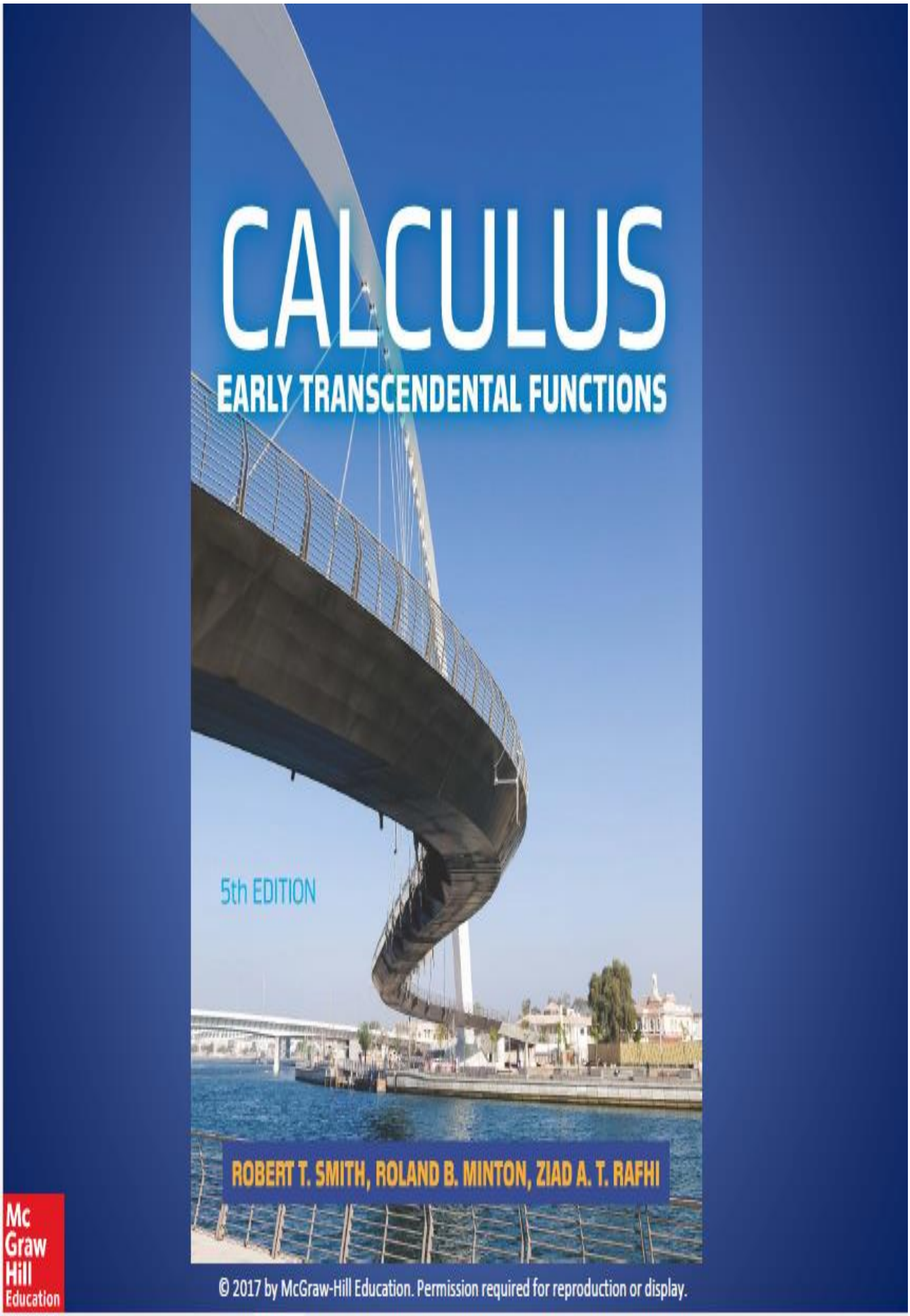

Methods of Integration

1- Integration by Parts

2- Integration by substitution

i) Algebraic substitutions

ii) Trigonometric substitutions

3- Integration by partial fractions

$$\int \frac{p(x)}{q(x)}\, dx\ ,$$

$$p(x)\ and\ q(x)\ are\ polynomials$$

The degree of the numerator is less than the degree of the denominator

a)*The denominator consists of linear factors*

b)*The denominator has repeated linear factors*

c)*A factor of second degree that does not decompose*

Example $$\int \frac{2x^2 - 5x + 2}{x^3 + x}\, dx$$

$$\frac{2x^2 - 5x + 2}{x^3 + x} =$$

$$= \frac{2x^2 - 5x + 2}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

$$\rightarrow 2x^2 - 5x + 2 = A(x^2 + 1) + (Bx + C)x$$

$$\rightarrow 2x^2 - 5x + 2 = Ax^2 + A + Bx^2 + Cx$$

$$\rightarrow 2x^2 - 5x + 2 = (A + B)x^2 + Cx + A$$

$$\rightarrow A = 2$$

$$\rightarrow C = -5$$

$$\rightarrow A + B = 2$$

$$\rightarrow B = 2 - A = 2 - 2 \rightarrow B = 0$$

$$\rightarrow \frac{2x^2 - 5x + 2}{x(x^2 + 1)} = \frac{2}{x} + \frac{-5}{x^2 + 1}$$

$$\rightarrow \int \frac{2x^2 - 5x + 2}{x(x^2 + 1)}\, dx =$$

$$= \int \frac{2}{x}\, dx + \int \frac{-5}{1 + x^2}\, dx$$

$$= 2\int \frac{1}{x}\, dx - 5\int \frac{1}{1 + x^2}\, dx$$

$$= 2ln|x| - 5\, tan^{-1}x + c$$

The degree of the numerator is more than or equal to the degree of the denominator

Example $$\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\, dx$$

$$\begin{array}{r} 2x \\ x^2 - 2x - 8 \overline{)\ 2x^3 - 4x^2 - 15x + 5} \\ \underline{2x^3 - 4x^2 - 16x} \\ x + 5 \end{array}$$

$$\frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} = 2x + \frac{x + 5}{x^2 - 2x - 8}$$

$$\frac{x + 5}{x^2 - 2x - 8} =$$

$$= \frac{x + 5}{(x + 2)(x - 4)} = \frac{A}{x + 2} + \frac{B}{x - 4}$$

$$\rightarrow x + 5 = A(x - 4) + B(x + 2)$$

$$at \quad x = -2 \quad \rightarrow \quad 3 = \ -6A \quad \rightarrow \quad A = -\frac{1}{2}$$

$$at \quad x = 4 \quad \rightarrow \quad 9 = \ 6B \quad \rightarrow \quad B = \frac{3}{2}$$

$$\rightarrow \frac{x+5}{x^2-2x-8} = \frac{-\frac{1}{2}}{x+2} + \frac{\frac{3}{2}}{x-4}$$

$$\rightarrow \frac{2x^3-4x^2-15x+5}{x^2-2x-8} =$$

$$= 2x + \frac{-\frac{1}{2}}{x+2} + \frac{\frac{3}{2}}{x-4}$$

$$\rightarrow \int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\, dx =$$

$$= 2\int x\, dx - \frac{1}{2}\int \frac{1}{x+2}\, dx + \frac{3}{2}\int \frac{1}{x-4}\, dx$$

$$= x^2 - \frac{1}{2} ln|x+2| + \frac{3}{2} ln|x-4| + c$$