



# CALCULUS

## EARLY TRANSCENDENTAL FUNCTIONS

5th EDITION

**ROBERT T. SMITH, ROLAND B. MINTON, ZIAD A. T. RAFHI**

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## Integration

$$\begin{array}{ccc}
 & \text{Differentiation} & \\
 f & \rightleftharpoons & f' \\
 & \text{Integration(Antiderivative)} &
 \end{array}$$

### Types of Integral

**Indefinite Integral:**

$$\int f(x) dx$$

**Definite Integral:**

$$\int_a^b f(x) dx$$

### Rules of Definite Integrals

$$1. \quad \int dx = x + c$$

$$2. \quad \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

*Example*  $\int x^3 dx = \frac{x^4}{4} + c$

$$3. \quad \int a f(x) dx = a \int f(x) dx$$

**Example** 
$$\int 4 x^3 dx = 4 \int x^3 dx$$

$$= 4 \left( \frac{x^4}{4} \right) + c = x^4 + c$$

$$4. \quad \int (a f(x) \pm b g(x)) dx = a \int f(x) dx \pm b \int h(x) dx$$

**Example** 
$$\int (x^2 - 2x + 3) dx$$

$$= \int x^2 dx - 2 \int x dx + 3 \int dx$$

$$= \frac{x^3}{3} - x^2 + 3x + c$$

$$5. \quad \int f' (f)^n dx = \frac{f^{n+1}}{n+1} + c$$

**Example** 
$$\int (x + 5)^3 dx = \frac{(x + 5)^4}{4} + c$$

**Example**

$$\begin{aligned}
 & \int x(x^2 + 5)^3 dx \\
 &= \frac{1}{2} \int 2x(x^2 + 5)^3 dx \\
 &= \frac{1}{2} \frac{(x^2 + 5)^4}{4} + c = \frac{(x^2 + 5)^4}{8} + c
 \end{aligned}$$

**Example**

$$\begin{aligned}
 & \int \frac{(\sqrt{x} + 1)^4}{\sqrt{x}} dx \\
 &= \int \frac{1}{\sqrt{x}} (\sqrt{x} + 1)^4 dx = 2 \int \frac{1}{2\sqrt{x}} (\sqrt{x} + 1)^4 dx \\
 &= (2) \frac{(\sqrt{x} + 1)^5}{5} + c = \frac{2(\sqrt{x} + 1)^5}{5} + c
 \end{aligned}$$

6.

$$\int f f' dx = \frac{f^2}{2} + c$$

**Example**

$$\int \sin x \cos x dx = \frac{\sin^2 x}{2} + c$$

**Example**

$$\begin{aligned}
 \int \frac{\ln x}{x} dx &= \int \frac{1}{x} \ln x dx \\
 &= \frac{(\ln x)^2}{2} + c
 \end{aligned}$$

7.

$$\int \frac{f'}{f} dx = \ln|f| + c$$

*Example*

$$\int \frac{1}{x} dx = \ln|x| + c$$

*Example*

$$\begin{aligned} \int \frac{x}{x^2 + 1} dx &= \frac{1}{2} \int \frac{2x}{x^2 + 1} dx \\ &= \frac{1}{2} \ln|x^2 + 1| + c \end{aligned}$$

8.

$$\int \frac{f'}{\sqrt{f}} dx = 2\sqrt{f} + c$$

*Example*

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$$

*Example*

$$\begin{aligned} \int \frac{x}{\sqrt{x^2 + 1}} dx &= \frac{1}{2} \int \frac{2x}{\sqrt{x^2 + 1}} dx \\ &= \frac{1}{2} \left( 2\sqrt{x^2 + 1} \right) + c = \sqrt{x^2 + 1} + c \end{aligned}$$

$$\begin{aligned} 9. \quad \int \frac{f'}{(f)^n} dx &= \int f'(f)^{-n} dx \\ &= \frac{f^{-n+1}}{-n+1} + c \end{aligned}$$

**Example**

$$\begin{aligned} \int \frac{1}{(x+2)^3} dx &= \int (x+2)^{-3} dx \\ &= \frac{(x+2)^{-2}}{-2} + c \\ &= \frac{-1}{2(x+2)^2} + c \end{aligned}$$

## **Exponential functions**

1. 
$$\int f' e^f dx = e^f + c$$

**Example** 
$$\int e^x dx = e^x + c$$

**Example** 
$$\int e^{2x} dx = \frac{1}{2} \int 2e^{2x} dx$$

$$= \frac{1}{2} e^{2x} + c$$

**Example** 
$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$$

$$= 2 \int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = 2e^{\sqrt{x}} + c$$

2.

$$\int f' a^f dx = \frac{a^f}{\ln a} + c$$

**Example**

$$\int 3^x dx = \frac{3^x}{\ln 3} + c$$

**Example**

$$\int 3^{2x} dx = \frac{1}{2} \int 2(3^{2x}) dx$$

$$= \frac{1}{2} \frac{3^{2x}}{\ln 3} + c = \frac{3^{2x}}{2 \ln 3} + c$$