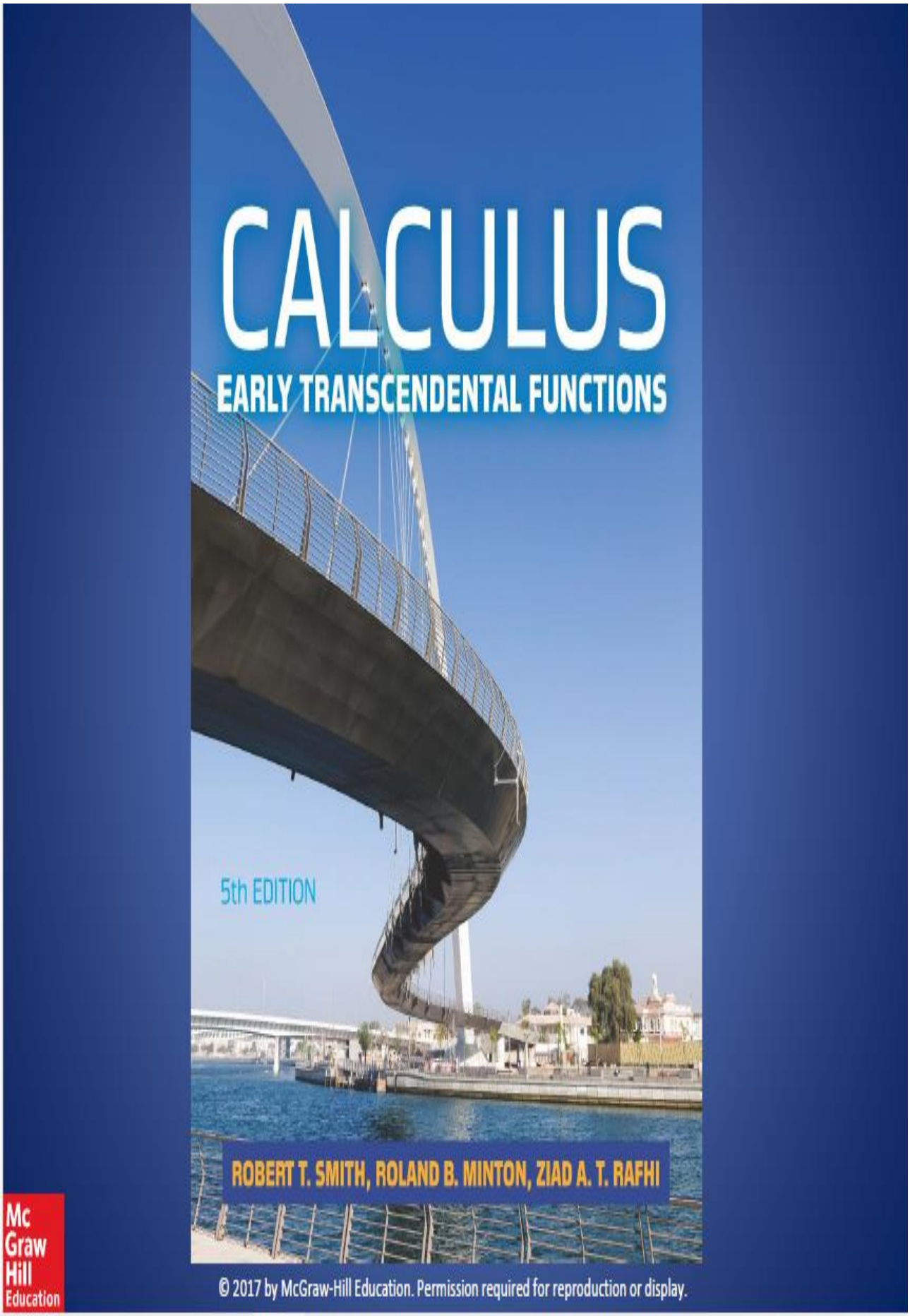

Complex numbers:

1-The standard form

$$z = x + yi$$

x is the real part, y is the imaginary part, and $i = \sqrt{-1}$

Example:

$$z = 2 + 3i \rightarrow x = 2,\ \ y = 3$$

$$z = 2 - 3i \rightarrow x = 2,\ \ y = -3$$

$$z = -2 + 3i \rightarrow x = -2,\ \ y = 3$$

$$z = -2 - 3i \rightarrow x = -2,\ \ y = -3$$

$$z = 3i \rightarrow x = 0,\ \ y = 3$$

$$z = 2 \rightarrow x = 2,\ \ y = 0$$

2- $i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 i = -i$

$i^4 = i^2 i^2 = (-1)(-1) = 1$

$i^5 = i^4 i = (1)(i) = i$

$i^6 = i^4 i^2 = (1)(-1) = -1$

$i^7 = -i\,, i^8 = 1\,,\ i^9 = i$

3- Conjugate of a complex number $\bar{z}$

$$z = x + yi \rightarrow \bar{z} = x - yi$$

Example:

$$z = 2 + 3i \rightarrow \bar{z} = 2 - 3i$$

$$z = 2 - 3i \rightarrow \bar{z} = 2 + 3i$$

$$z = 3i \rightarrow \bar{z} = -3i$$

$$z = 2 \rightarrow \bar{z} = 2$$

4- Addition, subtraction, and multiplication

Example: If $z_1 = 2 + 3i$, $z_2 = 3 - 4i$, then

$$a)\ z_1 + z_2 = 2 + 3i + 3 - 4i = 5 - i$$

$$b) \quad z_1 - z_2 = 2 + 3i - (3 - 4i)$$

$$= 2 + 3i - 3 + 4i = -1 + 7i$$

$$c) \quad z_1 z_2 = (2 + 3i)(3 - 4i)$$

$$= 6 - 8i + 9i - 12i^2$$

$$= 6 - 8i + 9i - 12(-1)$$

$$= 6 - 8i + 9i + 12$$

$$= 18 + i$$

$$d)\ 2z_1 + 3z_2 = 2(2 + 3i) + 3(3 - 4i)$$

$$= 4 + 6i + 9 - 12i$$

$$= 13 - 6i$$

Remark:

$$z\bar{z} = (x + yi)(x - yi)$$

$$= x^2 - xyi + xyi - y^2 i^2$$

$$= x^2 - xyi + xyi - y^2(-1)$$

$$= x^2 + y^2$$

Example: $(2 + 3i)(2 - 3i) = 2^2 + 3^2 = 13$

5-Division

Example: Write $\frac{1+2i}{2+3i}$ in the standard form

$$\frac{1+2i}{2+3i} = \frac{(1+2i)(2-3i)}{(2+3i)(2-3i)}$$

$$= \frac{2-3i+4i-6i^2}{2^2+3^2}$$

$$= \frac{2-3i+4i+6}{2^2+3^2}$$

$$= \frac{8+i}{13} = \frac{8}{13} + \frac{1}{13}i$$

6-Magnitude of the complex number $|z|$ or r

$$z = x + yi \rightarrow |z| = \sqrt{x^2 + y^2}$$

Example:

$$\text{If } z = 3 + 4i, then\ |z| = \sqrt{3^2 + 4^2} = 5$$

Example:

$$\text{If } z_1 = 2 + 3i,\ z_2 = 3 - 4i, \text{ then}$$

$$|2z_1 + 3z_2| = |2(2 + 3i) + 3(3 - 4i)|$$

$$= |4 + 6i + 9 - 12i|$$

$$= |13 - 6i|$$

$$= \sqrt{13^2 + 6^2}$$

$$= \sqrt{205}$$

7- Argument of the complex number $\arg(z)$ *or* θ

a) First quarter (x *is positive and* y *is positive*)

$$arg(z) = \theta = tan^{-1}\left|\frac{y}{x}\right|$$

Example: If $z = 1 + i$

$x = 1\,,\ \ y = 1$ first quarter

$$\theta = tan^{-1}\left|\frac{1}{1}\right| = tan^{-1}1$$

$$= \frac{\pi}{4}$$

b) Second quarter ($x\ is\ negative\ and\ y\ is\ positive$)

$$arg(z) = \theta = \pi - tan^{-1}\left|\frac{y}{x}\right|$$

Example: If $z = -1 + i$

$x = -1\ ,\ \ y = 1$ Second quarter

$$\theta = \pi - tan^{-1}\left|\frac{1}{-1}\right|$$

$$= \pi - tan^{-1}1$$

$$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

c) Third quarter ($x\ is\ negative\ and\ y\ is\ negative$)

$$arg(z) = \theta = tan^{-1}\left|\frac{y}{x}\right| - \pi$$

Example: If $z = -1 - i$

$x = -1\ ,\ \ y = -1$ Third quarter

$$\theta = tan^{-1}\left|\frac{-1}{-1}\right| - \pi$$

$$= tan^{-1}1 - \pi$$

$$= \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$$

d)Fourth quarter ($x\ is\ positive\ and\ y\ is\ negative$)

$$arg(z) = \theta = -tan^{-1}\left|\frac{y}{x}\right|$$

Example: If $z = 1 - i$

$x = 1\ ,\ \ y = -1$ Fourth quarter

$$\theta = -tan^{-1}\left|\frac{-1}{1}\right|$$

$$= -tan^{-1}1$$

$$= -\frac{\pi}{4}$$

8-Polar form (trigonometric form)

$$z = r\, e^{\theta i}$$

Or

$$z = r\,(cos\theta + i\, sin\theta)$$

Example:

$$z = 1 + \sqrt{3}\, i$$

$x = 1\,,$ $y = \sqrt{3}$ First quarter

$$r = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

$$\theta = tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$$

$$= tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

$$z = 2\, e^{\frac{\pi}{3}i}$$

or $z = 2\,(cos\frac{\pi}{3} + i\, sin\frac{\pi}{3})$

Example:

$$z = \sqrt{3} + i$$

$$x = \sqrt{3}\,, \qquad y = 1 \quad \text{First quarter}$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\theta = tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$$

$$= tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$z = 2\,e^{\frac{\pi}{6}i}$$

or $\qquad z = 2\,(cos\frac{\pi}{6} + i\,sin\frac{\pi}{6})$

Example:

$$z = 1 + i$$

$$x = 1\,, \qquad y = 1 \quad \text{First quarter}$$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = tan^{-1}\left|\frac{1}{1}\right|$$

$$= tan^{-1}1 = \frac{\pi}{4}$$

$$z = \sqrt{2}\, e^{\frac{\pi}{4}i}$$

or $\qquad z = \sqrt{2}\,(cos\frac{\pi}{4} + i\, sin\frac{\pi}{4})$