Lecture 1

Probability

Textbook: Statistics for Engineers and Scientists

Fifth Edition

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Section 2.1: Basic Ideas

Examples:

- rolling a die.
- tossing a coin.
- weighing the contents of a box of cereal.

Definition: An **experiment** is a process that results in an outcome that cannot be predicted in advance with certainty.

Sample Space

Definition: The set of all possible outcomes of an experiment is called the **sample space** for the experiment.

Examples:

- For rolling a six-sided die, the sample space is {1, 2, 3, 4, 5, 6}.
- For a coin toss, the sample space is {Heads, Tails}.

More Terminology

- Definition: A subset of a sample space is called an **event**.
- For any sample space, the empty set $\varnothing$ is an event, as is the entire sample space.
- A given event is said to have occurred if the outcome of the experiment is one of the outcomes in the event. For example, if a die comes up 2, the events {2, 4, 6} and {1, 2, 3} have both occurred, along with every other event that contains the outcome "2."

Example 1 [1]

An electrical engineer has on hand two boxes of resistors, with four resistors in each box. The resistors in the first box are labeled 10 Ω (ohms), but in fact their resistances are 9, 10, 11, and $12\,\Omega$. The resistors in the second box are labeled 20 but in fact their resistances are 18, 19, 20, and $21\,\Omega$. The engineer chooses one resistor from each box and determines the resistance of each.

Example 1 2

Let *A* be the event that the first resistor has a resistance greater than 10, let *B* be the event that the second resistor has resistance less than 19, and let *C* be the event that the sum of the resistances is equal to 28.

1. Find the sample space for this experiment.
2. Specify the subsets corresponding to the events *A*, *B*, and *C.*

We will denote this sample space by $\boldsymbol{S}$.

$$\boldsymbol{S} = \{(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$

The events A, B, and C are given by

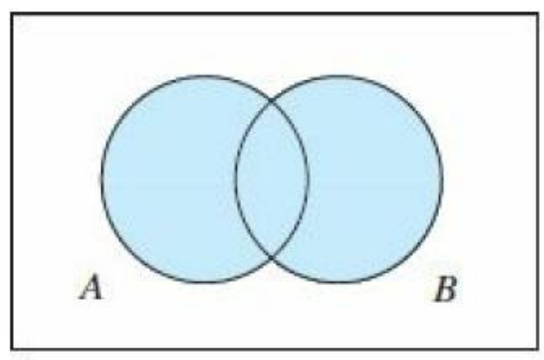
$$A = \{(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$
$$B = \{(9, 18), (10, 18), (11, 18), (12, 18)\}$$
$$C = \{(9, 19), (10, 18)\}$$

Combining Events

The **union** of two events *A* and *B*, denoted $A \cup B$, is the set of outcomes that belong either to *A*, to *B*, or to both.

In words, $A \cup B$ means "*A* or *B.*" So the event "*A* or *B*" occurs whenever either *A* or *B* (or both) occurs.

$A \cup B$



Example 2

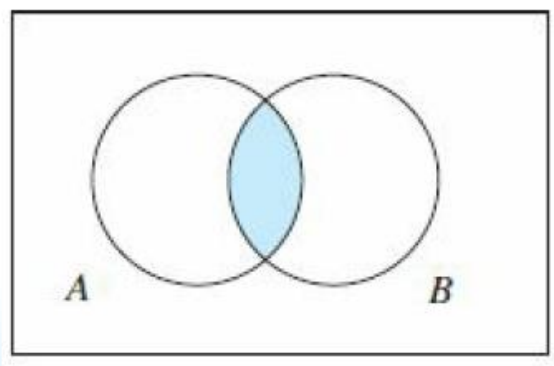
Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$.

What is $A \cup B$?

Intersections

- The **intersection** of two events *A* and *B*, denoted by $A \cap B$, is the set of outcomes that belong both to *A* and to *B*.
- In words, $A \cap B$ means "*A* and *B.*" Thus the event "*A* and *B*" occurs whenever both *A* and *B* occur.

$$A \cap B$$



Example 3

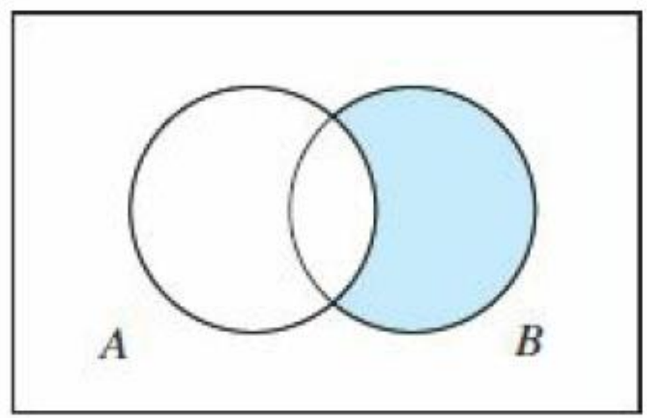
Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$.

What is $A \cap B$?

Complements

- The **complement** of an event *A*, denoted A^c, is the set of outcomes that do not belong to *A*.
- In words, A^c means "not *A.*" Thus the event "not *A*" occurs whenever *A* does **not** occur.

$$B \cap A^c.$$



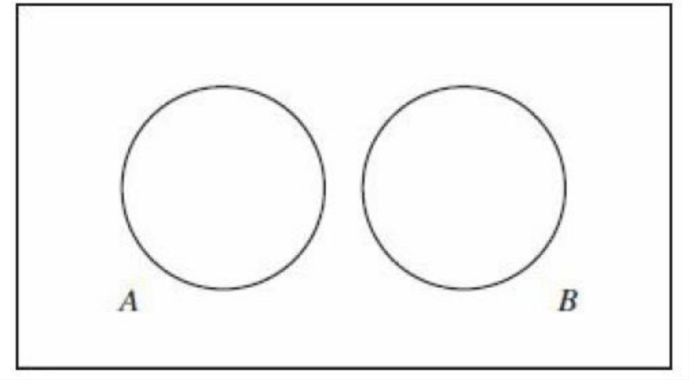
Example 4

Consider rolling a six-sided die. Let *A* be the event: "rolling a six" = {6}.

What is A^c in words? What outcomes are in A^c?

Mutually Exclusive Events

- Definition: The events *A* and *B* are said to be **mutually exclusive (**disjoint**)** if they have no outcomes in common.
- More generally, a collection of events $A_1, A_2, \ldots, A_n$ is said to be mutually exclusive if no two of them have any outcomes in common.



Back to Example 1

Suppose the experiment with the resistors is performed.

- Is it possible for events A and B both to occur?
- How about B and C?
- A and C?
- Which pair of events is mutually exclusive?

$$S = \{(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$

Let A be the event that the first resistor has a resistance greater than 10

let B be the event that the second resistor has a resistance less than 19

let C be the event that the sum of the resistances is equal to 28.

The events A, B, and C are given by

$$A = \{(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$
$$B = \{(9, 18), (10, 18), (11, 18), (12, 18)\}$$
$$C = \{(9, 19), (10, 18)\}$$

Solution

If the outcome is (11, 18) or (12, 18), then events A and B both occur. If the outcome is (10, 18), then both B and C occur. It is impossible for A and C both to occur, because these events are mutually exclusive, having no outcomes in common.

Probabilities

Definition: Each event in the sample space has a **probability** of occurring. Intuitively, the probability is a quantitative measure of how likely the event is to occur.

Given any experiment and any event A:

- The expression $P(A)$ denotes the probability that the event A occurs.
- $P(A)$ is the proportion of times that the event A would occur in the long run, if the experiment were to be repeated over and over again.

Axioms of Probability

1. Let ***S*** be a sample space. Then *P*(***S***) = 1.
2. For any event *A*, $0 \le P(A) \le 1$.
3. If *A* and *B* are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$. More generally, if $A_1, A_2, \ldots$ are mutually exclusive events, then

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

A Few Useful Things

- For any event *A*,

$$P\left(A^C\right) = 1 - P\left(A\right).$$

- Let $\varnothing$ denote the empty set. Then

$$P\left(\varnothing\right) = 0.$$

- General Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 6

In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01.

1. What is the probability that a randomly chosen can has a flaw?
2. What is the probability that it has no flaw?
3. What is the probability that a can has a flaw on the top but not on the side?

Solution

We are given that P(flaw on side) = 0.02, P(flaw on top) = 0.03, and P(flaw on side and flaw on top) = 0.01. Now P(flaw) = P(flaw on side or flaw on top). Using Equation (2.5),

$$\begin{aligned} P(\text{flaw on side or flaw on top}) &= P(\text{flaw on side}) + P(\text{flaw on top}) \\ &\quad - P(\text{flaw on side and flaw on top}) \\ &= 0.02 + 0.03 - 0.01 \\ &= 0.04 \end{aligned}$$

To find the probability that a can has no flaw, we compute

$$P(\text{no flaw}) = 1 - P(\text{flaw}) = 1 - 0.04 = 0.96$$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

We know that $P(A) = 0.03$ and $P(A \cap B) = 0.01$. Therefore $0.03 = 0.01 + P(A \cap B^c)$, so $P(A \cap B^c) = 0.02$.

بعض القوانين الهامة :-

مجموع الاحتمالات فى $S = 1$

1- $P(A^C) = 1 - P(A)$ احتمال عدم حدوث A

2- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

احتمال حدوث A او B او كليهما

3- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

احتمال حدوث A و B معا

4- $P(A - B) = P(A \cap B^C) = P(A) - P(A \cap B)$

احتمال حدوث A وعدم حدوث B

5- $P(B - A) = P(B \cap A^C) = P(B) - P(A \cap B)$

احتمال حدوث B و عدم حدوث A

6- (from De Morgane's law)

$P(A^C \cap B^C) = P(A \cup B)^C = 1 - P(A \cup B)$

عدم حدوث الاثنين معا

7- (from De Morgane's law)

$P(A^C \cup B^C) = P(A \cap B)^C = 1 - P(A \cap B)$

عدم حدوث احدهما على الاقل