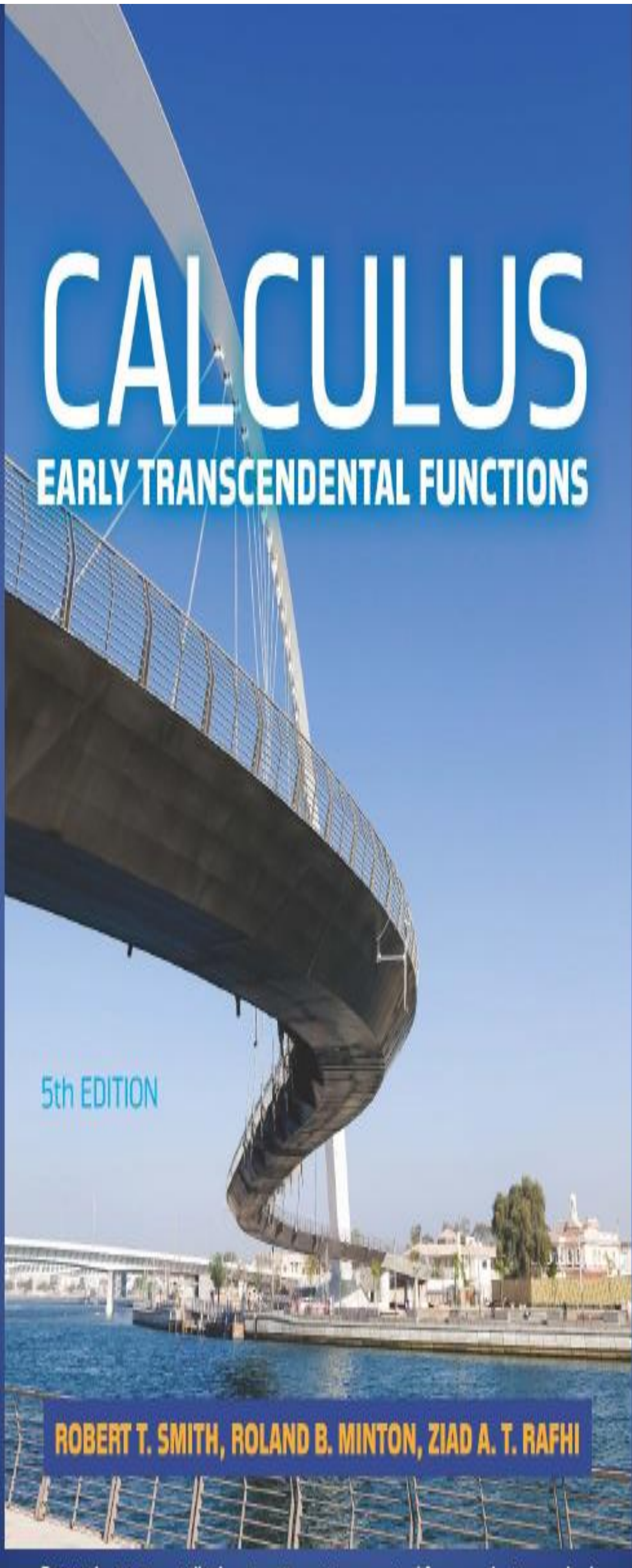
CALCULUS
EARLY TRANSCENDENTAL FUNCTIONS
5th EDITION
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Methods of Integration

Integration by Parts

$$\int u\,dv = u\,v - \int v\,du$$

Selection order of u

1. $Inverse\ trigonometric\ functions\ or\ logarithmic\ functions.$

2. $Polynomials.$

3. $Free\ choice.$

Example $$\int x\, e^x\, dx$$

$$u = x \quad \rightarrow \quad \frac{du}{dx} = 1 \quad \rightarrow \quad du = dx$$

$$dv = e^x\, dx \rightarrow \int dv = \int e^x\, dx \rightarrow \quad v = e^x$$

$$\int x\, e^x\, dx = x\, e^x - \int e^x\, dx$$

$$= x\, e^x - e^x + c$$

Example $$\int x\, e^x\, dx$$

	$differentiation$	$Integration$
$+$	x	e^x
$-$	1	e^x
$+$	0	e^x

$$\int x\, e^x\, dx = x\, e^x - e^x + c$$

Example $$\int x\,sinx\,dx$$

$$u = x \quad \rightarrow \frac{du}{dx} = 1 \rightarrow \quad du = dx$$

$$dv = sinxdx \rightarrow \int dv = \int sinx\,dx \rightarrow \quad v = -cosx$$

$$\int x\,sinx\,dx = x\,(-cosx) - \int (-cosx)\,dx$$

$$= -x\,cosx + \int cosx\,dx$$

$$= -x\,cosx + sinx + c$$

Example $$\int x\,sinx\,dx$$

	$differentiation$	$Integration$
$+$	x	$sinx$
$-$	1	$-cosx$
$+$	0	$-sinx$

$$\int x\,sinx\,dx = -x\,cosx + sinx + c$$

Example $$\int lnx\, dx$$

$$u = lnx \quad \rightarrow \frac{du}{dx} = \frac{1}{x} \rightarrow \quad du = \frac{1}{x} dx$$

$$dv = dx \rightarrow \int dv = \int dx \rightarrow \quad v = x$$

$$\int lnx\, dx = x\, lnx - \int x\, \frac{1}{x}\, dx$$

$$= x\, lnx - \int dx$$

$$= \; x\, lnx - x + c$$

Example $$\int x\, lnx\, dx$$

$$u = lnx \quad \rightarrow \frac{du}{dx} = \frac{1}{x} \rightarrow \quad du = \frac{1}{x} dx$$

$$dv = x\, dx \rightarrow \int dv = \int x\, dx \rightarrow \quad v = \frac{x^2}{2}$$

$$\int x\, lnx\, dx = \frac{x^2 lnx}{2} - \int \frac{x^2}{2} \frac{1}{x}\, dx$$

$$= \frac{x^2 lnx}{2} - \frac{1}{2} \int x dx$$

$$= \frac{x^2 lnx}{2} - \frac{x^2}{4} + c$$

Example $$\int \sin^{-1}x\,dx$$

$$u = \sin^{-1}x \rightarrow \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \rightarrow \quad du = \frac{1}{\sqrt{1-x^2}}dx$$

$$dv = dx \rightarrow \int dv = \int dx \rightarrow \quad v = x$$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x \; - \int \frac{x}{\sqrt{1-x^2}}dx$$

$$= x\sin^{-1}x \; - \left(\frac{1}{-2}\right)\int \frac{-2x}{\sqrt{1-x^2}}dx$$

$$= x\sin^{-1}x \; + \frac{1}{2}\left(2\sqrt{1-x^2}\right) + c$$

$$= x\sin^{-1}x \; + \sqrt{1-x^2} + c$$

Example $$\int x^2\, e^x\, dx$$

$$u = x^2 \quad \rightarrow \frac{du}{dx} = 2x \rightarrow \quad du = 2x\, dx$$

$$dv = e^x\, dx \rightarrow \int dv = \int e^x\, dx \rightarrow \quad v = e^x$$

$$\int x^2\, e^x\, dx = x^2\, e^x - \int 2xe^x dx$$

$$\int x^2\, e^x\, dx = x^2\, e^x - 2\int xe^x dx \qquad (1)$$

Finding $\int xe^x dx$

$$u = x \quad \rightarrow \frac{du}{dx} = 1 \rightarrow \quad du = dx$$

$$dv = e^x\, dx \rightarrow \int dv = \int e^x\, dx \rightarrow \; v = e^x$$

$$\int x\, e^x\, dx = x\, e^x - \int e^x dx$$

$$\int x\, e^x\, dx = x\, e^x - e^x \qquad (2)$$

From (2) into (1), we get

$$\int x^2\, e^x\, dx = x^2\, e^x - 2(x\, e^x - e^x) + c$$

$$\int x^2\, e^x\, dx = x^2\, e^x - 2x\, e^x + 2e^x + c$$

$$\int x^2\, e^x\, dx = (x^2\, e^x - 2x\, + 2)e^x + c$$

Example $\int x^2\, e^x\, dx$

	$differentiation$	$Integration$
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
$-$	0	e^x

$$\int x^2\, e^x\, dx = x^2\, e^x - 2x\, e^x + 2e^x + c$$

Example $$\int x^2 \, sinx \, dx$$

	$differentiation$	$Integration$
$+$	x^2	$sinx$
$-$	$2x$	$-\, cosx$
$+$	2	$-\, sinx$
$-$	0	$cosx$

$$\int x^2 \, sinx \, dx = -x^2 \, cosx + 2xsinx + 2cosx + c$$

Method of Substitution

$$\int f(x)\,dx \qquad (1)$$

Let

$$x = h(w)$$

$$\frac{dx}{dw} = h'(w) \qquad (2)$$

$$dx = h'(w)dw$$

From (2) into (1), we get

$$\int f\big(h(w)\big)\,h'(w)dw$$

Algebraic substitutions

Example $$\int x(x+1)^8\,dx$$

Let

$$x+1=w$$

$$x=w-1$$

$$\frac{dx}{dw}=1$$

$$dx=dw$$

$$\int x(x+1)^8\,dx=\int (w-1)(w-1+1)^8\,dw$$

$$=\int (w-1)w^8\,dw$$

$$=\int (w^9-w^8)\,dw$$

$$= \int w^9 \, dw - \int w^8 \, dw$$

$$= \frac{1}{10} w^{10} - \frac{1}{9} w^9 + c$$

$$= \frac{1}{10} (x+1)^{10} - \frac{1}{9} (x+1)^9 + c$$

Example $$\int \frac{1}{(x+2)\sqrt{x+1}}\,dx$$

Let

$$x + 1 = w^2$$

$$x = w^2 - 1$$

$$\frac{dx}{dw} = 2w$$

$$dx = 2w\,dw$$

$$\int \frac{1}{(x+2)\sqrt{x+1}}\,dx =$$

$$= \int \frac{1}{(w^2 - 1 + 2)\sqrt{w^2 - 1 + 1}}\,2w\,dw$$

$$= \int \frac{1}{(w^2 + 1)w}\,2w\,dw$$

$$= 2\int \frac{1}{1 + w^2}\,dw$$

$$= 2\,tan^{-1}w + c$$

$$= 2\,tan^{-1}\sqrt{x+1} + c$$