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Tutorial 1

بعض القوانين الهامة :-

مجموع الاحتمالات فى S = 1

1- $P(A^C) = 1 - P(A)$ احتمال عدم حدوث A

2- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

احتمال حدوث A او B او كليهما

3- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

احتمال حدوث A و B معا

4- $P(A - B) = P(A \cap B^C) = P(A) - P(A \cap B)$

احتمال حدوث A وعدم حدوث B

5- $P(B - A) = P(B \cap A^C) = P(B) - P(A \cap B)$

احتمال حدوث B و عدم حدوث A

6- (from De Morgane's law)

$P(A^C \cap B^C) = P(A \cup B)^C = 1 - P(A \cup B)$

عدم حدوث الاثنين معا

7- (from De Morgane's law)

$P(A^C \cup B^C) = P(A \cap B)^C = 1 - P(A \cap B)$

عدم حدوث احدهما على الاقل

A section of an exam contains four True-False questions. A completed exam paper is selected at random, and the four answers are recorded.

a. List all 16 outcomes in the sample space.

b. Assuming the outcomes to be equally likely, find the probability that all the answers are the same.

c. Assuming the outcomes to be equally likely, find the probability that exactly one of the four answers is "True."

d. Assuming the outcomes to be equally likely, find the probability that at most one of the four answers is "True."

Ans.

(a) The outcomes are the 16 different strings of 4 true-false answers. These are {TTTT, TTTF, TTFT, TTFF, TFTT, TFTF, TFFT, TFFF, FTTT, FTTF, FTFT, FTFF, FFTT, FFTF, FFFT, FFFF}.

(b) There are 16 equally likely outcomes. The answers are all the same in two of them, TTTT and FFFF. Therefore the probability is 2/16 or 1/8.

(c) There are 16 equally likely outcomes. There are four of them, TFFF, FTFF, FFTF, and FFFT, for which exactly one answer is "True." Therefore the probability is $4/16$ or $1/4$.

(d) There are 16 equally likely outcomes. There are five of them, TFFF, FTFF, FFTF, FFFT, and FFFF, for which at most one answer is "True." Therefore the probability is $5/16$.

The probability that a bolt meets a strength specification is 0.87. What is the probability that the bolt does not meet the specification?

Ans.

P(A) = 0.87

Probability that the bolt does not meet specifications is $P(A^c)$

$P(A^c) = 1 - 0.87 = 0.13$

Among the cast aluminum parts manufactured on a certain day, 80% were flawless, 15% had only minor flaws, and 5% had major flaws. Find the probability that a randomly chosen part

a. has a flaw (major or minor).

b. has no major flaw.

Ans.

$$a.\ P(\textit{has a flaw (major or minor).}) = p(major) + p(minor) = 0.15 + 0.5 = 0.2$$

$$b.\ P(\textit{has no major flows}) = 1 - p(major) = 1 - 0.05 = 0.95$$

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A system contains two components, A and B. The system will function only if both components function. The probability that A functions is 0.98, the probability that B functions is 0.95, and the probability that either A or B functions is 0.99. What is the probability that the system functions?

Ans.

The system will function if both A and B function

$$p(A \cap B) = p(A) + p(B) - p(A \cup B)$$

$$p(A \cap B) = 0.98 + 0.95 - 0.99 = 0.94$$

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Let S be the event that a randomly selected college student has taken a statistics course, and let C be the event that the same student has taken a chemistry course. Suppose $P(S) = 0.4$, $P(C) = 0.3$, and $P(S \cap C) = 0.2$.

a. Find the probability that a student has taken statistics, chemistry, or both.

b. Find the probability that a student has taken neither statistics nor chemistry.

c. Find the probability that a student has taken statistics but not chemistry.

Ans.

$$\boldsymbol{a.\ P(S \cup C) = P(S) + P(C) - P(S \cap C) = 0.4 + 0.3 - 0.2 = 0.5}$$

$$\boldsymbol{b.\ P(S^c \cap C^c) = 1 - P(S \cup C) = 1 - 0.5 = 0.5}$$

$$\boldsymbol{c.\ P(S - C) = P(S \cap C^c) = P(S) + -P(S \cap C) = 0.4 - 0.2 = 0.2}$$

True or false: If A and B are mutually exclusive,

a. $P(A \cup B) = 0$

b. $P(A \cap B) = 0$

c. $P(A \cup B) = P(A \cap B)$

d. $P(A \cup B) = P(A) + P(B)$

Ans.

a. False

b. True

c. False

d. True

Let V be the event that a computer contains a virus, and let W be the event that a computer contains a worm. Suppose $P(V) = 0.15$, $P(W) = 0.05$, and $P(V \cup W) = 0.17$.

a. Find the probability that the computer contains both a virus and a worm.

b. Find the probability that the computer contains neither a virus nor a worm.

c. Find the probability that the computer contains a virus but not a worm.

Ans.

(a) $$\begin{aligned} P(V \cap W) &= P(V) + P(W) - P(V \cup W) \\ &= 0.15 + 0.05 - 0.17 \\ &= 0.03 \end{aligned}$$

(b) $P(V^c \cap W^c) = 1 - P(V \cup W) = 1 - 0.17 = 0.83.$

(c) We need to find $P(V \cap W^c)$. Now $P(V) = P(V \cap W) + P(V \cap W^c)$ (this can be seen from a Venn diagram). We know that $P(V) = 0.15$, and from part (a) we know that $P(V \cap W) = 0.03$. Therefore $P(V \cap W^c) = 0.12$.

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Try yourself

All the fourth-graders in a certain elementary school took a standardized test. A total of 85% of the students were found to be proficient in reading, 78% were found to be proficient in mathematics, and 65% were found to be proficient in both reading and mathematics. A student is chosen at random.

a. What is the probability that the student is proficient in mathematics but not in reading?
b. What is the probability that the student is proficient in reading but not in mathematics?
c. What is the probability that the student is proficient in neither reading nor mathematics?

Try yourself

A system contains two components, A and B. The system will function so long as either A or B functions. The probability that A functions is 0.95, the probability that B functions is 0.90, and the probability that both function is 0.88. What is the probability that the system functions?

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Try yourself

Six hundred paving stones were examined for cracks, and 15 were found to be cracked. The same 600 stones were then examined for discoloration, and 27 were found to be discolored. A total of 562 stones were neither cracked nor discolored. One of the 600 stones is selected at random.

a. Find the probability that it is cracked, discolored, or both.

b. Find the probability that it is both cracked and discolored.

c. Find the probability that it is cracked but not discolored.

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Thank You