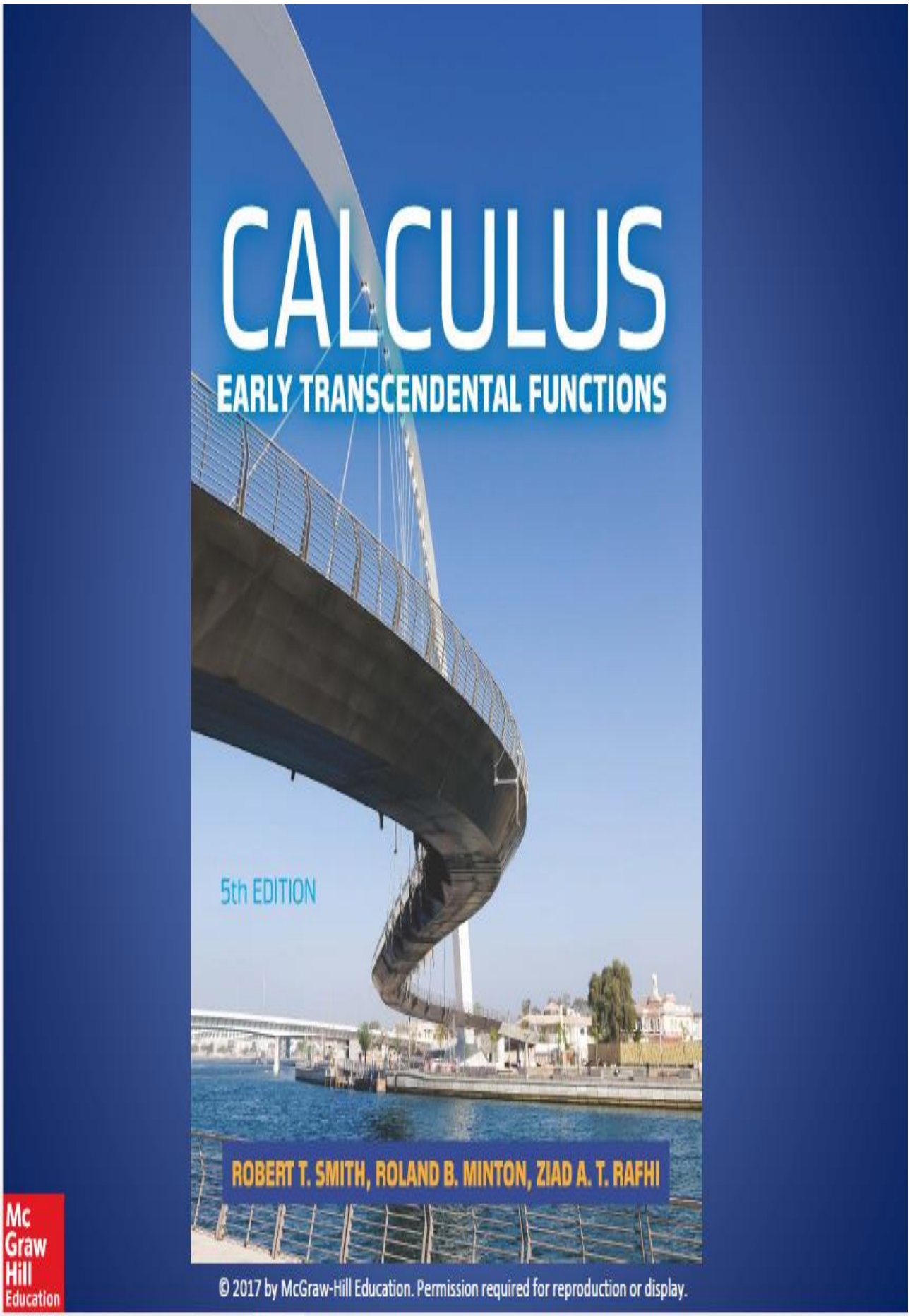

Methods of Integration

1- Integration by Parts

2- Integration by substitution

i) Algebraic substitutions

ii) Trigonometric substitutions

3- Integration by partial fractions

$$\int \frac{p(x)}{q(x)}\, dx\ ,$$

$$p(x)\ and\ q(x)\ are\ polynomials$$

If the degree of numerator < the degree of denominator

a) *The denominator consists of linear factors*

Example $\int \frac{1}{x^2+x-2}\,dx$

$$\frac{1}{x^2+x-2} =$$

$$= \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \quad (1)$$

$$\rightarrow 1 = A(x+2) + B(x-1) \quad (2)$$

$$at \quad x = 1 \quad \rightarrow \quad 1 = 3A \quad \rightarrow \quad A = \frac{1}{3}$$

$$at \quad x = -2 \quad \rightarrow \quad 1 = -3B \quad \rightarrow \quad B = -\frac{1}{3}$$

In (1) →

$$\rightarrow \frac{1}{x^2+x-2} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}}{x+2}$$

$$\rightarrow \frac{1}{x^2+x-2} = \frac{1}{3}\frac{1}{x-1} - \frac{1}{3}\frac{1}{x+2}$$

$$\rightarrow \int \frac{1}{x^2+x-2}\,dx =$$

$$= \frac{1}{3}\int \frac{1}{x-1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx$$

$$= \frac{1}{3}ln|x-1| - \frac{1}{3}ln|x+2| + c$$

b)*The denominator has repeated linear factors*

Example $$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

$$\frac{5x^2+20x+6}{x^3+2x^2+x}=\frac{5x^2+20x+6}{x(x^2+2x+1)}$$

$$=\frac{5x^2+20x+6}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2} \quad (1)$$

$$\rightarrow 5x^2+20x+6=A(x+1)^2+Bx(x+1)+Cx \quad (2)$$

$$at \quad x=0 \quad \rightarrow \quad 6=A$$

$$\rightarrow \quad A=6$$

$$at \quad x=-1 \quad \rightarrow \quad 5-20+6=-C$$

$$\rightarrow C=9$$

$$at\ \ x = 1 \rightarrow\ 5 + 20 + 6 = 4A + 2B + C$$

$$\rightarrow\ B = -1$$

in ① →

$$\rightarrow \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} = \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$$

$$\rightarrow \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\, dx =$$

$$= 6\int \frac{1}{x}\, dx - \int \frac{1}{x+1}\, dx + 9\int \frac{1}{(x+1)^2}\, dx$$

$$= 6\int \frac{1}{x}\, dx - \int \frac{1}{x+1}\, dx + 9\int (x+1)^{-2}\, dx$$

$$= 6ln|x| - ln|x+1| + 9\frac{(x+1)^{-1}}{-1} + c$$

$$= 6ln|x| - ln|x+1| - \frac{9}{x+1} + c$$