Lecture 4

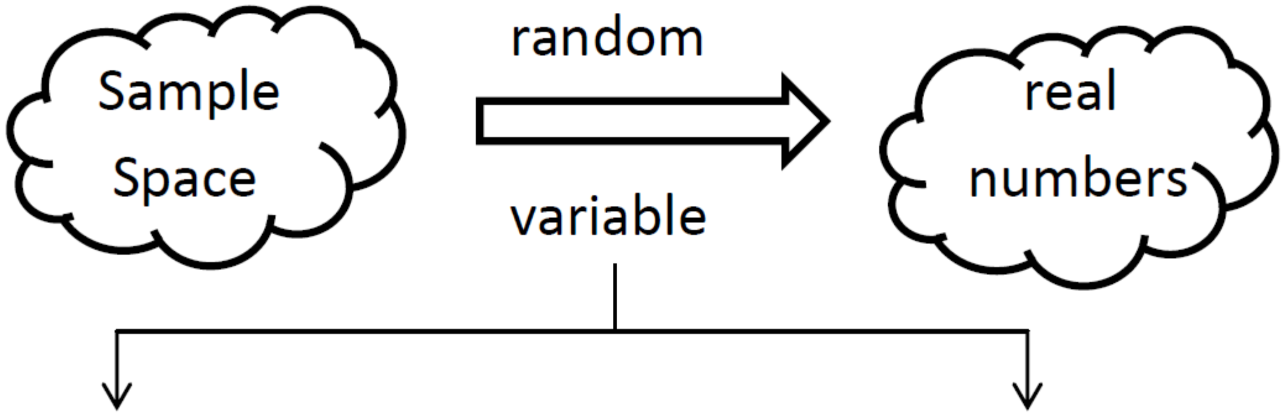
Random variable

Random Variables

- Definition: A **random variable** assigns a numerical value to each outcome in a sample space.
- Definition: A random variable is **discrete** if its possible values form a discrete set.

This means that if the possible values are arranged in order, there is a gap between each value and the next one. The set of possible values may be infinite; for example, the set of all integers is a discrete set.

المتغير العشوائى يقوم بنقل جميع العناصر الموجوده فى sample space الى ارقام على خط الاعداد (يحول جميع نتائج التجربه الاحتماليه الى ارقام)



Discrete	Continuous
عدد عناصر محدده	عدد عناصره Infinite
finite	عدد لا نهائى من القيم

Example

The number of flaws in a 1-inch length of copper wire manufactured by a certain process varies from wire to wire. Overall, 48% of the wires produced have no flaws, 39% have one flaw, 12% have two flaws, and 1% have three flaws. Let X be the number of flaws in a randomly selected piece of wire.

Then $P(X = 0) = 0.48$, $P(X = 1) = 0.39$, $P(X = 2) = 0.12$, and $P(X = 3) = 0.01$. The list of possible values 0, 1, 2, and 3, along with the probabilities of each, provide a complete description of the population from which X was drawn.

Example

Computer chips often contain surface imperfections. For a certain type of computer chip, 9% contain no imperfections, 22% contain 1 imperfection, 26% contain 2 imperfections, 20% contain 3 imperfections, 12% contain 4 imperfections, and the remaining 11% contain 5 imperfections. Let Y represent the number of imperfections in a randomly chosen chip. What are the possible values for Y? Is Y discrete or continuous? Find $P(Y = y)$ for each possible value y.

Solution

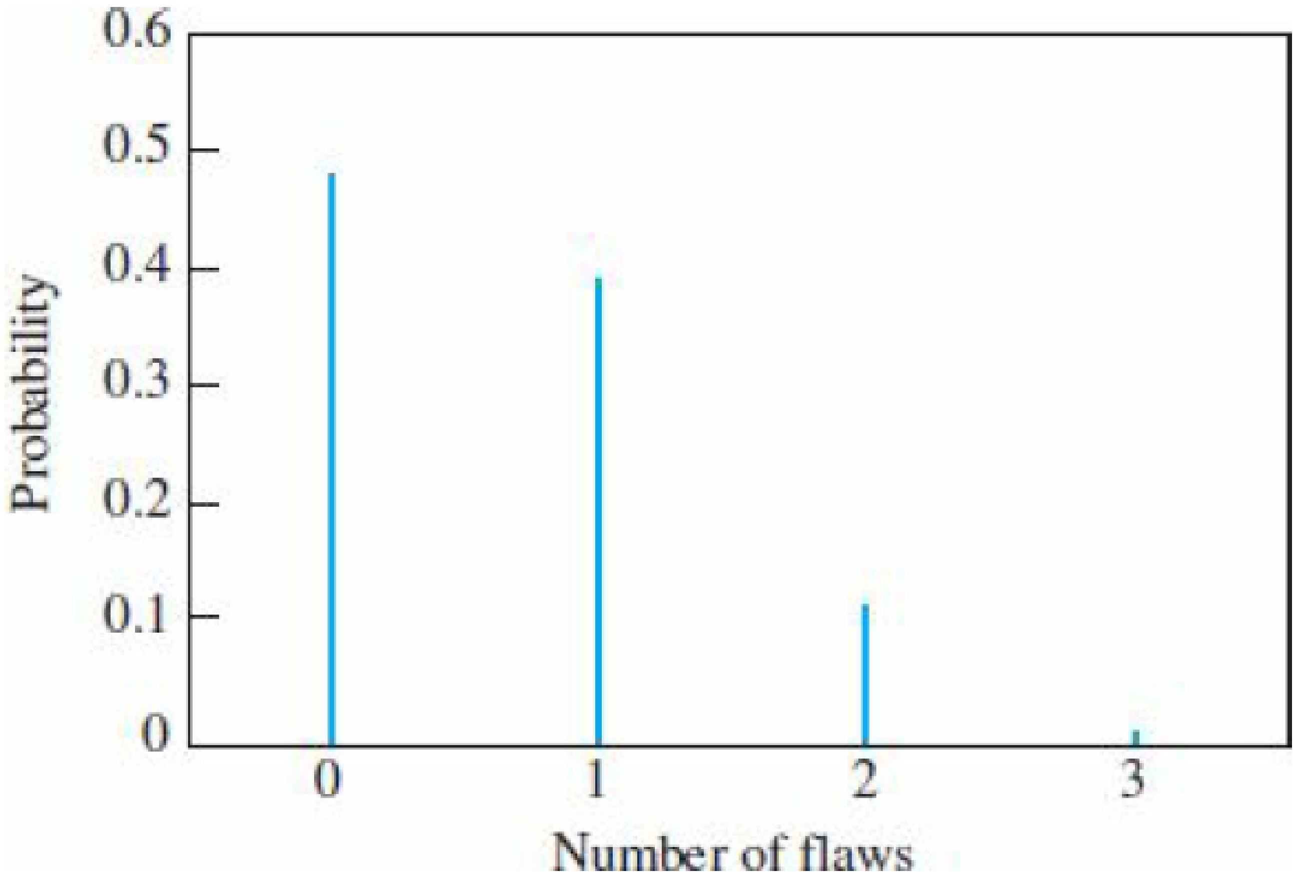
The possible values for Y are the integers 0, 1, 2, 3, 4, and 5. The random variable Y is discrete, because it takes on only integer values. Nine percent of the outcomes in the sample space are assigned the value 0. Therefore $P(Y = 0) = 0.09$. Similarly $P(Y = 1) = 0.22$, $P(Y = 2) = 0.26$, $P(Y = 3) = 0.20$, $P(Y = 4) = 0.12$, and $P(Y = 5) = 0.11$.

Probability Mass Function

- The description of the possible values of X and the probabilities of each has a name: the probability mass function.
- Definition: The **probability mass function** (pmf) of a discrete random variable X is the function

$$p(x) = P(X = x).$$

- The probability mass function is sometimes called the **probability distribution**.



0.6
0.5
0.4
0.3
0.2
0.1
0
Probability
0
1
2
3
Number of flaws

Cumulative Distribution Function

- The probability mass function specifies the probability that a random variable is *equal to* a given value. A function called the **cumulative distribution function** (cdf) specifies the probability that a random variable is *less than or equal to* a given value.
- The cumulative distribution function of the random variable *X* is the function $F(x) = P(X \leq x)$.

Summary for Discrete Random Variables

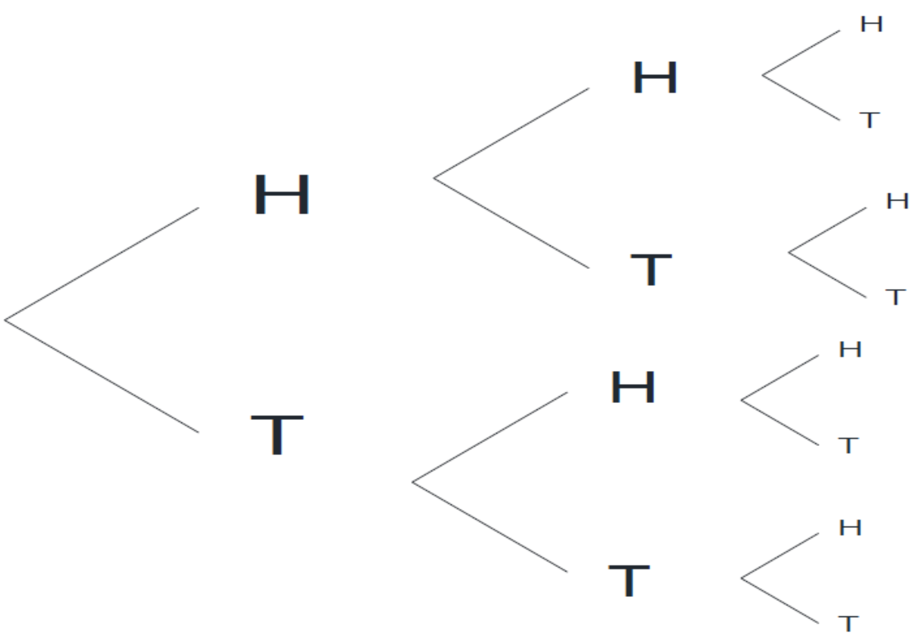
Let X be a discrete random variable. Then,

- The probability mass function of X is the function $p(x) = P(X = x)$.
- The cumulative distribution function of X is the function $F(x) = P(X \leq x)$.
- $F(x) = \sum_{t \leq x} p(t) = \sum_{t \leq x} p(X = 1).$
- $\sum_{x} p(x) = \sum_{x} P(X = x) = 1,$

  where the sum is over all the possible values of X.

Ex 1:- Coin is tossed 3 times . let x be random variable that denotes the number of heads appear . Find probability density function and commulative function and sketch them

sol



$n(S) = 8$

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

X = is number of head appear

0 H   1 H   2 H   3 H

X = 0 , 1 , 2 , 3

	x	0	1	2	3
Density function	P (x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
Cumulative function	F_X (x)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{8}{8} = 1$

$\frac{4}{8} = \frac{1}{8} + \frac{3}{8}$

$\frac{7}{8} = \frac{1}{8} + \frac{3}{8} + \frac{4}{8}$

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قيمه = 1

Mean for Discrete Random Variables

- Let X be a discrete random variable with probability mass function $p(x) = P(X = x)$.
- The **mean** of X is given by

$$\mu_X = \sum_x xP(X = x),$$

where the sum is over all possible values of X.

- The mean of X is sometimes called the expectation, or expected value, of X and may $E(X)$ or by μ.

Variance for Discrete Random Variables

- Let X be a discrete random variable with probability mass function $p(x) = P(X = x)$.
- The **variance** of X is given by

$$\sigma_X^2 = \sum_x (x - \mu_X)^2 P(X = x)$$
$$= \sum_x x^2 P(X = x) - \mu_X^2 .$$

- The variance of X may also be denoted by $V(X)$ or by σ^2.
- The standard deviation is the square root of the variance: $\sigma_X = \sqrt{\sigma_X^2}$.

Example

A certain industrial process is brought down for recalibration whenever the quality of the items produced falls below specifications. Let X represent the number of times the process is recalibrated during a week, and assume that X has the following probability mass function.

x	0	1	2	3	4
$p(x)$	0.35	0.25	0.20	0.15	0.05

Find the mean and standard deviation of X.

Thank You