Tutorial 2

At a certain car dealership, 20% of customers who bought a new vehicle bought an SUV, and 3% of them bought a black SUV. Given that a customer bought an SUV, what is the probability that it was black?

Let S denote the event that a person buys an SUV, and let B denote the event that a person buys a black SUV. Then

$$P(B \mid S) = \frac{P(B \cap S)}{P(S)}$$

$$= \frac{P(B)}{P(S)}$$

$$= \frac{0.03}{0.20}$$

$$= 0.15$$

Ex 28:- A fair 4 sided die is rolled twice , find probability that the maximum of two numbers is 4 given that minimum is 2 ?

A ≡ max of two no s. is 4

B ≡ min of two no s. is 2

$P(A) = \dfrac{7}{16}$ $P(B) = \dfrac{5}{16}$

$P(A \cap B) = \dfrac{2}{16}$

$P(A / B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{2/16}{5/16} = \dfrac{2}{5}$

1,1	1,2	1,3	1,4
2,1	2,2	2,3	2,4
3,1	3,2	3,3	3,4
4,1	4,2	4,3	4,4

If A and B are independent events, prove that the following pairs of events are independent: A^c and B, A and B^c, and A^c and B^c.

We will prove that A^c and B^c are independent the student

tries, in the same manner, to prove the other two cases.

$$p(A^c \cap B^c) = p(A \cup B)^c = 1 - p(A \cup B)$$
$$= 1 - \big(p(A) + p(B) - p(A \cap B) \big)$$
$$= 1 - p(A) - p(B) + p(A \cap B)$$
$$= 1 - p(A) - p(B) + p(A).p(B)$$
$$= \big(1 - p(A) \big) - p(B)\big(1 - p(A) \big)$$
$$= \big(1 - p(A) \big)\big(1 - p(B) \big)$$
$$= p\big(A^c \big) \; p\big(B^c \big)$$

A population of 600 semiconductor wafers contains wafers from three lots. The wafers are categorized by lot and by whether they conform to a thickness specification. The Page 86 following table presents the number of wafers in each category. A wafer is chosen at random from the population.

Lot	Conforming	Nonconforming
A	88	12
B	165	35
C	260	40

a. If the wafer is from Lot A, what is the probability that it is conforming?
b. If the wafer is conforming, what is the probability that it is from Lot A?
c. If the wafer is conforming, what is the probability that it is not from Lot C?

Let the probability that semiconductor is conforming is P(C)

Let the probability that semiconductor is conforming is P(N)

a. P(C|A) = 88/(88+12) = 0.88

b. $P(A|C) = \dfrac{88/600}{(88+165+260)/600} = \mathbf{0.1715}$

c. If $P(C^c|C) = \dfrac{(88+165)/600}{(88+165+260)/600} = \mathbf{0.4931}$,

An urn I contains 2 white balls and 3 black balls, urn II contains 4 white balls and 1 black balls and urn III contains 3 white balls and 4 black balls. An urn is selected at random and the ball drawn at random is found to be white. Find the probability that urn I was selected?

So, $p(A_1 / A) = \dfrac{p(A_1)p(A/A_1)}{\displaystyle\sum_{k=1}^{3} p(A_k)p(A/A_k)}$

$$p(A_1 / A) = \frac{\left(\dfrac{1}{3}\right)\left(\dfrac{2}{5}\right)}{\left(\dfrac{1}{3}\right)\left(\dfrac{2}{5}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{4}{5}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{3}{7}\right)} = \frac{19}{57}$$

You enter a chess tournament (بطولة شطرنج) where your probability of wining (المكسب) a game is 0.3 against (ضد) half the players (call them type 1) , 0.4 against a quarters of the players (call them type 2) and 0.5 against the remaining quarter (الربع المتبقى) of the players (call them type 3) . You play a game against a randomly chosen opponent (خصم او منافس) . What is the probability of winning (المكسب) ?

Suppose (افترض) that you win (فزت) . What is the probability that you had an opponent of type 1 ?
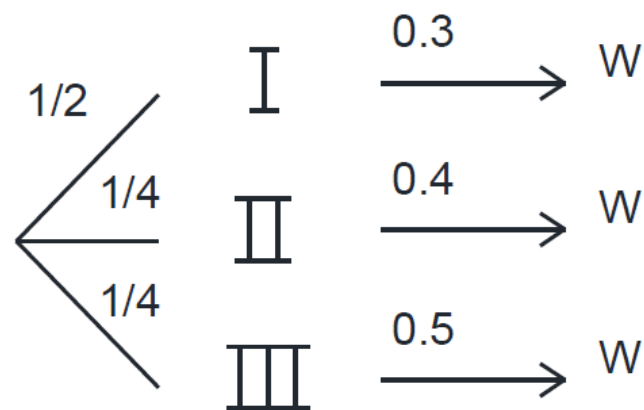
(بتلاعب خصم من النوع الاول)

$$P(W) = \frac{1}{2} * 0.3 + \frac{1}{4} * 0.4 + \frac{1}{4} * 0.5$$

$$= 0.375$$

$$P(I / W) = \frac{P(I \cap W)}{P(W)} = \frac{P(I)P(W/I)}{P(W)}$$

$$= \frac{1/2 * 0.3}{0.375} = 0.4$$

Ex 30 :- Stores A , B and C have 50 , 75 , 100 employees and respectively 50 , 60 , 70 percent of these are women . one employee resigns (يستقيل) and this is a woman. What is the probability that she works in store C .

$$= 0.49125$$

A company has three manufacturing plants to produce an

electrical relays (سويتشات كهربائيه) with 50 , 30 and 20 percent,

respectively, of its product. Suppose (بفرض) the probability
that a relay manufactured (المصنعه) by three plants is

defective (معيوبه) are 0.02 , 0.05 and 0.01 , respectively .

a- If a relay is selected at random from the output of the
    company , what is the probability that it is defective?

Ans 0.027

b- If a relay selected at random is found to be defective,
    what is the probability that it was manufactured by plant
    2 ?

Ans 5/9

Problems

Ex 26:- What is the probability that the sum of points obtaining in rolling two dice   زهر النرد   is 7 given that the dice shown at least 3 point

Ans 1/8

Ex 27:- We toss a fair coin three successive times, we wish to find the conditional probability P (A / B) when A and B are the events

A = { more heads than tail come up }

And B = { 1 st toss is a head }

Ans 3/4

Thank You