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Lecture 2

Counting Principals

Counting Methods

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- **The Fundamental Principle of Counting**:

  Assume that *k* operations are to be performed. If there are n_1 ways to perform the first operation, and if for each of these ways there are n_2 ways to perform the second calculation, and so on, then the total number of ways to perform the sequence of *k* operations is

$$n_1 n_2 \cdots n_k$$

Example

A certain make of automobile is available in any of three colors: red, blue, or green, and comes with either a large or small engine. In how many ways can a buyer choose a car?

Solution

There are three choices of color and two choices of engine. A complete list of choices is written in the following 3×2 table. The total number of choices is $(3)(2) = 6$.

	Red	Blue	Green
Large	Red, Large	Blue, Large	Green, Large
Small	Red, Small	Blue, Small	Green, Small

Example

When ordering a certain type of computer, there are 3 choices of hard drive, 4 choices for the amount of memory, 2 choices of video card, and 3 choices of monitor. In how many ways can a computer be ordered?

Permutations

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- A **permutation** is an ordering of a collection of objects. The number of permutations of *n* objects is

$$n(n-1)(n-2)\cdots(3)(2)(1)$$

- This is the product of the integers from 1 to *n* and can be written as *n*!, read "*n* factorial."

  Note: We define 0! = 1.
- So, the number of permutations of *n* objects is *n*!.

عدد الترتيبات التي يمكن الحصول عليها من ترتيب n من العناصر

Permutations

- The number of permutations of *k* objects chosen from a group of *n* objects is

$$\frac{n!}{(n-k)!}$$

- When order matters, use permutations.

عملية اختيار مجموعه عددها k من مجموعه عددها n بشرط الترتيب

Example

- Five people stand in line at a movie theater. Into how many different orders can they be arranged?

Ans. 5!

- Five lifeguards are available for duty one Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations?

Ans. 5!/(5-3)!

Combinations

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- In some cases, when choosing a set of objects from a larger set, we don’t care about the ordering of the chosen objects; we care only which objects are chosen.
- Each distinct group of objects that can be selected, without regard to order, is called a **combination**.

Combinations

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- The number of combinations of *k* objects chosen from a group of *n* objects is

$$\frac{n!}{k!(n-k)!}$$

Combinations

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- This can be extended to more than two groups.
- The number of ways to divide a group of *n* objects into groups of $k_1, \ldots, k_r$ objects where $k_1 + \cdots + k_r = n$ is

$$\frac{n!}{k_1! \cdots k_r!}$$

Example

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At a certain event, 30 people attend, and 5 will be chosen at random to receive door prizes. The prizes are all the same, so the order in which people are chosen does not matter. How many different groups of five people can be chosen?

Ans. 30!/(5!*25!)

Example

A six-sided die is rolled 20 times. Given that three of the rolls came up 1, five came up 2, four came up 3, two came up 4, three came up 5, and three came up 6, how many different arrangements of the outcomes are there?

$$\frac{20!}{3!\,5!\,4!\,2!\,3!\,3!}$$

انواع السحب

With replacement سحب مع ارجاع

Without replacement سحب بدون ارجاع

مسائل السحب يمكن حلها باستخدام قاعدة ضرب الاحتمالات المباشر ضرب احتمال كل عملية سحب في الأخرى مع مراعاة الترتيب

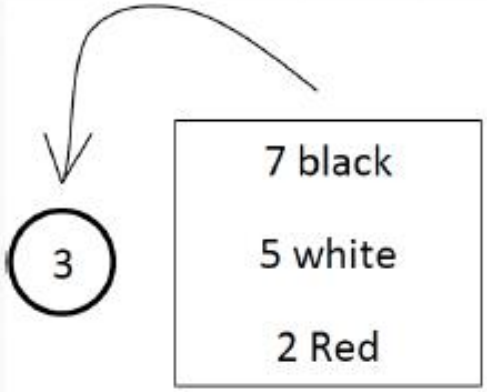
Ex 18:- An urn (وعاء) contains seven black balls , five white balls and two red balls . If we draw three balls without replacement what is the probability that they are of the same colour.

P (all ball are the same colour).

= p (3B or 3W or 3Red)

= p (3B) + P (3W) + P (3R)

$$= \frac{\binom{7}{3}\binom{5}{0}\binom{2}{0}}{\binom{14}{3}} + \frac{\binom{7}{0}\binom{5}{3}\binom{2}{0}}{\binom{14}{3}} + \frac{(0)}{\binom{14}{3}}$$

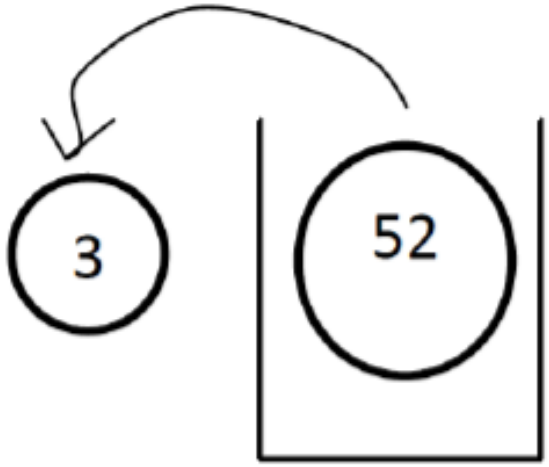


هنا الترتيب غير مهم

Ex 19:- Three cards are drawn from ordinary 52 cards check (الكوتشينه) with-out replacement , what is the probability that none of the three cards is a heart.

p (none of 3 cards heart)

$$= \frac{\binom{13}{0}\binom{39}{3}}{\binom{52}{3}}$$



4-Independent events

احداث يمكن حدوثها فى نفس الوقت لكن حدوث
حدث لا يؤثر على حدوث الاخر

A and B independent

$P(A \cap B) = P(A) \cdot P(B)$

$P(A^C \cap B^C) = P(A^C) \cdot P(B^C)$

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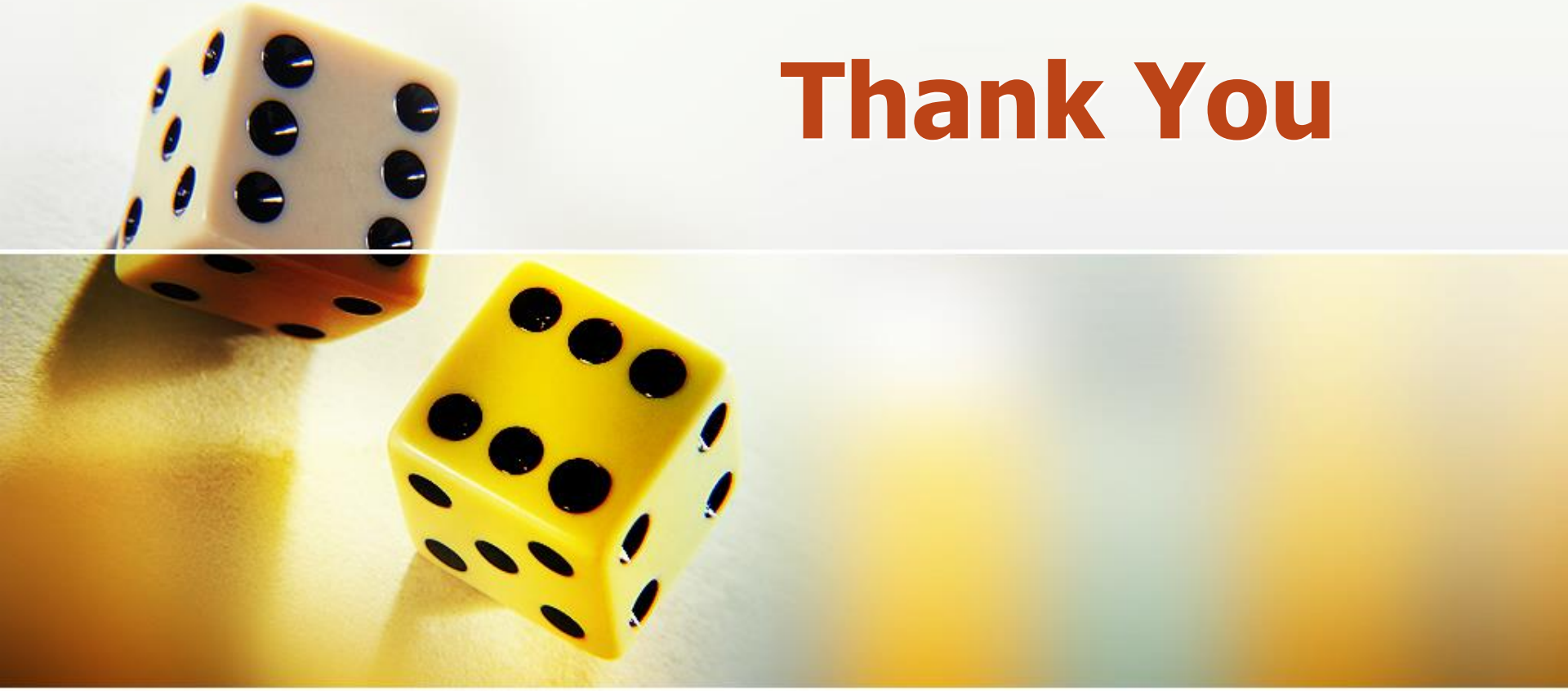

Thank You