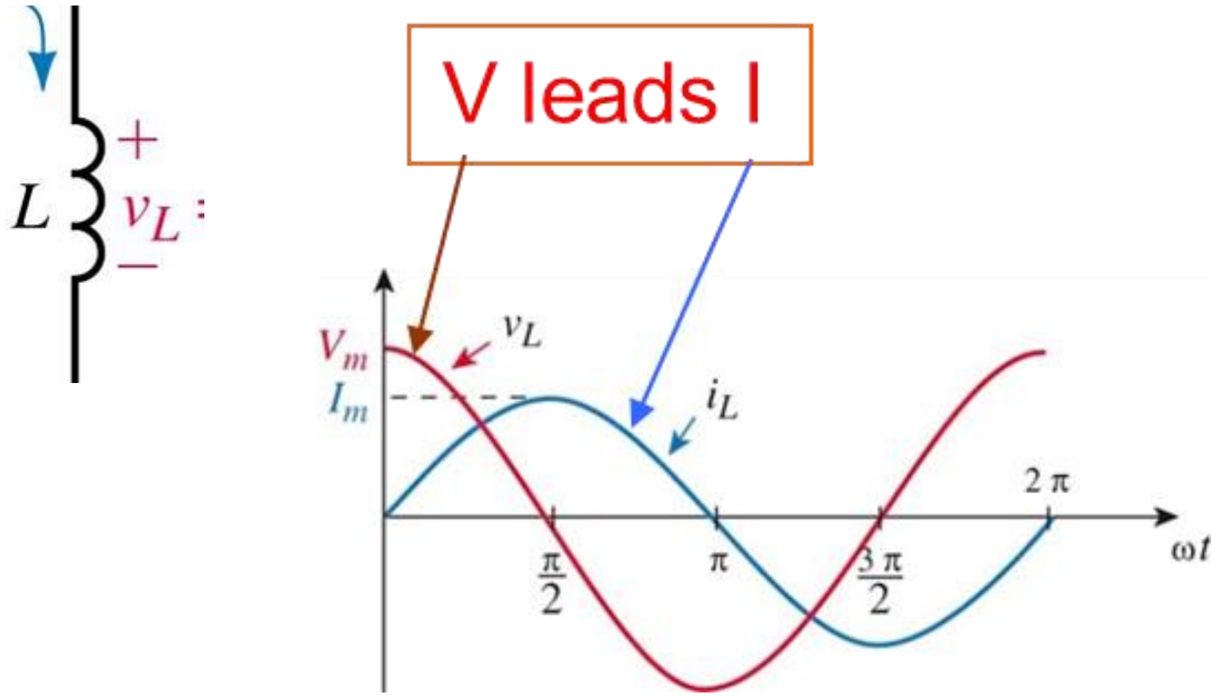
$$\mathbf{T} = \frac{1}{f} = \frac{1}{3183Hz} = 314\mu s$$

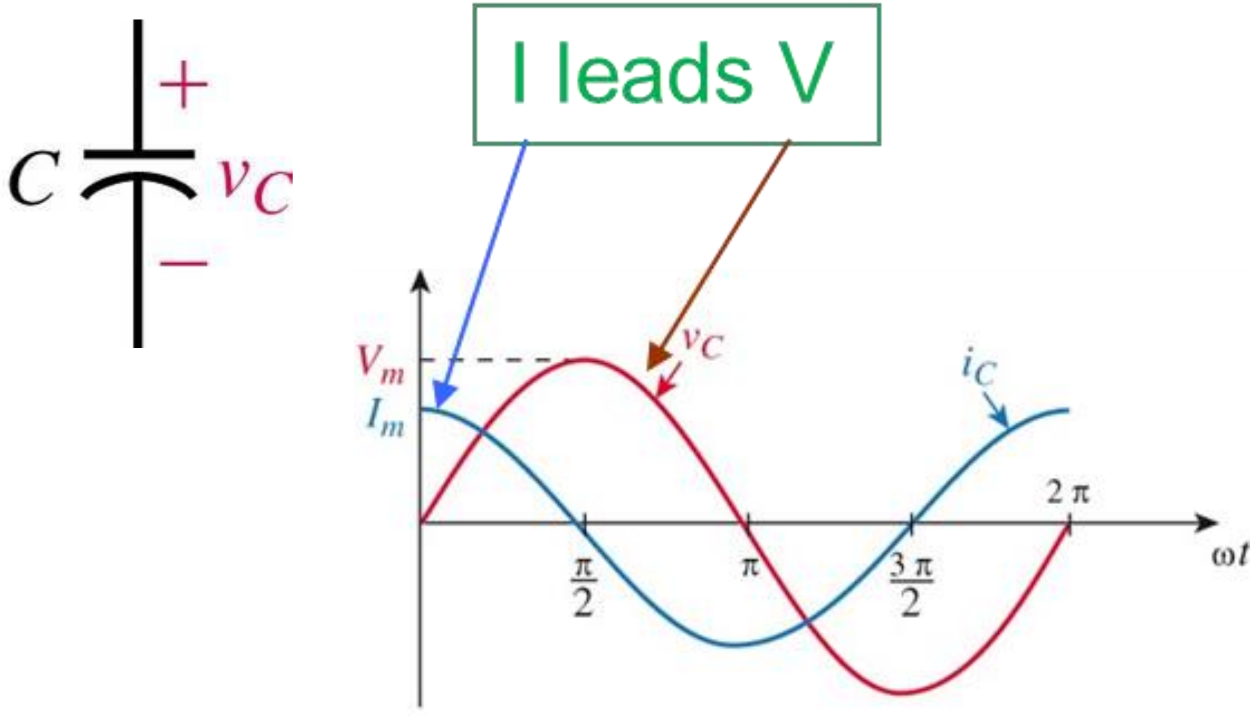
VLI & ICV

VLI & ICV

Voltage Inductance Current

Current Capacitance Voltage



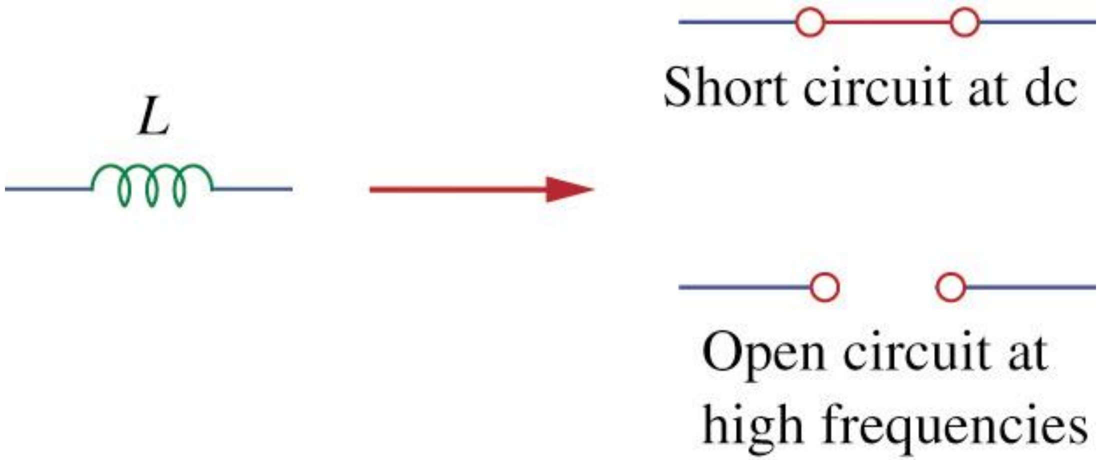


When voltage is applied to an inductor, it resists the change of current. The current builds up more slowly, lagging in time and phase.

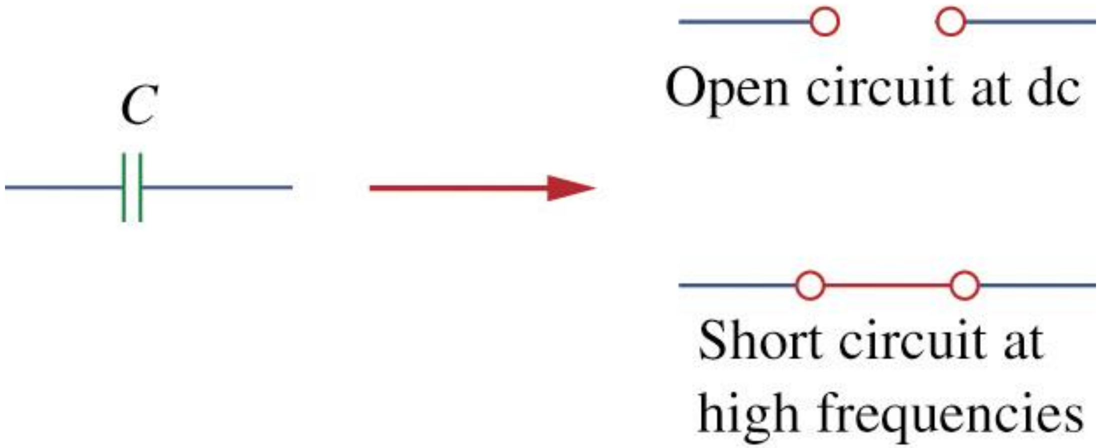
Since the voltage on a capacitor is directly proportional to the charge on it, the current must lead the voltage in time and phase to conduct charge to the capacitor plate and raise the voltage

Frequency Dependency

- **Inductors:**



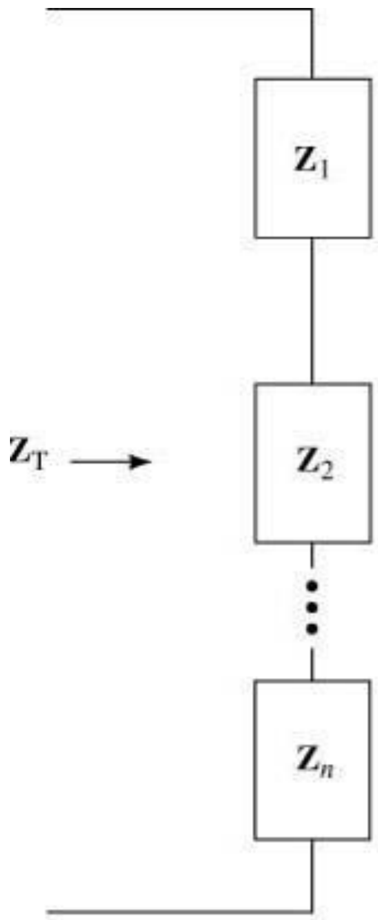
- **Capacitors:**



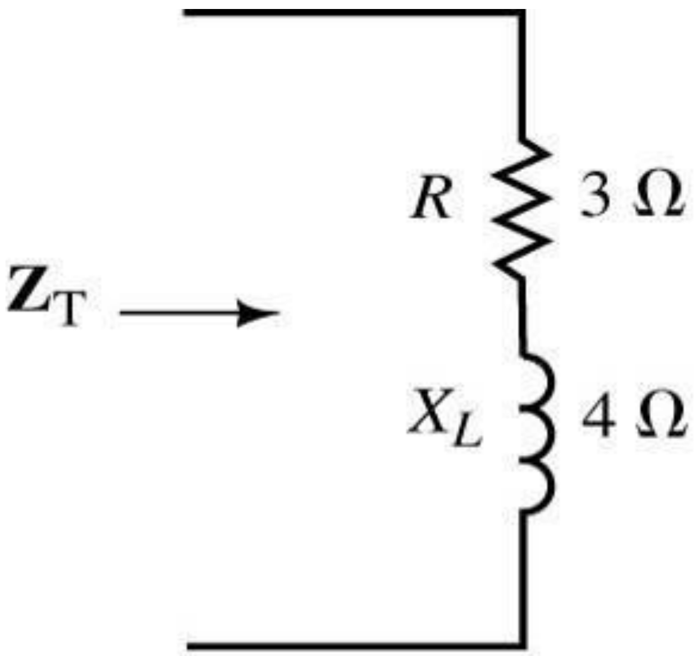
Series AC Circuits

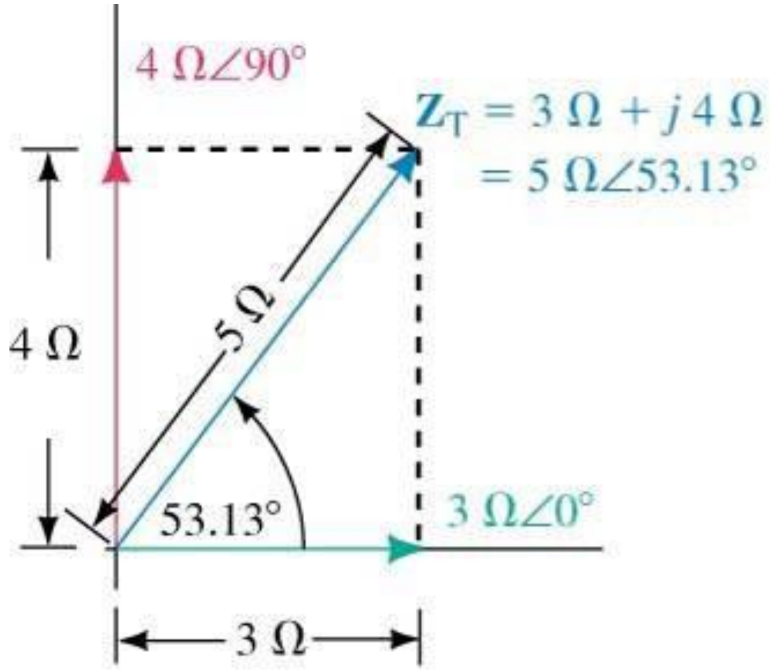
- Total impedance is sum of individual impedances.
- Also note that current is the same through each element.

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_n$$



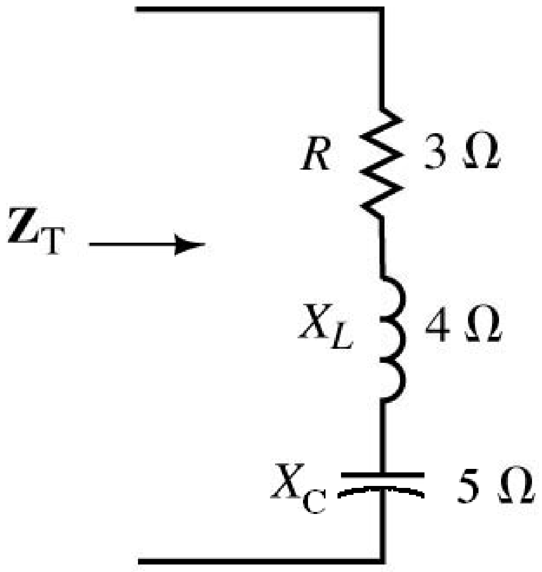
Impedance Example





Impedance Example

For the circuit below find Z_T.



Recall:

$\mathbf{Z}_R = R\angle 0^o = R + j0 = R$

$\mathbf{Z}_L = X_L\angle 90^o = 0 + jX_L = jX_L$

$\mathbf{Z}_C = X_C\angle -90^o = 0 - jX_C = -jX_C$

$$Z_T = Z_R + Z_L + (Z_C) = R + jX_L + (-jX_C)$$

$$3 + j4 - j5 = 3 - j1\ \Omega = \underbrace{\boxed{3.2\angle -18.4^\circ\ \Omega}}$$

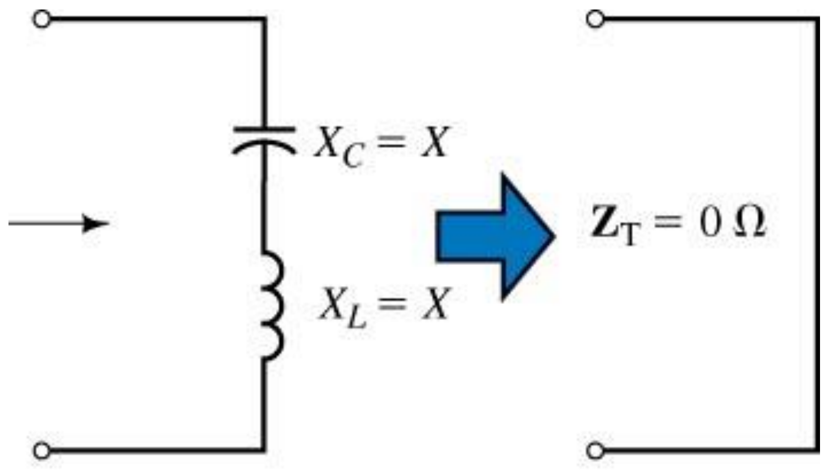
In case you were wondering:

$$C = \sqrt{a^2 + b^2} = \sqrt{3^2 + 1^2} = 3.16$$

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{-1}{3} = -18.43°$$

Special case of Impedance

- Whenever a ***capacitor and inductor of equal reactances*** are placed in series, the equivalent circuit is a ***short circuit***.
- If the total impedance has only a real component, the circuit is said to be **resistive** ($X = 0$ or $\theta = 0°$).
- *But since*
  - If $\theta > 0°$, the circuit is **inductive**. **VLI**
  - If $\theta < 0°$, the circuit is **capacitive**. **ICV**

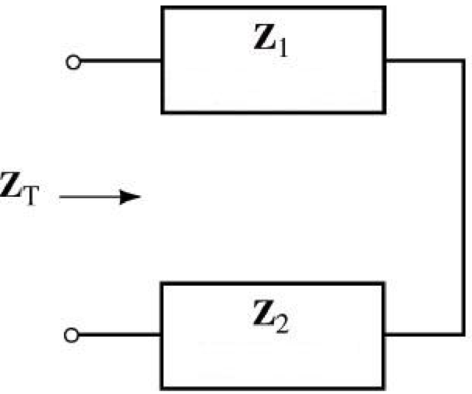


$$\mathbf{Z}_T = jX_L - jX_C = jX - jX = 0\ \Omega$$

Example Problem 9

A network has a total impedance of $\mathbf{Z}_T$=24.0kΩ∠-30˚ at a frequency of 2 kHz.

If the network consists of two series elements, what types of components are those and what are their R/L/C values?



$$Z_T = Z_R + Z_L + (Z_C) = R + jX_L + (-jX_C)$$

$$Z_T = 24\angle -30^\circ\, k\Omega$$

Convert the polar to rectangular:

$Z_T = 24\angle -30^\circ\, k\Omega \rightarrow$

$a = 24\cos(-30) = 20.78$

$b = 24\sin(-30) = -12$

$Z_T = (20.78 - j12)k\Omega$

Remember; capacitors are negative (-).

Knowing that the imaginary portion represents capacitance, find the value of the capacitor:

$X_C = -12k \rightarrow -12k = \dfrac{1}{2\pi fC}$

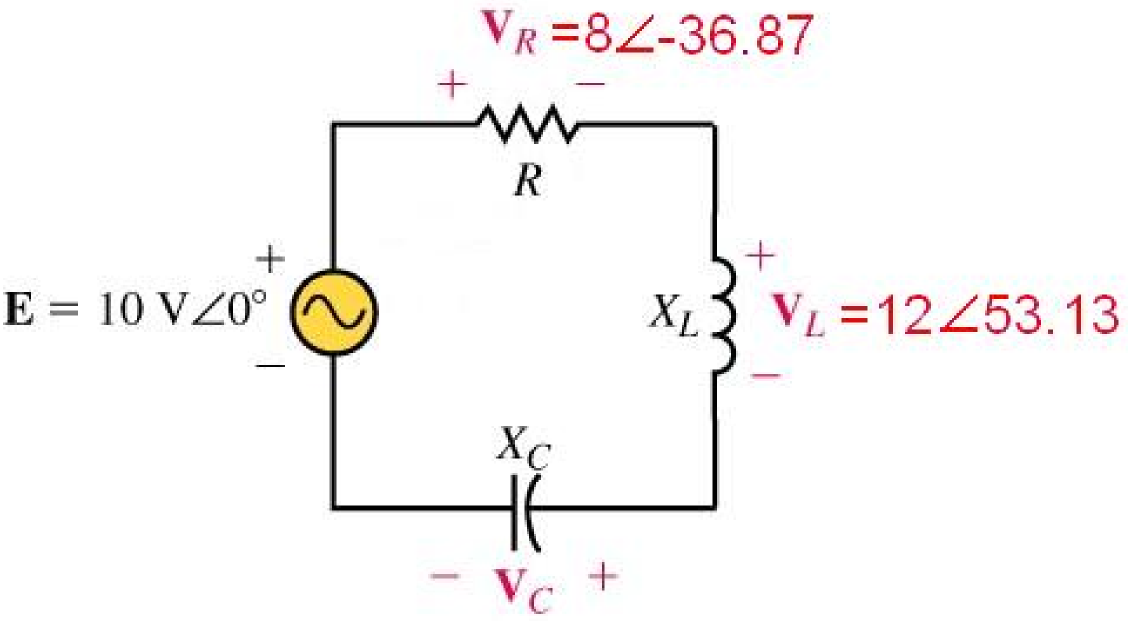
$C = \dfrac{1}{2\pi(2kHz)(-12k\Omega)} = \boxed{6.63nF}$

Now, knowing that the real portion represents resitstance:

$\boxed{R = 20.78k\Omega}$

Kirchhoff's Voltage Law (KVL)

- *The phasor sum of voltage drops and rise around a closed loop is equal to zero.*



$\mathbf{C} = a + jb$ (rectangular form)

$\mathbf{C} = C\angle\theta$ (polar form)

$$a = C\cos\theta$$

$$b = C\sin\theta$$

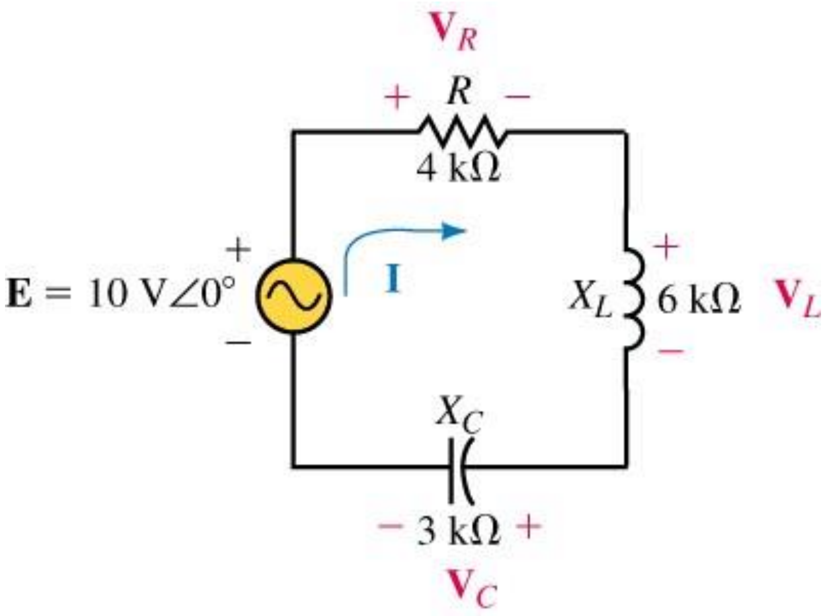
$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

<u>KVL</u> $V_c = ??????$

$$\begin{aligned}\mathbf{V}_c &= E - V_R - V_L \\ &= (10\angle 0) - (8\angle -36.87) - (12\angle 53.13) \\ &= 6\angle -126.87° V\end{aligned}$$

Voltage Divider Rule (VDR)



$$V_c = ??????$$

$$\mathbf{V}_x = \frac{\mathbf{Z}_x}{\mathbf{Z}_T}\mathbf{E} \qquad E = \frac{V_m}{\sqrt{2}}$$

<u>*VDR*</u>

$$\mathbf{V}_c = E\left(\frac{Z_C}{Z_R + Z_L + Z_C}\right)$$

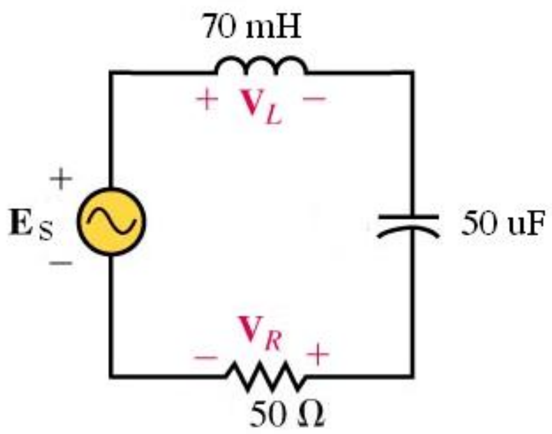
$$= 10\angle 0 \left(\frac{-3j}{4+6j-3j}\right)$$

$$= 6\angle -126.87$$

Example Problem 10

$e_s(t)$=170 sin (1000t + 0) V.

- Determine Z_{TOT}
- Determine total current I_{TOT}
- Determine voltages V_R, V_L, and V_c
- Verify KVL for this circuit
- Graph E, V_L, V_C, V_R in the time domain



$$V_{RMS} = \frac{170V}{\sqrt{2}}\angle 0° = \boxed{120\angle 0° V}$$

$$Z_L = j\omega L = j(1000)(70mH) = \boxed{j70\Omega}$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{1000 * 50\mu F} = \boxed{-j20\Omega}$$

$$Z_T = Z_R + Z_L + (Z_C) = R + jX_L + (-jX_C)$$

$$50 + j70 - j20 = 50 + j50\ \Omega = \boxed{70.7\angle 45°\ \Omega}$$

$$\boxed{\begin{array}{l} C = \sqrt{a^2 + b^2} = \sqrt{50^2 + 50^2} = 70.7 \\ \theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{50}{50} = 45° \end{array}}$$

$$I_{RMS} = \frac{V_{RMS}}{\mathbf{Z}_T} = \frac{120\angle 0°}{70.7\angle 45°} = \boxed{1.70\angle -45° A}$$

$$\mathbf{V}_x = I_T * Z_x$$

$$\mathbf{V}_R = I_T * Z_R = (1.70\angle -45°)(50\angle 0°) = \boxed{84.8\angle -45° V}$$

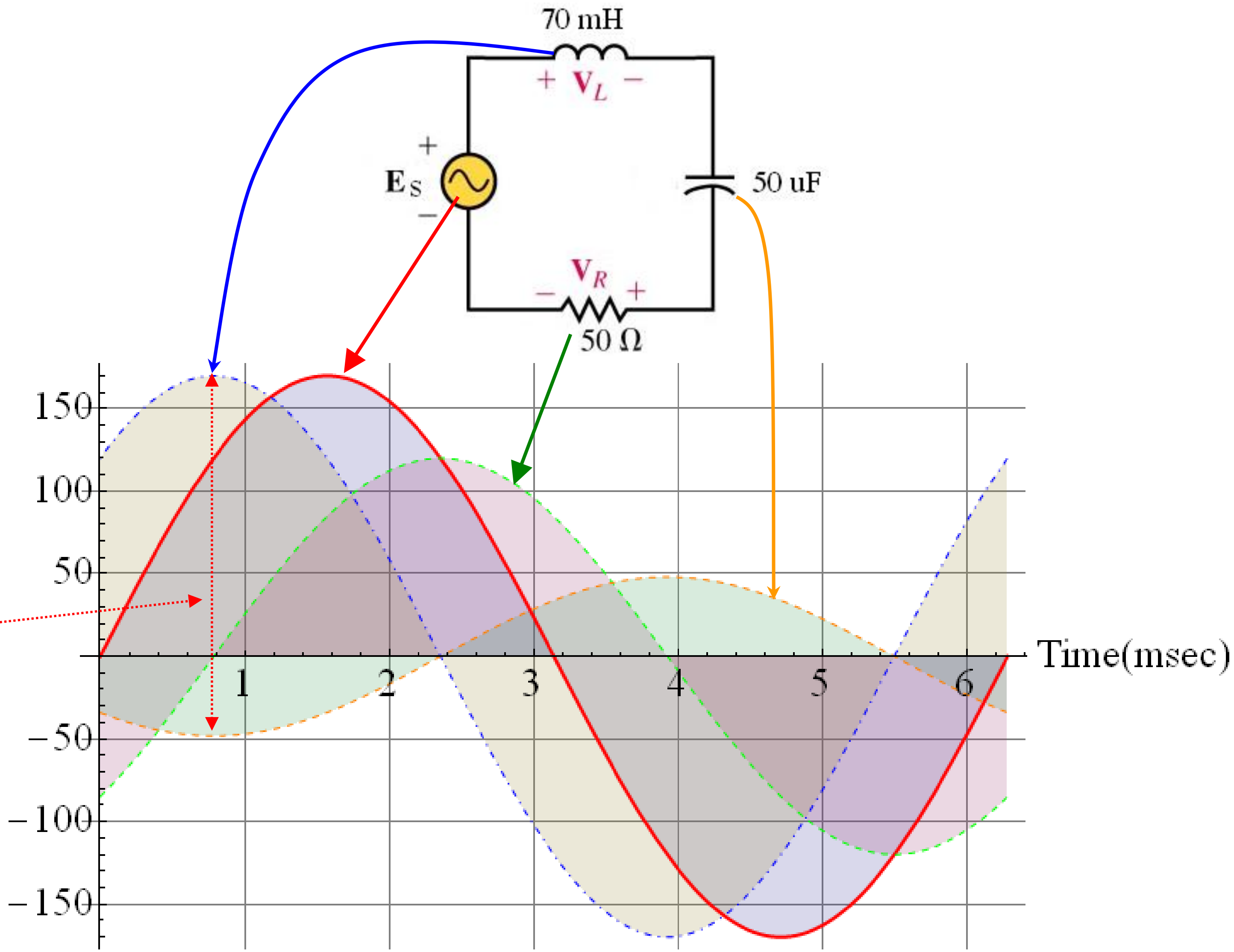
$$\mathbf{V}_L = I_T * Z_L = (1.70\angle -45°)(70\angle 90°) = \boxed{119\angle 45° V}$$

$$\mathbf{V}_C = I_T * Z_C = (1.70\angle -45°)(20\angle -90°) = \boxed{33.9\angle -135° V}$$

$KVL:$

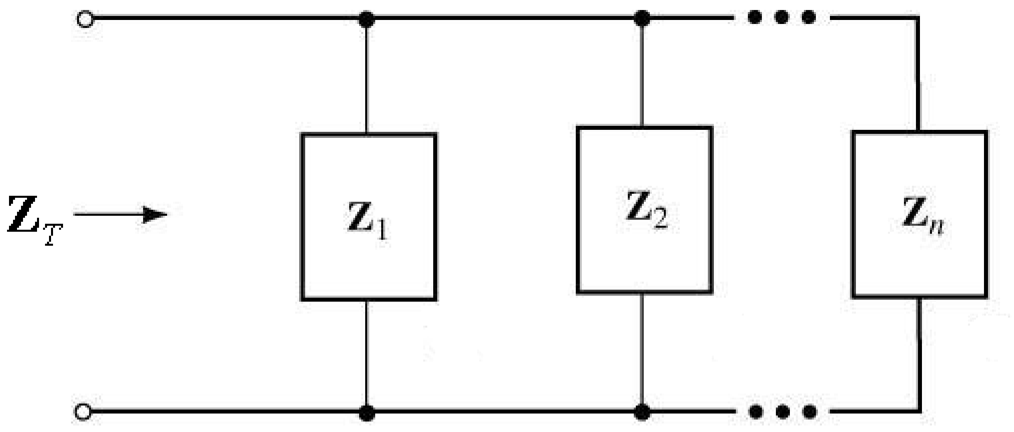
$$E = V_R + V_L + \mathbf{V}_c = 120\angle 0°$$

Example Problem 10



180° phase difference between L and C

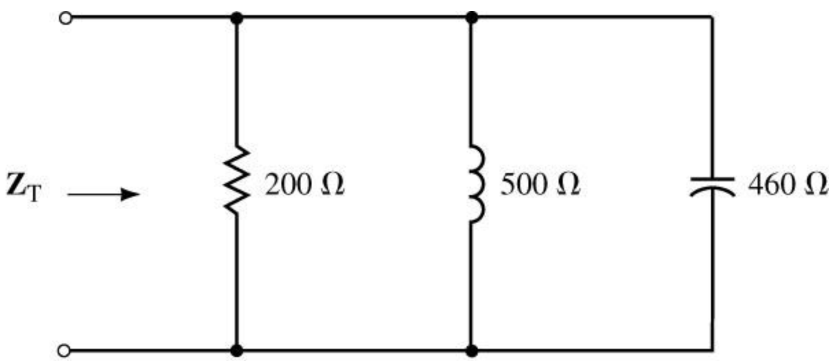
Parallel AC Circuit



$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2} + \cdots + \dfrac{1}{\mathbf{Z}_n}} \qquad (\Omega)$$

Example Problem 11

Determine Z_T for the following circuit:



(a)

$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2} + \cdots + \dfrac{1}{\mathbf{Z}_n}} \quad (\Omega)$$

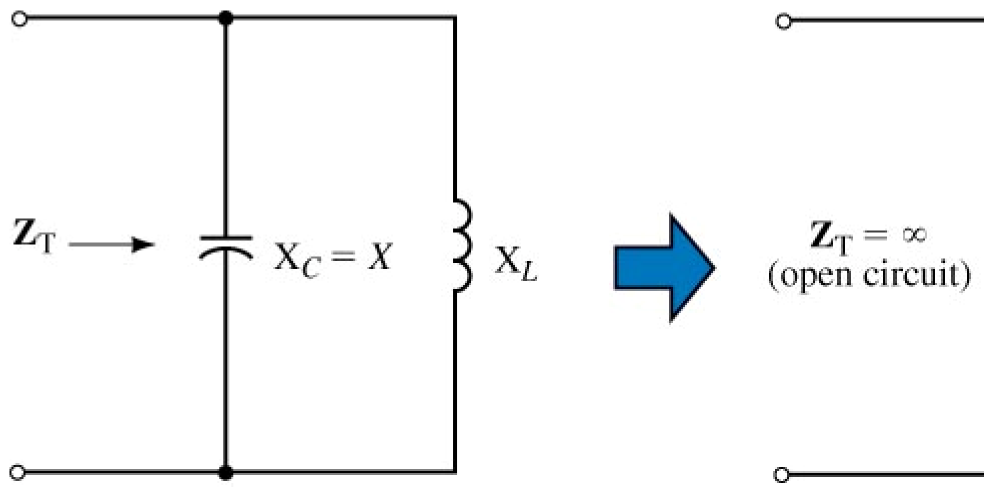
$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_R} + \dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_C}} \quad (\Omega)$$

$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{200} + \dfrac{1}{j500} + \dfrac{1}{-j460}} \quad (\Omega)$$

$$\boxed{\mathbf{Z}_T = 199.9\angle -2^\circ \quad (\Omega)}$$

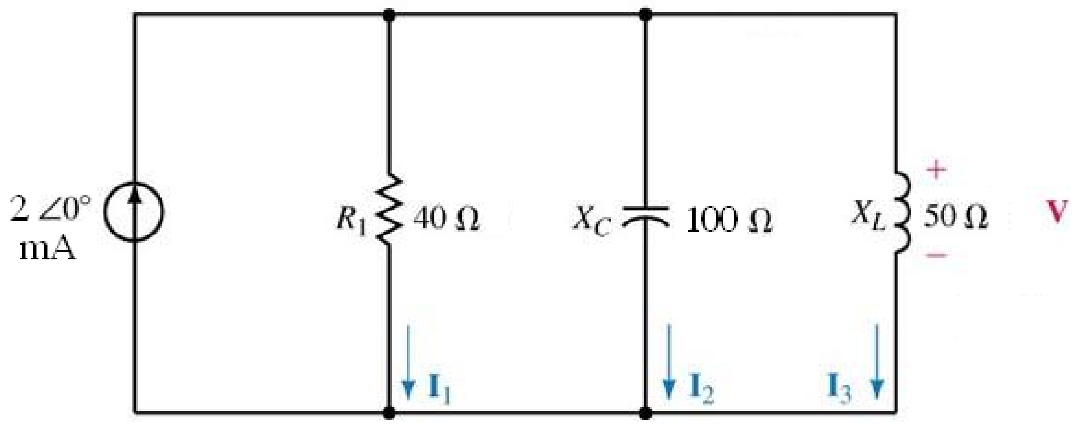
Impedances in Parallel (special case)

- What if $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are ***equal*** inductive and capacitive reactances?
- The resulting total impedance is infinite, so the equivalent is an ***open circuit***.



Kirchhoff's Current Law (KCL)

- ***The summation of current phasors entering and leaving a node is equal to zero.***



$IF:$

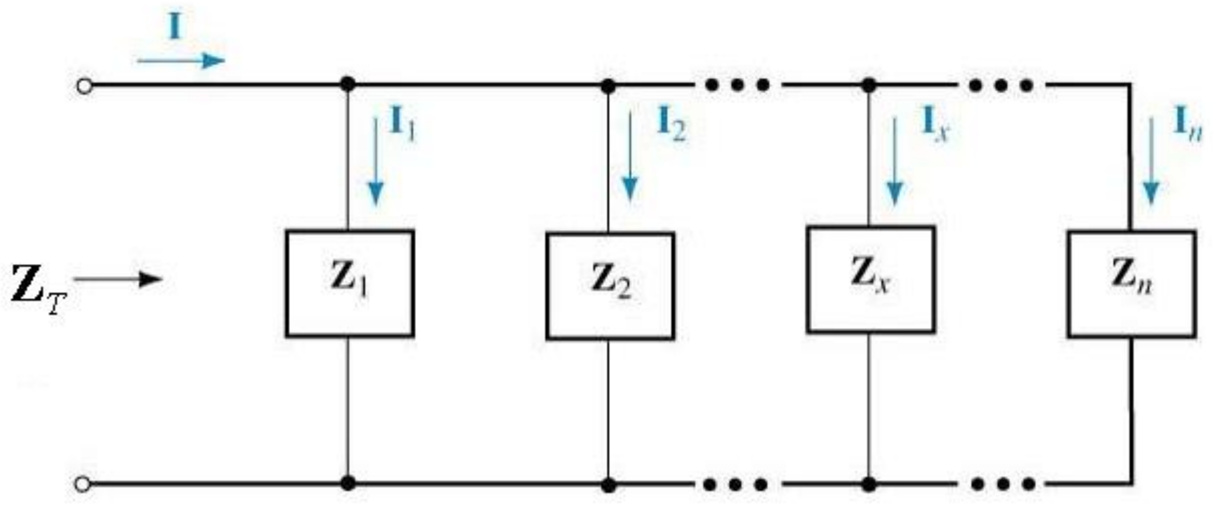
$I_1 = 1.86\angle 21.8 mA$

$I_2 = 743\angle 111.8 \mu A$

$\underline{KCL}$

$I_3 = I_T - I_1 - I_2 = 2\angle 0 - 1.86\angle 21.8 - 0.74\angle 111.8 mA = \boxed{1.49\angle -68.2 mA}$

Current Divider Rule



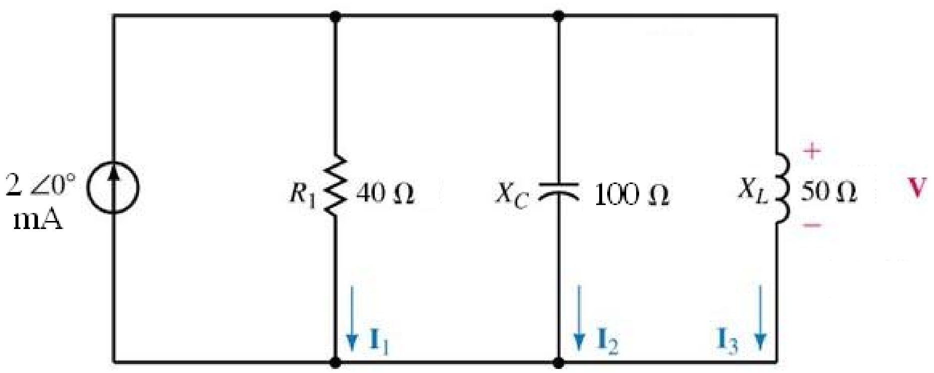
$$\boxed{\mathbf{I}_x = \frac{\mathbf{Z}_T}{\mathbf{Z}_x}\mathbf{I}}$$

Current Divider Rule

$$Z_T = \left(\frac{1}{R_1} + \frac{1}{Z_c} + \frac{1}{Z_L}\right)^{-1}$$

$$Z_T = \left(\frac{1}{40} + \frac{1}{100\angle -90} + \frac{1}{50\angle 90}\right)^{-1}$$

$$Z_T = 37.1\angle 21.8 \;\; \Omega$$



CDR: $\mathbf{I}_x = \dfrac{\mathbf{Z}_T}{\mathbf{Z}_x}\mathbf{I}$

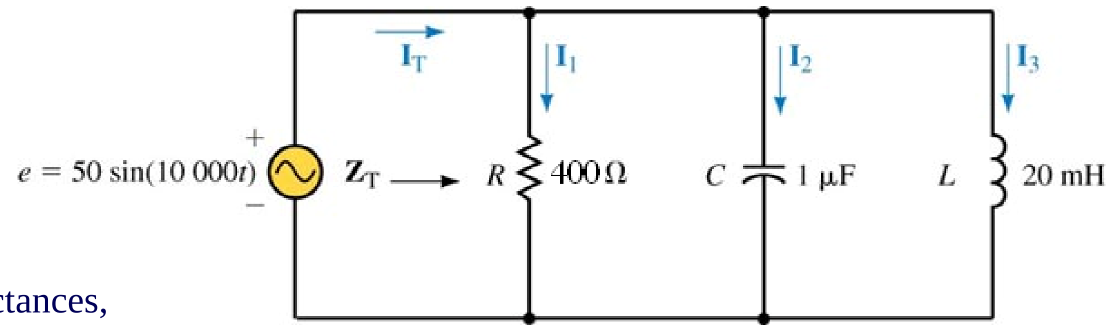
$$I_1 = I_T\left(\frac{Z_T}{R_1}\right) = 2\angle 0^\circ\, mA\left(\frac{37\angle 21.8}{40\angle 0}\right) = 1.86\angle 21.8 mA$$

$$I_2 = I_T\left(\frac{Z_T}{Z_C}\right) = 2\angle 0^\circ\, mA\left(\frac{37\angle 21.8}{100\angle -90}\right) = 0.743\angle 111.8 mA = 743\angle 111.8 \mu A$$

$$I_3 = I_T\left(\frac{Z_T}{Z_L}\right) = 2\angle 0^\circ\, mA\left(\frac{37\angle 21.8}{50\angle 90}\right) = 1.49\angle -68.2 mA$$

Example Problem 12

Find Z_T, I_T, I_1, I_2, I_3



$$Z_T = \left(\frac{1}{R_1} + \frac{1}{Z_c} + \frac{1}{Z_L}\right)^{-1}$$

However, Z_C and Z_L are not given in reactances, they are given as capacitance and inductance respectively. We need to convert to reactance:

$$Z_C = \frac{1}{\omega C}\angle -90^\circ = \frac{1}{(10000)(1*10^{-6})}\angle -90^\circ = 100\angle -90^\circ\Omega$$

$$Z_L = \omega L\angle 90^\circ = 10000*20*10^{-3}\angle 90^\circ = 200\angle 90^\circ\Omega$$

$$Z_R = R\angle 0^\circ = 400\angle 0^\circ\Omega$$

$$Z_T = \left(\frac{1}{400\angle 0^\circ} + \frac{1}{100\angle -90^\circ} + \frac{1}{200\angle 90^\circ}\right)^{-1}$$

$$\boxed{Z_T = 178.9\angle -63.43^\circ}$$

$$I_T = \frac{\left(\frac{50V}{\sqrt{2}}\right)}{178.9\angle -63.43^\circ\Omega} = \boxed{197.6\angle 63.43^\circ mA}$$

NOTE :

We could have used $\vec{I}_x = \dfrac{\vec{E}}{\vec{Z}_x}$

Now using CDR find I_1, I_2 and I_3:

$$I_1 = I_T\left(\frac{Z_T}{R_1}\right) = 198\angle 63.5^\circ mA\left(\frac{178.9\angle -63.43^\circ}{400\angle 0^\circ}\right) = \boxed{88.56\angle 0^\circ mA}$$

$$I_2 = I_T\left(\frac{Z_T}{Z_C}\right) = 198\angle 63.5^\circ mA\left(\frac{178.9\angle -63.43^\circ}{100\angle -90}\right) = \boxed{354.2\angle -90^\circ mA}$$

$$I_3 = I_T\left(\frac{Z_T}{Z_L}\right) = 198\angle 63.5^\circ mA\left(\frac{178.9\angle -63.43^\circ}{200\angle 90}\right) = \boxed{177.1\angle 90 mA}$$

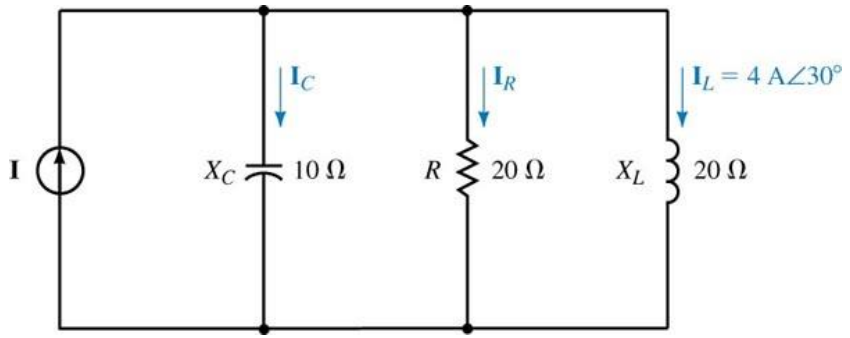
Example Problem 13

a. Find the total impedance $\mathbf{Z}_T$.

b. Determine the supply current ($\mathbf{I}_T$,) using the current divider rule

c. Calculate $\mathbf{I}_C$ and $\mathbf{I}_R$.

d. Verify KCL.



$$Z_T = \left(\frac{1}{R_1} + \frac{1}{Z_c} + \frac{1}{Z_L}\right)^{-1}$$

$$Z_T = \left(\frac{1}{20} + \frac{1}{-j10} + \frac{1}{j20}\right)^{-1} = \boxed{14.14\angle -45°}$$

$$I_L = I_T \frac{Z_T}{Z_L}$$

$$4\angle 30° A = I_T \frac{14.14\angle -45°\Omega}{j20\Omega}$$

$$I_T = \frac{4\angle 30° A}{\left(\frac{14.14\angle -45°\Omega}{j20\Omega}\right)} = \boxed{5.66\angle 165° A}$$

Now using CDR find I_C and I_R:

$$I_R = I_T\left(\frac{Z_T}{R_1}\right) = 5.66\angle 165^\circ A\left(\frac{14.14\angle -45°}{20\angle 0°}\right) = \boxed{4\angle 120° A}$$

$$I_C = I_T\left(\frac{Z_T}{Z_C}\right) = 5.66\angle 165^\circ A\left(\frac{14.14\angle -45°}{10\angle -90°}\right) = \boxed{8\angle -150° A}$$

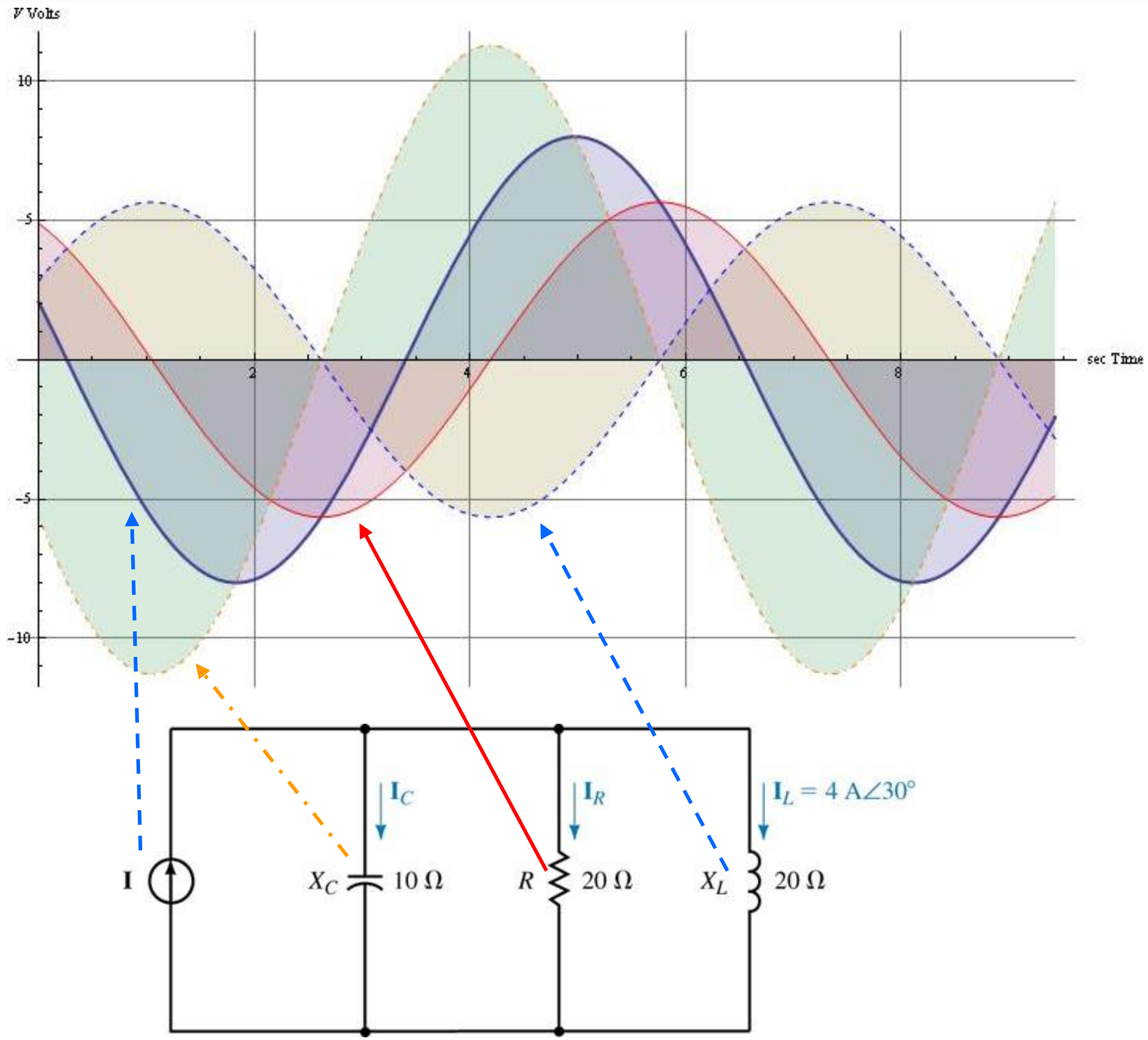
Same as +210º

KCL Verification:

$$I_T = I_R + I_C + I_L$$

$$5.66\angle 165° A = (4\angle 120° + 8\angle -150° + 4\angle 30°)A \rightarrow \boxed{Verified!}$$

Example Problem 13



Impedance and Admittance

The **admittance Y** is the reciprocal of impedance, measured in siemens (S).

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$ **is the ratio of** $\dfrac{\boxed{\textbf{the phasor current through it}}}{\boxed{\textbf{the phasor voltage across it}}}$

As a complex quantity

$$\boxed{\mathbf{Y} = G + jB} \longrightarrow \boxed{G + jB = \frac{1}{R + jX}}$$

G = Re Y is called the ***conductance***

B = Im Y is called the ***susceptance.***

unit

Admittance, conductance, and susceptance are all expressed in the unit of siemens

Impedance and Admittance

$$\boxed{G + jB = \frac{1}{R + jX}}$$

By rationalization,

conjugate

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

Equating the real and imaginary parts gives

$$\boxed{G = \frac{R}{R^2 + X^2}}, \qquad \boxed{B = -\frac{X}{R^2 + X^2}}$$